Emitter Localization by Using Least Square Method

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Abstract— **The purpose of this project is to estimate the position of a specific signal source with sensors distributed on the plane. The effect of the sensor distribution, number of sensors, and noise level on the accuracy of the emitter localization have examined. The entire study has created and implemented on MATLAB.**

Keywords—Emitter localization, PDOA, SNR, MATLAB, least square method,

I. INTRODUCTION

Determine localization of emitters is one of the important topics of the electronics engineering. In this study the PDOA model have used to determine the emitter localization.
The entire project has developed and executed using MATLAB. First, on 2D plane, the effect of number of sensors, the effect of distribution of sensors and the effect of SNR value on accuracy of emitter localization have examined. The model for 2D has rearranged and a model for emitter localization in a 3D environment was derived. To approximate real-life scenarios, the noise values of the sensors are determined by the SNR value that entered by the user.

II. BACKGROUND INFORMATION

A. 2D Model

There are a lot of technique to find localization of emitter such as "The direction of arrival ", "The signal's propagation time ", "frequency difference of arrival", "The received signal power". In this study "non-linear least square" method that is an algorithm of power difference of arrival have used. The fundamental idea behind the power difference of arrival (PDOA) is that signal power level data from a variety of sensors can be used to infer a position by applying an appropriate path loss model. This Technical Report uses both generated and measured data to examine the accuracy of several algorithms for PDOA geolocation solution evaluation [1]. There are other algorithms that used for determining the localization of emitters by used power difference of arrival, Beside the "non-linear least square" algorithm. These are [1]:

- Maximum Likelihood
- The Discrete Probability Density Method
- Intersection Density
- Modified Y.T. Chan's Least Squares Method
- S. Wang's Least Squares Method
- L. Zhu's Least Squares Method

non-linear least square algorithm:

To determine the power difference between sensors, The distance between sensors and emitter should be find. The (1.1) formula have used to find distance between nth sensor and emitter.

$$d_n = \sqrt{(x - x_n)^2 + (y - y_n)^2} \qquad 1.1$$

d_n = The distance between the emitter and nth sensor.
x, y = The coordinates of the emitter.
x_n, y_n = The coordinates of the nth sensors.

The distance between the emitters and sensor have used to calculated measured signal by nth sensor by the (1.2) formula:

$$P_n = P_0 - 10 * \alpha * log_{10}(\frac{d_1}{d_0}) \qquad 1.2$$

P_n = Power measured by the nth sensor.
P0 = Reference power.
d0 = Reference distance.
α = Path loss exponent. Generally, between 2 and 4.

The value of reference power and reference distance are not important. These variables going to be eliminated in following equations. The power difference between n^{th} and $(n+1)^{th}$ sensors is determined by equation (1.3).

$$P_{difference} = P_n - P_{n+1} = 10\,\alpha\,log_{10}(\frac{d_{n+1}}{d_n}) \qquad 1.3$$

After substitution of equation 1.2 into equation 1.3 the general formula to find power difference between n^{th} and $(n+1)^{th}$ sensors have been found:

$$P_{difference} = P_n - P_{n+1} = 5\,\alpha\,log_{10}(\frac{(x - x_{n+1})^2 + (y - y_{n+1})^2}{(x - x_n)^2 + (y - y_n)^2}) \qquad 1.4$$

To determine the localization of emitter, the power difference between nth and $(n+1)^{th}$ sensors are not enough. An error function that shows the relation between actual power difference between sensors and the power difference that theoretically calculated should be created. The 'ERROR(x,y)' is equal to:

$$\sum_{n<n+1} P_{actual} - 5\alpha log(\frac{(x - x_{n+1})^2 + (y - y_{n+1})^2}{(x - x_n)^2 + (y - y_n)^2}) \qquad 1.5$$

The coordinates where the error function gives the minimum value give the estimated coordinates of the emitter.

B. 3D Model

To derive a 3D model for 2D model the Z axis have been added to the formulas.

$$d_n = \sqrt{(x - x_n)^2 + (y - y_n)^2 + (z - z_n)^2} \qquad 1.6$$

After implantation the d_n value into other formulas the model for the 3D have been appeared. The 'ERROR(x,y)' is equal to:

$$\sum_{n<n+1} P_{actual} - 5\alpha log(\frac{(x - x_{n+1})^2 + (y - y_{n+1})^2 + (z - z_{n+1})^2}{(x - x_n)^2 + (y - y_n)^2 (z - z_n)^2}) \qquad 1.5$$

III. REALIZATION OF SIMULATIONS

A visual interface has been created to facilitate project use. If you answer "yes" on the first screen, another screen will appear asking for the sensor number and SNR value. The positions of emitters and sensors are determined randomly. If the answer to the first question is "no", another interface is opened so that the user can enter the positions of the emitters and sensors specifically.
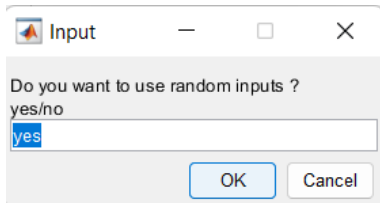
The interface for the first question.
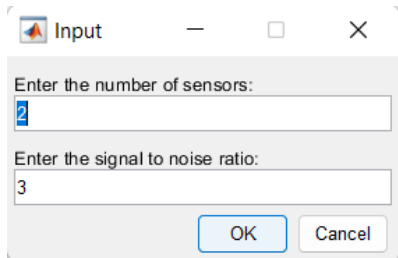
The interface after answering "yes", The interface demanding only number of sensors and SNR value.
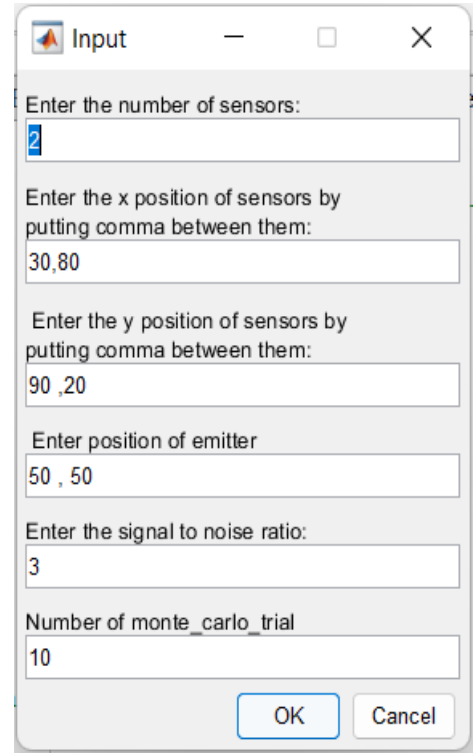
interface that allows the user to customize all simulation settings after answering "no" to the first question.
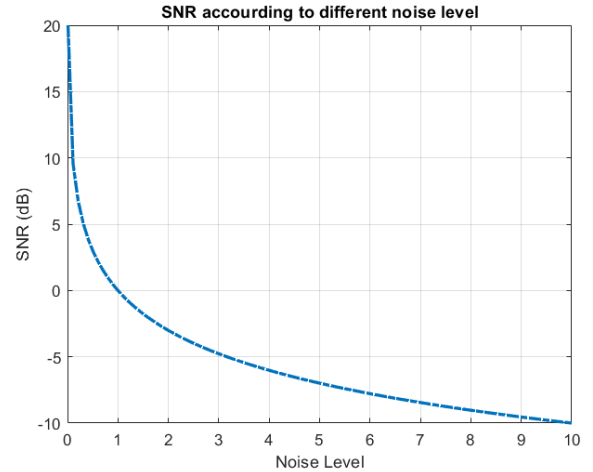
It is recommended to examine the graph containing the noise values corresponding to the SNR level before entering the SNR values. As the noise level increases, the accuracy of finding the location of the emitter decreases.

### A. Effect of the number of sensors on location accuracy

In this section, when the SNR value is taken as a constant of 3, the effect of the number of sensors on the accuracy of estimating the position of the emitter is examined. Simulations were carried out with the number of sensors being 2, 3, 5, 10. By performing 100 Monte Carlo experiments, the accuracy of the estimating is determined by the mean square error method. The distribution of each trial on the plane was visualized with a 3D histogram.
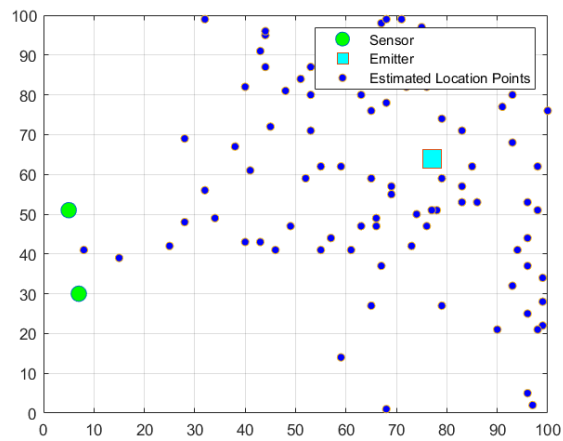
2 Sensors:
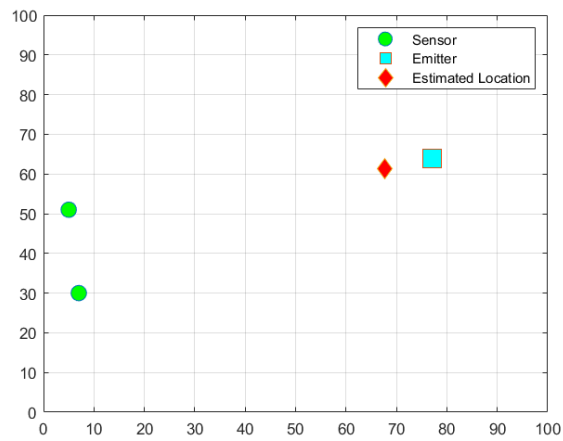
3 sensors:

Mean of 100 Monte Carlo trials Estimated localization. The mean square error value is 1177.49.
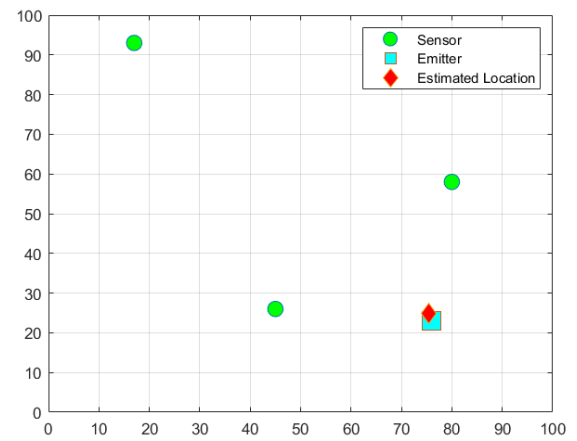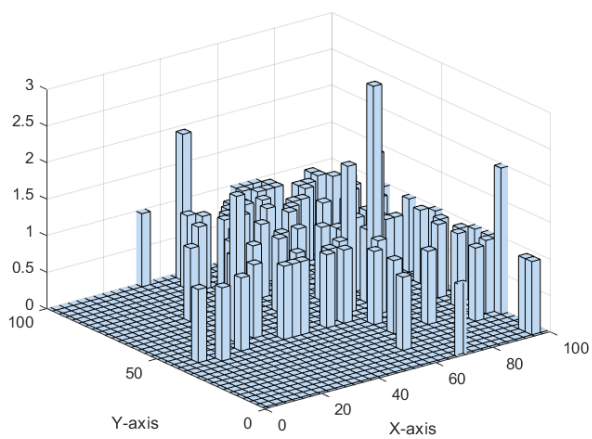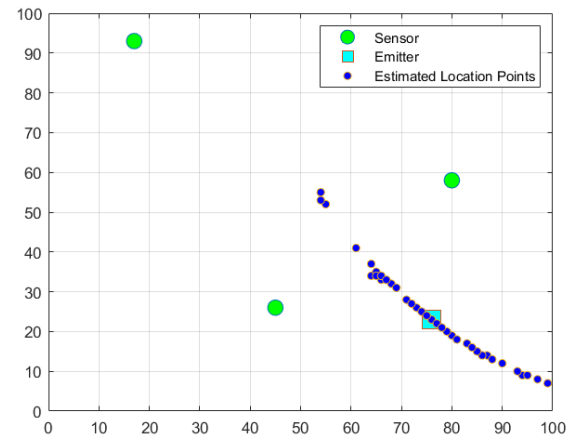
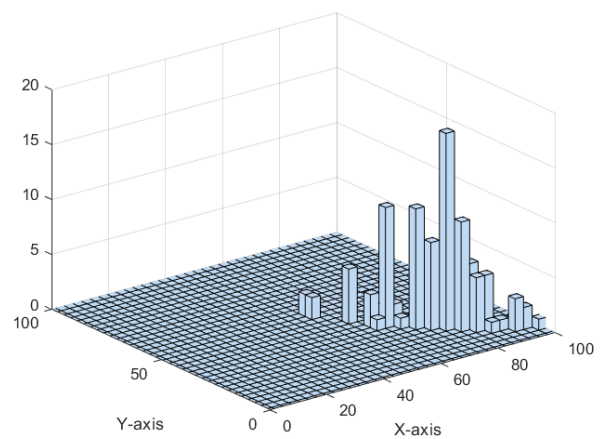Mean of 100 Monte Carlo trials Estimated localization. The mean square error value is 198.64.

Distribution of estimated points.

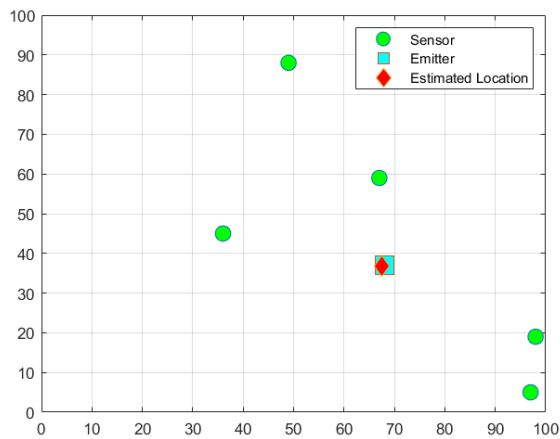Distribution of estimated points.

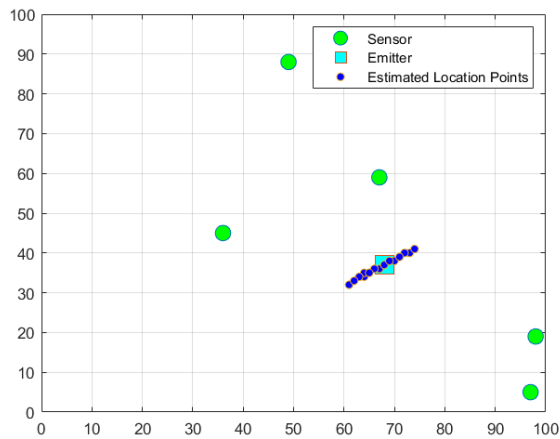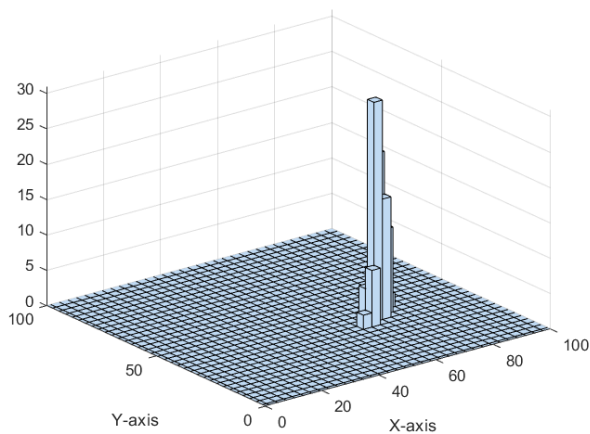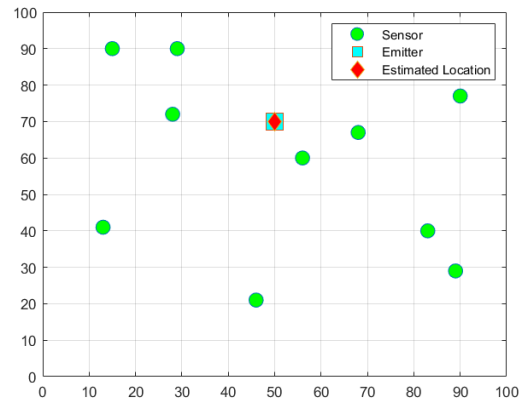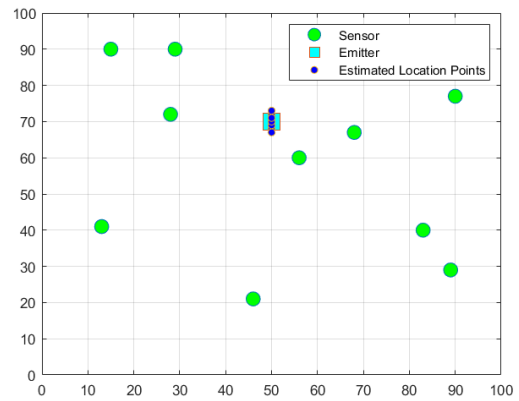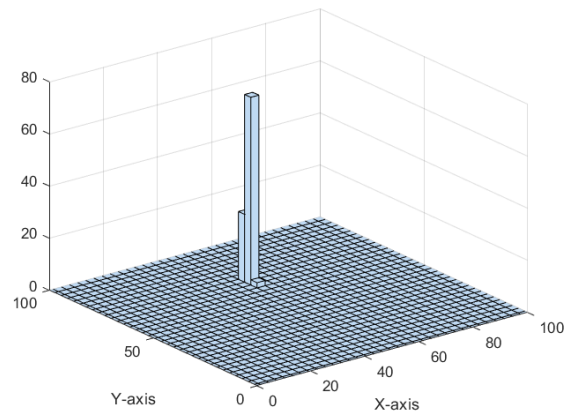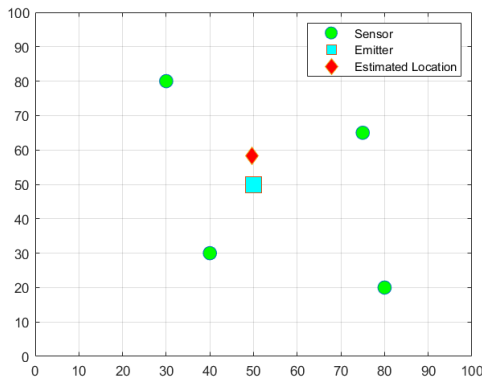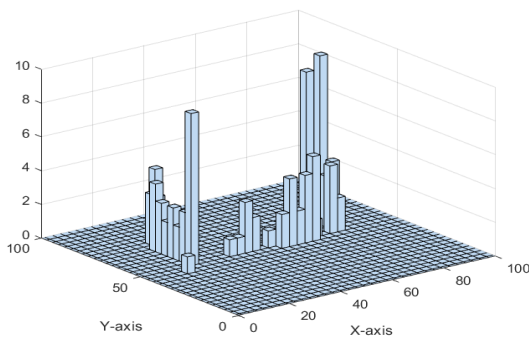The histogram of distribution of estimated points.

The histogram of distribution of estimated points.

5 Sensors:

Mean of 100 Monte Carlo trials Estimated localization. The mean square error value is 11.66.

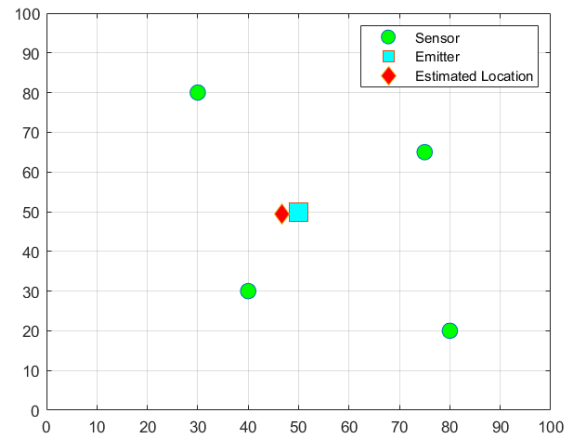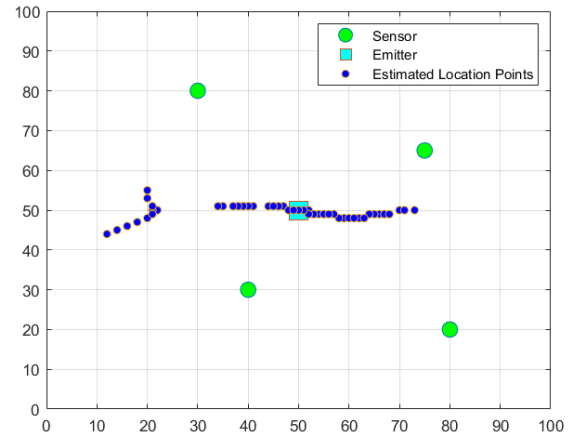Distribution of estimated points.

The histogram of distribution of estimated points.

10 sensors:

Mean of 100 Monte Carlo trials Estimated localization. The mean square error value is 1.37.

Distribution of estimated points.

The histogram of distribution of estimated points.

The simulation clearly demonstrates that as the number of sensors on the plane increases, the distance to the actual position decreases. The decrease in the "mean square error" value with an increasing number of sensors also confirms this phenomenon.

B. Effect of the SNR value on location accuracy

In this section, the number of sensors and positions is taken as a constant, to examine the effect of the SNR value on the accuracy of estimating the position of the emitter. The number of sensors is 4 and the location of emitters are S1(30,80), S2(40,30), S3(80,20), S4(75,65). Simulations were carried out with the different SNR value of -10dB, -3dB, 0dB, 3dB, 10dB. By performing one hundred Monte Carlo experiments, the accuracy of the estimating is determined by the mean square error method. The distribution of each trial on the plane visualized with a 3D histogram.

SNR = -10 dB

Mean of one hundred Monte Carlo trials Estimated localization.
The mean square error value is 743.56.
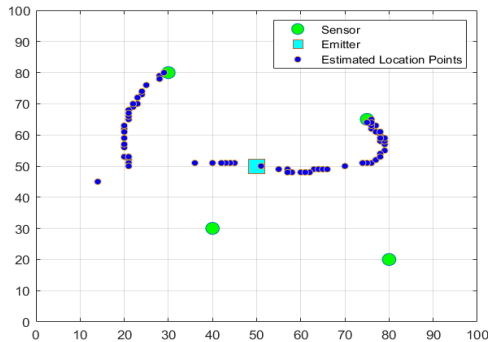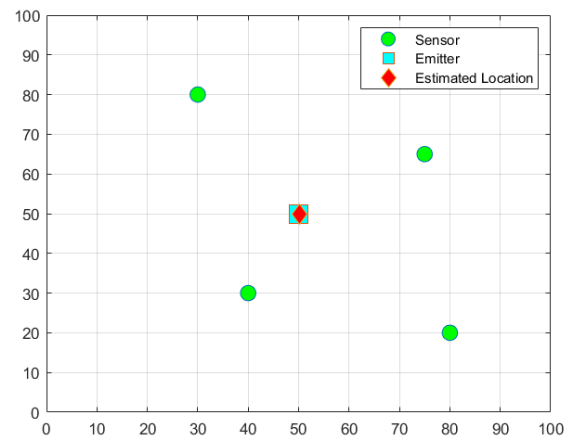

Distribution of estimated points.

The histogram of distribution of estimated points.

SNR = -3 dB

Mean of one hundred Monte Carlo trials Estimated localization.
The mean square error value is 257.56.

Distribution of estimated points.
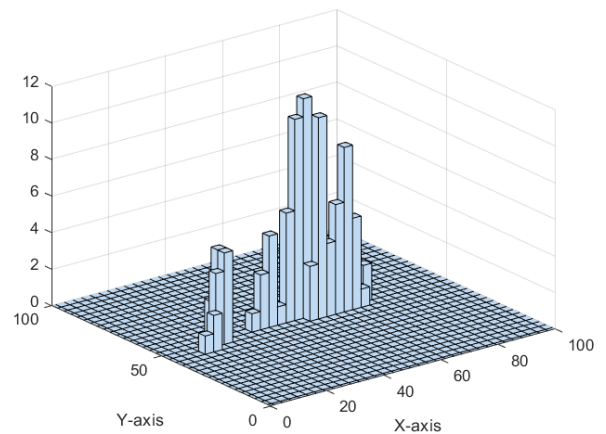
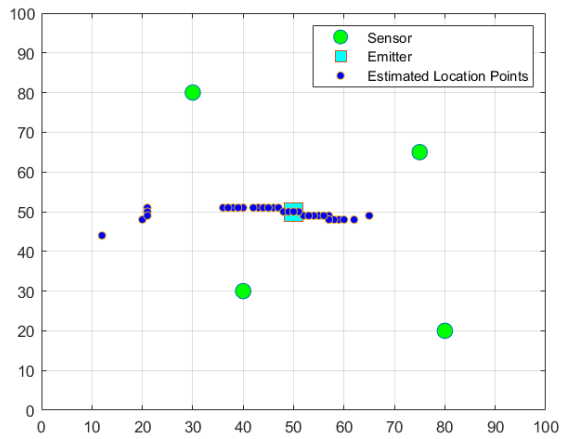The histogram of distribution of estimated points.

SNR = 0 dB
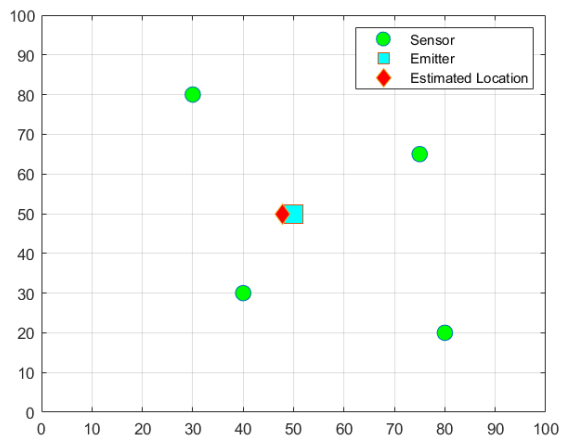
Mean of one hundred Monte Carlo trials Estimated localization. The mean square error value is 95.83.
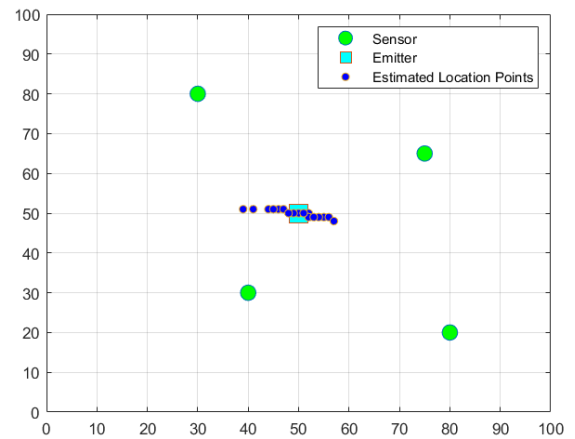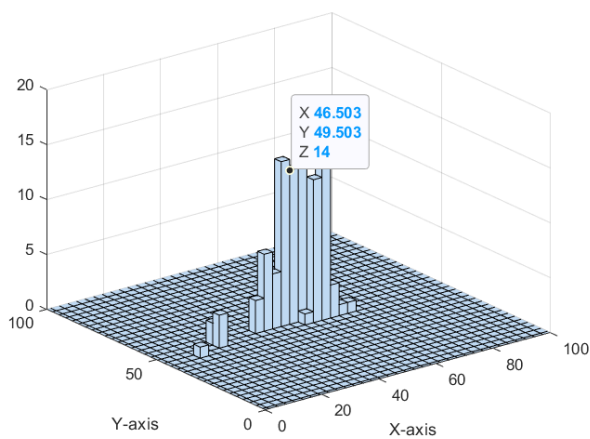
SNR = 3dB

Mean of one hundred Monte Carlo trials Estimated localization. The mean square error value is 11.23.
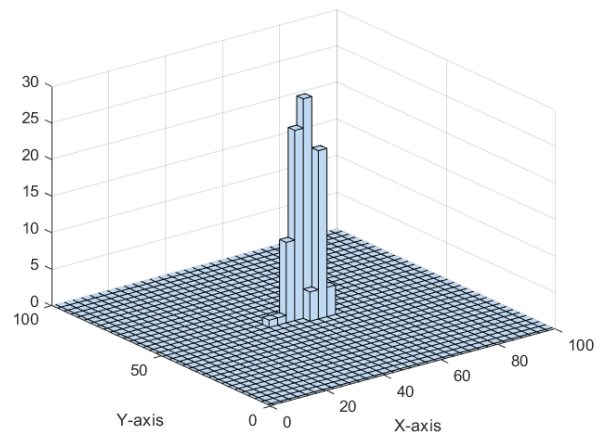

Distribution of estimated points.
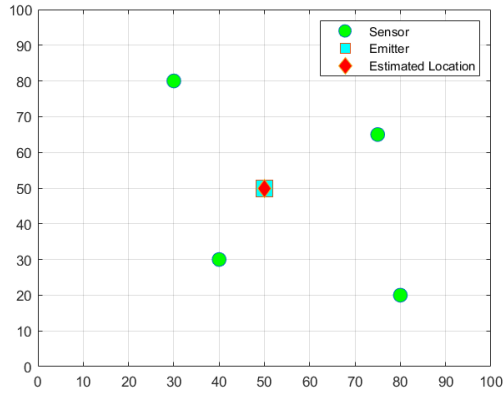

Distribution of estimated points.

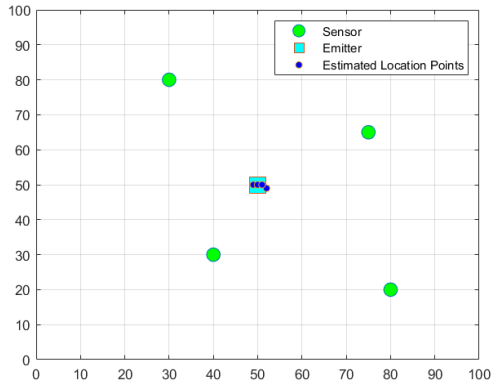The histogram of distribution of estimated points.
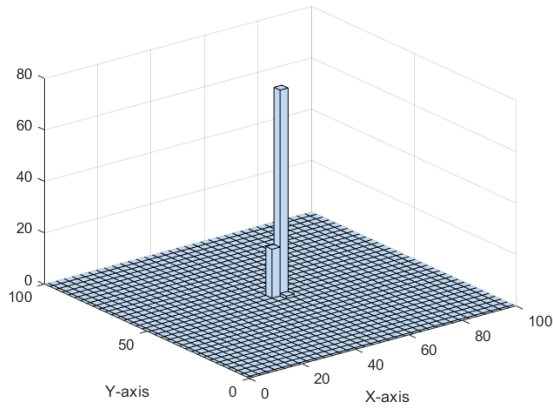

The histogram of distribution of estimated points.

SNR = 10dB

Mean of one hundred Monte Carlo trials Estimated localization. The mean square error value is 0.44.

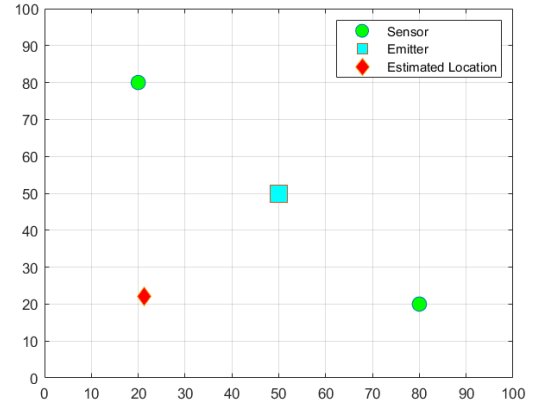Distribution of estimated points.

The histogram of distribution of estimated points.

As can be seen from the simulations, decreasing the SNR value significantly reduces the accuracy of the points estimated on the plane. As a result, the more the SNR value increases, the more accurate the estimations become. This phenomenon can be understood from the 'mean square error' value.

C. Effect of the sensors disturubiton on location accuracy

In this section, the impact of distributing four sensors evenly at the same distance from the transmitter on the accuracy of finding the transmitter's position was investigated, as well as the effect of randomly distributing four sensors. The SNR value is constant for 3 dB.
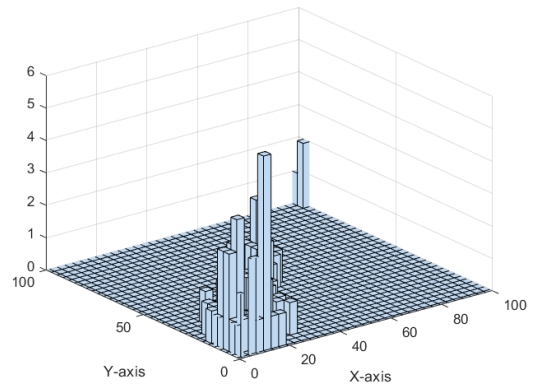
2 sensors distributed at same distance to emitter:

Mean of one hundred Monte Carlo trials Estimated localization. The mean square error value is 2330.54.
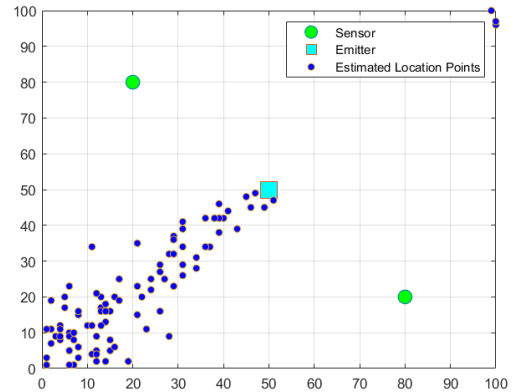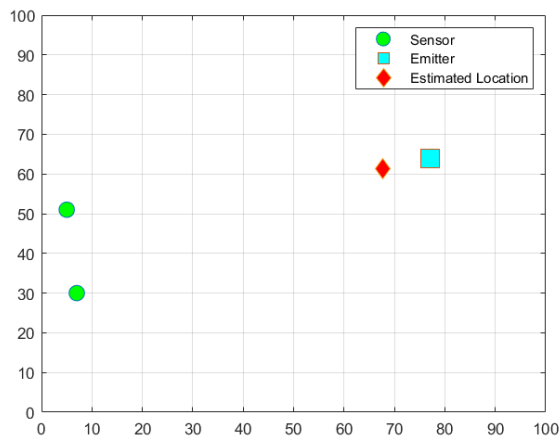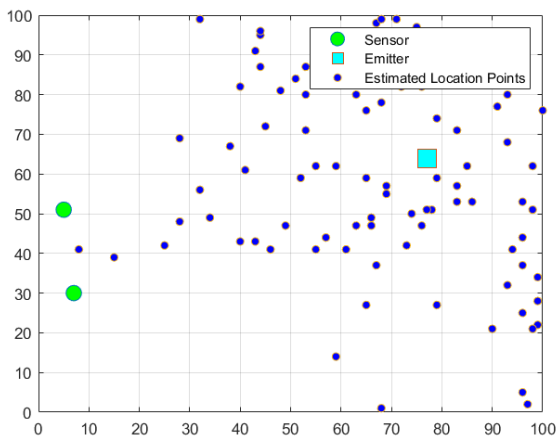

Distribution of estimated points.

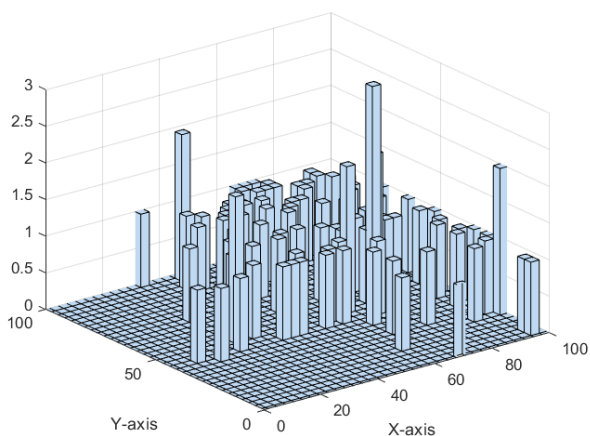The histogram of distribution of estimated points.

2 sensors distributed randomly on plane:

4 sensors distributed at same distance to emitter:

Mean of 100 monte Carlo trials Estimated localization. The mean square error value is 1177.49.

Mean of 100 Monte Carlo trials Estimated localization. The mean square error value is 11.14.
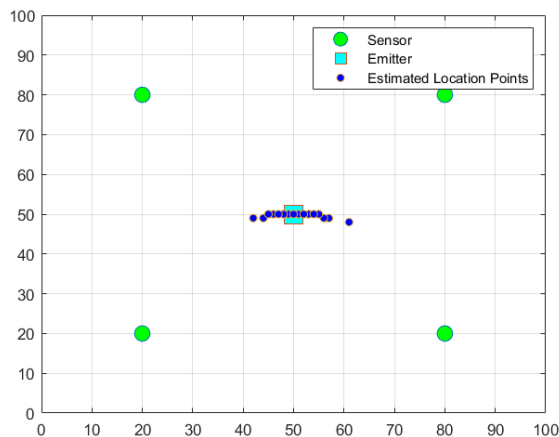

Distribution of estimated points.

Distribution of estimated points.

The histogram of distribution of estimated points.

The histogram of distribution of estimated points.

4 sensors distributed randomly on plane:

Mean of one hundred Monte Carlo trials Estimated localization. The mean square error value is 97.58.

Distribution of estimated points.
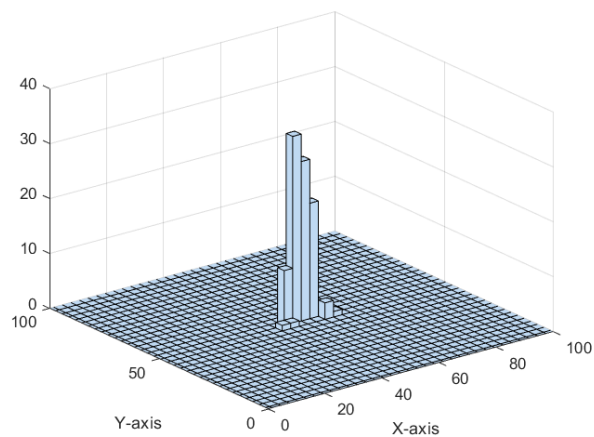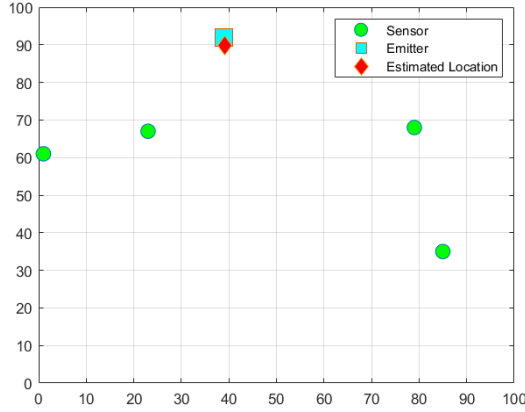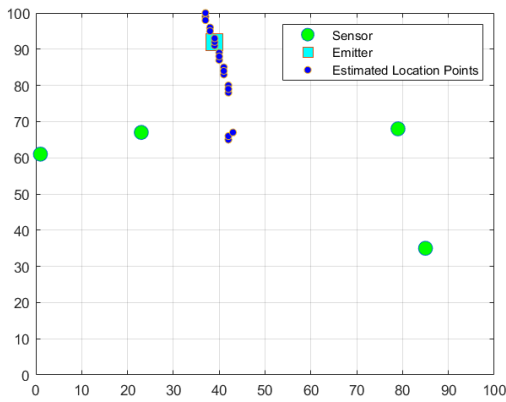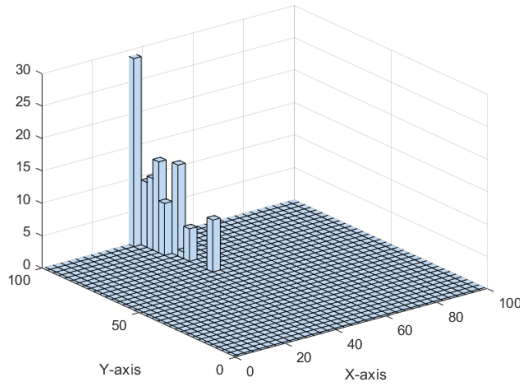

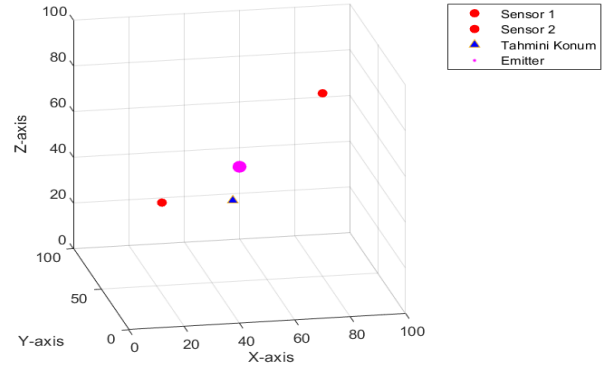The histogram of distribution of estimated points.

According to the results obtained for the two sensors, in both cases the 'mean square error' value is too high for the estimation to be considered accurate. But when the sensors and emitter are arranged linearly, it is observed that the 'mean square error' value is much higher. For these reasons, when using 2 sensors, the sensors and emitter should not be placed in the same direction.

When we look at the 4 sensors, the estimation made by the sensors placed properly around the emitter gives more accurate results than the sensors placed randomly.

D. 3D Model

In this section, the estimated points, and their accuracy in case of using 2, 4, and 6 sensors in 3D space are examined. SNR value is taken as constant 3dB. Similarities between these distributions in 3D space and the distributions in the 2D plane have also been observed. Additionally, the predictions that would emerge if the sensor arrays were in a plane shape were observed.
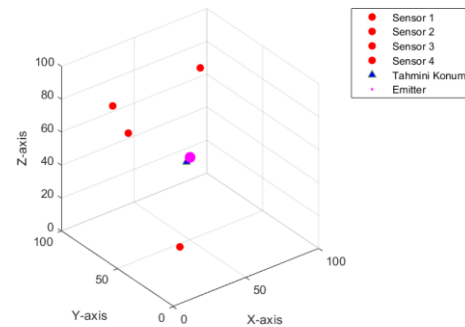
2 Sensors:

Mean of one hundred Monte Carlo trials Estimated localization. The mean square error value is 2595.02.
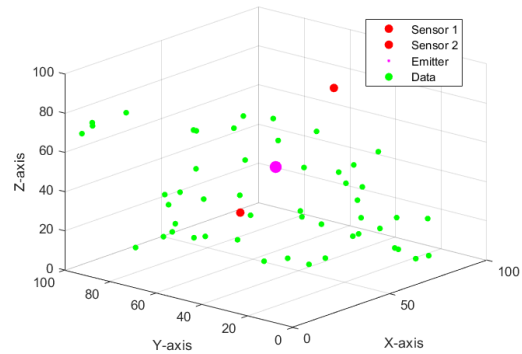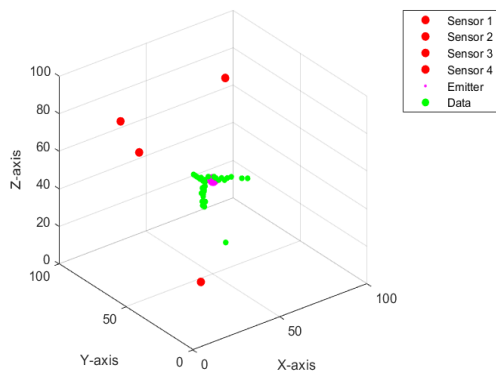

Distribution of estimated points.
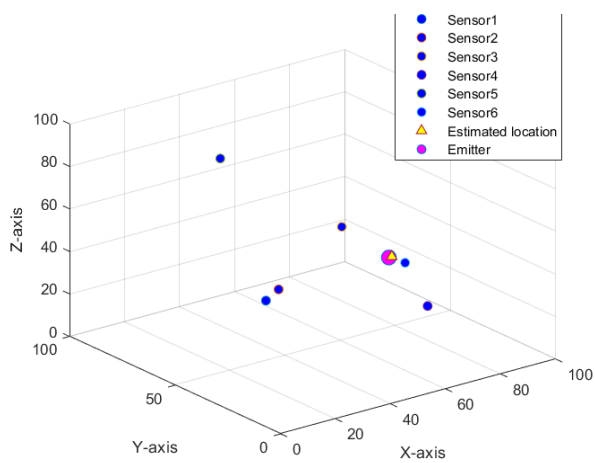
4 Sensors:

Mean of one hundred Monte Carlo trials estimated localization. The mean square error value is 199.52.
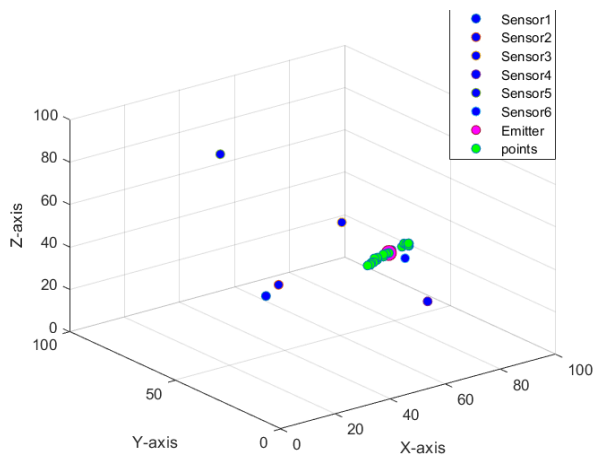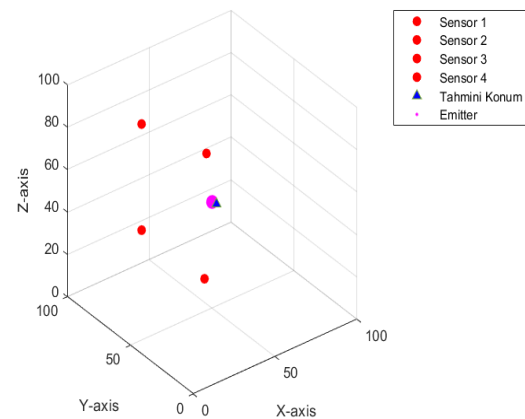
Distribution of estimated points.

6 sensors:

Mean of the hundred Monte Carlo trials estimated localization. The mean square error value is 33.67.
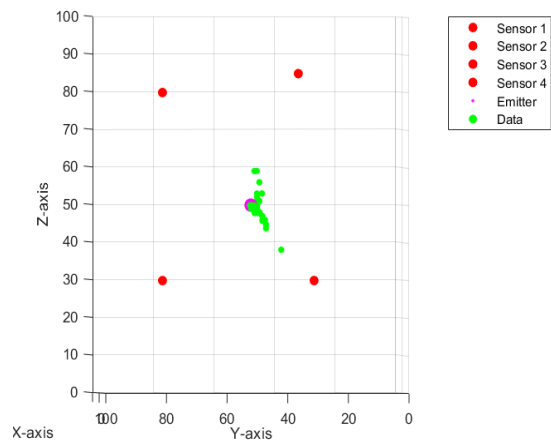

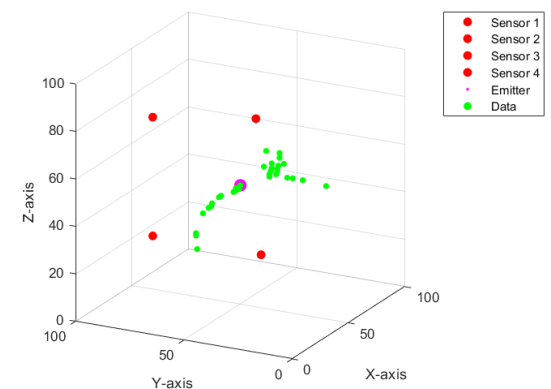Distribution of estimated points.

PLANE CONFIGURATION

Mean of one hundred Monte Carlo trials Estimated localization. The mean square error value is 577.76.

Distribution of estimated points. When look 2D perspective.

Distribution of estimated points. When look 3D perspective.

In the 3D model, as in the 2D model, the prediction accuracy increases as the number of sensors increases. It is understood that in the simulations made using 2 and 4 sensors in the 3D model, the mean square value is very high, meaning the prediction accuracy is poor. In addition, it has been observed that the prediction made when 6 sensors are used reaches a usable level of accuracy. It has been noticed that when the point cloud in 3D space is viewed from a two-dimensional perspective, it is similar to the point arrays formed in the 2D plane.

IV. CONCLUSION

In our study, we observed that the factors affecting the accuracy of the emitter location in both 2D, and 3D models are similar and that these factors affect the result in similar ways. Increasing the number of sensors in the two models directly increased the estimated location accuracy. This situation was observed by examining the mean square error values.
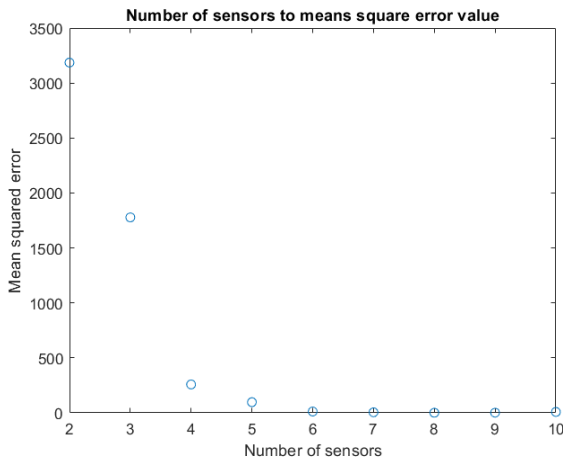
It has been observed that as the SNR value decreases and the noise values of the sensors increase, the positioning accuracy decreases.
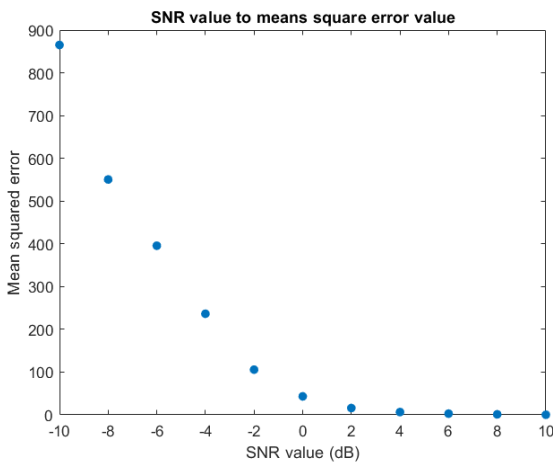
As a result, in order to achieve accurate emitter localization, the number of sensors should be kept as many as possible and noise should be suppressed as much as possible.

REFERENCES

[1] B.R. Jackson., S. Wang, &R. Inkol (2011). "Emitter geolocation estimation using power difference of arrival. An algorithm comparison for non-cooperative emitters ". URL:
https://cradpdf.drdc-rddc.gc.ca/PDFS/unc110/p534968_A1b.pdf