Anticipatory Networks

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Contents

Abstract

This chapter presents a network structure for a class of anticipatory systems in Rosen's sense linked by causal and information transfer relations. They form a multigraph termed an *anticipatory network* (AN). The ANs were first defined in order to model a compromise solution selection process in multicriteria optimization problems. The nodes in such a network, termed optimizers, are capable of

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selecting a nondominated solution taking into account the anticipated consequences of a decision to be made. Specifically, to make a decision in a problem associated to an optimizer, the decision-maker should take into account the anticipated outcomes of each future decision problem linked by a causal relation with the current one. The ANs presented in this chapter are based on an assumption that constraints and preference structures in nodes associated to future decision problems may depend on the values of criteria that result from solving preceding problems. The nodes of a hybrid AN may correspond to optimizers, random, conflicting, interactive, or predetermined (nonautonomous) decision problems. We will overview most relevant types of ANs, their solution concepts, and constructive solution algorithms. It will be pointed out that the structure of an anticipatory network imposes the superanticipatory property for its components. We will also present timed AN as well as further extensions of ANs with various information exchange relations. The discussion section contains a survey of real-life applications, including coordination of autonomous robotic formations, foresight scenario filtering, strategy building, and others.

Keywords

Anticipatory networks · Anticipatory systems · Multicriteria optimization · Superanticipatory systems · Scenarios · Foresight · Multicriteria decision-making · Preference modeling

Introduction

This chapter is devoted to investigating the causal and information transfer relations between anticipatory systems in the sense of Rosen (1985). The anticipatory decision systems correspond to nodes of the network, while the relations between them form a multigraph termed an *anticipatory network* (AN). A special relation, which is fundamental to the theory presented in this chapter, is called *anticipatory feedback*. It models decision-makers' wishes to confine the scope of future decisions of a given agent to a favorite subset of all admissible decisions. Each anticipatory network must contain at least one causal and one *anticipatory feedback* relation.

The decision-making processes modeled by ANs extend earlier approaches to selecting a solution to a multicriteria optimization problem based on direct multistage modeling the future consequences of decisions to be made. To select the so-called best compromise (Skulimowski 1985, 1986), the most favorable consequences and the corresponding decision should be identified. This early model restricted the anticipatory decision making to the initial decision problem only, modeled as the unique initial node in a chain of problems. In addition, this approach did not allow the decision-makers to take into account different decision-making scenarios. These ideas have been studied further and applied in real-life situations, such as ensuring a long-term energy security or strategic planning of multistage marketing activities.

Although the above-cited papers (Skulimowski 1985, 1986) were inspired by the multicriteria decision theory and multilevel optimization and written without any prior knowledge of Rosen's anticipatory system theory, the relevant links between these theories were discovered after extending the multistage modeling of future consequences to decision networks. The fundamental question is whether the decision units in an anticipatory network can be actually regarded as anticipatory systems. The reply is affirmative under some natural assumptions, but it is not straightforward. Even if the nodes of an anticipatory network correspond to human decision-makers, one should take into account that although the latter possess capabilities to act as anticipatory systems, nevertheless many of them abstain to think anticipatorily. A more detailed discussion of these issues, referring to the notions of rationality and predictability, is provided in the next section. Generally, the concept of anticipation applied in the theory of anticipatory networks is strongly embedded into the context of different types of forward-looking activities such as forecasting, foresight, or backcasting and their interrelations. The reader is referred to the discussion of these notions in the introduction to this handbook (Poli 2017).

There exist also interesting relations of the problem of constructing the decision network to the notion of impredicativity of most complex real-life phenomena discussed in Poli (2017). The elicitation of decision node characteristics can be regarded as predicative modeling, while the anticipation of their decisions involves cognitive processes. These relations may shed more light on the idea of anticipatory networks as their impredicativity can both be seen as inherited from the theory of anticipatory systems and its fundamental feature in its own right. This follows from a limited ability to estimate the parameters of future decision problems which are used to building the network.

The ANs provide a more general approach to the compromise solution selection process in causally dependent multicriteria problems (Skulimowski 2014a). Each node in such a network is an *optimizer* capable of selecting a nondominated solution of an associated multicriteria optimization problem taking into account the anticipated consequences of a decision to be made. Optimizers can be regarded as models of decision-making aspects of simple anticipatory systems. Specifically, to make a decision in an initial problem in the network, the decision-maker responsible for generating the problem solution with the corresponding optimizer should take into account the anticipated outcomes of each future decision problem linked by a causal relation with the current one. The first class of causal relations studied in the above quoted papers fulfilled an assumption that constraints and preference structures in future decision problems may depend on the values of criteria yielded in the preceding problems. This model has been further extended to the networks of hybrid objects (Skulimowski 2012), including the nodes modeling random, conflicting, interactive, or automatic (nonautonomous) decision problems.

In the next two sections, we will present basic notions related to ANs. Then, in sections "Anticipatory Chains" and "Anticipatory Trees and General Networks," we will overview different types of anticipatory networks, their properties, solution principles, and constructive algorithms to generate anticipatory decisions. Their extensions may contain further information exchange relations. These are presented in section "Generalizations and Extensions of Anticipatory Networks." Following (Skulimowski 2014b), it will be shown that the structure of an anticipatory network imposes the superanticipatory property for its components. We will also present timed ANs and a survey of real-life applications. The latter include coordination of autonomous robotic formations (Skulimowski 2016b), foresight scenario filtering (Skulimowski et al. 2013), strategy building for a regional creativity support center (Skulimowski 2014c), and others.

Anticipatory Decision-Making in Multicriteria Problems

The problem of selecting a solution to a multicriteria optimization problem based on supplementary information on the decision-maker's preferences has been considered by many authors who have proposed a variety of preference models capable of exploring different types of additional information supplementing the preference structure generated by the criteria in the original problem formulation, cf., e.g., Steuer (1987), Ehrgott (2005), Kaliszewski (2006), and Skulimowski (2017b). One very common approach assumes that when making a choice, the decision-maker optimizes a real-valued function ν that is uncertain or cannot be expressed explicitly. This function is called a utility or value function (Debreu 1959). Specifically, the information about the future consequences of selecting an outcome is represented by a single value of ν. Once the values of ν are known, the feasible outcomes of the multicriteria problem can be linearly ordered, and the problem is converted into a scalar optimization problem.

A multilevel model of future consequences of decisions made contained in ANs constitutes an essentially different preference structure in a multicriteria optimization problem than those provided by other multicriteria decision-making methods. It extends the approach proposed in the above-cited paper Skulimowski (1985).

Its main idea can be formulated as follows:

> *Anticipated future consequences of a decision made and their assessments are used as sources of additional preference information to solving multicriteria decision problems.*

The above principle can be regarded as an exemplification of *anticipatory feedback introduced in* (Skulimowski 2014a). Its exploitation is possible if the following two conjugate assumptions are fulfilled.

(i) The decision-maker responsible for solving a decision problem D included in the network knows the way how the solutions to preceding problems influence the parameters of decision problems causally dependent on D.
(ii) The decision-maker knows the estimates (forecasts or scenarios) of future decision problem formulations, their specific parameters and solution rules, as

well as the relations binding their anticipated outcomes with feasible solutions to the current problem.

In particular, the first assumption is fulfilled by the present-time decision maker at the starting node in the network. This allows us to model the consequences of a decision to be made when solving any problem in the network.

The above model of consequences includes a theory of networked decision units, which model the decision problem to be solved and the solution process. If the decision maker solves an optimization problem then the decision units are termed *optimizers* (Skulimowski 2014a). By definition, an *optimizer* O acts on a set of feasible decisions U and selects its subset X according to the rule P that is characteristic for this decision unit. Throughout this chapter, it will be assumed that a selection rule P is equivalent to optimizing a vector function $F{:}U \to E$ on U, where E is a vector space with a partial order p:

$$(F : U \to E) \to \min(p). \tag{1}$$

The outcome subset resulting from the action of an optimizer O on U and F will be denoted as $X \coloneqq O(U, F, p)$. If the selection rule P yields the set of Pareto-optimal points $\Pi(U, F, p)$ in the problem (1), i.e., if

$$X \coloneqq \Pi(U, F, p) = \{u \in U : [\forall x \in U : x\ p\ u \Rightarrow x = u]\}$$

then we will omit the term X in the definition of this optimizer and denote it simply by $O{:=}(U, F, p)$.

Let us observe that if the model of consequences in a multicriteria decision problem appears as a chain of linked multicriteria optimizers:

$$\begin{aligned} &O_1 \to^{\varphi(1)} O_2 \to^{\varphi(2)} O_3 \to^{\varphi(3)} \ldots \to^{\varphi(n-1)} O_n, \\ &O_i := X(F_i, U_i, P_i) \text{for } i = 1, \ldots, n, \end{aligned} \tag{2}$$

where the multifunctions $\varphi(i)$ influence the constraints in the problem O_{i+1} only then (2) is equivalent to a multilevel multicriteria programming problem (cf. Skulimowski 1985, 2014a; Nishizaki and Sakawa 2009). Furthermore, let us note that the above-defined optimizers can model decision-making processes irrespectively whether the decisions are made by a single decision-maker or by a cooperating group. The latter case leads to admitting a Pareto equilibrium which – from the formal point of view – is just an element of the set $\Pi(U, F, p)$. In both cases, nondominated decisions can be regarded *rational*, i.e., any other decision would be worse in at least one aspect from that just admitted in a rational procedure.

Optimal decisions are rational and most appropriate to model the economic and technological development processes, so the rationality assumption was already admitted in the theory presented in Skulimowski (1985, 2014a). Nevertheless, real-life decision-makers' behavior frequently cannot be classified as rational, which motivated us to include other types of decision-making models into an anticipatory network (Skulimowski 2012). If a decision-maker is rational in the above sense, i.e., if decisions are optimal with respect to a well-defined set of criteria, this assumption is a necessary prerequisite for the predictability. The latter is granted if the decision-maker's criteria and/or goals are explicitly known to an external observer. Otherwize the expectations concerning the decision choice by an agent in the future as a response to present actions could not be based on the constructive assumptions (i) and (ii).

It is to be noted that to plan informed decisions with anticipatory networks, all its decision units should be predictable. Predictability can be ensured in the networks including optimizers as well as other decision units such as conflicting games and algorithmic (nonautonomous) decision units, both deterministic or stochastic, cf. section "Hybrid Networks." Optimizers and game units will be termed *active* to emphasize the role of their corresponding decision-makers in selecting a decision autonomously (cf. this chapter, section "Hybrid Networks"; Skulimowski 2011).

The active decision units in an anticipatory network can actually be regarded as models of anticipatory systems since a rational decision-maker is always selecting a nondominated decision and uses an information about own preferences to select a compromise. It implies existence of a model of itself. This needs not be the case when decisions result from an optimization of a single criterion, where the objective is usually imposed by external actors or external circumstances and neither own preferences are taken into account nor any freewill is in play. Thus the "rationality" assumption and the multicriteria analysis framework turned out to be keys to the unification of the Rosen's ASs and ANs. The other Rosen's condition, concerning a knowledge of the environment and its future state, is fulfilled by the definition of AN.

From the point of view of the anticipatory decision theory, the nature of anticipatory phenomena that occur in the optimizers and other autonomous and predictable decision units in an anticipatory network and their subsequent algorithmic treatment are similar, if not identical. This is why when we refer in the following to a network of optimizers, the results will be valid for other predictable decision units in a general anticipatory network as well, unless it is explicitly mentioned.

From this point on, the term *causal network* will refer to the graph of a causal relation that links the criteria, constraints and preference structures in some problems to the outcomes of preceding problems with respect to the above relation. Causal networks of optimizers thus model linked multicriteria optimization problems. The causal relations between other predictable decision problems can be modeled in the same way. We assume that the functions φ linking decision units in a causal network model real-life causal dependence relations between linked problems. Moreover, the paths in the above causal network follow the decision-making process in a temporal order this will be termed the *temporal alignment* assumption. In this Chapter, the causal relations describing the decision consequences reduce to influencing the constraints U_i by outputs from the problems preceding O_i. The evolution of compromise selection mechanisms ψ_i in an anticipatory network of multicriteria optimizers can be modeled in a similar way as the influence on constraints and included in the same model as an additional type of causal relations.

Thus, the anticipatory networks model the information feedback and apply it to selecting a solution to the decision problem at a starting element in the network, taking into account the future consequences of this decision modeled by the entire anticipatory structure. An AN is termed *basic* if all its nodes are optimizers and there is exactly one node with no predecessors. From among all optimizers included in a basic anticipatory network, a real-life decision-maker characterized by a decision freedom (Skulimowski 2011) is associated to the initial problem, which has no predecessors, while the other optimizers in the network are merely models producing "would-be" decisions. This changes when considering sustainability decision units (see section "Interactive Optimizers and Sustainable Decisions") and timed anticipatory networks (see section "Timed Anticipatory Networks").

It can be observed that feedback expressed as a loop in a causal network fulfilling the temporal alignment assumption could violate the physical causality principle (e.g., the "the time travel paradox"). In macroscopic systems, where it is most likely that the causality principle holds, we assume that the causal restriction imposed by the time arrow cannot be overcome. The latter is equivalent to the assumption that the future cannot influence the past. Therefore causal networks of decision units are modeled as acyclic digraphs (cf., e.g., Christofides 1975). The influence of anticipated future optimization results on the decisions made at earlier stages can be modeled by another relation that describes an information feedback based on forecasting, foresight, expectation or anticipation. This relation corresponds to anticipation in the theory of anticipatory systems proposed by Rosen (1985) and will be termed *anticipatory feedback*. It can be defined as the *information flow concerning the anticipated output from a decision unit O_i regarded as an input to the unit O_j that precedes O_i in the causal order*. When the decision units are optimizers, the above information can be then used to specify a selection rule of the optimizer O_j. The relations of the causal influence and the anticipatory feedback, when considered jointly for the same set of decision nodes, form an *anticipatory network* of decision units.

For the sake of presentation's clarity, the impact of different models of uncertainty on the theory of anticipatory networks is not studied in this chapter. The stochastic, fuzzy, intuitionistic, or possibilistic extensions of causal and anticipatory relations, criteria, and constraints, as well as different types of additional information exchange between decision units (cf. Skulimowski 2016a) are left as a subject of further research.

Basic Notions of the Anticipatory Network Theory

Definition of a real-life network of any kind must include an explanation what do nodes (or vertices) and edges (arcs) mean. In the previous section, we defined nodes as decision units identified with the decision problem they solve at different moments of time. Here, we will describe the relations that bind the decision units and are depicted as edges of anticipatory networks.

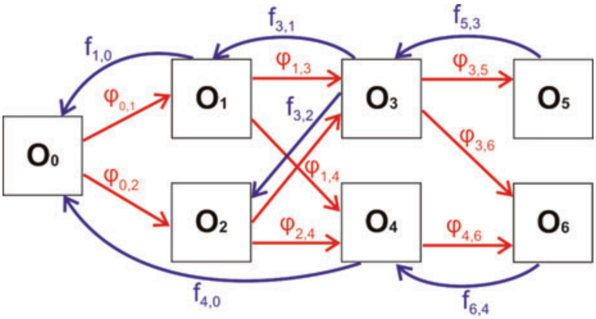


Fig. 1 An example of an anticipatory network of decision units: *red lines* denote causal relations; *blue lines* denote anticipatory feedback

Definition 1 *Any two active decision units O_m=X_1(U, F, P) and O_n=X_2(W, G, R) are in the causal influence relation "O_m influences O_n" if there exist two different outputs from O_m, x_1, $x_2 \in X_1$, such that either the choice of a decision in O_n is restricted to two different subsets of W that depend on choice of x_1 or x_2 in O_m or if the preference structure R or the choice criteria G are modified in a specified manner depending on the choice of x_1 or x_2.* ■

All anticipatory networks presented in this chapter are built according to the assumption that the outcomes of preceding problems can influence constraints in subsequent problems. Other types of causal relations, for example, the impact of past decisions on preference structures of future decision-makers, can also be taken into account in a similar way, but – for the brevity of presentation – these relations are not considered here. Observe that the decision problem expressed as X_2*(W, G, R)* is *more general* than any *influenced problem* in the sense that the influence restricts the decision scope at O_n. This principle needs not be preserved for other types of influence, for example, an influence relation may extend the decision scope by relaxing or removing constraints.

An example of a basic anticipatory network is given in Fig. 1. We have assumed that the preference structures of all decision units cannot be influenced by previous decisions; therefore they are not shown in the diagram.

In the network diagram presented above, the red lines annotated $\varphi_{i,j}$ denote *direct causal influences*. The blue lines annotated $f_{\mathrm{i,j}}$ denote the anticipatory information feedback between the decision-maker at the *j*-th node and anticipated outputs of the decision-making process at node *i*.

Besides the direct influence, we will define the *weak (or potential) causal dependence relation*, which is the negation of the relation "*O_n cannot be influenced by O_m*." Two decision units, O_m and O_n are weakly causally dependent iff these units are connected by a path of direct causal influences, termed a causal path.

The existence of a causal path from O_m to O_n does not ensure that the Definition 1 is fulfilled (Skulimowski 2014a). Specifically, if O_j influences O_m and O_m influences O_n it does not imply that O_j has an influence on O_n. When building an anticipatory network, it is convenient to assume that the weak causal influence relation is defined as the transitive closure (cf., e.g., Christofides 1975, for basic notions and algorithms from graph theory) of the actually identified influence relation. This is justified by the following facts:

(a) It is usually easier to prove the existence of influence than to exclude it.

(b) The number of pairs in a set of n decision units that are in an influence relation is usually considerably smaller than the quantity of all pairs of decision units, excluding self-influence, i.e., smaller than $n(n - 1)/2$.

A *general weak causal influence relation* is the transitive closure of the union of all actually observed partial causal influences of different types and of the intersection of complements of all identified influence exclusions.

If a weak causal influence relation c_w is defined first, then the causal component r of an anticipatory network can be represented as a subgraph of its transitive reduction. Since in a real-life situation the nature of future causal influences is uncertain, r can be regarded as a transitive reduction of certain unknown causal influence relation (cf. Definition 1). The latter does not need to be transitive itself, so it contains no more elements than the transitive closure of r, less than the weak causal relation c_w, but at least as much as its transitive reduction as shown in Fig. 1.

The narrow blue lines in Fig. 1 denote the anticipatory feedback relation. A blue arrow between O_n and O_m means that O_m takes into account the anticipated decision of O_n when selecting an own decision. This relation needs not be transitive.

As already announced, for simplicity's sake, in this chapter, we will provide a survey of anticipatory networks with one type of causal influence and one type of future information feedback only.

The discussion on causality in the first part of this section is of relevance for the construction of anticipatory networks and for the verification of their correctness. Specifically, if two decision units O_m and O_n are not connected by any causal relation then no anticipatory information feedback between them makes sense. Even if there exists an anticipatory information flow between O_n and O_m, then any attempt to use it will fail. Consequently, when analyzing networks with anticipatory feedback relations, it is usually sufficient to take into account only *relevant information feedbacks* (*RIF*); by definition O_n and O_m are linked by a RIF relation iff there exist both an information feedback between O_n and O_m and a weak causal influence between O_m and O_n. However, in general, for given decision units O_k, O_n, and O_m, where both O_n and O_m influence O_k, but there does not exist any direct influence between O_n and O_m, there may exist a different, indirect way of influencing the decision made at O_m by O_n (or vice versa), based on forcing O_m (or) by choosing at O_n such a way of influencing O_k so that O_m has to make a restricted choice to attain its goals at O_k. This phenomenon, called *induced anticipatory feedback*, is discussed in more detail in (Skulimowski 2016a, p.29).

An anticipatory network is termed *solvable* if the subsequent taking into account all anticipatory information feedbacks to restrict the decision choice accordingly yields a nonempty solution set at the starting node (or nodes). To provide constructive algorithms to solving ANs of decision units, we will admit the following assumptions:

- The network is finite.
- If a decision problem O_p is directly influenced by n_p preceding problems and it influences m_p subsequent problems, then n_p, m_p, and all influence characteristics are assumed known.

- For each decision units influenced by more than one predecessor and for each type of influencing function (cf. section "Generalizations and Extensions of Anticipatory Networks"), the rules aggregating different influencing factors are defined and assumed known.

- Any decision problem O_p in the network may be linked by a finite number j_p of anticipatory information feedbacks with anticipated solutions of influenced future problems, and all feedback parameters are known.

In the next two sections we will provide a short overview of the solution methodology for the above-defined class of anticipatory decision problems.

Any anticipatory network can be decomposed into chains, which makes it possible to apply aggregated chain rules iteratively. Anticipatory trees reserve a special attention since they possess the property max $n_p = 1$ for all P active nodes in the network. If more than one predecessor can influence a decision unit, the influence aggregation rules should be defined, e.g., as intersection or union of the sets of alternatives made feasible by different preceding decision units with the causal relation (cf. Skulimowski 2016a). Similarly, the anticipatory feedback aggregation rules are defined for each optimizer or game unit linked by both anticipatory feedback and causal relationships, with more than one subsequent decision unit. They are given usually in form of a logical multiplication of the conditions imposed on the selection of decisions at O_p coming from different future decision units. In section "Anticipatory Chains" we will define an additional quantitative measure of partial satisfaction of future conditions that can be used if it is impossible to fulfill all feedback rules.

We will also present an algorithm (Skulimowski 2014a), which generates a constructive solution to multicriteria decision problems represented as a chain of optimizers. This algorithm will then be applied in section "Anticipatory Trees and General Networks" to solve anticipatory trees and general anticipatory networks.

Anticipatory Chains

A simplest possible nontrivial anticipatory network with the linear causal relation is called an *anticipatory chain*. If such a chain contains only optimizers as active decision units, it will be called a *chain of optimizers*, being thus a generalization of the problem (2) with constraints in sets U_i defined as values of multifunctions

$$Y_i : F_{i-1}(U_{i-1}) \to 2^{U_i}. \tag{3}$$

From (2) and (3), it follows that

$$\varphi(i) := Y_i \circ F_{i-1}. \tag{4}$$

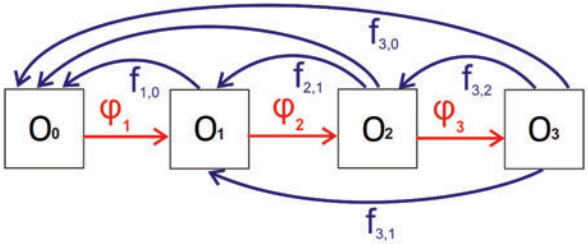


Fig. 2 An example of a chain of optimizers with anticipatory feedback consisting of four systems $O_i=(U_i,F_i, p_i)$, $i = 0,1,\ldots 3$, with $F_i:=\mathrm{id}_{Ui}$, therefore $\varphi(i) \equiv Y_i$, U_i ordered by the partial order p_i, causal relations defined by multifunctions $\varphi(i)$, and six anticipatory feedback relations $f_{j,i}$ (*blue arrows*). The temporal order is determined by causal relations (*red arrows*)

Analogously, the dependence of preference relations P_i on the outcomes of previous problems in a causal chain of decision units can be modeled by the function

$$\psi : X(U_{i-1},F_{i-1},P_{i-1}) \ni x \rightarrow P_i. \tag{5}$$

An anticipatory chain is thus a certain number of anticipatory information feedbacks along a simple causal graph of decision units where each node may have at most one successor and one predecessor.

Figure 2 contains an example of a chain of four optimizers. All optimizers except (perhaps) O_3 are anticipatory systems and select their solutions based on the outcomes of other optimizers. The decision-maker at the initial problem O_0 makes the decision based on the model of future decision-making processes at O_1, O_2, and O_3, while a decision-maker at $O_1=(U_1, F_1, P_1)$ will make its decision taking into account the anticipated outcomes at O_2 and O_3, and O_2 takes into account the anticipated decisions at O_3.

Although chains are the simplest class of anticipatory networks, they are important since a large class of anticipatory networks can be reduced to a sequential analysis of all its subchains.

Two algorithms for solving the anticipatory decision choice problem at the initial node $O_0 := (U_0, F_0, P_0)$ in a chain of optimizers O_i, $i = 0, 1, \ldots, N$, with discrete admissible decision sets U_i, have been given in Skulimowski (2014a). We have merged them and provide in this section as Algorithm 1. Causal relations between optimizers are defined as multifunctions $\varphi(i)$ that constrain the scope of admissible decisions of O_i and depend on solutions of previous problems O_{i-1}, for $i = 1,\ldots,N$. The following definitions will be helpful to describe the solution procedure in a more rigid manner.

Definition 2 *Let O_i, $i = 0,1,\ldots,N$ be an anticipatory chain. For each $i = 0, \ldots, N-1$, let $J(i) \subset \{i+1, \ldots, N\}$ be the index set of future decision problems (it may be empty) that points out which anticipated outcomes are considered when making a decision at the i-th stage. J(i) will be called the sets of feedback indices for the i-th decision unit.* ■

Remark 1 From the above definition it follows that if $m \in J(i)$ then there exist information feedback between the m-th and i-th nodes. Moreover, the overall theory remains valid if we allow self-feedback loops, i.e., if i may be an element of $J(i)$, which describe interactive decision-making at the same decision unit. A discussion of this case is given in section "Interactive Optimizers and Sustainable Decisions."

Definition 3 *Let O_i $i = 0,1,\ldots,N$, be an anticipatory chain with causal relations defined as multifunctions $\varphi(i)$ pointing out the scope of admissible decisions in U_{i+1} in such a manner that if $u_i \in U_i$ is admissible then u_{i+1} is admissible in U_{i+1}* iff $u_{i+1} \in \varphi(i)(u_i)$. *Moreover, let us assume that all elements of U_0 are admissible. Then any sequence of admissible solutions $(u_{0,\ m(0)}, u_{1,m(1)}, \ldots, u_{N,\ m(N)})$ will be termed an* ***admissible chain***. ■

The set of all admissible chains in an anticipatory network G will be denoted by $A(G)$. If all sets $U_0,\ldots,U_N$ are finite, $U_i := \{x_{i,1},\ldots,x_{i,k(i)}\}$, then $A(G)$ can be constructed by listing subsequently the elements of U_0, then replicating each row of the list corresponding to $u_{0,i}$ c_{1i}-times, where c_{1i} is the cardinality of $\varphi(1)(u_{0,i})$, and adding to each row of the extended list the values of $\varphi(1)(u_{0,i})$. This procedure should be repeated recursively, replicating at the i-th step each row corresponding to the j-th element of U_i c_{ij} times, where c_{ij} is the cardinality of $\varphi(i)(u_{i-1,j})$, until the elements of U_N are added to the list. An example of the digraph which illustrates the above construction is shown in Fig. 3 (left).

In the above chain of optimizers $\varphi(i)$ is defined by (4), i.e., $\varphi(i) := Y_i \circ F_{i-1}$. By virtue of the following Lemma 1 in (Skulimowski 2014a), which is based on a construction of $A(G)$ for a given finite anticipatory chain G, the enumeration of the set $A(G)$ can be accomplished without constructing all admissible chains.

Lemma 1 *Let M be the number of all admissible chains in a chain of decision units $O_0,\ldots,O_N$ with decision sets*

$$U_0 := \{u_{0,1}, \ldots, u_{0,k(0)}\}, \ldots, U_N := \{u_{N,1}, \ldots, u_{N,k(N)}\} \tag{6}$$

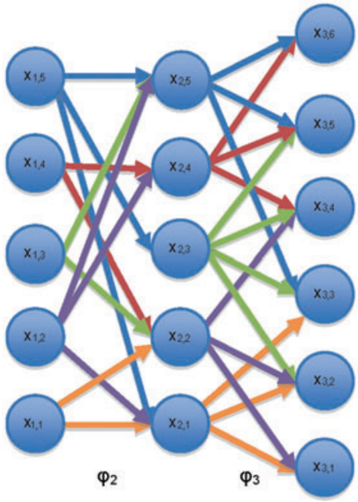
and let d_{ij} be the number of partial admissible chains starting at U_0 and ending at $u_{ij} \in U_i$ for $i = 1,\ldots,N$, $j = 1,\ldots,k(i)$. Then

$$d_{ij} = \sum_{1 \le p \le b_{ij}} d_{i-1,p}, \tag{7}$$

where b_{ij} is the cardinality of the set

$$\varphi(i)^{-1}(u_{ij}) := \{u_{i-1,p} \in U_{i-1} : u_{ij} \in \varphi(i)(u_{i-1,p})\}. \tag{8}$$

Hence,



(a)

		U_2				
U_1	φ_2	$x_{2,1}$	$x_{2,2}$	$x_{2,3}$	$x_{2,4}$	$x_{2,5}$
	$x_{1,1}$	1	1	0	0	0
	$x_{1,2}$	1	0	0	1	1
	$x_{1,3}$	0	1	0	0	1
	$x_{1,4}$	0	1	0	1	0
	$x_{1,5}$	1	0	1	0	1

(b)

		U_3					
U_2	φ_3	$x_{3,1}$	$x_{3,2}$	$x_{3,3}$	$x_{3,4}$	$x_{3,5}$	$x_{3,6}$
	$x_{2,1}$	1	1	1	0	0	0
	$x_{2,2}$	1	1	0	1	0	0
	$x_{2,3}$	0	1	1	1	1	0
	$x_{2,4}$	0	0	0	1	1	1
	$x_{2,5}$	0	0	1	0	1	1

Fig. 3 An example of an explicit graphical definition of multifunctions φ_2 and φ_3 for O_1, O_2, and O_3, *O_i=(U_i, F_i, P_i), U_i :={$x_{i,1}$,...,$x_{i,k(i)}$}, i = 1,2,3 (left). Arrows* starting from an element x_{ij} indicate the admissible decisions in a causally dependent unit which are determined when selecting the decision x_{ij}. On the *right*, we show the corresponding incidence matrices of the causal relations between O_1 and O_2 *(top)* and O_2 and O_3 *(bottom)*

$$M = \sum_{1 \le p \le k(N)} d_{N,p}. \tag{9}$$

■

A proof of this lemma - based on mathematical induction - is given in Skulimowski (2014a).

From Lemma 1, it follows that in the worst case when $\varphi(i)(u_{i-1,j}) = U_i$ for $i = 1, \ldots, N$, the number of all admissible chains is equal to $\Pi_{0 \le i \le N}\, k(i)$. In real-life situations, N is usually much smaller than the number of admissible decisions at each decision unit. Consequently, the computational effort necessary to compute the set *A(G)* in most problems with discrete sets is moderate enough to get the solution with a PC in a reasonable time.

By imposing additional anticipatory conditions specifying the properties of future decisions, the set of admissible chains A will be confined to a smaller set of all decision sequences that fulfill the anticipatory requirements. In the following Definition 4 and Assumption 1, we propose a specific form of such requirements that is suitable for constructive filtering of admissible decision chains to yield decisions with desired properties. It is to be noted that this form is one of a plethora of different anticipatory requirements that may occur in real-life problems.

Definition 4 *Let O_k, $k = 0,1,\ldots,N$, be a chain of active decision units and let for a fixed i,* $0 \le i < N$, $\{V_{ij}\}_{j \in J(i)}$ *be a family of subsets such that $V_{ij} \subset U_j$ for all $j \in J(i)$,* *where $J(i)$ is an index set that points out the outcomes of future decision problems that are relevant for $O_{i,j\ \in\ J(i)}$. Let us denote future decisions to be made at O_j by u_j. Then the anticipatory feedback at O_i is the requirement that*

$$\forall j \in J(i) : u_j \in V_{\mathrm{ij}} \subset \cdot U_j. \tag{10}$$

In discrete problems, V_{ij} are usually defined as explicit lists. The existence of an active anticipatory feedback means that the decision-maker at its target node O_i selects the solution such that the condition (10) is satisfied for all $j \in J(i)$. If it is impossible, the decision-maker admits relaxation rules ensuring a maximum fulfillment of (10). Such rules are discussed later in this section.

We will assume that the anticipatory feedback condition (10) is given in a constructive form, according to the following assumptions (cf. Skulimowski 2014a):

Assumption 1 The decision-makers at units $O_i = X(U_i, F_i, P_i)$ strive to select such decisions $u_i \in U_i$ that the decisions made at O_j for $j \in J(i)$, causally dependent on $F_i(u_i)$, reach or exceed certain attainable reference levels $q(i, j)$ of criteria $F_j{:}U_j \rightarrow E_j$, i.e.,

$$V_{ij} := \{u \in U_j : F_j(u) \le q(i,j)\} \text{for certain}\, q(i,j) \in E_j. \tag{11}$$

Assumption 2 Decision-makers at all units O_i such that $J(i)$ is nonempty select a nondominated decision from among those admissible at O_i according to the anticipatory feedback requirements (11) defined for $j \in J(i)$, at the same time taking into account the preference structure P_i. The preference structures P_j are known to the decision-makers preceding O_j in the causal order.

Assumption 3 The sets $J(i)$ and the associated subset family V_{ij} are nonempty for at least one i. Moreover, if $J(i) \neq \emptyset$, then $V_{ik} \neq U_k$ for at least one $k \in J(i)$.

Assumption 4 The network of decision units is *nonredundant*. By definition, it means that all causally final units O_N (i.e., no other unit O_k depends on O_N) are starting nodes for an anticipatory feedback.

Networks that do not fulfill assumption 3 are termed *trivial*, those that do not satisfy Assumption 4 are *redundant*. Observe that causal relations influencing units beyond last starting node for an anticipatory feedback have no effect on any anticipatory decision. A similar observation can be made for initial nodes, so without a loss of generality, we can assume that $J(0) \neq \emptyset$.

In a nontrivial AN, the following anticipatory decision problems can be formulated:

Problem 1 *Find the set of all admissible chains that fulfill anticipatory feedback condition* (10–11). ■

As already mentioned, the inclusions (10) are often too restrictive. If no feasible solutions to Problem 1 exist, the condition (10) may be relaxed by allowing its partial fulfillment. The Problem 2 below takes into account such relaxation for the anticipatory feedback conditions defined by reference values *q(i,j)* (11).

Problem 2 *Find the set of all admissible chains* $(u_1,\ldots,u_N)$ *that minimize the following function:*

$$g(u_0,\ldots,u_N) := \sum_{i \in J(0)} h(u_i, q(0,i)) w_{0i} \tag{12}$$

and such that if $J(i) \neq \emptyset$, $1 \leq i < N$, *then the truncated admissible chain* $(u_i,\ldots,u_N)$ *minimizes the function*

$$g_i(u_i, \ldots, u_N) := \sum_{j \in J(i)} h\Big(u_j, q(i,j)\Big) w_{\mathrm{ij}}, \tag{13}$$

where h is certain quantitative measure of satisfaction of (11), *and* w_{ij} *are positive relevance coefficients associated to the anticipatory feedbacks between the decision units* O_i *and* O_j, *for* $j \in J(i)$. ■

For example, as *h* in the above Problem 2 one can apply the distance function

$$h\Big(u_i, q(i,j)\Big) := \min\{\| F_i(u_i) - y \| : y \in F_i(U_i) \text{and } y \leq q(i,j)\}. \tag{14}$$

A key role in the solution process of both above problems is played by the following notion:

Definition 5 *A sequence of decisions* $u_{0,m(0)}, \ldots, u_{N,\, m(N)}$, *fulfilling* (10–11) *or maximizing* (12–14)*, will be termed an* ***anticipatory chain*** *of type 1 or 2, respectively.* ■

Anticipatory chains of type 1 ot 2 are regarded as solutions to Problems 1 or 2, respectively. From their recursive construction one can derive the following statement.

Proposition 1 (Skulimowski 2014a)**.** *Suppose that* $\{u_{k,\, m(k)}, \ldots, u_{N,\, m(N)}\}$ *is a truncated anticipatory chain for the decision unit* O_k *in a nonredundant anticipatory chain. If J(n) is nonempty for a certain* $n \in [k+1 : N]$ *then* $\{u_{n,\, m(n)}, \ldots, u_{N,\, m(N)}\}$ *is a truncated anticipatory chain for* O_n. ■

The proof of Prop. 1 was given in (Skulimowski 2014a). It allows us to apply a solution procedure based on the dynamic programming principle and on the following additional assumptions:

Assumption 5 The solutions made by decision units with a shorter anticipation horizon are calculated first.

Assumption 6 All functions (13) can be additively aggregated with (12), and the coefficients w_{ij} are independent on the first index, i.e. $w_{ij}:=w_j$.

Observe that in general, there may exist arbitrary prioritizations of anticipatory feedbacks and/or decision units that determine the order of taking them into account in the solution process. The following algorithm solving the above problems is based on Algorithms 1 and 1′ from (Skulimowski 2014a).

Algorithm 1

Input data structure: *A chain of optimizers with* $N + 1$ *elements* $O_i \coloneqq ((F_i : U_i \to E_i) \to \min(P_i))$, *for* $i = 0,1,...N$, *and the multifunctions* $\varphi(i) \coloneqq Y_i \circ F_{i-1}$ *are given explicitly in form of an array* Φ *of elements of* U_{i+1} *parameterized by the elements of* U_i. *For each* $i = 0,\ldots,N\text{-}1$, *let* $J(i)$ *be the set of feedback indices for the i-th optimizer.*

Output: ***All anticipatory chains of type 1 or 2,*** $(\boldsymbol{u}_{a0},\ldots \boldsymbol{u}_{aN})$.

Step 1. *Find* $\Pi_0 := \Pi(U_0,F_0,P_0)$ *– the set of nondominated points in the problem* $(F_0 : U_0 \to E_0) \to \min(P_0)$ *denoted by* O_0. *Order the elements of* $\Pi_0 = \{u_{0,1}, \ldots, u_{0,\,k(0)}\}$ *in an arbitrary way.*

Step 2. *Find all admissible chains starting from elements of* Π_0 *by a forward step-by-step enumeration of values of* $\varphi(i)$ *in* Φ. *Reorder the elements in* Φ *so that its i-th column contains the i-th element of all admissible chains, ordered lexicographically according to the order of their predecessors*.

Step. 3. *Let* $M \coloneqq \{m \in [0 : N - 1] : J(m) \neq \emptyset\}$. *Find* m *– the largest element of* M.

Step 4m. *Find the smallest element of* $J(m)$, $s(m)$. *Find the set* Ω_m *of all admissible chains* $C(m)$ *starting at certain* $u \in U_m$ *u such that the feedback condition is satisfied at* $O_{s(m)}$.

Repeat *for all* $n \in J(m)$.

Find the set $\Omega_{m,n}$ *of all admissible chains starting at certain* $u \in U_m$ *such that the feedback condition is satisfied at* O_n.

Set $\Omega_m \coloneqq \Omega_m \cap \Omega_{m,\,n}$.

If Ω_m *is empty,* ***then***

for all admissible chains starting at elements of U_m, *calculate the function* $g_m(u_m, \ldots, u_N) \coloneqq \sum_{j\,\in\,J(m)} h(u_j, q(m, j))w_j$.

Find the surrogate anticipatory chain as an admissible chain $(u_m,\ldots,u_N)_{\max}$ *with the minimum value of* g_m.

Step 5m,n. *Remove m from M,*

If $M=\emptyset$, ***stop***

else

Find $m_{\max}$ *– the largest element of* M.

Find the minimum value of $\sum_{j\in[m_{\max};m]} h\left(u_j, q(m_{\max},j)\right)w_j$ *on* U_{mmax}

Set

$$g_{max}(u_{\max},\ldots,u_m,\ldots,u_N) := g_m(u_m,\ldots,u_N) + \sum_{j\in[m_{max}:m]} h\left(u_j, q(m_{\max},j)\right)w_j$$

Set the anticipatory chain as $(u_{max},\ldots,u_m,\ldots,u_N)$, a concatenation of previous – and current – step anticipatory chains.
Go to Step 4m.

else
Step 6m. *Remove m from M.*

If $M=\emptyset$, ***stop***
else *let Find m_{max} – the largest element of M.*
Set $\Omega_m := C(m_{max}, m) \oplus \Omega_m$, where $C(m_{max}, m)$ *is the set of all admissible chains starting at O_{mmax} and truncated at O_m.*
Go to Step 4m

end ■

The operation "$\oplus$" in the Step 6m above is the concatenation of sequences applied to admissible and anticipatory chains. Algorithm 1 is capable of solving both Problems 1 and 2, starting from solving Problem 1 and switching to Problem 2 when Problem 1 does not admit any solutions. It can be easily generalized to decision units different than optimizers. Recall that Problem 2 is a relaxed version of Problem 1 that allows to find admissible chains that partly satisfy the feedback conditions, where the intersection operation in Step 4m is replaced by calculating the functions (12–14).

An anticipatory chain starting at O_0 found as a solution to Problems 1 or 2 can be regarded as a compromise solution to the multicriteria optimization problem (1) that fits best the anticipated consequences of an initial decision. The simplifying assumption $w_{ij}:=w_j$ guarantees that this property is valid for all decision stages in the anticipatory chain.

The application of the above-presented anticipatory preference structure leads in most cases to a considerable reduction of the number of compromise alternatives considered for selection in a multicriteria decision-making problem with an anticipatory preference structure. An illustrative example that demonstrates the action of Algorithm 1 in the chain of five optimizers can be found in (Skulimowski 2014a, Sec. 4).

Anticipatory Trees and General Networks

In this section, we will show how the algorithms presented in section "Anticipatory Chains" can be applied to solve anticipatory networks where each decision unit may influence multiple decisions to be made in the future. If we additionally assume that each decision unit except the initial one is directly influenced by exactly one predecessor then one can observe that the causal subgraph of the anticipatory network so arisen is a *causal tree* of decision units. Then we will investigate a situation where each decision may directly depend on the outcomes of multiple decision units. Both cases combined define general *causal networks.* Any anticipatory network can be represented as a general causal network with anticipatory feedbacks.

To provide a formal definition of anticipatory trees we will define first the bifurcation decision units:

Definition 6 *Let a decision unit O_i influence causally the decisions in two causally independent units O_k and O_m and let O_t be a decision unit with the following properties:*

(a) *O_t is causally dependent on O_i.*
(b) *O_k and O_m are both causally dependent on O_t.*
(c) *If O_p is causally dependent on O_t, then O_k and O_m cannot both causally depend on O_p.*

*Then O_t will be termed a **bifurcation unit** for O_i, O_k, and O_m.* ■

Now, we can can formulate the following:

Definition 7 *An **anticipatory tree** is a finite network of decision units that contains at least one bifurcation unit and at least one anticipatory feedback, and such that no decision unit in the network directly depends on two or more causally independent units.* ■

Any unit in *T* that does not causally influence any other unit will be termed an *end node* of T. The *final branch* of *T* is a chain that starts from a bifurcation unit and does not contain any other bifurcation units.

From Definition 7, it follows that for given decision units O_i, O_k, and O_m in an anticipatory tree configured as in Definition 6, the bifurcation unit O_t always exists and is unique. An example of four bifurcation units in a simple anticipatory tree is given in Fig. 4. The following property, proven in (Skulimowski 2014a), makes possible the reduction of the analysis of anticipatory trees to the subsequent analysis of anticipatory chains in the tree.

Proposition 2 Skulimowski (2014a)**.** *Assume that the decision unit O_i influences two causally independent units O_k and O_m in an anticipatory tree T, and let O_t be the bifurcation unit for $O_{i,}$, O_k, and O_m. Furthermore, let C_k and C_m be the sets of admissible chains starting at O_i and ending at O_k and O_m, respectively, and let C_k' and C_m' contain the elements of C_k and C_m, respectively, starting at O_i and truncated at the bifurcation unit O_t. Then all admissible chains with respect to both O_k and O_m starting at O_i can be generated as extensions of the elements of the intersection of C_k' and C_m'. Specifically, an extension of any such sequence of decisions starting at O_i and ending at O_t is to be concatenated with an arbitrary subsequence starting at O_t of an admissible chain that was truncated at prior to the intersection of C_k' and C_m'.* ■

Let A_1 and A_2 be the sets of admissible chains in two anticipatory chains $\{O_i,\ldots, O_j,\ldots,O_k\}$ and $\{O_i,\ldots,O_j,\ldots,O_m\}$, respectively, where O_j is their bifurcation unit.

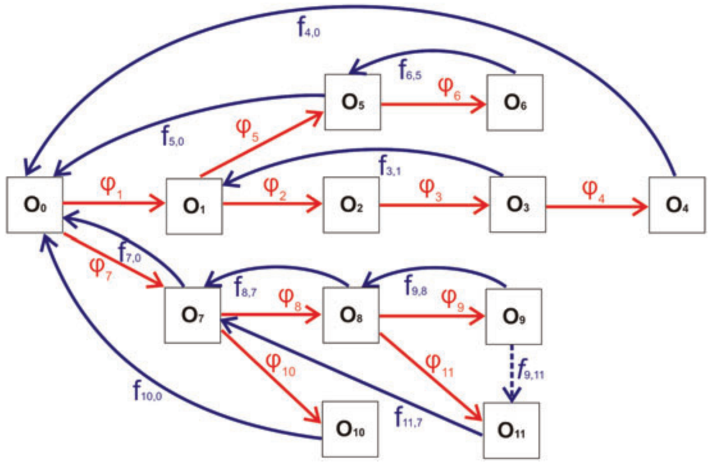


Fig. 4 An example of an anticipatory tree consisting of 12 nodes $O_i=(U_i, F_i, P_i)$, $i = 0,1,\ldots,11$, and five chains. All nodes model optimizers, causal relations are defined by multifunctions φ_j (*red arcs*), and nine relevant anticipatory feedback relations are denoted as $f_{j,i}$ (*blue arrows*). The *dotted arrow* between O_9 and O_{11} is an irrelevant anticipatory feedback, because there is no causal relation between these decision units

Only those elements of A_1 and A_2 that overlap on the common branch $\{O_i,\ldots,O_j\}$ of the anticipatory tree $T \coloneqq \{O_i, \ \ldots, O_k\} \cup \{O_i, \ \ldots, O_m\}$ can be prolonged to chains admissible with respect to causally independent consequences represented by the decisions made at O_k and O_m.

A similar statement, derived from the properties of the logical product of feedback conditions and the corresponding intersection of chains applies also to the anticipatory chains. Another straightforward observation is that anticipatory feedback between causally independent O_k and O_m in an anticipatory tree is always irrelevant, i.e., by definition there is no action at O_k that might influence the output of O_m. From the above observations, one can derive the following algorithm to find all anticipatory chains in an anticipatory tree. It is based on a survey of all bifurcation units and corresponding chains of decision units ending at final branches of the tree. Similarly as in Algorithm 1, *J(i)* denotes the set of anticipatory feedback indices for the *i*-th decision unit.

Algorithm 2 Skulimowski (2014a).

Data structure. *Let us consider an anticipatory tree T of optimizers with $N + 1$ elements*

$$O_i := (U_i, F_i, P_i), \tag{15}$$

for $i = 0,1,\ldots,N$, *and let the values of the multifunctions $\varphi(i) \coloneqq Y_i \circ F_{i-1}$ be given as an array A of elements of U_{i+1} with rows parameterized by the elements of U_i for* $i = 0,\ldots,N\text{-}K\text{+}1$, where K is the number of end nodes in T.

Step 1. *Find $\Pi_0 := \Pi(U_0, F_0, P_0)$ – the set of nondominated points in the problem* $O_0 \coloneqq (F_0 : U_0 \to E_0) \to \min(P_0)$. *Order the elements of $\Pi_0 = \{u_{0,1}, \ldots, u_{0,k(0)}\}$ in an arbitrary way.*

Step 2. *Eliminate all irrelevant anticipatory feedbacks from the tree T (*cf. Fig. 4*).* *Set $T_1 := T$, $B := \emptyset$.*

Step 3. *Repeat until T_1 contains only one chain.*

Find the end nodes $O_{e1}, \ldots, O_{en}$, the final branches ending at O_{ei}, and the set of corresponding bifurcation units B_1 of the tree T_1.

Remove the units forming all final branches from T_1 except the elements of B_1.

Set $B := B \cup B_1$.

If *T_1 is a chain, go to Step 4.*

Step 4*. Order the bifurcation units in B according to the causal order in T so that they form a subtree B in T.*

Step 5k. *Survey all bifurcation units in B according to the causal order, starting from the end nodes of B. For an arbitrary chain corresponding to the element $b \in B$ just considered, apply the Proposition 2 to find all admissible chains starting from elements of Π_0 and ending in a final branch of the anticipatory tree. Apply the ordering and immediate admissible predecessors coding as in Algorithm 1.*

Step. 6k. *For each k-th final branch β_k corresponding to the bifurcation unit $b \in B$, apply Steps 4m and 5m,n of Algorithm* 1 *to find the anticipatory chains starting at Π_0 and ending at the end node of β_k.*

Remove from the tree T the final branch of this chain of optimizers for which the anticipatory chains have been found.

If *T* consist of O_0 only, **stop**

else go to Step 5k. ■

An application of Algorithm 2 to find all anticipatory chains in a simple tree of optimizers is given in Skulimowski (2014a). The Algorithm 2 can be extended to any decision units by replacing the set of nondominated decisions with another decision selection rule.

Example. Let us consider the anticipatory tree *T* shown in Fig. 4 below. Observe that the Step 1 of Algorithm 2 need not be illustrated in this example because we analyze the optimal decisions only, without taking into account how these were derived from, constraints, the criteria functions and preference structures. To eliminate irrelevant feedback relations, all anticipatory feedbacks are checked whether they link weakly causally dependent decision units in an appropriate order. For instance, if an anticipatory feedback links O_3 and O_1, this can be accomplished by an iterative search in the list of causal predecessors of O_3 until either O_1 or O_0 is reached. In the latter case, the feedback would be irrelevant. In Fig. 4, the feedback between O_9 and O_{11} is identified as irrelevant and eliminated from further consideration. There are five end nodes in the tree, namely, O_4, O_6, O_9, O_{10}, and O_{11}. According to Definition 6, O_1 is the bifurcation unit for the end nodes O_4 and O_6, while O_8 is the bifurcation unit for O_9 and O_{10} and O_7 is the bifurcation unit for O_9, O_{10}, and O_{11}. After surveying all end nodes we find out that $B = \{O_1, O_7, O_8\}$.

Pursuing the procedure outlined in Step 3 of Algorithm 2, we remove the final branches of T which correspond to the bifurcation units identified so far, or – equivalently – all decision units causally dependent only on either O_1, O_7, or O_8.

The only bifurcation decision unit in the tree so reduced which is now added to B is O_0. Thus B consists of four bifurcation optimizers: O_1, O_7, O_8, and O_0. In Step 4 of Algorithm 2, they are ordered according to the causal order inherited from T. The further calculations in Steps 5k and 6k follow this order. ■

Generally, in a network of active decision units, there may exist nodes that are directly causally influenced by two or more predecessors (cf. Figs. 1 and 7). Such problems emerge often when, for example, an input to an investment decision comes from two or more independent economic, social, or technological processes, which are optimized with respect to the same criteria. In order to deal with such a situation, observe first that the causal dependences in form of constraints on the set of admissible decisions in a subsequent problem O_k, which come from two or more causally independent decision units yield, in fact, just an intersection of constraints. For example, if the units are optimizers $O_i=(U_i, F_i, P_i)$ and $O_j=(U_j, F_j, P_j)$ influencing another unit with the constraint multifunctions Y_i and Y_j, respectively, their joint influence can be represented as a new multifunction Y defined on the Cartesian product of $F_i(U_i)$ and $F_j(U_j)$, namely,

$$Y(u_{\mathrm{ip}}, u_{\mathrm{jr}}) := Y_i(u_{\mathrm{ip}}) \bigotimes Y_j(u_{\mathrm{jr}}), \tag{16}$$

where "$\bigotimes$" denotes an arbitrary set theoretical combination of the arguments of Y, characteristic to their joint influence (e.g. the union of sets corresponds to a "permissive" influence, while the intersection occurs in a restrictive Y). Based on this observation, in case of arbitrary networks, one has to survey all decision units which are causally influenced by two or more predecessors. The above process can be repeated iteratively in a similar way as Algorithms 1 and 2. The Problem 1 can be regularized for a case where no solution exists, by solving the relaxed Problem 2 as proposed in section "Anticipatory Chains."

From the above remarks, we conclude that a solution scheme based on Algorithms 1 and 2 as subroutines can be used to solve general anticipatory networks.

Generalizations and Extensions of Anticipatory Networks

The aim of this section is to present further ideas concerning anticipatory network theory and basic hints on how to analyze them. There exists a variety of network architectures, potential anticipatory feedback rules, and resulting solution rules. Below we discuss some of these extensions and directions for further research.

Interactive Optimizers and Sustainable Decisions

The principal application of the decision model presented in this chapter is to assist the decision-maker in choosing a compromise decision at an initial decision unit O_0. This decision will be selected taking into account all the anticipatory information encoded in the network's structure.

The above theory can also be applied to model interactive solution process in multicriteria decision problems. The latter can be represented as a self-feedback loop in a causal graph provided that a multicriteria decision unit O_χ is endowed with an interactive solution selection mechanism

$$\chi : \Pi(U,F,P) \times I \to \Pi(U,F,P), \tag{17}$$

where $\Pi(U, F, P)$ is the set of nondominated solutions for the problem (1) and I is the set states of knowledge of an external agent. These evolve according to the equation $s_{i+1} := \zeta(s_i, \pi_i)$, where ζ is a transition function that assigns a new state of knowledge s_{i+1} based on the former state s_i and on the assessment of the current nondominated output π_i generated by the multicriteria optimizer O_χ. The actions of O_χ and ζ are repeated iteratively for different states of knowledge. For an appropriately defined transition function ζ the sequence of nondominated solutions thus generated converges to a compromise solution in $\Pi(U, F, P)$. There may be multiple external agents ('advisors') involved in the decision making process as well as the latter may depend on the random states of nature **N**. The unit O_χ will be termed a *repetitive optimizer*. Its scheme is shown in Fig. 5.

Another class of decision units related to interactive optimizers is referred to as *sustainability units*. Instead of modeling an interaction on a separate time scale, assuming the identity preservation of the decision-makers, sustainability units are stretched in time, and the parameters of the subsequent-stage units evolve following predefined modification rules. Unlike as in the classical anticipatory network model, it is assumed that the decision-makers responsible for future decisions in a sustainability unit may be the same as at the previous stages. The decision-makers may also vary but remain closely related to their predecessors. Only the outputs from the last-stage units are taken into account in the optimization process and as sources of anticipatory feedback information for other units in the network.

Hybrid Networks

Hybrid anticipatory networks have been introduced in Skulimowski (2012a). They may contain nodes modeling different types of future decision problems, apart from optimization problems. Hybrid network nodes can model the choice of a mixed strategy in games with conflicts that may eventually lead to Nash equilibria, subset selection problems, rankings, or predetermined, random, or irrational decisions. Although hybrid anticipatory networks may contain nodes of all above types, their structure is similar to networks of optimizers as the nodes are connected by edges

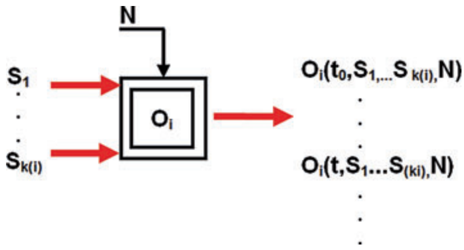


Fig. 5 A repetitive optimizer – an interactive anticipatory decision-making agent in an AN

modeling causal and anticipatory feedback relations. All nodes in an anticipatory network are termed *decision units*, while optimizers and game units are additionally termed *active*. Decision units of all kinds produce output decisions based on the inputs fed by units preceding them in the causal order. In Fig. 6, we present the schemes of the most frequently occurring decision problems that can be modeled as nodes in hybrid anticipatory networks.

Similarly as in networks of optimizers, the inputs to all units presented in Fig. 6 depend on the outputs from other units except the initial node, which has no inputs unless it is influenced by the external environment (the nature) $\boldsymbol{N}$. This dependence may have the form of a multifunction φ that defines an influence relation r, or another relation that, e.g., modifies the preference structure of a decision unit. Every decision unit has two functions:

(i) It transforms the input signals into the mapping that modifies the parameters of the decision problem to be solved; this transformation is a characteristic feature of a particular decision unit.
(ii) It solves its decision problem with modified parameters and produces the output, which can be identified with the solution of the decision unit's characteristic problem; the output is unique except the game units, where the number of outputs equals the number of players.

For example, the aggregation of input signals represented by multifunctions $\varphi_1, \ldots, \varphi_k$ that restrict the choice of an admissible decision at a decision unit O_i by imposing additional constraints may be defined as an intersection $\varphi = \varphi_1 \cap \ \ldots \ \cap \varphi_k$. If U_i is the decision set at O_i and the outputs at the decision units causally preceding O_i are $x_1, \ldots, x_k$ then $\varphi(x_1, \ \ldots, x_k) = \varphi_1(x_1) \cap \ \ldots \ \cap \varphi_k(x_k) \subset U_i$. In general, aggregations can be defined by arbitrary Boolean and algebraic operations, depending on modeling purposes.

Recall that a decision unit O_i is termed *active* if a decision-maker associated to O_i is able to perform a free choice (Skulimowski 2011). Otherwise it is termed *passive*. The operations of decision units, including those described in section "Interactive Optimizers and Sustainable Decisions," are summarized in Table 1.

An example of a hybrid anticipatory network where the influence relations form the causal graph Γ is shown in Fig. 7 below.

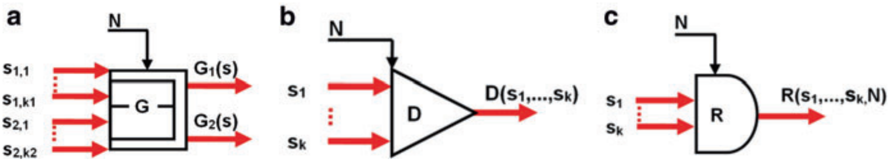


Fig. 6 Decision units occurring in hybrid anticipatory networks: **a** two-player game unit with $s = (s_{1,1}, \dots, s_{1,k1}, s_{2,1}, \dots, s_{2,k2})$, **b** a predetermined algorithmic decision unit with no freewill, and **c** random decisions with a known distribution function. Output from all units may depend on the states of nature N (source: Skulimowski 2012)

Table 1 Decision units occurring in hybrid anticipatory networks

Decision unit	Type	Internal parameters	Output function(s)
Multicriteria optimizer	Active	The feasible decision set U, the (vector) criterion F, preference structure P	A single optimal solution or a subset of the Pareto set
n-player game unit	Active	Strategy sets for all players, information sets, payoff functions	The values of payoff functions G_i for all players
Algorithmic decision unit	Passive	The deterministic output function D. The value of D on the decision set U is calculated with a known algorithm	A unique value of D on $V \subset U$, or a subset of $D(U)$ determined by the inputs
Random decision unit	Passive	Probability distributions describing the random decision generation	A random number or a random output subset
Repetitive decision unit	Active	An interaction algorithm to assess anticipated own decisions (cf. section "Interactive Optimizers and Sustainable Decisions")	Compromise decision made within an interaction
Sustainability unit	Active	A unit is stretched in time, the parameters of the subsequent-stage units follow a modification rule	Outputs from the last-stage units only
Other units	Active or passive	Uncertainty characteristics (fuzzy, possibilistic, fuzzy-random variables, etc.) and decision sets	Different types of outputs, depending on the specificity of the decision unit

Source: classification based on Skulimowski (2012), *extended*

Timed Anticipatory Networks

Timed anticipatory networks have been introduced and analyzed in Skulimowski (2016b, 2017a). Their definition requires two time scales: one is for the anticipatory network evolution; the other one is required for modeling the anticipatory solution process at each time step of the first scale. Consequently, we will define first the discrete solution time interval T and assume that there exists an optimization problem to be solved for all $t \in T$ modeled as an initial node of the network. Moreover, in a *timed anticipatory networks (TAN)*, the anticipatory multigraph may vary in time. The formal definition of a TAN is given below (Skulimowski 2016b).

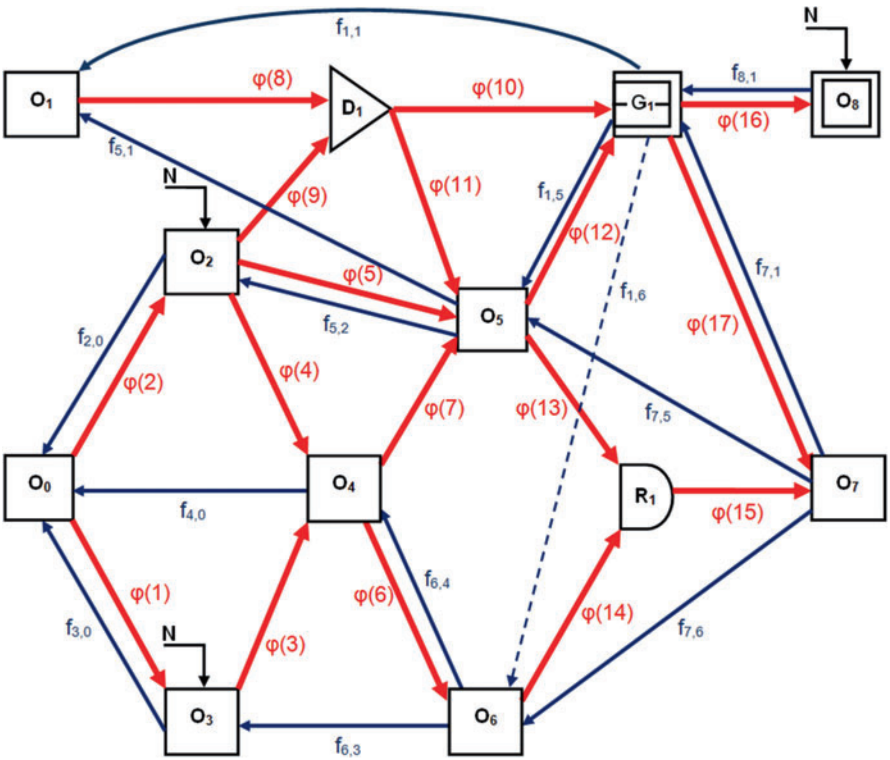


Fig. 7 An example of a hybrid anticipatory network with 12 units. The *dotted line* shows an induced anticipatory feedback (cf. Skulimowski 2016a). Observe that the unit O_8 is a repetitive optimizer (based on Skulimowski 2012)

Definition 8 *A timed anticipatory network A(t) is a directed-multigraph-valued time series defined for* $t \in T = \{t_0, t_1, \ldots, t_n\}$, $t_{i-1} < t_i$ *for* $i = 1,\ldots,n$, *such that*

(a) *For each* $t \in T$, *A(t) is an anticipatory network where each decision is to be made within a prescribed time interval termed decision time horizon.*

(b) *For each i,* $i = 0,\ldots,n-1$, t_{i+1} *corresponds to certain decision time horizon in A(t).*

(c) *The decisions made and solutions implemented in the network* $A(t_i)$ *by all agents until the time* t_{i+1} *comply with the structure of the network* $A(t_{i+1})$.

(d) *The initial node and at least one other node in* $A(t_{i+1})$ *inherit the multigraph structure from* $A(t_i)$. ■

There are many more specific structures of TANs modeling different decision situations. In a TAN considered in the following sections, any unit's decision is a sequence of simple decisions made by the same evolving agent at subsequent moments of time. The decisions made by future agents at a moment t_i become known to their predecessors at time t_{i+1} and may be used for supervisory learning of decision rules, supplementing the anticipatory feedback information.

The following principles, derived from real-life interpretations of timed ANs as anticipatory robot formations, will serve to model the evolution and compute the optimal solutions of structured TANs:

- A finite discrete time interval $T = [t_0, t_f]$ plays the role of evolution period.
- For every $t \in T$, the causal subgraph of $A(t)$ is embedded in a given structure graph $S(A)$ that is characteristic for this TAN, $S(A)$ is a finite digraph with no cycles, and non-isolated nodes with no predecessors are termed *initial nodes*.
- The functions of TAN's units are characterized by their positions in the structure graph, i.e., different physical units and decision-makers may be substituted for the same specific node in the structure graph during the network evolution.
- All TAN units, $V_i(t_k)$, $i = 1,\ldots,N$, $k:=k(i)$, are initially homogeneous and anonymous; this may change for $t_k > t_0$.
- At least one decision, namely, at an initial node, is made, and at least one causal impact is executed at each time step t (*nontrivial progress principle*).
- If a unit $V_i(t_k)$ remains in the network at time t_{k+1}, then its native decision scope U_i and performance criteria F_i remain unchanged (*network stationarity principle*).
- Some units may be deleted, and some new ones may be admitted to the network as time changes from t_k to t_{k+1}.
- The decision-maker responsible for the unit $V_i(t_k)$ can remain in the network at time t_{k+1} as the decision-maker at a new unit $V_j(t_{k+1})$.

As an example, we describe the situation where each of the anticipatory decision problems modeled by the networks $A(t)$, $t = t_0, \ldots, t_f$ is solved either independently from problems occurring at the following moments or it takes into account the resources needed by the next-step problem only. The latter situation may happen exclusively if the decision-maker corresponding to the initial node at t_k knows which unit will play the role of the robot network coordinator at the next moment t_{k+1}. This case is investigated in more detail in Skulimowski (2016b).

Observe that the above-presented rules are less restrictive than the incremental evolutionary building of ANs, where the *causality preservation* principle must be satisfied, i.e., units along a causal chain cannot change their order from the t_k-*th* to the t_{k+1}-*th* modeling step.

The solution concept of a TAN is in a natural way a combination of solutions of $A(t)$ for $t \in T$. One can consider additive, multiplicative (in case of relative growth criteria), and Bayesian probability combination rules.

Now, we will provide the following three anticipatory optimization problem formulations for a TAN with corresponding solution principles:

(**P1**) A temporal combination of momentary criteria values is calculated for the initial node acting as an optimizer for all $t \in T$, irrespectively whether which different V_i's play the role of $V_0(t)$. All or only some selected criteria G of $V_0(t)$ are taken into account.

(**P2**) A temporal combination of momentary criteria values is calculated for the same physical unit that preserves its identity for different moments of time in a TAN. This procedure may be performed for all or only for some selected units.

(**P3**) The performance criteria are split into two groups: for each $t \in T$, the first group $G=(G_1,\ldots,G_{n1})$ is optimized on the set of admissible decisions of $A(t)$, and a temporal combination of their momentary nondominated values is calculated; the second group of criteria, $F_i=(F_{i1},\ldots,F_{i,m})$, describes the performance of V_i irrespective of its current role in $A(t)$; the F_i and their optimal (nondominated) values for $A(t)$ are assigned to the fixed physical units and combined for all $t \in T$.

An example of solving an anticipatory robot swarm optimization problem with the mixed principle P3 has been presented in Skulimowski (2016b). There exist a variety of multigraph-transforming rules $A(t + 1) = a(A(t))$ that satisfy the principles P1, P2, or P3 and such that for each $t \in T \cdot A(t) \subset S(A)$. The rules of TAN evolution based on the above-cited example are illustrated in Fig. 8.

Observe that a temporal combination of nondominated momentary criteria values needs not be nondominated. Therefore a post-optimal selection of Pareto-optimal decisions should be executed to find an optimal solution for the entire evolution period.

Vehicle routing, traffic coordination, supervision, and search for efficient equilibrium algorithms (cf., e.g., Huang et al. 2016) can benefit from the anticipatory decision-making principles in TANs; see Skulimowski (2016b) for further references.

Nested Anticipation and Superanticipatory Systems

Nested properties of anticipation are directly implied by the definition of the modern anticipatory systems (Rosen 1985). They were noted and discussed since the dawn of the anticipatory system theory. Rosen's definition assumes the existence of a *model* of itself and of the environment. If another anticipatory system is contained in the environment, its model should contain the first system, including its model. Here, we will show that a quantitative analysis of nested and recursive properties of anticipatory systems leads to the idea of superanticipation (Skulimowski 2014b, 2016a) that is closely related to anticipatory networks and can be described in formal and quantitative terms. In anticipatory networks, when making a decision, decision-maker(s) at a starting decision node can take into account the network of its successors and their models. Thus, the model of future environment available to a decision-maker contains models of agents acting in the future, including their own future models. The formal definition of a superanticipatory system can be formulated in the following way (Skulimowski, 2014b, 2016a).

Definition 9 *A* ***superanticipatory system*** *is an anticipatory system that contains at least one model of another future anticipatory system.* ■

By the above definition, the notion of superanticipation is idempotent, i.e., the inclusion of other superanticipatory systems in the model of the future of a superanticipatory system does not extend this class of systems since every superanticipatory system is also anticipatory. However, superanticipatory systems can be distinguished by a grade of nested superanticipation.

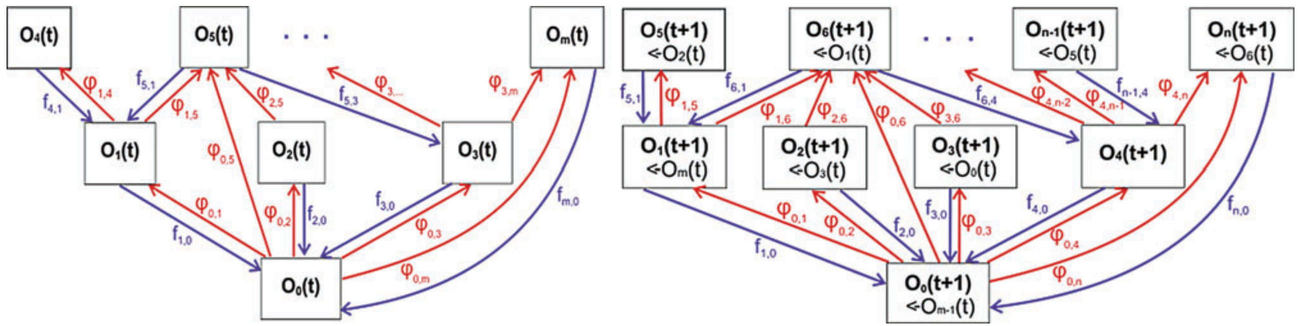


Fig. 8 An example of evolution of a timed anticipatory network from t (*left*) to $t + 1$ (*right*)

Definition 10 *A superanticipatory system S is* ***of grade n*** *if it contains a model of an external superanticipatory system of grade n-1 and does not contain any model of a superanticipatory system of grade n except itself. An anticipatory system which does not contain any model of another anticipatory system may be termed super-anticipatory of grade 0.* ■

The estimation of the grade n of a superanticipatory system S depends on its knowledge of the grade of other anticipatory systems modeled by S. When constructing its model of the environment, S is likely to underestimate the actual power of other system models. Thus, according to Definition 6, the grade of superanticipation of S derived from its anticipatory model is a grade of this model and an estimation from below of the actual grade of S as a biological, social, or an artificial autonomous system. In conflicting real-life environments without a perfect information exchange, modeled systems may have models of a higher grade than S could estimate. The exact value of the grade can be attributed to artificial collaborative systems only.

It turns out that anticipatory networks constitute a relevant subclass of super-anticipatory systems. Specifically, Thm.1 in (Skulimowski 2014b, 2016a) states that a finite anticipatory network is a superanticipatory system of grade n if it contains an anticipatory chain with at least n decision units and exactly n nested anticipatory feedbacks, and n is maximal with this property. For example, an anticipatory chain on n optimizers, each one linked with all its causal predecessors with an anticipatory feedback, is a superanticipatory system of grade n. The converse theorem is not true, for instance, two coupled superanticipatory systems of grade 1, S_1, and S_2, where S_1 contains a model of S_2 and vice versa, do not form an anticipatory network because if an anticipatory chain from S_1 to S_2 exists, then by acyclicity of anticipatory networks, there is no such a chain from S_2 to S_1.

Further development of superanticipatory systems, and nested anticipation in general, can greatly benefit from the relations to the notion of self-referentiality and other fundamental ideas of autonomous system theory (cf. Varela 1979).

Applications of Anticipatory Networks

The principal motivation to admit assumptions used in the definitions of networked discrete optimizers presented in the previous sections of this chapter came from the need to formalize scenario building and roadmapping processes in technological foresight (cf. Skulimowski 2014c) based on the identification of future decision-making processes. Anticipatory networks can model both hierarchical decision structures and horizontal links between decision problems, taking into account a large number of economic, environmental, social, and technological criteria. The other motivating application areas were:

- Multicriteria decision-making theory, where the use of scalar utility values is often insufficient
- Decision algorithms for artificial autonomous systems (AADS, cf. Skulimowski 2014d, 2016b)
- Financial decisions related to long-term resource allocation and their impact models

Following Skulimowski (2014a, b, c, d), two of them are presented in this chapter (sections "Timed Anticipatory Networks" and "Scenario Building and Other Foresight Applications"). Discussion of further anticipatory networks' applications is given in section "Discussion and Further Applications" below.

Scenario Building and Other Foresight Applications

The notion of scenario referred to in this section should be meant as alternative visions of the future similarly as foresight or strategic planning scenarios (Godet 2001; Skulimowski 2014c). The actual appearance of a specific scenario depends on the choice of economic, technological, or political decisions made by external agents in one or more present and/or future problems or on random or external trends and events, where the decision agents cannot be identified uniquely. This is why scenarios can be regarded as conditional forecasts.

When decision units with forecasted parameters are included in an anticipatory network, they allow us to generate decision rules taking into account alternative network structures depending on different variants of external influence. The influence of the decision made at O_i on future constraints and preferences can itself generate scenarios identified with the sequences of plausible outcomes from future decision problems following O_i. Assuming that each decision unit is rational and there are no conflicts with other units, one can apply multicriteria optimization methods to find all potential variants of anticipated rational future problem outcomes as either a nondominated point or Pareto equilibria in cooperative games. The admissible chains can be regarded as elementary scenarios (Skulimowski 2008) of the future values of optimization criteria and other indicators that characterize decision units in an anticipatory network. The anticipatory feedbacks can be used to filtering out non-plausible elementary scenarios, contributing thus to an efficient and rational building of foresight scenarios.

Game-theoretic models can also be applied to select and filter elementary scenarios defined in the same way as for the optimizers based on multicriteria optimization principles.

The Computational Methods for Anticipatory Decision-Making

As estimated from computational evidence – the Algorithms 1 and 2 based on forward-looking surveys of anticipatory feedbacks allow us to efficiently manage problems with up to 10^3 alternatives in up to 10^2 optimizers. The analysis of larger or more specialized problems can be accomplished with tailored decision support systems capable of using anticipatory information.

A discussion of the "anytime" property of Algorithms 1 and 2 is given in Skulimowski (2014a). This property can also be regarded as an ability to include more and more predicative aspects of a real-life system into a computational model. When referring to engineering and many economic applications, one can assume that this computational property is fulfilled.

A related problem of searching in a dynamic graph considered by Likhachev et al. (2008) is of particular importance to the problem of planning autonomous mobile robot activity, where the external circumstances change in real time. The autonomous agent that models anticipatory systems in the network should be capable of acquiring the information about the environment and other systems and to update their models in real time. The design of such an algorithm for real-time implementations of anticipatory planning of robot operations requires an estimation of the expected information flow from the environment and other robots in the swarm and to confront it with the computational and communication capacities of the mobile robots. The qualitative analysis of network topology, including discovery of new units, emergence of new anticipatory feedbacks, and causal influences, has to be accomplished in real time as well. A more profound analysis of cooperating robot systems is presented in the paper (Skulimowski 2016b), devoted to anticipatory autonomous vehicle swarms.

Discussion and Further Applications

Apart from the abovementioned inspirations coming from the potential uses in technological foresight, sustainable knowledge platform planning (Skulimowski 2017a), medicine, finance, and environmental modeling, as well as applications of timed networks to adaptive robot control systems with feedback, discussed briefly in section "The Computational Methods for Anticipatory Decision-Making," there are further potential areas of basic and applied research, where the ANs may provide competitive solutions. These include:

- Modeling the behavioral decision-making mechanisms in systems biology with nested anticipatory systems (cf. Baliga 2008; Terenzi 2008; Skulimowski 2014b)
- Extending the theory of n-level multicriteria programming (Nishizaki and Sakawa 2009) and merging it with the variable and flexible contraints optimization
- Investigating relations of anticipatory network models to predictive and anticipatory control (Kaczorek 2002)
- Analysis of anticipation phenomena in queuing networks as well as their applications in telecommunications

The latter field attracts an ample attention of researchers recently. For example, Mayora and Osmani (2014) defined the concept of Human-Aware Networking (HAN). The main idea behind HAN is to utilize the sensed context and the recognized human behaviors mostly based on the use of mobile devices in order to feed the network with the interaction patterns of users to better set up and optimize network parameters. With HAN, the goal is to maximize the relationship between provided quality of service. The AN-based models provide a new approach to modelling decision making, forecasting and foresight processes (Skulimowski 2014c). Skulimowski (2014c, 2017b) presented an application of anticipatory networks to build the development strategy for a regional creativity support center. Another real-life application is the sustainable exploitation strategy planning for an innovative knowledge platform elaborated within a Horizon 2020 research project (cf. www.moving-project.eu), where anticipatory networks are used to define the framework of the roadmapping process of the platform.

Further potential real-life applications of the model extensions presented in this chapter include:

- Modeling the impact of social innovations, including open innovation and knowledge platforms (Skulimowski 2017a)
- Planning long-term financial investment decisions, cf. Sec. 6.4 in (Skulimowski 2014a) for a more detailed discussion of this class of applications
- Environmental sustainability modeling, where the anticipatory feedback corresponds to the classical "ensuring a sustainable future for next generations" principle

ANs may also be useful to invent novel analysis approaches and new architectures of multilayer artificial neural networks, where active decision units and other anticipatory systems modeled in an AN coexist with neurons and fed them anticipatory information to adaptively optimize their structure and parameters. Hybrid networks could also be useful to model the anticipatory functions of the brain (cf. Antle and Silver 2009; Ghajar and Ivry 2009; Pezzulo and Rigoli 2011; Hassabis et al. 2014), where AN components are black box models of anticipatory functions evidenced in brain but with unknown or imprecise location in the cortex (Fiebach and Schubotz 2006).

We expect that further application in the medicine and health care will emerge in the next years, along with further progress in the AN theory. Anticipatory planning clinical therapy paths belong to the relevant motivating applications of the overall anticipatory network theory. ANs can be particularly useful to model therapy results and to select the best clinical path when patients will undergo different medical interventions and diagnostic examinations at hospital departments, as well as ambulant and home treatments. In such a case, the results of a recommended therapy depend to a large extent to the trust of anticipated rational behavior of the patient, in a networked environment of influences from other medical personnel as well as from home and working conditions impact.

Summary

The theory of anticipatory networks combines the features of causal and influence modeling (cf., e.g., Sloman and Hagmayer 2006) with the anticipatory system theory and new concepts of anticipatory feedback relation and superanticipation. The well-

known ideas of modeling linked decision problems has been endowed with additional modeling power by adding the qualitative and quantitative characteristics of anticipation. This has made possible first to define new methods of anticipatory preference modeling to solving multicriteria optimization problems linked by causal relations. Then, it emerged a number of further fruitful applications of this theory, specifically in the areas of autonomous systems, foresight, and strategic planning.

Applying the anticipatory models presented in this chapter needs heterogeneous analytic techniques, combining forecasting to define the parameters of future problems, with foresight-based techniques, optimization, uncertainty analysis, and knowledge engineering. This will push the researchers to build complex models that are likely to require new insights into the decision theory, computational methods, and cognitive science.

Unlike as in the supervisory models of multicriteria decision-making, with just one source of recommendations, the decision-maker embedded in an anticipatory network may update the decision choice depending on the anticipated decisions and anticipatory behavior of other decision units. This is why a key role in an anticipatory network is played by trust between decision agents. This is particularly relevant in networks modeling cooperating autonomous robots together with some human operators. An in-depth analysis of different anticipation and trust levels, partial anticipation, and further information flows in the network should shed new light on the trust, confidence, and cooperation theory of autonomous agents. A better cooperation of robots, either individual, such as mobile rehabilitation robots cooperating with a handicapped human, robot formations, and swarms, using supervisory control to form ad hoc anticipatory networks as presented in Skulimowski (2016b) can bring new benefits from anticipatory network modeling.

For the sake of parsimony, this chapter did not refer to those anticipation aspects and anticipation-related concepts which do not have an interpretation in the AN theory yet. As an exception, we will mention a speculative yet potentially high-impact application area of anticipatory networks in its contribution to modeling of quantum consciousness (Hameroff and Penrose 2014; Skulimowski 2013). The hypothesis concerning quantum entanglement across time (cf., e.g., Olson and Ralph 2011) can interplay with the synchronization phenomena in macroscopic physical systems (Dubois 2001; Sun and Bollt 2014). Further extensions of the anticipatory network models, including continuous and stochastic decision problems, as well as a possibility of constructing a time-space model consisting of linked elementary decision problems, by an analogy to entangled qubit models in quantum information theory may provide new challenges to further researchers, see Tuszynski (2006, ed.) for a review of related ideas.

Finally, we have to stress that the concept of anticipation encompasses many different aspects and application fields, and not all of them can be or are intended to be converted into a computational science area. This is why the approach to quantify the anticipation presented in this chapter cannot be regarded neither as universal nor unique. As emphasized in sections "Introduction" and "Anticipatory Decision-Making in Multicriteria Problems," building a computable anticipatory network presumes an availability of forecasts and the knowledge of parameters that can be taken for granted in some areas of application only, related predominantly to ICT, engineering, economy, and finance. In humanities, literature, or some branches of social sciences, anticipation may play a relevant role despite a lack of quantitative models. In addition, every model referring to human decisions is by its nature approximate, as it merely simulates some aspects of human thinking that are fundamental for the decision-making process. Nevertheless, the above disclaimer may gradually lose its validity along with the future development of artificial intelligence tools and artificial creativity that are sometimes supposed to invade the areas traditionally restricted to human creative activity during the next century, cf. Skulimowski (2014d). As the theory of anticipatory networks will be further developed, one can expect that more fruitful relations to other areas of the science of anticipation will be discovered, more constructive tools will be created, and more real-life applications found.

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