Intro to AI and ML
MATRIX PROJECT

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Problem Statement

Original Question
Find the equation of the circle which is the mirror image of the circle

$$x^2 + y^2 - 2x = 0 \tag{1}$$

about the line

$$y = 3 - x \tag{2}$$

Problem Statement

Matrix Form

Find the equation of the circle, which is the mirror image of the circle

$$\mathbf{x}^T\mathbf{x} - \begin{pmatrix} 2 & 0 \end{pmatrix}\mathbf{x} = 0 \tag{3}$$

in the line

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = 3 \tag{4}$$

Table of Contents

Desired Answer

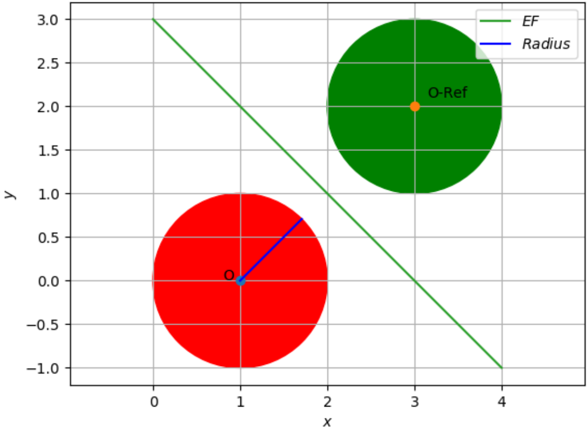

Figure: Reflection of circle about a line

Table of Contents

Solution

Let $\mathbf{c}$ be the center and r be the radius of the circle respectively.

$$\|(\mathbf{x}-\mathbf{c})\|^2 = r^2 \tag{5}$$

$$\Rightarrow (\mathbf{x}-\mathbf{c})^T(\mathbf{x}-\mathbf{c}) = r^2 \tag{6}$$

$$\Rightarrow \mathbf{x}^T\mathbf{x} - 2\mathbf{c}^T\mathbf{x} = r^2 - \mathbf{c}^T\mathbf{c} \tag{7}$$

Comparing with eqn(1),

$$\mathbf{c} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{8}$$

$$r^2 - \mathbf{c}^T\mathbf{c} = 0 \Rightarrow r = 1 \tag{9}$$

Solution

We have the equation of line as

$$\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} = 3 \tag{10}$$

this can be written in the form

$$\mathbf{Nx} = C \tag{11}$$

where N is the normal to the line and C is a constant.
Comparing with eqn(8),

$$\mathbf{N} = \begin{bmatrix}1\\1\end{bmatrix} \tag{12}$$

Intersection of line (passing through center $\mathbf{c}$ *and* $\mathbf{c} + \mathbf{0.1N}$*) with the given line gives the foot of perpendicular on the given line from* $\mathbf{c}$*.*

Solution

Let $\mathbf{f}$ *and* $\mathbf{c'}$ *be the foot of perpendicular and image of center respectively. Then we have*

$$\frac{\mathbf{c}+\mathbf{c'}}{2} = \mathbf{f} \tag{13}$$

$$\Rightarrow \mathbf{c'} = 2\mathbf{f} - \mathbf{c} \tag{14}$$

Since the radius remains same after reflection, we have equation of reflected circle as

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{c'}^T\mathbf{x} = r^2 - \mathbf{c'}^T\mathbf{c'} \tag{15}$$

Conclusion

So,the reflected circle is

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{c'}^T\mathbf{x} = r^2 - \mathbf{c'}^T\mathbf{c'} \tag{16}$$

Table of Contents

Walkthrough of the code 1(Functions)

function norm_vec(AB) //returns the normal vector of line AB.
function mid_pt(B,C) //calculates the mid point of two given points.
function line_intersect_normal_form(N,P) //creates a line from normal form.
function reflection_normal_form(n1,p1,A)//returns reflection of a point about a line.

Walkthrough of the code 1(Main Section)

MAIN SECTION

```
// centre of the circle from A
cen=np.matmul(cenM,A.T)
// constant term for the circle
D=0
// Reflected centre
refCen=reflection_normal_form(B,C,cen)
// Radius of the circle
radius=(cen[0]**2+cen[1]**2-D)**0.5
// Foot of perpendicular of the center to the line
E=(cen+refCen)/2
```

Table of Contents

Using slope and intercept of the line with linear tranformation

Solution

Consider the line $L : y = mx$ *that passes through origin.*
Let $\boldsymbol{A}$ *be the matrix representation of reflection*(T) *with respect to the standard basis* $\{e1, e2\}$ *about the line* L*.*
Any vector on the line L *does not move under the linear transformation* T*.*
Since the vector

$$\boldsymbol{V1} = \begin{bmatrix} 1 \\ m \end{bmatrix} \tag{17}$$

is on the line L*, it follows that*

Solution

$$\boldsymbol{A} * \boldsymbol{V1} = \boldsymbol{V1} \tag{18}$$

Vector $\boldsymbol{V2}$ *of the form*

$$\boldsymbol{V2} = \begin{bmatrix} -m \\ 1 \end{bmatrix} \tag{19}$$

is perpendicular to L and we have

$$\boldsymbol{A} * \boldsymbol{V2} = -\boldsymbol{V2} \tag{20}$$

Hence, We have

$$\boldsymbol{A} * [\boldsymbol{V1} \quad \boldsymbol{V2}] = [\boldsymbol{V1} \quad -\boldsymbol{V2}] \tag{21}$$

Solution

$$A \begin{bmatrix} 1 & -m \\ m & 1 \end{bmatrix} = \begin{bmatrix} A \begin{bmatrix} 1 \\ m \end{bmatrix} & A \begin{bmatrix} -m \\ 1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 1 & m \\ m & -1 \end{bmatrix}. \tag{22}$$

Using inverse, we have

$$A = \frac{1}{1+m^2} * \begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}. \tag{23}$$

For $L1 : y = mx + c$, *we translate the coord. system* $(0,0) - > (0, -intercept)$ *and then apply the transformation and then again translate* $(0, -intercept) - > (0,0)$