

### 9.3 (b) Extrinsic semiconductors

A material whose charge carriers originate from impurity atoms is called an impurity semiconductor or extrinsic semiconductor. In these the conduction is mainly due to addition of impurities which increases the conductivity of a semiconductor. The phosphorus atom is said to donate electron to the semiconductor and this energy level of the fifth electron is called the donor level. Since the elements such as P, As or Sb donate negative charges (electrons), these are called N type impurities and the semiconductors doped with them are called N type semiconductors (Figure 9.3(a)).

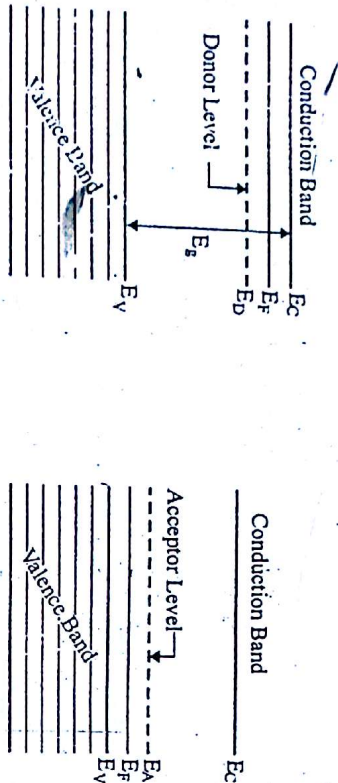


Figure 9.3 (a) N type semiconductor

Figure 9.3 (b) P type semiconductor

Excitation of electron from the donor level into the conduction band takes place much more readily than across the energy gap from the valence band. The donor level is so close to the bottom of the conduction band and most of the donor level electrons are excited into the conduction band at room temperature and become the majority charge carriers. So in N type semiconductor's electrons are majority charge carriers and holes are minority charge carriers.

Consider the impurities like Al, Ga or In in semiconductors. Aluminium has three electrons in outer shell and to complete its bond it requires one more electron, that is supplied by a semiconductor atom like Silicon, thereby creating a vacant electron site or hole in the valence band of semiconductor.

Since aluminium accepts one extra electron, the energy level of this is called acceptor level which is just above the valence band. So Al, Ga and In are called acceptor impurities and the semiconductors doped with acceptor impurities are called P-type semiconductors (Figure 6.3(b)). The majority charge carriers in P type semiconductors are holes since most of the holes are free for conduction purposes at room temperature in P-type semiconductor. The minority charge carriers are electrons in the P-type semiconductors. Unlike intrinsic semiconductors, the concentration of holes and electrons are unequal in extrinsic semiconductors.

### 9.4 Carrier concentration in intrinsic semiconductors

9.5

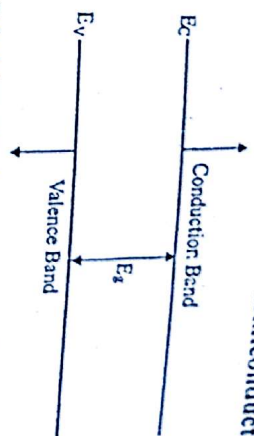


Figure 9.4 Calculation of carrier concentration in intrinsic semiconductor

The number of electrons available between energy interval  $E$  and  $E + dE$  is given by

$$dn = P(E) g(E) dE \quad \dots 9.1$$

where  $g(E) dE$  is the density of states in the energy interval  $E$  and  $E + dE$  and  $P(E)$  is the probability that a state of energy  $E$  is occupied. The conduction band does not have all states of the same energy. The energy is  $E_c$  at the bottom of the conduction band and extends to higher values of energy as we consider the top of the conduction band. The number of electrons in the conduction band should therefore be calculated by integrating equation 9.1 from  $E_c$  to the energy corresponding to the top of the conduction band

$$\text{i.e.,} \quad n = \int_{E_c}^{\text{top energy}} P(E) g(E) dE$$

We know that the density of states (number of electrons per unit volume) for a potential free electron is

$$g(E) dE = 2 \times \frac{4\pi}{h^3} m_e^{3/2} (2E)^{1/2} dE \quad \dots 9.2$$

The factor '2' is introduced here following Pauli's exclusion principle. The electron in the conduction band is not a free electron, but it is a free or conduction electron with effective mass  $m^*$  due to the movement of electron in a periodic potential provided by the crystal lattice. Therefore, the density of states of electrons in the conduction band is

$$g(E) dE = \left( \frac{4\pi}{h^3} \right) 2^{3/2} (m_e^*)^{3/2} E^{1/2} dE \quad \dots 9.3$$

If we consider the minimum energy of the electrons to be  $E_c$  and call  $m_e^*$  the effective mass of electrons in the conduction band, then

$$g(E) dE = \left( \frac{4\pi}{h^3} \right) 2^{3/2} (m_e^*)^{3/2} (E - E_c)^{1/2} dE \quad \dots 9.4$$

$$\text{Now } P(E) = \frac{1}{\exp \left( \frac{E - E_f}{kT} \right) + 1} \quad \dots 9.5$$

where  $E_f$  is called the fermi energy level and  $k$  is called the Boltzmann constant. Therefore

$$dn = P(E) g(E) dE = \frac{\left(\frac{4\pi}{h^3}\right) (2m_e^*)^{3/2} (E - E_c)^{1/2} dE}{\exp\left(\frac{E - E_f}{kT}\right) + 1} \quad \dots 9.6$$

Integrating, we get

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \int_{E_c}^{\infty} \frac{(E - E_c)^{1/2} dE}{\exp\left(\frac{E - E_f}{kT}\right) + 1} \quad \dots 9.7$$

Here the upper limit of the integration has been replaced by an infinite energy. This has been done to simplify the integration and there is only a slight difference between the results obtained with an upper limit of infinite energy (calculated simply) and actual upper limit (calculated in a complicated way). Assume at room temperature the fermi level is lower than the bottom of the conduction band by more than  $4kT$ . That is,

$$E_c - E_f \geq 4kT \approx 0.1 \text{ eV.}$$

Hence for the entire integration limit at room temperature

$$E - E_f \geq 4kT \text{ and } \exp\left(\frac{E - E_f}{kT}\right) \gg 1$$

$$\text{Therefore } \frac{1}{\exp\left(\frac{E - E_f}{kT}\right) + 1} \approx \exp\left(-\frac{E - E_f}{kT}\right) = \exp\left(\frac{E_f - E}{kT}\right)$$

$$\text{Therefore } n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \int_{E_c}^{\infty} (E - E_c)^{1/2} \exp\left(\frac{E_f - E}{kT}\right) dE \quad \dots 9.8$$

To solve the integral, put  $E = E_c + x$

$$\text{Therefore } n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \exp\left(\frac{E_f}{kT}\right) \int_0^{\infty} x^{1/2} e^{-x/kT} e^{-x/kT} dx$$

$$\text{or } n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \exp\left(\frac{E_f - E_c}{kT}\right) \int_0^{\infty} x^{1/2} e^{-x/kT} dx$$

It can be easily shown with the help of gamma function that

$$\int_0^{\infty} x^{1/2} e^{-x/kT} dx = (kT)^{3/2} \frac{\pi^{1/2}}{2} \quad \dots 9.9$$

$$\text{Therefore } n = 2 \left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} \exp\left(\frac{E_f - E_c}{kT}\right) \quad \dots 9.10$$

In an intrinsic semiconductor, the number of electrons in the conduction band must equal the number of vacancies or holes in the valence band. So the probability that a state of energy  $E$  is unoccupied in the valence band is

$$1 - P(E) = 1 - \left(\exp\left(\frac{E - E_f}{kT}\right) + 1\right)^{-1} = \frac{\exp\left(\frac{E - E_f}{kT}\right)}{1 + \exp\left(\frac{E - E_f}{kT}\right)} \quad \dots 9.11$$

Assuming the Fermi level lies at an energy distance  $\gg 4kT$  from  $E_v$ , the top of the valence band, we get

$$\exp\left(\frac{E - E_f}{kT}\right) \ll 1 \text{ and hence } 1 - P(E) = \exp\left(\frac{E - E_f}{kT}\right)$$

The number of holes or vacancies in the energy interval  $E$  and  $E + dE$  in the valence band is

$$dp = g(E) (1 - P(E)) dE \quad \dots 9.12$$

Taking the effective mass of holes as ' $m_h^*$ '

$$dp = \frac{4\pi}{h^3} (2m_h^*)^{3/2} E^{1/2} \exp\left(\frac{E - E_f}{kT}\right) dE \quad \dots 9.13$$

The highest vacant state can have energy corresponding to the top of the valence band. The lowest energy for the sake of mathematical simplification may be taken as  $-\infty$ .

Integrating we get

$$p = \int_{-\infty}^{E_v} \frac{4\pi}{h^3} (2m_h^*)^{3/2} (E_v - E)^{1/2} \exp\left(\frac{E - E_f}{kT}\right) dE \quad \dots 9.14$$

Equation (9.14) can be solved with the help of gamma functions and then

$$p = 2 \left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} \exp\left(\frac{E_v - E_f}{kT}\right) \quad \dots 9.15$$

This equation gives the number of holes or vacancies in the valence band.

In intrinsic semiconductors,  $n = p$

$$\text{Therefore } 2 \left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} \exp\left(\frac{E_f - E_c}{kT}\right) = 2 \left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} \exp\left(\frac{E_v - E_f}{kT}\right) \quad \dots 9.16$$



$$(or) m_e^{*1/2} \exp\left(\frac{E_f - E_c}{kT}\right) = m_h^{*1/2} \exp\left(\frac{E_v - E_f}{kT}\right)$$

$$\text{Rearranging we get } \left(\frac{m_h^*}{m_e^*}\right)^{3/2} = \exp\left(\frac{E_f - E_c - E_v + E_f}{kT}\right) = \exp\left(\frac{2E_f - E_v - E_c}{kT}\right)$$

Taking logarithms on both sides we get

$$\frac{3}{2} \log\left(\frac{m_h^*}{m_e^*}\right) = \left(\frac{2E_f - E_v - E_c}{kT}\right)$$

$$\frac{3}{2} kT \log\left(\frac{m_h^*}{m_e^*}\right) = 2E_f - (E_c + E_v)$$

Simplifying we get

$$E_f = \frac{E_c + E_v}{2} + \frac{3}{4} kT \log \frac{m_h^*}{m_e^*} \quad \dots 9.17$$

which shows that if  $m_h^* = m_e^*$ ,  $E_f = \frac{E_c + E_v}{2}$

That is, the fermi level is located half way between the valence and conduction bands and its position is independent of temperature. But in general  $m_h^* > m_e^*$  so the fermi level is a function of temperature and is raised slightly with temperature.

Since  $n = p$ , we assume

$n = p = n_i$  where  $n_i$  is called the intrinsic carrier concentration.

$$\text{Therefore } n_i^2 = np = 4 \left(\frac{2\pi kT}{h^2}\right)^3 (m_e^* m_h^*)^{3/2} e^{-E_g/2kT}$$

$$\text{Therefore } n_i = 2 \left(\frac{2\pi kT}{h^2}\right)^{3/2} (m_e^* m_h^*)^{3/4} e^{-E_g/4kT} \quad \dots 9.18$$

where  $E_g$  is the energy gap and it is equal to  $E_c - E_v$ .

$$\text{The electrical conductivity } \sigma = (ne\mu_e + pe\mu_h) \quad \dots 9.19$$

where  $\mu_e$  and  $\mu_h$  are the mobility of electron and hole respectively. Mobility is defined as the extra velocity acquired by the electron or hole under potential gradient. In the case of intrinsic semiconductor,

$$\sigma = n_i e(\mu_e + \mu_h)$$

$$\sigma = (\mu_e + \mu_h) 2e \left(\frac{2\pi kT}{h^2}\right)^{3/2} (m_e^* m_h^*)^{3/4} e^{-E_g/2kT} \quad \dots 9.20$$

So the electrical conductivity depends upon the negative exponential of the forbidden energy gap between the valence band and conduction band and on the mobility of both holes and electrons. The mobility in a pure semiconductor is determined by the interaction of the electron with lattice waves or phonons. In such case  $\mu_e$  and  $\mu_h$  are both proportional  $T^{-3/2}$ .

Therefore the electrical conductivity in the intrinsic semiconductor can be written as

$$\sigma_i = A e^{-E_g/2kT} \quad \dots 9.21$$

where A is a constant. From the graph we know that conductivity increases with temperature (Figure 9.4(a)).

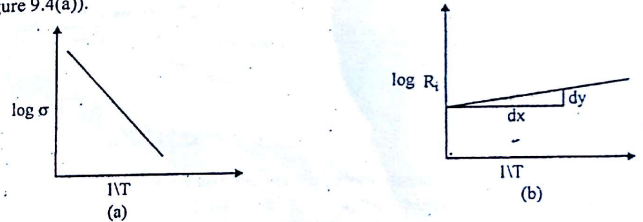


Figure 9.4

- (a) Variation of  $\log \sigma$  with reciprocal of temperature in intrinsic semiconductor  
(b) Variation of Resistance with temperature in intrinsic semiconductor

#### Determination of band gap (or) energy gap in intrinsic semiconductor

We know that  $\sigma_i = A e^{-E_g/2kT}$

Therefore the resistivity of the intrinsic semiconductor

$$\rho_i = \frac{1}{\sigma_i} = \frac{1}{A} e^{E_g/2kT}$$

$$\frac{R_i a}{L} = \frac{1}{A} e^{E_g/2kT}$$

where  $R_i$  the resistance,  $a$ , the area of cross section and  $L$ , the length of intrinsic semiconductor. Since  $a$  and  $L$  are constants,

$$R_i = C e^{E_g/2kT} \quad \text{where } C = \frac{L}{aA}$$

Taking logarithms on both sides  $\log R_i = \log C + \frac{E_g}{2kT}$

The above equation suggests us a method of determining the energy gap of an intrinsic material. If we find the resistance of the intrinsic semiconductor using Post Office Box or

Carey Foster's Bridge at various temperatures, we can plot a curve between  $\frac{1}{T}$  and  $\log R_p$ .

Then the slope of that curve will give the value of  $\frac{E_g}{2k}$ .

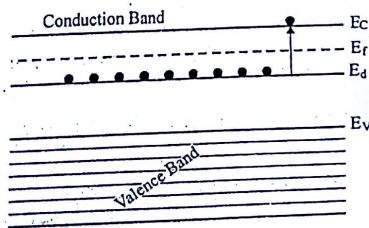
From curve  $\frac{dy}{dx} = \frac{E_g}{2k}$

Therefore  $E_g = 2k \frac{dy}{dx}$  where  $k$  is Boltzmann constant.

### 9.5 Carrier concentration in n type semiconductors and variation of fermi level with temperature and concentration of donor atoms

Referring figure 9.5, below the conduction band there are  $N_d$  donor levels per  $m^3$  of energy  $E_d$ . At low temperatures small fraction of donors will be ionised and practically all donor levels will be filled with electrons. Let us assume that  $(E_c - E_f) > 4kT$  then in that case density of electrons in conduction band will be given by equation

$$n = 2 \left( \frac{2\pi m_e^* kT}{h^2} \right)^{3/2} \exp \left( \frac{E_f - E_c}{kT} \right) \quad \dots 9.22$$



- $E_c$  = Bottom energy level in conduction band
- $E_f$  = Fermi energy level
- $E_d$  = Donor energy level
- $E_v$  = Top energy level in valence band.

Figure 9.5 N-type semiconductor

If we assume that  $E_f$  lies more than a few  $kT$  above the donor level then the density of empty donors is given by

$$N_d (1 - P(E_d)) \approx N_d \exp \left( \frac{E_d - E_f}{kT} \right) \quad \dots 9.23$$

At very low temperatures, practically no electrons get sufficient energy for excitation from the valence band into the conduction band. Therefore the density of empty donors

should be the same as the density of electrons in the conduction band. Here the electrons in the conduction band are due to donated electrons from the donor level.

$$2 \left( \frac{2\pi m_e^* kT}{h^2} \right)^{3/2} \exp \left( \frac{E_f - E_c}{kT} \right) = N_d \exp \left( \frac{E_d - E_f}{kT} \right) \quad \dots 9.24$$

Taking logarithm and rearranging, we get

$$\left( \frac{E_f - E_c}{kT} \right) - \left( \frac{E_d - E_f}{kT} \right) = \log N_d - \log 2 \left( \frac{2\pi m_e^* kT}{h^2} \right)^{3/2}$$

$$E_f = \frac{E_d + E_c}{2} + \frac{kT}{2} \log \frac{N_d}{2 \left( \frac{2\pi m_e^* kT}{h^2} \right)^{3/2}} \quad \dots 9.25$$

At  $T = 0 \text{ K}$

$$E_f = \frac{E_d + E_c}{2}$$

which shows that fermi level lies exactly halfway between donor level and bottom of conduction band. As  $T$  increases fermi level drops. As the temperature is gradually increased from a low temperature the contribution of electron to the conduction band from the valence band increases and at very high temperature (500 K in Germanium) it far exceeds the donor concentration and the intrinsic behaviour predominates at higher temperatures.

In the figure 9.6,  $E_i$  is the centre of forbidden gap (or) fermi level position in intrinsic semiconductor.

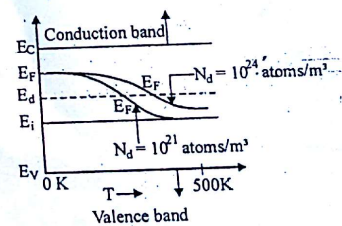


Figure 9.6 Variation of Fermi level with temperature and for various concentrations in 'n' type semiconductor

Putting the value of  $E_f$  from 9.25 into  $\exp \left( \frac{E_f - E_c}{kT} \right)$

The displacement of spot on the screen is

$$d = \frac{l}{2S} \frac{L}{V_a} V_d = \frac{2 \times 12 \times 30}{2 \times 0.5 \times 2000} \text{ cm} = 0.36 \text{ cm.}$$

### EXERCISE 50

1. With a neat diagram and necessary theory explain how  $e/m$  of an electron is obtained by Dunnington's method.  
[Bangalore, April 1992]
2. With a neat diagram describe a cathode ray oscilloscope. Obtain an expression for the deflection sensitivity of the electrostatic type.  
[Bangalore, Nov. 92]