

**Theorem 1.** Suppose  $(P, w)$  contains  $(Q, w')$  as a convex subposet. If  $(Q, w')$  has multiplicity, then so does  $(P, w)$ .

*Proof.* Let  $L_{(Q, w')}$  be the set of all linear extensions of  $(Q, w')$ .

Since  $(Q, w')$  has multiplicity, there exist  $l_1, l_2 \in L_{(Q, w')}$  such that they have the same  $F$  function.

*Case 1:* Every labeled node in  $(P, w)$  belongs in  $(Q, w')$  and thus,  $(P, w) = (Q, w')$ . Hence,  $l_1, l_2 \in L_{(P, w)}$  and  $(P, w)$  has multiplicity.

*Case 2:* There exists labeled node(s) in  $(P, w)$  that don't belong in  $(Q, w')$ .

□