CSC 211: Computer Programming
(Recursive) Backtracking

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THINK BIG WE DO

Administrative Announcements

- **MC05 due 07/10**

- **A03 due 07/09**

- **Exam#02 - 07/13**

- No make up day this Friday

Recursion Reminder

- Problem solving technique in which we solve a task by reducing it to smaller tasks (of the same kind)
  - then use same approach to solve the smaller tasks

- Technically, a recursive function is one that calls itself

- General form:
  - **base case**
    - solution for a **trivial case**
    - it can be used to stop the recursion (prevents "*stack overflow*")
    - every recursive algorithm needs at least one base case
  - **recursive call(s)**
    - divide problem into **smaller instance(s)** of the **same structure**

Recursion Reminder

- Recursive Checklist:

  - **Find what information we need to keep track of.** What inputs/outputs are needed to solve the problem at each step?

  - **Find our base case(s).** What are the simplest (nonrecursive) instance(s) of this problem?

  - **Find our recursive step.** How can this problem be solved in terms of one or more simpler instances of the same problem that lead to a base case?

  - **Ensure every input is handled.** Do we cover all possible cases? Do we need to handle errors?

Recursion Reminder

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  - ✓ **Ensure every input is handled.** Do we cover all possible cases? Do we need to handle errors?

Backtracking

- Write a recursive function printAllBinary that accepts an integer number of digits and prints all binary numbers that have exactly that many digits, in ascending order, one per line
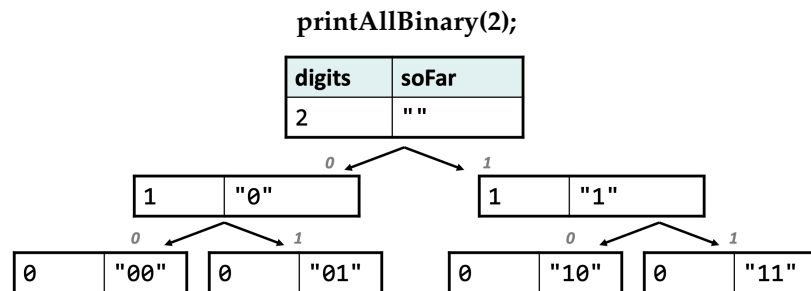
printAllBinary(2);	printAllBinary(3);
00	000
01	001
10	010
11	011
	100
	101
	110
	111

Decision Trees

printAllBinary(2);
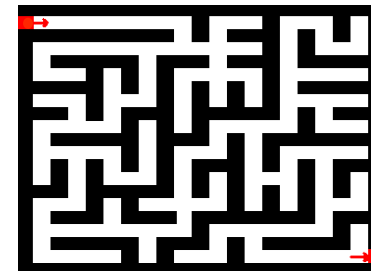
digits	soFar
2	""

- This kind of diagram is called a **call tree** or **decision tree**

- Think of each call as a choice or decision made by the algorithm:

  • Should I choose 0 as the next digit?
  • Should I choose 1 as the next digit?

- The idea is to try every permutation. For every position, there are 2 options, either '0' or '1'. **Backtracking** can be used in this approach to try every possibility or permutation to generate the correct set of strings.
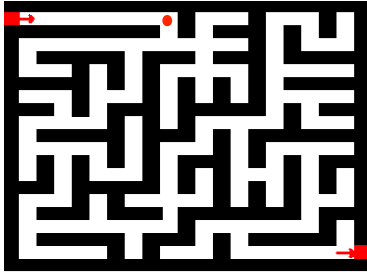
Backtracking

- **Recursive Backtracking**: using recursion to explore solutions to a problem and abandoning them if they are not suitable

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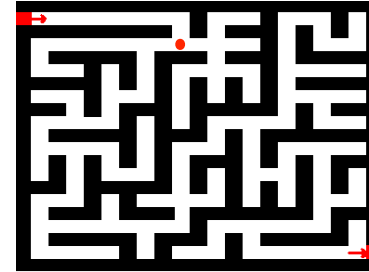
Backtracking

- **Recursive Backtracking**: using recursion to explore solutions to a problem and abandoning them if they are not suitable
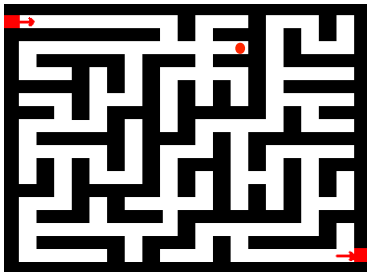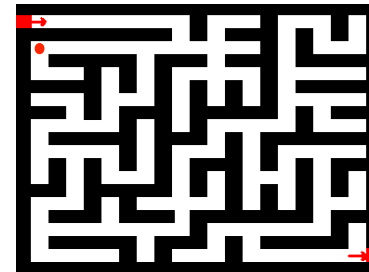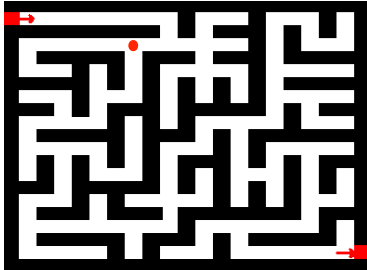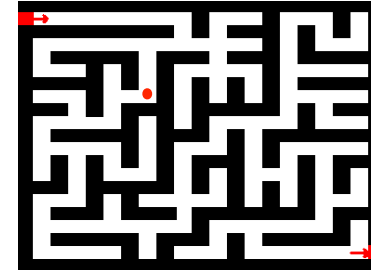
Backtracking

- **Recursive Backtracking**: using recursion to explore solutions to a problem and abandoning them if they are not suitable

Backtracking

- Let's take a look at a problem similar to the binarySequence problem.

- Write a recursive function diceRoll that accepts an integer representing a number of 6-sided dice to roll, and output all possible permutations of values that could appear on the dice.

diceRoll(2)

{1,1}	{3, 1}	{5, 1}
{1, 2}	{3, 2}	{5, 2}
{1, 3}	{3, 3}	{5, 3}
{1, 4}	{3, 4}	{5, 4}
{1, 5}	{3, 5}	{5, 5}
{1, 6}	{3, 6}	{5, 6}
{2, 1}	{4, 1}	{6, 1}
{2, 2}	{4, 2}	{6, 2}
{2, 3}	{4, 3}	{6, 3}
{2, 4}	{4, 4}	{6, 4}
{2, 5}	{4, 5}	{6, 5}
{2, 6}	{4, 6}	{6, 6}

Backtracking

- Backtracking Checklist:
  - **Find what choice(s) we have at each step.** What different options are there for the next step?

    For each valid choice:

    - **Make it and explore recursively.** Pass the information for a choice to the next recursive call(s).
    - **Undo it after exploring.** Restore everything to the way it was before making this choice.

    - **Find our base case(s).** What should we do when we are out of decisions?

Backtracking

- Backtracking Checklist:

  ✓ **Find what choice(s) we have at each step.** What different options are there for the next step?

  For each valid choice:

  - **Make it and explore recursively.** Pass the information for a choice to the next recursive call(s).

  - **Undo it after exploring.** Restore everything to the way it was before making this choice.

  ✓ **Find our base case(s).** What should we do when we are out of decisions?

  > What die value should I choose next?

Backtracking

- Backtracking Checklist:

  ✓ **Find what choice(s) we have at each step.** What different options are there for the next step?

  For each valid choice:

  - **Make it and explore recursively.** Pass the information for a choice to the next recursive call(s).

  - **Undo it after exploring.** Restore everything to the way it was before making this choice.

  ✓ **Find our base case(s).** What should we do when we are out of decisions?

  > We need to communicate the dice chosen so far to the next recursive call

---

Backtracking

- Backtracking Checklist:

  ✓ **Find what choice(s) we have at each step.** What different options are there for the next step?

  For each valid choice:

  - **Make it and explore recursively.** Pass the information for a choice to the next recursive call(s).

  - **Undo it after exploring.** Restore everything to the way it was before making this choice.

  ✓ **Find our base case(s).** What should we do when we are out of decisions?

  > We need to be able to remove the die we added to our first roll so far

---

Backtracking

- Backtracking Checklist:

  ✓ **Find what choice(s) we have at each step.** What different options are there for the next step?

  For each valid choice:

  - **Make it and explore recursively.** Pass the information for a choice to the next recursive call(s).

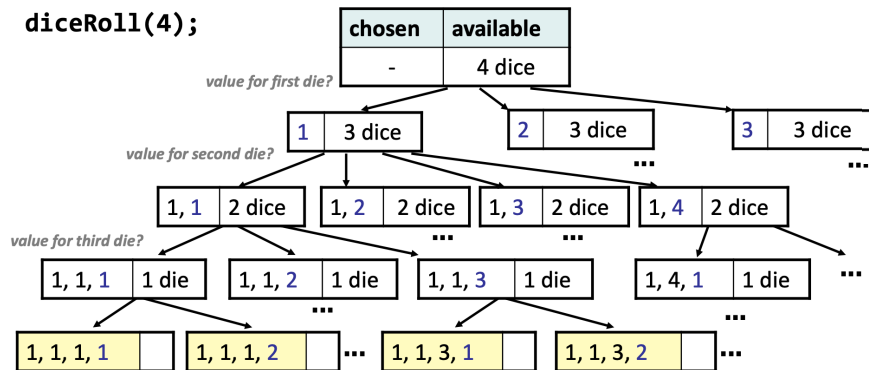 - **Undo it after exploring.** Restore everything to the way it was before making this choice.

  ✓ **Find our base case(s).** What should we do when we are out of decisions?

  > We have no dice left to choose, print them out

Backtracking

```
diceRoll(4);
```

chosen	available
-	4 dice

value for first die?

1	3 dice		2	3 dice		3	3 dice

...   **...**

value for second die?

1, 1	2 dice	1, 2	2 dice	1, 3	2 dice	1, 4	2 dice

...   **...**

value for third die?

1, 1, 1	1 die	1, 1, 2	1 die	1, 1, 3	1 die	1, 4, 1	1 die	**...**

...   **...**

| 1, 1, 1, 1 | | 1, 1, 1, 2 | | **...** | 1, 1, 3, 1 | | 1, 1, 3, 2 | | **...** |

- Observations?

- This is a really big search space.

- Depending on approach, we can make wasteful decisions.
  Can we optimize it? Yes. Will we right now? No.

Backtracking

- Let's us write flexible code, allowing us to make a decision and "backtrack" if we need to

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

Backtracking

- **Pseudocode**

- function diceRolls(dice, chosenArr):
      if dice == 0:
          Print current roll.
      else:
          // handle all roll values for a single die; let recursion do the rest.
          for each die value i in range [1..6]:
              choose that the current die will have value i
              // explore the remaining dice
              diceRolls(dice-1, chosenArr)
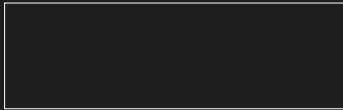   un-choose (*backtrack*) the value I

- Write a recursive function diceRoll that accepts an integer representing a number of 6-sided dice to roll, and output all possible combinations of values that could appear on the dice.

** Need to keep track of our choices somehow
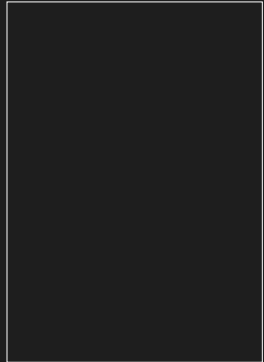

Code Demo

Recursive Backtracking Trace (Slide 25)

Output for diceRolls (2):

Stack

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```
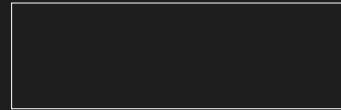
25

Recursive Backtracking Trace (Slide 26)

Output for diceRolls (2):

Stack

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [ 1 ]
```

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```
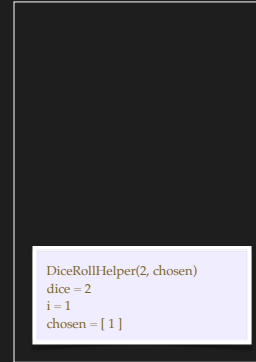
26

Recursive Backtracking Trace (Slide 27)

Output for diceRolls (2):

Stack

```
DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [ 1 , 1 ]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [ 1 ]
```

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
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                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```
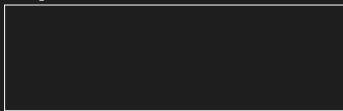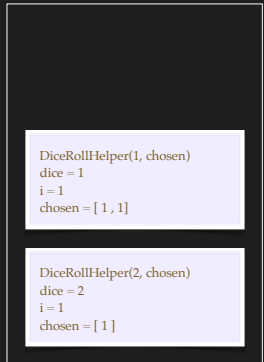
27

Recursive Backtracking Trace (Slide 28)

Output for diceRolls (2):

Stack

```
DiceRollHelper(0, chosen)
dice = 0
chosen = [ 1 , 1 ]
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [ 1 , 1 ]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [ 1 ]
```

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```
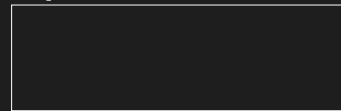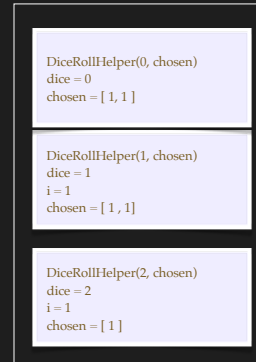
28

Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
```

Stack

DiceRollHelper(0, chosen)
dice = 0
chosen = [1, 1]

DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [1, 1]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

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        std::cout << "{";
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    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
```

Stack

DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [1, 1]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
```

Stack

DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [1,]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

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                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
```

Stack

DiceRollHelper(1, chosen)
dice = 1
i = 2
chosen = [1,]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
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                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

Recursive Backtracking Trace (Slide 33)

Output for diceRolls (2):

```
{1, 1}
```

Stack

```
DiceRollHelper(1, chosen)
dice = 1
i = 2
chosen = [ 1 , 2 ]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [ 1 ]
```
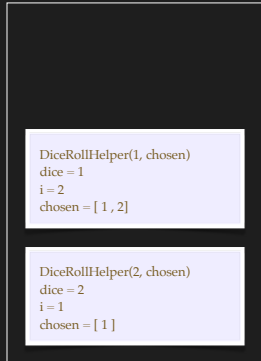
```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

Recursive Backtracking Trace (Slide 34)

Output for diceRolls (2):

```
{1, 1}
```

Stack

```
DiceRollHelper(0, chosen)
dice = 0
chosen = [ 1 , 2 ]
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 2
chosen = [ 1 , 2 ]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [ 1 ]
```
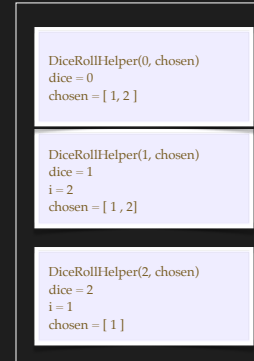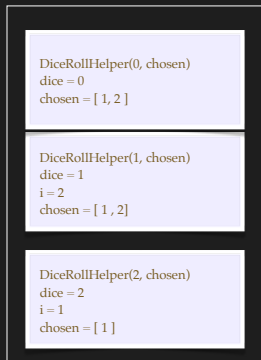
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void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

Recursive Backtracking Trace (Slide 35)

Output for diceRolls (2):

```
{1, 1}
{1, 2}
```

Stack

```
DiceRollHelper(0, chosen)
dice = 0
chosen = [ 1 , 2 ]
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 2
chosen = [ 1 , 2 ]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [ 1 ]
```
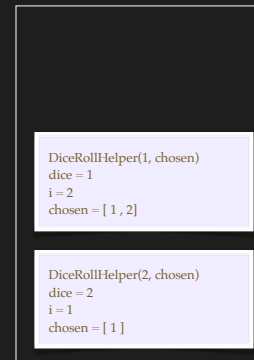
```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

Recursive Backtracking Trace (Slide 36)

Output for diceRolls (2):

```
{1, 1}
{1, 2}
```

Stack

```
DiceRollHelper(1, chosen)
dice = 1
i = 2
chosen = [ 1 , 2 ]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [ 1 ]
```

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
{1, 2}
```
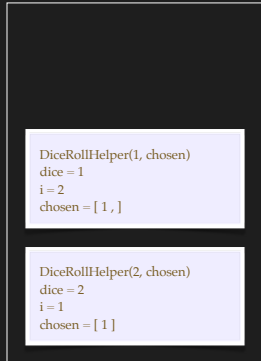
```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

Stack

```
DiceRollHelper(1, chosen)
dice = 1
i = 2
chosen = [ 1 , ]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [ 1 ]
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
{1, 2}
```
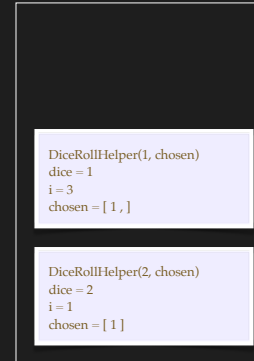
```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

Stack

```
DiceRollHelper(1, chosen)
dice = 1
i = 3
chosen = [ 1 , ]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [ 1 ]
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
{1, 2}
```
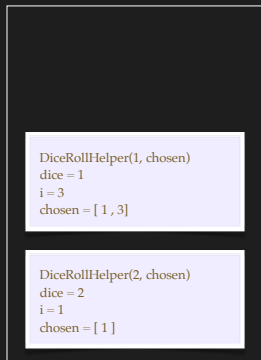
```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 3
chosen = [ 1 , 3 ]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [ 1 ]
```

Stack

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
{1, 2}
```
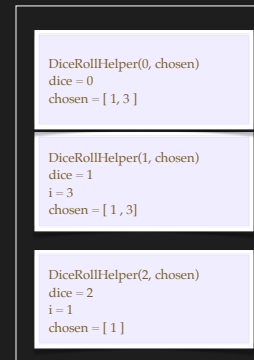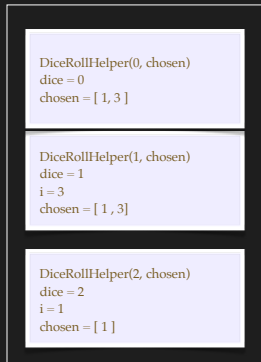
```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

```
DiceRollHelper(0, chosen)
dice = 0
chosen = [ 1 , 3 ]
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 3
chosen = [ 1 , 3 ]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [ 1 ]
```

Stack

Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
{1, 2}
{1, 3}
```

Stack:

```
DiceRollHelper(0, chosen)
dice = 0
chosen = [ 1, 3 ]
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 3
chosen = [ 1, 3 ]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [ 1 ]
```

Stack

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() −1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice − 1, chosen);
            chosen.pop_back();
        }
    }
}
```

Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
{1, 2}
{1, 3}
```

Stack:

```
DiceRollHelper(1, chosen)
dice = 1
i = 3
chosen = [ 1, 3 ]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [ 1 ]
```
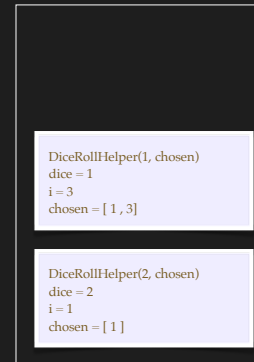
Stack

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() −1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice − 1, chosen);
            chosen.pop_back();
        }
    }
}
```
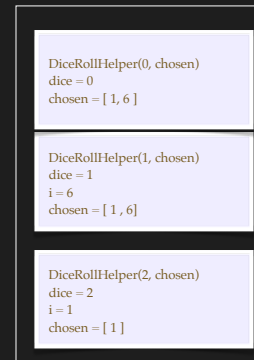

Recursive Backtracking Trace

Fastforward…

Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}    {1, 5}
{1, 2}
{1, 3}
{1, 4}
```

Stack:

```
DiceRollHelper(0, chosen)
dice = 0
chosen = [ 1, 6 ]
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 6
chosen = [ 1, 6 ]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [ 1 ]
```
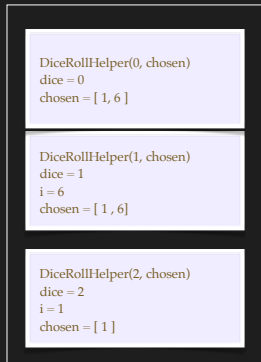
Stack

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() −1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice − 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}   {1, 5}
{1, 2}   {1, 6}
{1, 3}
{1, 4}
```

Stack:

```
DiceRollHelper(0, chosen)
dice = 0
chosen = [ 1 , 6 ]
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 6
chosen = [ 1 , 6 ]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [ 1 ]
```
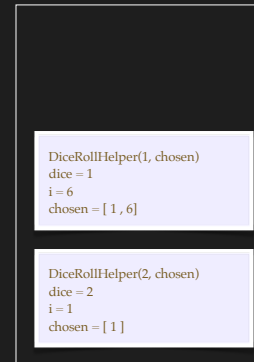
Stack

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}   {1, 5}
{1, 2}   {1, 6}
{1, 3}
{1, 4}
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 6
chosen = [ 1 , 6 ]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [ 1 ]
```
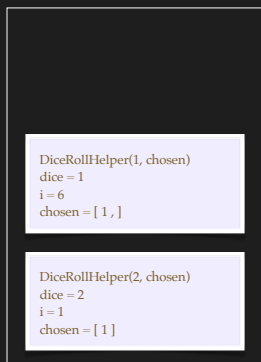
Stack

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}   {1, 5}
{1, 2}   {1, 6}
{1, 3}
{1, 4}
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 6
chosen = [ 1 , ]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [ 1 ]
```
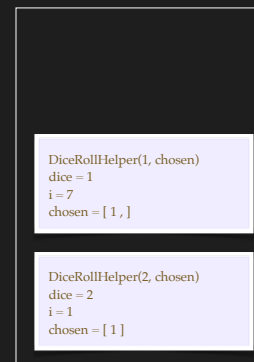
Stack

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}   {1, 5}
{1, 2}   {1, 6}
{1, 3}
{1, 4}
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 7
chosen = [ 1 , ]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [ 1 ]
```

Stack

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

Recursive Backtracking Trace (Slide 49)

Output for diceRolls (2):

```
{1, 1}    {1, 5}
{1, 2}    {1, 6}
{1, 3}
{1, 4}
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [ 1 ]
```

Stack

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() −1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice − 1, chosen);
            chosen.pop_back();
        }
    }
}
```

Recursive Backtracking Trace (Slide 50)

Output for diceRolls (2):

```
{1, 1}    {1, 5}
{1, 2}    {1, 6}
{1, 3}
{1, 4}
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [ ]
```

Stack

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() −1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice − 1, chosen);
            chosen.pop_back();
        }
    }
}
```

Recursive Backtracking Trace (Slide 51)

Output for diceRolls (2):

```
{1, 1}    {1, 5}
{1, 2}    {1, 6}
{1, 3}
{1, 4}
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 2
chosen = [ ]
```

Stack

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() −1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice − 1, chosen);
            chosen.pop_back();
        }
    }
}
```

Recursive Backtracking Trace (Slide 52)

Output for diceRolls (2):

```
{1, 1}    {1, 5}
{1, 2}    {1, 6}
{1, 3}
{1, 4}
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 2
chosen = [2]
```

Stack

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() −1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice − 1, chosen);
            chosen.pop_back();
        }
    }
}
```
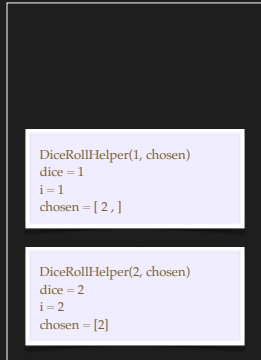
Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}    {1, 5}
{1, 2}    {1, 6}
{1, 3}
{1, 4}
```

Stack

```
DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [ 2 , ]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 2
chosen = [2]
```
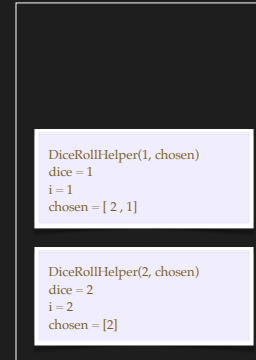
```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}    {1, 5}
{1, 2}    {1, 6}
{1, 3}
{1, 4}
```

Stack

```
DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [ 2 , 1]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 2
chosen = [2]
```
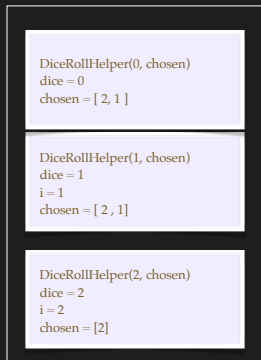
```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}    {1, 5}
{1, 2}    {1, 6}
{1, 3}
{1, 4}
```

```
DiceRollHelper(0, chosen)
dice = 0
chosen = [ 2, 1 ]
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [ 2, 1]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 2
chosen = [2]
```
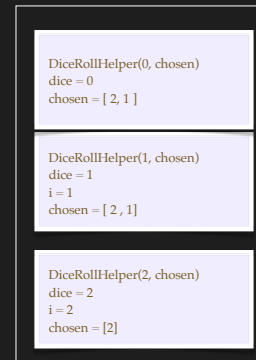
Stack

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}    {1, 5}
{1, 2}    {1, 6}
{1, 3}    {2, 1}
{1, 4}
```

```
DiceRollHelper(0, chosen)
dice = 0
chosen = [ 2, 1 ]
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [ 2, 1]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 2
chosen = [2]
```
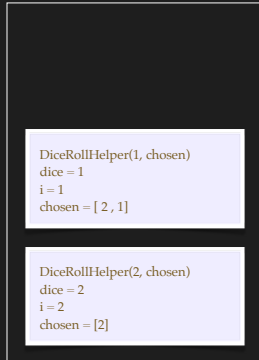
Stack

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

    // Base Case
    if (dice == 0) {

        //Print out contents of vector {1,1}
        std::cout << "{";
            for(int i=0; i < chosen.size(); i++){
                std::cout << chosen.at(i);
                if(i < chosen.size() -1){
                    std::cout << ",";
                }
            }
        std::cout << "} \n";
    }

    //Recursive case
    else {

        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

{1, 1}   {1, 5}
{1, 2}   {1, 6}
{1, 3}   {2, 1}
{1, 4}

```
DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [ 2 , 1]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 2
chosen = [2]
```

Stack

```cpp
void diceRollHelper(int dice, std::vector<int>& chosen) {

// Base Case
if (dice == 0) {

    //Print out contents of vector {1,1}
    std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
    std::cout << "} \n";
}

//Recursive case
else {

    for (int i = 1; i <= 6; i++) {
        chosen.push_back(i);
        diceRollHelper(dice - 1, chosen);
        chosen.pop_back();
    }
  }
}
```