Proof: Let's recombine this by the algebra.

$$n^2 + n + 1 = n(n + 1) + 1$$

Integer number can be even or odd only, there is two possible cases: let n is even, then n+1 is odd, even multiply by odd is always an even, then n(n+1)+1 is odd;
let n is odd, then n+1 is even, odd multiply by even is always an even, then n(n+1)+1 is odd.
Clearly, if a both of above cases return odd, then $n^2 + n + 1$ is always an odd, Q.E.D.