Intro to AI and ML

Matrix Project

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February 14, 2019

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Problem Statement

Original Question
The equation of the circle which is the mirror image of the circle

$$x^2 + y^2 - 2x = 0 \tag{1}$$

about the line

$$y = 3 - x \tag{2}$$

Problem Statement

Matrix Form

Find the equation of the circle, which is the mirror image of the circle

$$\boldsymbol{x}^T\boldsymbol{x} - (2 \quad 0)\boldsymbol{x} = 0 \tag{3}$$

in the line

$$(1 \quad 1)\boldsymbol{x} = 3 \tag{4}$$

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Desired Answer

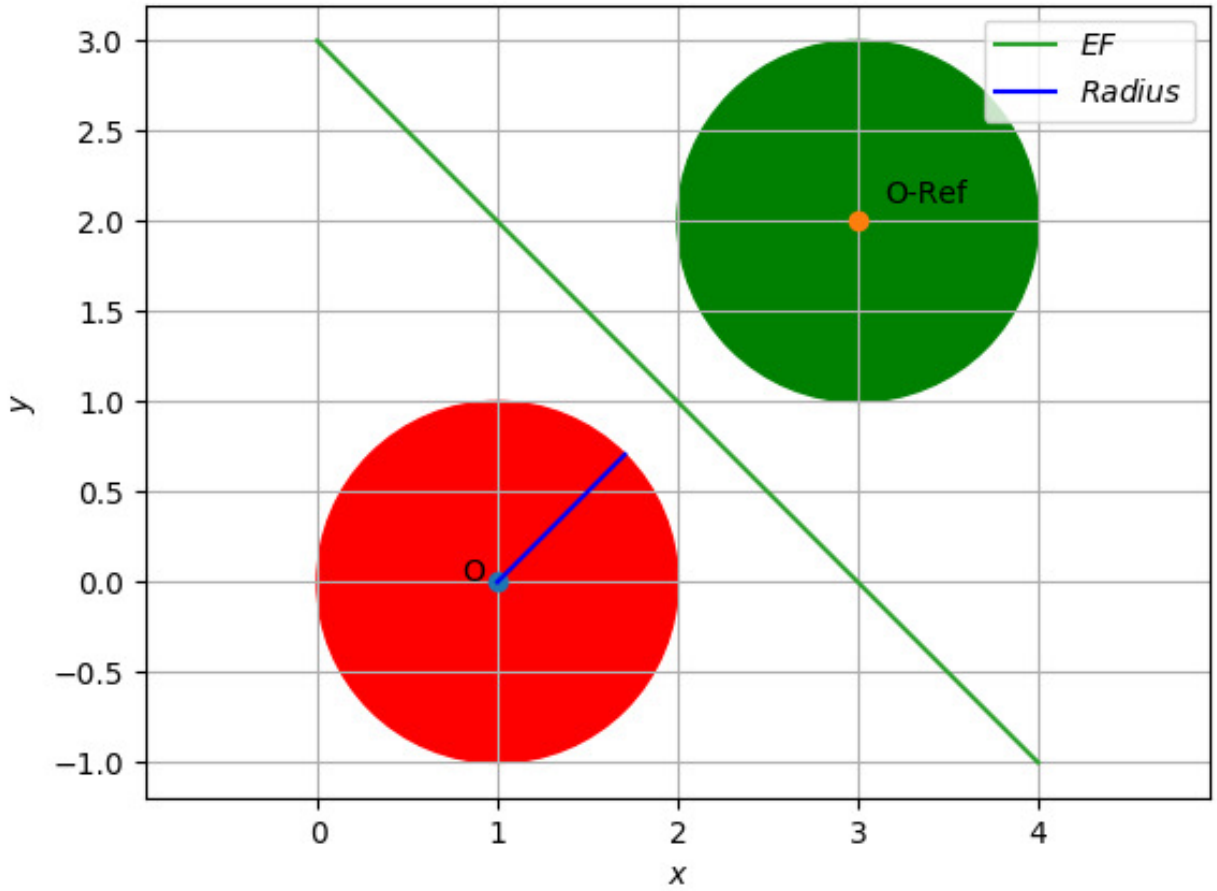

Figure: Reflection of circle about a line

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Solution

Let $\boldsymbol{c}$ be the center and r be the radius of the circle respectively.

$$\| (\boldsymbol{x} - \boldsymbol{c}) \|^2 = r^2 \tag{5}$$

$$\Rightarrow (\boldsymbol{x} - \boldsymbol{c})^T (\boldsymbol{x} - \boldsymbol{c}) = r^2 \tag{6}$$

$$\Rightarrow \boldsymbol{x}^T \boldsymbol{x} - 2\boldsymbol{c}^T \boldsymbol{x} = r^2 - \boldsymbol{c}^T \boldsymbol{c} \tag{7}$$

Comparing with eqn(1),

$$\boldsymbol{c} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{8}$$

$$r^2 - \boldsymbol{c}^T \boldsymbol{c} = 0 \Rightarrow r = 1 \tag{9}$$

Solution

We have the equation of line as

$$(1 \quad 1)\boldsymbol{x} = 3 \tag{10}$$

this can be written in the form

$$\boldsymbol{N}\boldsymbol{x} = C \tag{11}$$

where N is the normal to the line and C is a constant.
Comparing with eqn(8),

$$\boldsymbol{N} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{12}$$

Intersection of line (passing through center $\boldsymbol{c}$ *and* $\boldsymbol{c} + \mathbf{0.1}\boldsymbol{N}$*) with the given line gives the foot of perpendicular on the given line from* $\boldsymbol{c}$*.*

Solution

Let $\boldsymbol{f}$ *and* $\boldsymbol{c'}$ *be the foot of perpendicular and image of center respectively. Then we have*

$$\frac{\boldsymbol{c} + \boldsymbol{c'}}{2} = \boldsymbol{f} \tag{13}$$

$$\Rightarrow \boldsymbol{c'} = 2\boldsymbol{f} + \boldsymbol{c} \tag{14}$$

Since the radius remains same after reflection, we have equation of reflected circle as

$$\boldsymbol{x}^T\boldsymbol{x} - 2\boldsymbol{c'}^T\boldsymbol{x} = r^2 - \boldsymbol{c'}^T\boldsymbol{c'} \tag{15}$$

Conclusion

Conclusion

So,the reflected circle is

$$\boldsymbol{x}^{\boldsymbol{T}}\boldsymbol{x} - 2\boldsymbol{c}'^{\boldsymbol{T}}\boldsymbol{x} = r^2 - \boldsymbol{c}'^{\boldsymbol{T}}\boldsymbol{c}' \tag{16}$$

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Walkthrough of the code(Functions)

function norm_vec(AB) //returns the normal vector of line AB.
function mid_pt(B,C) //calculates the mid point of two given points.
function line_intersect_normal_form(N,P) //creates a line from normal form.
function line_intersect_normal_form(N,P) //returns reflection of a point about a line.

Walkthrough of the code(Main Section)

MAIN SECTION

// centre of the circle

cen=np.matmul(cenM,A.T)

// constant term for the circle

D=0

// Reflected centre

refCen=reflection_normal_form(B,C,cen)

// Radius of the circle

radius=(cen[0]2+cen[1]**2-D)**0.5**

// Foot of perpendicular of the center to the line

E=(cen+refCen)/2