MODAL QUESTIONS

L.A.Q.S

Unit-I

L.D.E

1) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^{3x} + \sin 2x$ $\boxed{\text{Ans:- 1)}}$ $y = C_1 e^{x} + C_2 e^{2x} + \frac{e^{3x}}{2}\left(x - \frac{3}{2}\right) + \frac{1}{20}(3\cos 2x - \sin 2x).$

2) Solve $(D^3 + 2D^2 + D)y = e^{2x} + x^2 + x + \sin 2x$ $\boxed{\text{Ans:- 2)}}$ $y = C_1 + C_2 e^{-x} + C_3 x e^{-x} + \frac{e^{2x}}{18} + \frac{x^3}{3} - \frac{3x^2}{2} + 4x + \frac{1}{50}(3\cos 2x - 4\sin 2x)$

3) Solve $(D^2 - 1)y = xe^{x}\sin x$

4) Solve $(D^2 + 9)y = xe^{2x}\cos x$

$\boxed{\text{Ans:- 3)}}$ Try

$\boxed{\text{Short-cut Tip}}$:- $xe^{x}\sin x = Im.[xe^{(1+i)x}]$

& $xe^{2x}\cos x = Re.[xe^{(2+i)x}]$

$\boxed{\text{Ans:- 4)}}$ $y = C_1\cos 3x + C_2 \sin 3x + \frac{e^{2x}}{400}\left[(30x - 11)\cos x + (10x - 2)\sin x\right]$

5) Solve $\frac{d^2y}{dx^2} - y = x\sin 3x + \cos x.$

$\boxed{\text{Ans:- 5}}$ $y = C_1 e^{x} + C_2 e^{-x} - \frac{1}{50}\left(5x\sin 3x + 3\cos 3x + 25\cos x\right)$

Cauchy's homogeneous L.D.E

Hint:- put $x = e^{z} \Rightarrow z = \log x$

$\frac{d}{dz} = \theta$, $xD = \theta$, $x^2D^2 = \theta(\theta - 1).$

6) Solve $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 12y = x^3\log x$

Ans:- $y = C_1 x^3 + C_2 x^{-4} + \frac{x^3}{98}\log x(7\log x - 2$

7) Solve $(x^2D^2 + xD + 1)y = \log x \sin(\log x)$

$\boxed{\text{Ans:- 7}}$ $y = C_1\cos z + C_2 \sin z - \frac{z^2}{2}\cos z - (z\cos z - \sin z)$

where $z = \log x$.

Unit-II

S.A.Qs

8) Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

Ans: Try

9) Solve $(D^2+4)y=0$

Ans:- Try

10) Solve $(D^3-14D+8)y=0$

Ans:- $y = C_1 e^{-4x} + \underbrace{C_2 e^{(2-\sqrt{2})x} + C_3 e^{(2+\sqrt{2})x}}_{\text{or}}$

$e^{2x}[C_2\cosh(x\sqrt{2}) + C_3\sinh(x\sqrt{2})]$

11) Find the particular Integral of $(D^2-D-2)y = e^{2x}$

Ans:- $y = \frac{xe^{2x}}{3}$

12) Find the particular Integral of $(D^3+a^2D)y = \sin ax$, where $a>0$

Unit-II

L.A.Qs

1) Define Laplace transform. Find the Laplace transform of f(t). of piecewise continuous functions.

(i) $f(t) = \begin{cases} 1, & 0 \le t < 1 \\ t, & 1 \le t < 2 \\ 0, & t > 2 \end{cases}$

Ans:- (i) $F(s) = \frac{1}{s} - \frac{2e^{-2s}}{s} - \frac{e^{-2s}}{s^2} + \frac{e^{-s}}{s^2}$

(ii) $f(t) = \begin{cases} \frac{t}{T} & \text{when } 0<t<T \\ 1 & \text{when } t>T \end{cases}$

Ans:- (ii) $F(s) = \frac{1-e^{-sT}}{Ts^2}$.

(iii) $f(t) = \begin{cases} 2, & 0<t<\pi \\ 0, & \pi<t<2\pi \\ \sin t, & t>2\pi \end{cases}$

Ans:- (iii) Try

Tip:- $\int e^{at}\sin bt = \frac{e^{at}}{a^2+b^2}[a\sin bt - b\cos bt]$

2) Define Laplace transform of periodic function $f(t)$.

(i) Find $L\{f(t)\}$ of period '2a' where $f(t) = \begin{cases} t, & 0<t<a \\ -t+2a, & a<t<2a. \end{cases}$

{ triangular wave of period $T=2a$ }

Ans: $L\{f(t)\} = \frac{1}{s^2}\frac{(1-e^{-as})}{(1+e^{-as})}$ or $\frac{1}{s^2}\tanh\left(\frac{as}{2}\right)$

(ii) Find $L\{f(t)\}$ of period '2b' where $f(t) = \begin{cases} K, & 0<t<a \\ -K & a<t<2a \end{cases}$ where K is constant

{ Square wave of period $p=2b$ }

Ans: $L\{f(t)\} = \frac{K}{s}\frac{(1-e^{-as})}{(1+e^{-as})}$ or $\frac{K}{s}\tanh\left(\frac{as}{2}\right)$

(iii) Find $L\{f(t)\}$ of period $\frac{2\pi}{\omega}$ where $f(t) = \begin{cases} E\sin\omega t, & 0<t<\frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega}<t<\frac{2\pi}{\omega} \end{cases}$

Ans: $L\{f(t)\} = \frac{E\omega}{(1-e^{-\pi s/\omega})(s^2+\omega^2)}$.

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3) Find i) $L\left\{\frac{\cos at-\cos bt}{t} + t\sin at\right\}$ (ii) $L\left\{\int_0^t e^{-t}\cos t\,dt\right\}$

(iii) $L\left\{e^{-3t}\int_0^t e^{t}\frac{\sin t}{t}\,dt\right\}$ (iii) Evaluate $\int_0^\infty t e^{-3t}\sin t\,dt$ Ans: $=\frac{3}{50}$

& $\int_0^\infty \frac{e^{-t}-e^{-3t}}{t}\,dt$ Ans: $=\log 3$.

Find

4) (i) $L^{-1}\left\{\frac{2s^2-6s+5}{s^3-6s^2+11s-6}\right\}$ Ans: $f(t) = \frac{1}{2}e^{t} - e^{2t} + \frac{5}{2}e^{3t}$

(ii) $L^{-1}\left\{\log\left(\frac{s+1}{s-1}\right)\right\}$ Ans: $f(t) = \frac{2\sinh t}{t}$.

(iii) $L^{-1}\left\{\frac{2s^2-1}{(s^2+1)(s^2+4)}\right\}$ using partial fractions. Any Try (iv) $L^{-1}\left\{\frac{s-5}{(s^2+3s+2)}\right\}$ Any Try

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5) State Convolution theorem. Find (i) $L^{-1}\left\{\frac{1}{(s^2+1)(s^2+9)}\right\}$

(ii) $L^{-1}\left\{\frac{s}{(s^2+4)(s^2+9)}\right\}$ (iii) $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$ (iv) $L^{-1}\left\{\frac{1}{s(s^2+9)(s^2+4)}\right\}$

(v) $L^{-1}\left\{\frac{1}{(s-2)(s+2)^2}\right\}$ using Convolution thm.

(vi) $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$

6) Solve by Laplace transform method:

(i) $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} - 12x = e^{3t}$, $x(0)=1$, $x'(0)=-2$

(ii) $y'' + 9y = \sin 3t$, $y(0)=0$, $y'(0)=0$.

(iii) $y'' + y' - 2y = t$, $y(0)=1$, $y'(0)=0$.

(iv) $y'' - 3y' + 2y = 4t + e^{3t}$, $y(0) = y'(0) = 1$.

(v) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = e^{2t}$ with $x(0)=2$, $\frac{dx}{dt} = -1$ at $t=0$.

Unit-II
S.A.Qs

Find

7) (i) $L\{t^2 e^{3t}\}$ (ii) $L\{\sinh t \cos t\}$ (iii) $L\{t \cos t\}$

(iv) $L\{t \cos 2t \sin 3t\}$ (v) $L\{\cosh^2 2t\}$ (vi) $L\{e^t(-3t(2\cos 5t - 3\sin 5t))\}$

(vii) $L\{\frac{\sin at}{t}\}$.

8) Find (i) $L^{-1}\left\{\frac{s^2-3s+4}{s^3}\right\}$ (ii) $L^{-1}\left\{\frac{s+2}{s^2-4s+13}\right\}$

(iii) $L^{-1}\left\{\frac{2s+6}{s^2+4}\right\}$ (iv) $L^{-1}\left\{\frac{1}{s^2-5s+6}\right\}$.

Unit-III

L.A.Qs & S.A.Qs

1) Define Beta function. prove $\boxed{\beta(m,n)=\beta(m+1,n)+\beta(m,n+1)}$

2) prove that (i) $\boxed{\beta(m,n)=\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx}$ (ii) $\boxed{\beta(m,n)=\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx}$

3) Show that $\int_a^b (x-a)^m(b-x)^n dx = (b-a)^{m+n+1}\beta(m+1,n+1)$.

4) Evaluate (i) $\boxed{\int_0^1 \frac{x^4 dx}{\sqrt{1-x^2}}}$ (ii) $\int_0^2 x(8-x^3)^{1/3}dx$ Ans:- (i) try (ii) $\frac{16\pi}{9\sqrt{3}}$

5) Evaluate (i) $\int_0^{\pi/2} \sin^{7/2}\theta\cos^{3/2}\theta\, d\theta$ (ii) $\int_0^{\pi/2} \sin^4\theta\cos^2\theta$ using β-function.

(iii) Show $\boxed{\int_0^{\pi/2}\sqrt{\cot\theta}\,d\theta = \frac{1}{2}\Gamma\tfrac{1}{4}\Gamma\tfrac{3}{4} \text{ or } \frac{\sqrt{2}\pi}{2}}$ (or) $\sqrt{\tan\theta}$

Ans:- (i) $\frac{1}{48\sqrt{\pi}}\left[\Gamma\left(\frac{1}{4}\right)\right]^2$ (ii) $\frac{\pi}{32}$

6) Define Gamma function. Evaluate (i) $\int_0^\infty x^{1/2}e^{-x/5}dx$

(ii) $\int_0^\infty e^{-x^2}x^{3/2}dx$ (iii) show $\Gamma(\tfrac{1}{2})=\sqrt{\pi}$ (iv) $\boxed{I=3\int_0^\infty \sqrt{x}\,e^{-x^3}dx=\sqrt{\pi}}$

7) prove $\boxed{\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}}$ where $m>0, n>0$

8) (i) $\int_0^\infty \frac{x\,dx}{(1+x^6)}$ using β-Γ functions $\leftarrow$ Ans:- (i) $\frac{\pi}{3\sqrt{3}}$

(ii) $\boxed{\int_0^\infty \frac{x^a}{a^x}dx = \frac{\Gamma(a+1)}{(\log a)^{a+1}}, a>1}$

9) $\boxed{\int_0^\infty \frac{x^2}{1+x^4}dx}$ using β-Γ functions Tip: $x=\sqrt{\tan\theta}$ Ans:- $\sqrt{2}\pi$

$x=0 \Rightarrow \theta=0$
$x\to\infty \Rightarrow \theta=\pi/2$

10) Evaluate (i) $\boxed{\int_0^1 \frac{x^a-x^b}{\log x}dx}$ using differentiation under integration Ans:- $\log\left(\frac{1+a}{1+b}\right)$

(ii) If $\int_0^\infty \frac{dx}{(x^2+\alpha)} = \frac{\pi}{2\sqrt{\alpha}}, \alpha>0$ then show $\int_0^\infty \frac{dx}{(x^2+\alpha)^2}=\frac{\pi}{4\alpha^{3/2}}$

(iii) $\int_0^\infty \frac{e^{-x}-e^{-7x}}{x}dx$ Ans:- $\log\left(\frac{7}{1}\right)$

(iv) $\int_0^\infty e^{-ax}\frac{\sin x}{x}dx = \tan^{-1}\left(\frac{1}{a}\right)$ hence show $\int_0^\infty \frac{\sin x}{x} = \frac{\pi}{2}$.

U-4

5 Marks

1) P.T. $\text{div}(\bar{F}\times\bar{G}) = \bar{G}\cdot(\nabla\times\bar{F}) - \bar{F}\cdot(\nabla\times\bar{G})$

2) P.T $\text{div}(f\bar{v}) = f\,\text{div}(\bar{v}) + (\text{grad}\,f)\cdot\bar{v}$ where is the scalar function

3) If $\vec{A}$ and $\bar{B}$ are irrotational then S.T $\bar{A}\times\bar{B}$ is solenoidal

4) Show $\text{div}(r^n\vec{r}) = (n+3)r^n$ where $\bar{r} = xi+yj+zk$ & $r = |\bar{r}|$.

5) Find directional derivative of $f(x,y,z) = x^2-y^2+2z^2$ at $P(1,2,3)$ in the direction of the line PQ, where Q is point $(5,0,4)$.

U-4 Vector Differen[tiation]

2 Marks

1) Find the Unit normal to the Surface $xy^3z^2=4$ at $(-1,-1,2)$

2) Find the Directional derivative of $f(x,y,z)=xy^3+yz^3$ at the point $(2,-1,1)$ in the direction of the Vector $i+2j+2k$

3) P.T. $\nabla(r^n)=nr^{n-2}\bar{r}$

4) Find the work done in moving a particle in the force field $\bar{F}=3x^2\bar{i}+(2xz-y)\bar{j}+z\bar{k}$ along the st. line from $(0,0,0)$ to $(2,1,3)$

5) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $x^2+y^2+z^2+4x-6y-8z+12=0$ at the point $(4,-3,2)$

6) S.T. $(x^2-yz)\bar{i}+(y^2-zx)\bar{j}+(z^2-xy)\bar{k}$ is irrotational and find its Scalar potential.

7) Find the directional derivative of x^2y^3+xy at $(2,1)$ in the direction of a Unit vector which makes an angle of $\frac{\pi}{3}$ with x-axis.

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UNIT - 5

Vector Integration

2 Marks

1) Evaluate $\int_C \bar{F} \cdot dr$ where (i) $\bar{F} = (x^2+y)\,\bar{i} + xy\,\bar{j}$ where C is $y = x^2$ from $(0,0)$ to $(2,4)$

(ii) $\bar{F} = 3xy\,i - y^2 j$ & C is arc of parabola $y = 2x^2$ from $(0,0)$ to $(1,2)$.

2) State Stokes theorem

3) State Green's theorem

4) State Gauss Divergence theorem

5) Evaluate $\int_C (x^2+xy)\,dx + (x^2+y^2)\,dy$ where C is the square formed by the lines $y = \pm 1$ and $x = \pm 1$

5 marks

1) Apply Gauss Divergence theorem to Evaluate $\iint_S \bar{F}\cdot\bar{n}\,ds$ where $\bar{F} = (x^2-yz)\bar{i} + (y^2-xz)\bar{j} + (z^2-xy)\bar{k}$ and S is the surface of the rectangular parallelopiped bounded by $0\le x\le a$, $0\le y\le b$, $0\le z\le c$.

Repeated ; Verify G.D-Theorem for above problem.

2) Using the stokes theorem evaluate $\oint_C \bar{F}\cdot d\bar{r}$ where $\bar{F} = (x^2+y^2)\bar{i} - 2xy\,\bar{j}$ and C is the rectangle bounded by $x=a$, $x=-a$; $y=0$, $y=b$

3) Evaluate $\iint_S \bar{A}\cdot\bar{n}\,ds$ where $\bar{A} = z\bar{i} + x\bar{j} - 3y^2z\,\bar{k}$ and S is the surface of the cylinder $x^2+y^2=16$ and S is the surface of the cylinder including in the first octant between $z=0$ & $z=5$

4) Verify Green's theorem for $f(x,y) = e^{-x}\sin y$, $g(x,y) = e^{-x}\cos y$ and C is a square with vertices at $(0,0)$ $(\frac{\pi}{2},0)$, $(\frac{\pi}{2},\frac{\pi}{2})$ and $(0,\frac{\pi}{2})$

5) Evaluate $\iiint_V \bar{A}\,dv$ where $\bar{A} = 2xz\bar{i} - x\bar{j} + y^2\bar{k}$ and V is the region bounded by the surface $x=0$, $y=0$, $x=2$, $y=6$, $z=x^2$ and $z=4$.

6) If $\bar{F} = (2x^2-3z)\bar{i} - 2xy\bar{j} - 4x\bar{k}$ then Evaluate $\iiint_V (\nabla \times \bar{F})\, dV$ where V is the closed region bounded by $x=0, y=0, z=0$ and $2x+2y+z=4$

7) Verify stoke's theorem for $\bar{F} = (x^2+y^2)\bar{i} - 2xy\bar{j}$ taken round the rectangle bounded by the lines $x=\pm 1$, $y=0$, $y=2$

8) Verify Stoke's thm for $F = (y-z+2)i + (yz+4)j - xzk$ where S is surface of the cube $x=0, y=0, z=0, x=1, y=1, z=1$ above the x-y plane.

Extra Questions

1) Verify Green's theorem in a plane for (i) $\int_C (3x^2-8y^2)dx + (4y-6xy)dy$ where C is the region bounded by $y=\sqrt{x}$ to $y=x^2$

(ii) $\int_C (2xy-y^2)dx + (x^2+y^2)dy$ where C is the closed curve bounded by $y=x^2$ & $y^2=x$

(i) A: $\frac{3}{2}$
Try → (ii) A=

2) Verify Green's theorem in the xy-plane for the integral $\int_C (2x-y^3)dx + xy\,dy$; where C is the boundary of the region enclosed by the circles $x^2+y^2=1$ & $x^2+y^2=9$

3) Evaluate by Stoke's theorem the integral $\int_C (x+y)dx + (2x-z)dy + (y+z)dz$, where C is the triangle with the vertices $(2,0,0), (0,3,0)$ $(0,0,6)$

A: 21

4) Verify G.D. Stoke's theorem for a Vector field defined by $\vec{A} = (xz)\bar{i} - y\bar{j} + x^2y\bar{k}$ where S is the surface of the region bounded by $x=0, y=0, z=0$ & $2x+y+2z=8$ which is not included in the xz-plane.

5) Verify G.D. Theorem for $\bar{F} = 4xz\bar{i} - y^2\bar{j} + yz\bar{k}$ over a cube bounded by $x=0, y=0, z=0$ and $x=y=z=a$

A:

6) Verify G.D.T for $\bar{F} = 4x\bar{i} - 2y^2\bar{j} + z^2\bar{k}$ taken over the region bounded by $x^2+y^2=4$, $z=0$ and $z=3$

7) Evaluate $\iint_S \bar{F}\cdot\bar{n}\, ds$ where S is the surface of the sphere $x^2+y^2+z^2=1$ which lies in the first quadrant and $\vec{F} = yz\bar{i} + zx\bar{j} + xy\bar{k}$

8) Evaluate $\iint_S \bar{F}\cdot\bar{n}\, ds$ where S is $x^2+y^2=16$, $z=0$ and $z=5$ which lies in the first octant if $\bar{F} = z\bar{i} + x\bar{j} - 3y^2z\bar{k}$