UNIT - IV

EME

BE 1/4 ECE-I

A) Kinematics.

B) Gear trains

C) Belt drives

D) Fluid Mechanics.

(35Ω)

Text books:-

A) Kinematics
B) Gear trains
C) Belt drives
→ Theory of Machines By
1.) R.S. Khurmi
& 2.) S.S. Rattar
etc

D) Fluid Mechanics
↳ 1.) R.K. Bansal
2.) R.S. Khurmi.

A) Kinematics:-

Introduction:-

The "theory of machines" may be defined as that "branch of Engineering-science, which deals with the study of relative motion between the various parts of a machine".

→ Theory of machines may be sub-divided into the following four branches.

1.) **Kinematics:-**
It is that branch of theory of machines, which deals with the relative motion between the various parts of the machines.

2.) **Dynamics:-**
It is that branch of theory of machines, which deals with the forces and their effects, while acting upon the machine parts in motion.

3.) **Kinetics:-**
It is that branch of theory of machines, which deals with the inertia forces which arise from the combined effect of the mass and motion of the machine parts.

4) **Statics:-**
It is that branch of theory of machines, which deals with the forces and their effects while the machine parts are at rest. The mass of the parts is assumed to be negligible.

Machine:-

"A machine is a device, which receives energy and transfo[rms] it into some ~~usefut~~ useful work."

→ A machine consists of a number of parts or bodies.

→ We shall study the mechanisms of the various parts or bodies from which the machine is assembled.

This is done by making one of the parts as fixed, and the relative motion of other parts is determined with respect to the fixed part.

⇒

Link.
↓
pair
↓
kinematic pair
↓
Chain.
↓
kinematic chain
↓
mechanism
↓
machine

Kinematic Link (or) Element:-

Def:- Each part of a machine, which moves relative to some other part is known as kinematic link (or simply link) or element.

(or)

It is the smallest mechanical element which moves with respect to someother link (or) moves relatively with others links.

→ A link or element need not to be a rigid body, but it must be a resistant body.

→ Link should have the following two characteristics.

1.) It should have relative motion, and

2.) It must be a resistant body.

Resistant body:-

A body is said to be a resistant body if it is Capable of transmitting the required forces with negligible deformation. Eg:- Connecting rod etc

Types of links:-

In order to transmit motion, the driver and the follower may be Connected by the following three types of links.

1.) Rigid link:-

A rigid link is one, which does not undergo any deformation while transmitting motion.

-> Strictly Speaking, In universe there is no perfect rigid link.

Eg:- Connecting rod, Crank etc

2.) Flexible link:-

A flexible link is one which is partly deformed in a manner not to affect the transmission of motion.

Eg:- Belts, ropes and chains etc

3.) Fluid link:-

A fluid link is one which is formed by havin a fluid in a receptacle and the motion is transmitt through the fluid by pressure or Compression only.

Eg:- Hydraulic brakes, presses etc.

Pair:-

When two links are joined with each other, it is called a pair.

Kinematic pair:-

The two links or elements of a machine, when in Contact with each other, are said to form a pair. If the relative motion between them is Completely or Successfully Constrained (ie. in a definite direction), the pair is known as kinematic pair

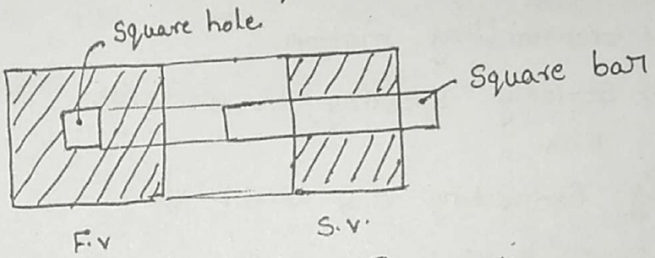
Types of Constrained Motions:-

The important types of motion of pairs are.

① Completely Constrained motion:- [only one type of motion]

When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is said to be Completely Constrained motion.

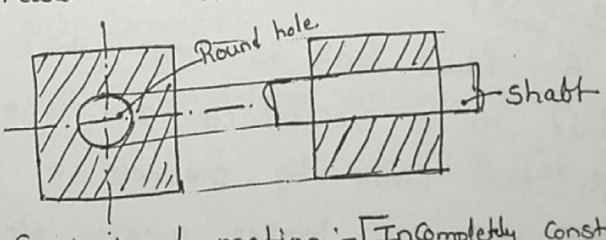
Eg:- The motion of square bar in a square hole.



② In Completely Constrained motion :- [more than one type of motion]

When the motion between a pair can take place in more than one direction, then the motion is called an Incompletely Constrained motion.

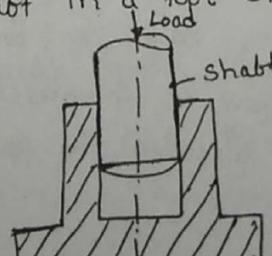
Eg:- Circular shaft or bar in a circular hole.



③ Successfully Constrained motion :- [Incompletely Constrained motion Convert into Completely Constrained motion]

When the motion between the elements, forming a pair is such that, the Constrained motion is not Completed by itself, but by some other means, then the motion is said to be Successfully Constrained motion.

Eg:- Shaft in a foot step bearing,



Classification of kinematic pairs:

Kinematic pair

- Acc to type of Relative motion
  - Eg:-
  - Sliding pair
  - Rolling pair
  - Turning pair
  - Screw pair
  - Spherical pair etc
- Acc. to type of Surface incontact
  - Eg:-
  - Lower pair (Surface Contact)
  - Higher pair. (line or point contact)
- Acc. to the type of closur
  - Eg:-
  - Self closer pair
  - Force-clos pair etc

Chain:-

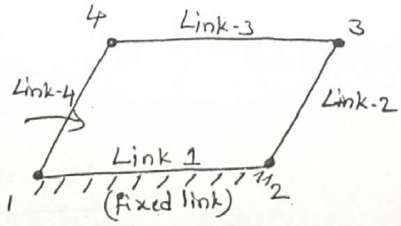
Chain is formed by joining two pairs in such a way that first link is connected to the last link.

Kinematic chain:-

When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion.(ie, Completely or successfully constrained motion), it is called a kinematic chain.

Mechanism:-

When one of the links of a kinematic chain is fixed the chain is known as Mechanism. It may be used for transmitting or transforming motion.



Gear

B) Gear Trains :-

Unit-IV

Power transmission.

Modes of power transmission.

Transmission by friction.

-iction dias.

Flexible connection.

Eg:- Belt drive.

Transmission by mech

Direct Contact

Eg:-Toothed Gears

Flexible Connection

Eg:- chain drive.

-elt drives.

-elts.

Flat belts

V belts

Rope belts.

Belt drives.

1. Open 

2. Cross. 

derivations

1. Length of belt.

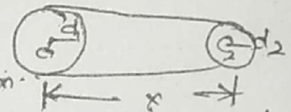
2. Ratio of belt tension.

3. Conditions for max. power transmission

Pulley

driver

driven.



Gears :- (Introduction)

→ Gears are used to transmit motion (or) power from one shaft to another, preferably if the Centre distance b/w two shafts is small.

→ The most Commonly used types of gears are

① Spur Gears ② Helical gears ③ Bevel gears ④ Spiral gears

⑤ Rack & pinion ⑥ Worm and worm wheel. etc

Advantages & dis-advantages of Gears :-

Adv :-

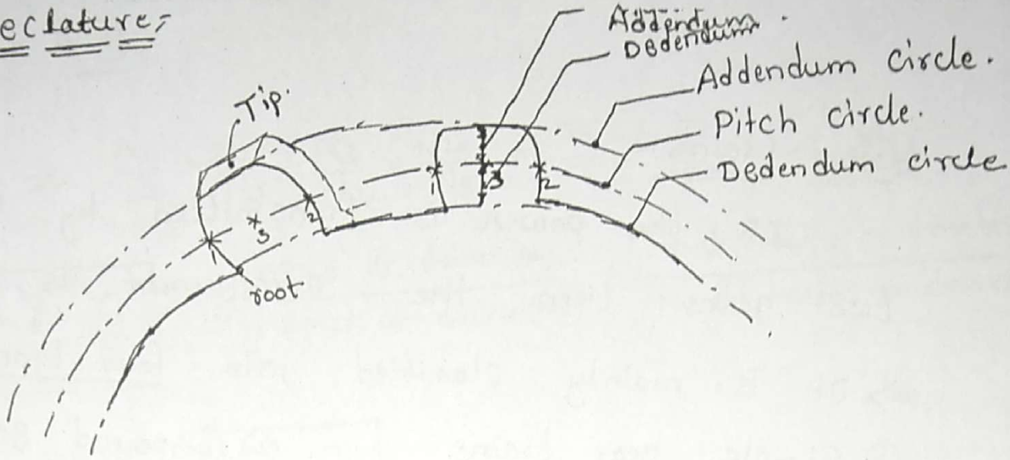
1. They are positive non-slip drives
2. Most Convenient for very small Centre distances.
3. In gear drives, unlike belt and chain, the velocity ratio will remain Constant throughout
4. They have very high transmision efficieny.

Dis-adv :-

1. They are not suitable for shafts of very large Centre distances.
2. They alway require Some kind of lubrication.
3. At high speeds noise and vibrations will be more etc.

Scanned by CamScanner

Gear Nomneclature;



Pitch circle;-

It is a imaginary circle having pure rolling motion, when two gears are mating to each other.

- Pitch circle diameter (P.C.D)

Addendum circle;-

The circle passing through the all the tip of the teeth

Dedendum circle;-

The circle passing through the all the roots of the teeth

⇒ **Addendum** = It is the radial distance between pitch circle. and addendum circle.

⇒ **Dedendum** ;- It is the radial distance between pitch circle and dedendum circle

Circular pitch ;-(P_c)

It is the distance between the two successive points measured along the pitch circle (Eg;- 1-1 (or) 2-2 (or) 3-3 the above fig.)

ie, $\boxed{P_c = \frac{\pi D}{T}}$; It is denoted by (P_c).

D = dia. of pitch circle.

T = no. of teeths on gear.

Module (m);-

It is the ratio of pitch circle diameter of the gear to the no. of teeth.

- It is denoted by "m" (units mm)

ie, $\boxed{m = \frac{D}{T}}$ (or) $\boxed{D = mT \quad R = \frac{mT}{2}}$

Note;- $\boxed{P_c = \pi m}$

Gear trains:-

If the power is transmitted by using more than two gears then the arrangement is called Gear train.

⇒ It is mainly classified into four types.

① Simple gear trains ② Compound gear trains.

③ Reverted gear trains ④ Epi-Cyclic gear trains. (or) planetary gear trains.

Velocity ratio (or) speed ratio:-(V.R)

$$V.R = \frac{\omega_{Driver}}{\omega_{Driven}} \quad (or) \quad V.R = \boxed{\frac{N_{Driver}}{N_{Driven}}} \quad ie \quad \boxed{\frac{N_1}{N_2} = \frac{T_2}{T_1} = \frac{d_2}{d_1}}$$

Train Value (T.V):- (It is reciprocal of velocity ratio)

$$T.V = \frac{1}{V.R} = \boxed{\frac{N_{Driven}}{N_{Driver}}}$$

① Simple Gear train:-

In a simple gear train, a series of gear wheels are mounted on different shafts between the driving and driven shafts, each shaft carrying only one gear.

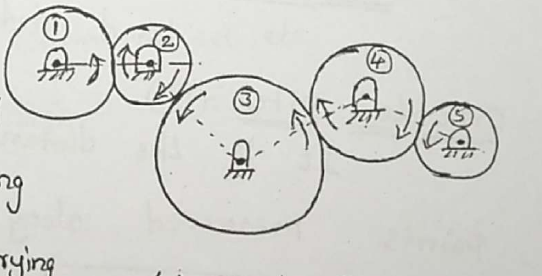


fig:- Simple G.T.

The gear ① is driver gear, gear ②, ③, ④ are intermediate (or) idler gears and gear ⑤ is a driven gear or follower.

Idlers:-

The intermediate gears have no effect on the speed ratio but they change the direction of rotation of the driven gear.

⇒ Even number of idler gears will rotate the driven gear in the direction opposite to that of the driving gear.

⇒ Odd Number of idler gears will rotate the driven gear in the

Speed ratio (V.R) :-

$$\frac{N_1 \text{(driver)}}{N_5 \text{(driven)}} \Rightarrow \frac{N_1}{N_2} \times \frac{N_2}{N_3} \times \frac{N_3}{N_4} \times \frac{N_4}{N_5} = \frac{\cancel{T_2}}{T_1} \times \frac{\cancel{T_3}}{\cancel{T_2}} \times \frac{\cancel{T_4}}{\cancel{T_3}} \times \frac{T_5}{\cancel{T_4}}.$$

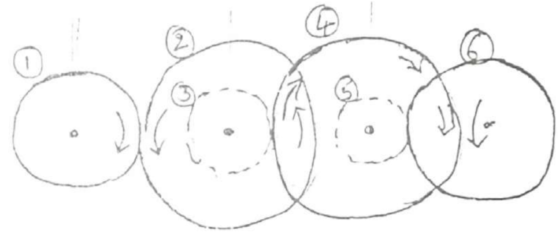
$$\boxed{\frac{N_1}{N_5} = \frac{T_5}{T_1}} \quad \text{ie,} \left\{ \frac{\text{speed of driver } (N_1)}{\text{speed of driven } (N_5)} = \frac{\text{No. of teeths on driven } (T_5)}{\text{No. of teeths on driver } (T_1)} \right\}$$

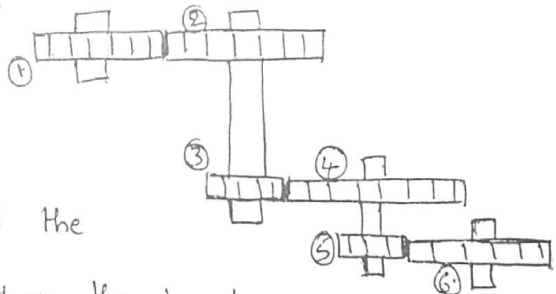
Train value (T.V) :- $\boxed{\frac{N_5}{N_1} = \frac{T_1}{T_5}}$

⇒ Centre distance :- Distance b/w driver & driven gears is called Centre distance.

② Compound Gear train :-

When a series of gears are connected in such a way that two or more gears rotate about an axis with the same angular velocity, it is known as Compound gear train.





In this type, Some of the intermediate shafts, ie, other than the input and the output shafts, Carry more than one gear.

$$N_2 = N_3 \quad \& \quad N_4 = N_5$$

Velocity ratio (V.R) :- $\frac{N_1}{N_6} = ?$

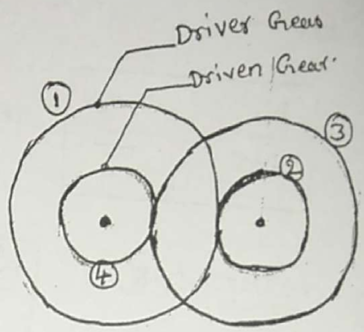
$$\Rightarrow \frac{N_1}{N_2} \times \frac{N_2}{N_3} \times \frac{N_3}{N_4} \times \frac{N_4}{N_5} \times \frac{N_5}{N_6}$$

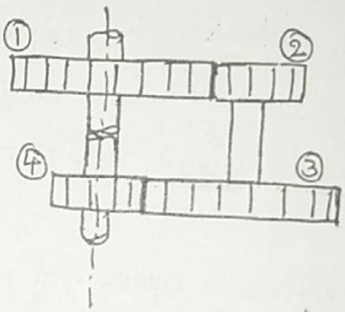
$$\Rightarrow \boxed{\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}}$$

③ Reverted Gear train :-

If the axis of the first and last wheels of a Compound gear Coincide, it is called a reverted gear train.

Eg:- used in clocks, simple lathes etc





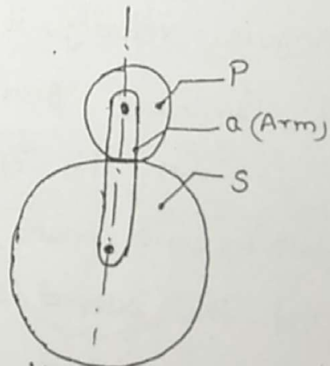
$V.R \Rightarrow \frac{N_1}{N_4} = ?$

ie, $\frac{N_1}{N_2} \times \frac{N_2}{N_3} \times \frac{N_3}{N_4}$

$$\boxed{\frac{N_1}{N_2} \cdot \frac{N_3}{N_4} = \frac{T_2}{T_1} \cdot \frac{T_4}{T_3}}$$

④ Epi-Cyclic (or) planetary Gear Train :-

When there exists a relative motion of axes in a gear train, it is Called a planetary (or) an epicyclic gear train (or simply epicyclic gear (or) train).



Thus ~~the~~ in an epicyclic train, the axis of atleast one of the gears also moves relative to the frame

Eg:- Automobile differential.

Analysis of Epicyclic gear train :-

⇒ Two methods are available.

① Tabular method.

② Relative velocity method.

(Tabular method only we discuss)

S.No.	Action.	Revolution of a	Rev. of S	Rev. of P.
1.	Arm "a" fixed, S +1 rev.	0	1	$-\frac{T_s}{T_P}(1)$
2.	Arm "a" fixed, S +x rev.	0	x	$-\frac{T_s}{T_P}(x)$
3.	Add 'y' (ie, arm revolving y revolutions)	y	x+y	$-\frac{T_s}{T_P}(x)+y$

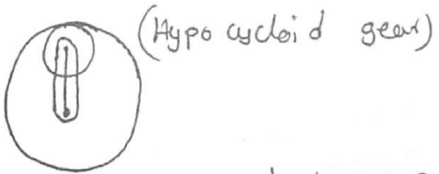
$\therefore$ Speed of arm (a) $= y$

$\therefore$ Speed of gear (S) $= y+x$

$\therefore$ Speed of gear (P) $= -\frac{T_s}{T_P}x + y$

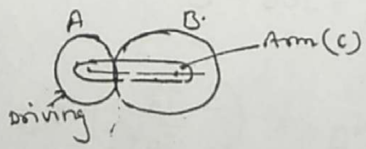
To find unknown Speed.

Note: Annular gear



Problem:- In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 rpm in the anticlockwise direction about the centre of gear A which is fixed, determine the speed of gear "B". If the gear A instead of being fi[xed] maked 300rpm in clockwise direction what will be the speed of gear "B". ?

Sol: Given data: $T_a = 36$, $T_b = 45$; $N_c = 150$ rpm.



S.No	Action.	Rev. of arc (C)	Rev. of gear (A)	Rev. (B)
1.	Arm fixed, "A" rotates through +1 rev. (anticlockw)	0	+1	$-\frac{T_A}{T_B}$
2.	Arm fixed, A rotates through +x rev (anticlock)	0	+x	$-\frac{T_A}{T_B} \times x$
3.	Add +y revs to all elements (anti)	+y	x+y	$\left(-\frac{T_A}{T_B} \times x\right)+y$

Note: clockwise is positive
&
Anti clock wise is Negative

Speed of gear 'B' when gear A is fixed:-

Speed of arm (N_C) is 150 rpm (A.C.W)

ie $y = -150$ rpm. (A.C.W)

Since gear "A" is fixed.

ie $x+y=0$

$x = -y$
$= -(-150)$
$x = 150$ rpm.

∴ speed of gear 'B',

ie $N_B = \left(-\frac{T_A}{T_B}\right)x + y$

$= \left(-\frac{36}{45} \cdot 150\right) - 150 = -270$ (A.C.W)

Speed of the gear "B" when gear 'A' makes 300 rpm:-

(~~Clockwise~~). $y = -150$ rpm A.C.W

$x+y = +300$ (C.W)

$x = 300 - y$
$= 300 - (-150) = +450$ rpm.

Speed of gear B,

$N_B = \left(-\frac{T_A}{T_B} x\right) + y$

$= \left(\frac{-36}{45}\right) 450 - 150$

$\boxed{N_B = -510 (A.C.W)}$

Rev. (13)

Problem 9.13

A gear wheel of 20 teeth drives another gear wheel having 36 teeth running at 200 rpm. Find the speed of the driving wheel and the velocity ratio.

Solution :

Driving Wheel :	*Driven Wheel :*	
$T_1 = 20$	$T_2 = 36$	
$N_1 = ?$	$N_2 = 200$	Velocity Ratio = ?

Velocity Ratio of Simple Gear Train,

$$\frac{N_1}{N_2} = \frac{T_2}{T_1}$$

$$N_1 = \frac{T_2}{T_1} \times N_2$$

$$= \frac{36}{20} \times 200$$

$$= 360 \text{ r.p.m}$$

$\frac{120}{\pi} = 2$

$$\text{Velocity Ratio} = \frac{N_1}{N_2}$$

$$= \frac{360}{200}$$

$$= 1.8 : 1$$

Problem 9.14

A compound gear train consists of 4 gears, A, B, C and D, and they have 20, 30, 40 and 60 teeth respectively. A is keyed to the driving shaft, and D is keyed to the driven shaft, B and C are compound gears, B meshes with A, and C meshes with D. If A rotates at 180 rpm, find the rpm of D. (VTU, July/August 2003)

Solution :

$T_A = 20, \quad T_B = 30 \quad T_C = 40 \quad T_D = 60 \quad N_A = 180 \text{ rpm} \quad N_D = ?$

Velocity Ratio of Compound Gear Train,

$$\frac{N_A}{N_D} = \frac{T_D}{T_C} \times \frac{T_B}{T_A}$$

$$N_D = \frac{T_C}{T_D} \times \frac{T_A}{T_B} \times N_A$$

$$= \frac{40 \times 20}{60 \times 30} \times 180$$

$$= 80 \text{ r.p.m}$$

Problem 9.15

A gear wheel has 50 teeth of module 5 mm. Find the pitch circle diameter and the circular pitch.

Solution :

$m = 5 \text{ mm}, \quad T = 50, \quad d = ? \quad p_c = ?$

$$\text{Module} = m = \frac{d}{T}$$

$$5 = \frac{d}{50}$$

$\therefore$ Pitch circle diameter = d = 5 × 50 = 250 mm

$$\text{Circular Pitch} = p_c = \frac{\pi d}{T}$$

$$= \frac{\pi \times 250}{50}$$

$$= 15.7 \text{ mm}$$

Problem 9.16

A simple gear train is made up of four gears A, B, C, and D having 20, 40, 60 and 70 teeth respectively. If gear A is the main driver rotating at 500 rpm clockwise, calculate the following:

1. Speeds of intermediate gears.

2. Speed and direction of the last follower

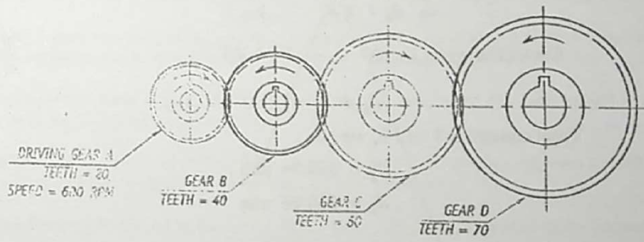
3. Train Value

Solution :

$N_A = 500 \text{ rpm}, \quad T_A = 20, \quad T_B = 40, \quad T_C = 60, \quad T_D = 70$

$\text{Train Value} = ? \quad N_B = ? \quad N_C = ? \quad N_D = ?$

The arrangement can be sketched as given in *Fig. 9.23* below:



Simple Gear Train Arrangement
Fig. 9.23

(i) *A drives B*

$\therefore \quad \frac{N_A}{N_B} = \frac{T_B}{T_A} \Rightarrow \frac{500}{N_B} = \frac{40}{20} \Rightarrow N_B = 250$ rpm

(ii) *B drives C*

$\therefore \quad \frac{N_B}{N_C} = \frac{T_C}{T_B} \Rightarrow \frac{250}{N_C} = \frac{60}{40} \Rightarrow N_C = 166.67$ rpm

(iii) C drives *D*

$\therefore \quad \frac{N_C}{N_D} = \frac{T_D}{T_C} \Rightarrow \frac{166.67}{N_D} = \frac{70}{60} \Rightarrow N_D = 142.86$ rpm

$\therefore$ Speed of the last follower = N_D = 142.86 rpm and its direction is anti-clockwise.

$$\text{Velocity Ratio} = \frac{N_A}{N_D} = \frac{500}{142.86} = 3.5 : 1$$

$$\text{Train Value} = \frac{1}{\text{Speed Ratio}} = \frac{1}{3.5} = 0.285$$

Problem 9.17

Two spur gears A and B connect two parallel shafts that are 500 mm apart. Gear A runs at 400 rpm and Gear B at 200 rpm. If the circular pitch is given to be 30 mm, calculate the number of teeth on gears A and B.

Solution :

N_A = 400 rpm, N_B = 200 rpm, p_C = 30 mm T_A = ?, T_B = ?

The arrangement can be sketched as given in *Fig. 9.24*.

Let d_A and d_B represent the diameters of gears A and B.

$$\text{Velocity Ratio} = \frac{N_A}{N_B} = \frac{d_B}{d_A}$$

$$\Rightarrow \frac{400}{200} = \frac{d_B}{d_A}$$

$$\Rightarrow d_B = 2\,d_A \quad \text{...Eq. 1}$$

$$\text{Center Distance} = 500 = \frac{1}{2}(d_B + d_A)$$

$$\Rightarrow d_B + d_A = 1000 \quad \text{...Eq. 2}$$

From equations 1 and 2, we get

d_A = 333.34 mm

d_B = 666.66 mm

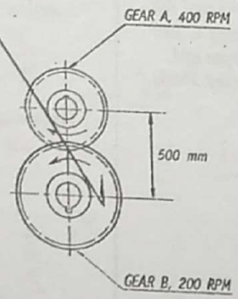


Fig. 9.24

$$\text{Number of teeth of gear A} = T_A = \frac{\pi d_A}{p_C} = \frac{\pi \times 333.34}{30} = 35$$

$$\text{Speed Ratio} = \frac{N_A}{N_B} = \frac{T_B}{T_A}$$

$$\text{Number of teeth of gear B} = T_B = \frac{N_A}{N_B} \times T_A = \frac{400}{200} \times 35 = 70$$

EXERCISES 9

Power Transmission

1. Name the different methods of power transmission.
2. State the differences between the application of belt, chain and gear drives.

Belt Drives

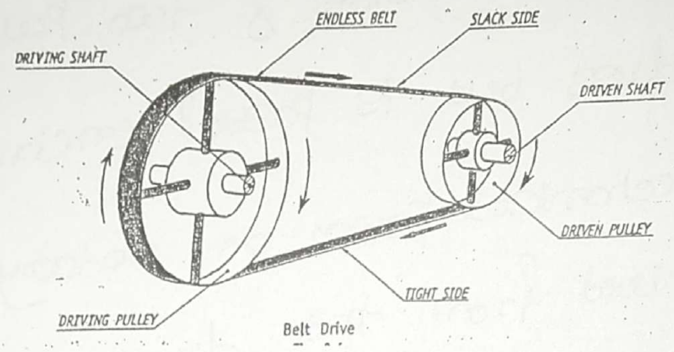
3. Explain how open and cross belt drives function. *(VTU, September 1999)*
4. What is the difference between open and crossed systems of belt drives ?
5. What is meant by speed cone ? Where is it used ?
6. What is the function of a stepped cone pulley ? Explain with a simple sketch. *(VTU, March 1999)*
7. Sketch neatly a speed cone pulley. Explain its working. How it is useful in practice?
8. Why Jockey pulley is used ? *(VTU, September 1999)*
9. How is the direction of rotation of the driven pulley in belt drives reversed ?
10. When are the fast and loose pulleys used ? Explain.
11. Define slip. Why it occurs ? Explain
12. What is creep in belt drives ? Explain.
13. What are the advantages and disadvantages of belt drives ?
14. What are the advantages of V-belts over flat belts ? *(VTU, September 2000)*

Scanned by CamScanner

c) Belt drives :- Unit-IV 1

Elements Of Mechanical Engineerin

Belt drives are one of the common methods generally employed whenever power or rotary motion is to be transmitted between two parallel shafts. A belt drive consists of two pulleys over which an endless belt is passed encircling both of them. The mechanical power or rotary motion is transmitted from the driving pulley to driven pulley because of the frictional grip that exists between the belt and the pulley surfaces. Although friction is undesired in general engineering practice, but in belt drives it serves as a helpful agent. Obviously, for the belt to move, the pull or tension on one of the sides of the belt should be more than the other side. The portion of the belt which is having less tension is called slack side and the one which has higher tension is a tight side.



Belt Drive

The portion of the belt between the two pulleys that become either slack or tight depends on the direction of rotation of the driving pulley. For clock wise rotation of the driving pulley, the lower side of the belt will be the tight side & the upper side will be the slack side. Since the tensions in the tight and slack sides ...

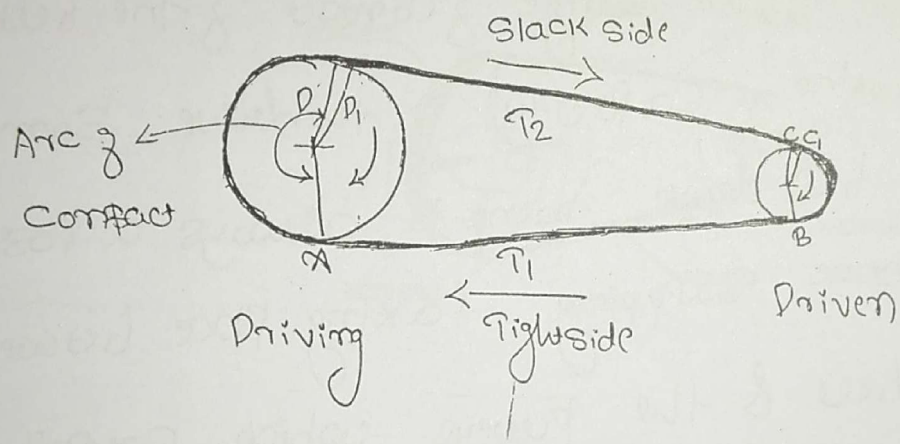
depends on the angle of contact, the belt drives have to be arraged such that the slack side comes above & the tight side come below the pulleys. This arrangement increases the angle of contact of the belt and also the capacity of the drive. Some time in a belt drive there is always a possibility of some slipping taking place between the belt & the pulleys which causes the driven pulley to rotate at a lesser speed, consequently reduces the power transmission. Hence belt drives are said to be not a positive type of power transmission system.

<u>Belt materials</u>

i) Leather ii) Rubber
iii) Canvas iv) Balata

Types of belt drives

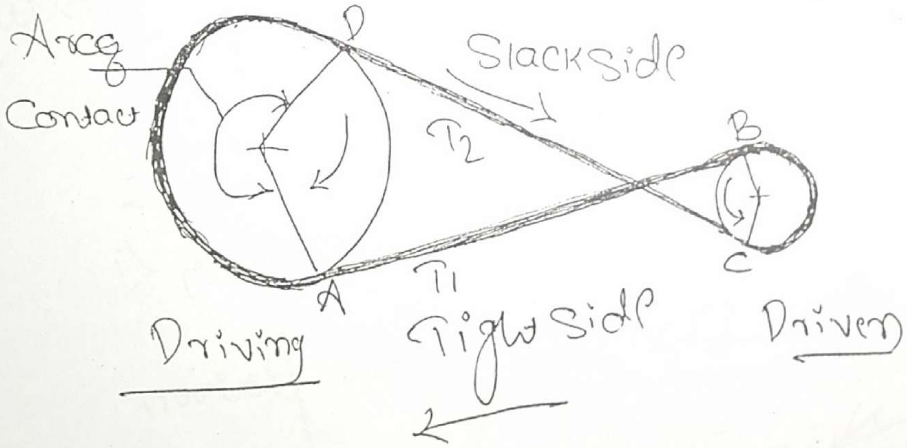
1.) Open Belt Drive



This type of belt drive is employed when the two parallel shafts have to rotate in the same direction. When the shafts are placed far apart, the lower side of the belt should be tight side and the upper side must be the slack side. This is because, when the upper side becomes the slack side, it will sag due to its own weight thus increase arc of contact.

Flat belt drives of the open system should always have their shaft axes either horizontal or inclined. They should never be vertical, for if so arranged, the centrifugal force developed in the belt combined with gravity force causes the belt to strech & tend to leave the pulley.

2.) Crossed Belt drive



This type of belt drive is used when two parallel shafts have to rotate in the opposite direction.

Scanned by CamScanner

At the junction where the belt crosses, it rubs against itself and wears off.

To avoid excessive wear, the shafts must be placed at a maximum distance from each other and operated at very low speeds.

Velocity Ratio of belt drive

It is the ratio between the velocities of the driver & driven pulleys.

Let,

$d_1 \Rightarrow$ Diameter of the driver

$d_2 \Rightarrow$ Diameter of the driven

$N_1 \Rightarrow$ Speed of the driver (RPM)

$N_2 \Rightarrow$ Speed of the driven (RPM)

The peripheral velocity of the belt on driving pulley.

$V_1 = \frac{\pi d_1 N_1}{\ldots}$ (m/s) $\Rightarrow$ (V_1 = ...

For driven pulley

$$v_2 = \frac{\pi d_2 N_2}{60} \quad (m/s)$$

When there is no slip

$$v_1 = v_2$$

$$\frac{\pi d_1 N_1}{60} = \frac{\pi d_2 N_2}{60} \Rightarrow \boxed{d_1 N_1 = d_2 N_2}$$

$$\boxed{\frac{N_1}{N_2} = \frac{d_2}{d_1}}$$

$$\cancel{\frac{N_2}{N_1} = \frac{d_1}{d_2}}$$

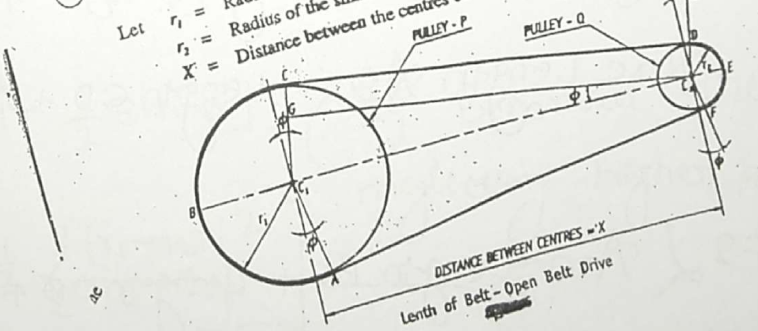
If thickness (t) of the belt is given

$$\boxed{\frac{N_1}{N_2} = \frac{d_2 + t}{d_1 + t}}$$

Length of the belt derivation :-

i) Open System

Let r_1 = Radius of the larger pulley P
r_2 = Radius of the smaller pulley Q
X = Distance between the centres of the two pulleys



Lenth of Belt - Open Belt Drive

Let the two pulleys P&Q be connected by an open belt as shown in fig. From the centre C_2 of the ~~[illegible]~~ smaller pulley draw a line C_2G parallel to CD.

Let,

r_1 = Radius of the larger pulley P

r_2 = Radius of the smaller pulley Q.

X = Distance between the centers of the two pulleys.

From the geometry of the belt drive shown in fig. the length of belt is given by,

L = Arc Length ABC + Length CD + Arc Length DEF + Length FA

= 2 (Arc Length BC + Length CD + Arc length DE)

$$= 2\left[\left(\frac{\pi}{2}+\phi\right) r_1 + \text{Length } CD + \left(\frac{\pi}{2}-\phi\right) r_2\right]$$

$$= 2\left[\left\{\frac{\pi}{2}+\phi\right\}r_1 + \text{length } GC_2 + \left\{\frac{\pi}{2}-\phi\right\}r_2\right] \quad (\because CD = GC_2)$$

$$= 2\left[\left\{\frac{\pi}{2}+\phi\right\}r_1 + x\cos\phi + \left\{\frac{\pi}{2}-\phi\right\}r_2\right] \quad \left(\because \frac{GC_2}{x} = \cos\phi\right)$$

$$= 2\left[\frac{\pi}{2}(r_1+r_2) + \phi(r_1-r_2) + x\cos\phi\right]$$

$$= \pi(r_1+r_2) + 2\phi(r_1-r_2) + 2x\cos\phi \quad \text{--- (1)}$$

From tringle GC_1C_2

$$\sin\phi = \frac{r_1-r_2}{x} \quad \left(\text{Since } \sin\phi = \frac{GC_1}{C_1C_2} = \frac{GC-CG}{x}\right)$$

$C_1C = r$ & $CG = D_{C_2}$

Since ϕ is very small

$$\phi = \frac{r_1-r_2}{x} \quad \text{--- (2)}$$

$$\cos\phi = [1-\sin^2\phi]^{1/2}$$

$$= \left[1-\frac{1}{2}\sin^2\phi\right] \quad (\text{By Binomial Theorem and neglecting higher powers})$$

$$= \left[1-\frac{1}{2}\left\{\frac{(r_1-r_2)}{x}\right\}^2\right] \quad \text{--- (3)}$$

Substituting (2) & (3) in (1)

$$L = \pi(r_1+r_2) + 2\left(\frac{r_1-r_2}{x}\right)(r_1-r_2) + 2x\left[1-\frac{1}{2}\left(\frac{r_1-r_2}{x}\right)^2\right]$$

$$= \pi(r_1+r_2) + 2\frac{(r_1-r_2)^2}{x} + 2x - \frac{(r_1-r_2)^2}{x}$$

$$\boxed{L = \pi(r_1+r_2) + \frac{(r_1-r_2)^2}{x} + 2x}$$

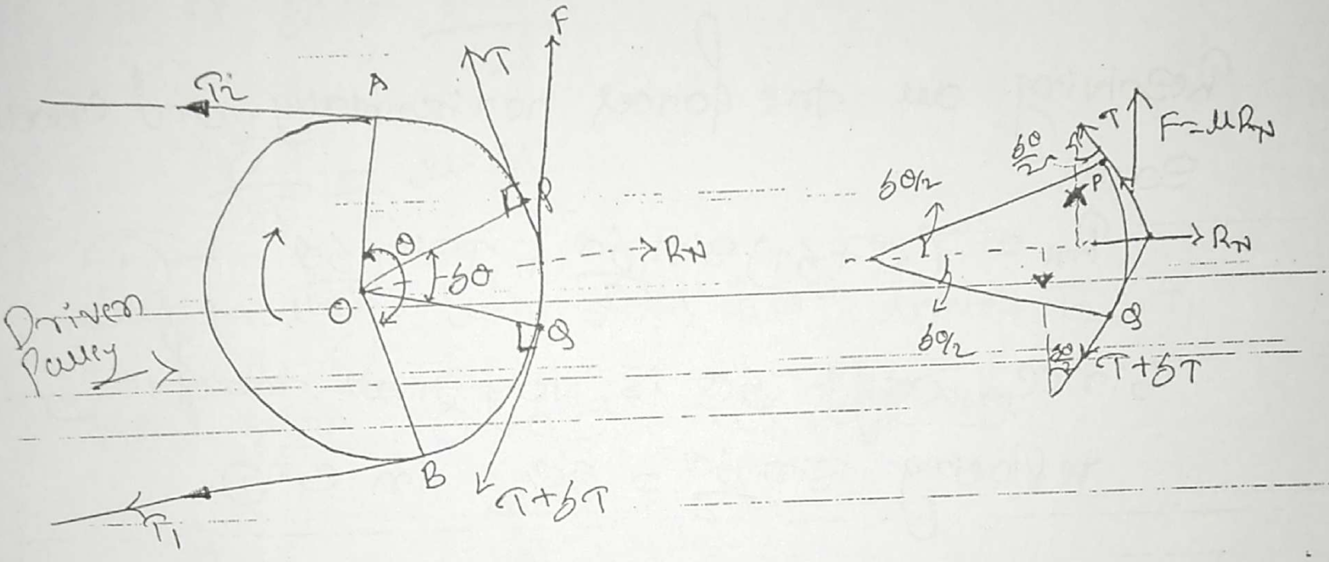
ii) For crossed Belt drive

$$\boxed{L = \pi(r_1+r_2) + \frac{(r_1+r_2)^2}{x} + 2x}$$

Scanned by CamScanner

Ratio of driving tensions for Flat Belt Drive derivation

Consider a driven pulley rotating in the clockwise direction as shown in fig.



T_1 = Tension on tight side

T_2 = Tension on slack side

θ = Angle of lap of the belt over pulley

μ = Co-efficient of friction between the belt & pulley

Now consider a small portion of the belt PQ, subtending an angle $\delta\theta$ at the center of the pully as shown in fig. PQ is in equilibrium under following forces.

Scanned by CamScanner

1. Tension 'T' in the belt at P,

2. Tension $(T+\delta T)$ in the belt at Q

3. Normal Reaction R_N

4. Frictional force, $F = \mu R_N$

Resolving all the forces horizontally and equating same.

$$R_N = (T+\delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2} \quad \text{— (1)}$$

Since, angle $\delta\theta$ is very small, therefore by replacing $\sin\frac{\delta\theta}{2} = \delta\theta/2$ in eqn (1)

$$R_N = (T+\delta T)\frac{\delta\theta}{2} + T \times \frac{\delta\theta}{2} = T\frac{\delta\theta}{2} + \frac{\delta T\,\delta\theta}{2} + T\frac{\delta\theta}{2}$$

$$\boxed{R_N = T\cdot\delta\theta} \quad \text{— (2)} \quad \left[\text{Neglecting } \frac{\delta T\cdot\delta\theta}{2}\right]$$

Now resolving the forces vertically

$$\mu R_N = (T+\delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2} \quad \text{— (3)}$$

Since, $\delta\theta$ is very small replace $\cos\frac{\delta\theta}{2} = 1$ in eqn (3)

$\mu \times R_N = T + \delta T - T = \delta T$

$$R_N = \frac{\delta T}{\mu} \quad — (4)$$

Equating the values of R_N from eqns (2) & (4)

$$T \cdot \delta\theta = \frac{\delta T}{\mu}$$

$$\frac{\delta T}{T} = \mu \cdot \delta\theta$$

Integrating both sides b/w limits T_2 & T_1 and from 0 to θ respectively

$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^{\theta} \delta\theta$$

$$\boxed{\ln\left[\frac{T_1}{T_2}\right] = \mu\theta \quad \text{or} \quad \left[\frac{T_1}{T_2}\right] = e^{\mu\theta}} \quad — (5)$$

eqn (5) can be expressed in terms of corresponding logarithm to the base 10.

$$\boxed{2.3 \log(T_1/T_2) = \mu\theta}$$

Power-Transmitted by a Belt drive :-

$$\boxed{P = (T_1 - T_2)\, v}\ (W)$$

T_1 = Tension on tight side (N)

T_2 = Tension on slack side (N)

v = Velocity of belt (m/sec)

If velocity is given in terms of (m/sec) then $\boxed{P = (T_1 - T_2)\, v}$

Initial Tension in the belt :-

Initially the belt is wrapped over the two pulleys tightly. Since the belt is made of elastic material owing to it's tight wrapping there always exists a uniform tension throughout the belt even when the drive is not functioning.

$T_0 = \frac{T_1 + T_2}{2}$ (N)

T_1 = Tight side (N)

T_2 = Tension on slack side (N)

T_0 = Initial tension

Centrifugal Tension

Since the belt continuously runs over the pulleys, therefore some centrifugal force is caused, whose effect is to increase the tension on both, tight as well as slack sides.

Centrifugal Tension $T_C = m \cdot v^2$

$m \Rightarrow$ Mass of the belt per unit length in kg,

$v \Rightarrow$ Linear velocity of belt in (m/s)

Maximum tension in the belt

$$\boxed{T = T_1 + T_C}$$

Condition for transmission of maximum Power : Derivation ;

W.K.T Power transmitted by a belt,

$$P = (T_1 - T_2)v \quad \text{—} (1)$$

$T_1 \Rightarrow$ Tension on tight side (N)

$T_2 \Rightarrow$ Tension on slack side (N)

$v \Rightarrow$ velocity of belt in (m/s)

We have seen that ratio of driving tensions is

$$\frac{T_1}{T_2} = e^{\mu\theta} \quad \text{or} \quad T_2 = \frac{T_1}{e^{\mu\theta}} \quad \text{—} (2)$$

Substitute T_2 in eqn (1)

$$P = \left(T_1 - \frac{T_1}{e^{\mu\theta}}\right)v = T_1\left(1 - \frac{1}{e^{\mu\theta}}\right)v = T_1 \cdot v \cdot C \quad \text{—} (3)$$

where, $C = 1 - \frac{1}{e^{\mu\theta}}$

$\boxed{\text{W.K.T} \quad T_1 = T - T_C}$ $T \Rightarrow$ max. tension to which ...

Substitute value of T_1 in eqn (3)

$$P = (T - T_C) v \cdot C = (T - mv^2) v \cdot C =$$

$$P = (Tv - mv^3) C \quad \ldots \text{(Substituting } T_C = mv^2\text{)}$$

For maximum power, differentiate the above expression with respect to v & equate to zero.

$$\frac{dP}{dv} = 0$$

$$\frac{d\,(Tv - mv^3) C}{dv} = 0$$

$$T - 3mv^2 = 0 \qquad (T_C = mv^2)$$

$$T - 3T_C = 0$$

$$\boxed{T = 3T_C} \qquad \boxed{V_{max} = \sqrt{\frac{T}{3m}}}$$

It shows that when the power transmitted is maximum, $(1/3^{rd})$ of the maximum tension is absorbed as centrifugal tension.

(The above statement can also be written as below

Therefore, for maximum power transmission, centrifugal tension in the belt must be equal to one third of the maximum available belt tension. And the belt should be on the point of slipping i.e maximum frictional force is developed in the belt.

Hence

$$\boxed{T_c = \frac{T}{3}}$$

$$\text{Also, } T_1 = T - T_c = \frac{2}{3}T$$

Note: Velocity ratio

$$\boxed{\frac{N_1}{N_2} = \frac{d_2}{d_1}}$$

$\Rightarrow$ $\boxed{\frac{N_1}{N_2} = \frac{d_2+t}{d_1+t}}$; If thickness is consider.

$\Rightarrow$ If slip is consider on both pulley.

$\boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1-\frac{S}{100}\right)}$ where $(S = S_1 + S_2 - 0.01\, S_1 S_2)$

$\Rightarrow$ Maximum Tension on belt (when Centrifugal Tension (T_c) consider)

ie $\boxed{T_{max} = T_1 + T_c}$

$\Rightarrow$ Minimum Tension on belt (when T_c consider).

ie $\boxed{T_{min} = T_2 + T_c}$. where, $\boxed{T_c = mv^2}$

$\Rightarrow$ Maximum Tension (T_{max}) $\Rightarrow$ $\boxed{T_1 + T_c = \sigma b t}$

Elements of Mechanical Engineering.

5R

9.18 Simple Problems on Belt Drives

Problem 9.1

Power is to be transmitted from a shaft to another by means of a belt drive. The diameter larger pulley is 600 mm and that of the smaller pulley is 300 mm. The distance between the cer the two pulleys is 3 metre. If the axes of the two shafts are in the same plane and parallel other, find the length of the belt required for : (i) open belt drive and (ii) crossed belt drive.

Solution :

Radius of Larger Pulley $r_1 = \frac{600}{2} = 300$ mm

Radius of Smaller Pulley $r_2 = \frac{300}{2} = 150$ mm

Distance between the axes of the driving and driven shafts $X = 3$ m $= 3000$ mm

Length of belt in the Open System :

$$L = \pi\,(r_1 + r_2) + \frac{(r_1 - r_2)^2}{X} + 2X$$

$$= \pi\,(300 + 150) + \frac{(300 - 150)^2}{3000} + 2 \times$$

$$= 7421.2 \text{ mm}$$

Length of belt in the Crossed System :

$$L = \pi\,(r_1 + r_2) + \frac{(r_1 + r_2)^2}{X} + 2X$$

$$= \pi\,(300 + 150) + \frac{(300 + 150)^2}{3000} + 2 \times$$

$$= 7481.2 \text{ mm}$$

Problem 9.2

An engine is driving a generator by means of a belt. The pulley on the driving diameter of 55 cm and runs at 276 rpm. If the radius of the pulley on the generator it 15 speed in r.p.m.

Solution :

Engine : Driving system
$d_1 = 55$ cm
$N_1 = 276$ rpm

Generator : Driven system
$d_2 = 2r_2$
$= 2 \times 15 = 30$ cm

Velocity Ratio $\frac{N_1}{N_2} = \frac{d_2}{d_1}$

$$N_2 = \frac{d_1}{d_2} \times N_1$$

$$= \frac{55}{30} \times 276$$

$$= 506 \text{ r.p.m}$$

Scanned by CamScanner

Problem 9.3

A motor running at 1750 rpm drives a line shaft at 800 rpm. If the diameter of the pulley on the motor shaft is 160 mm, find that of the pulley on the line shaft.

Solution :

Motor : Driving system

$N_1 = 1750$ rpm

$d_1 = 160$ mm

Line shaft : Driven system

$N_2 = 800$ rpm

$d_2 = ?$

$$\text{Velocity Ratio} \quad \frac{N_1}{N_2} = \frac{d_2}{d_1}$$

$$d_2 = d_1 \times \frac{N_1}{N_2}$$

$$= 160 \times \frac{1750}{800}$$

$$= 350 \text{ mm}$$

Problem 9.4

A shaft running at 100 rpm is to drive a parallel shaft at 150 rpm. The pulley on the driving shaft is 35 cm in diameter. Find the diameter of the driven pulley. Calculate the linear velocity of the belt and also the velocity ratio.

Solution :

Driving Shaft :

$N_1 = 100$ rpm

$d_1 = 35$ cm

Driven Shaft :

$N_2 = 150$ rpm

$d_2 = ?$

Linear Velocity of the Belt = ?

Velocity Ratio = ?

(a) Velocity Ratio $= \frac{N_1}{N_2}$

$$= \frac{100}{150}$$

$$= \frac{2}{3}$$

Velocity Ratio = 2 : 3

(b) Now $\frac{N_1}{N_2} = \frac{d_2}{d_1}$

$$d_2 = d_1 \times \frac{N_1}{N_2}$$

$$= 35 \times \frac{100}{150}$$

$$= 23.33 \text{ cm}$$

(c) Linear Velocity of the belt $= \pi d_1 N_1$

$$= \pi \times \frac{35}{100} \times 100$$

$$= 109.95 \text{ m / min}$$

Problem 9.5

The sum of the diameters of two pulleys A and B connected by a belt is 900 mm. If they run at 700 and 1400 rpm respectively, determine the diameter of each pulley.

Solution :

$d_A + d_B = 900$ mm $\quad N_A = 700$ rpm $\quad N_B = 1400$ rpm

Velocity Ratio $\frac{N_A}{N_B} = \frac{d_B}{d_A}$

$$\frac{700}{1400} = \frac{d_B}{d_A}$$

$$\therefore \quad d_A = 2d_B \quad \ldots 1$$

But $\quad d_A + d_B = 900 \quad \ldots 2$

Substituting d_A from equation *1* in equation 2

ie., $\quad 2d_B + d_B = 900$

$$d_B = 300 \text{ mm}$$

$$d_A = 2d_B$$

$$= 2 \times 300$$

$$= 600 \text{ mm}$$

Problem 9.6

In an open belt drive running in the clockwise direction, the tension in the tight side is 3000 N and the arc of contact is 150°. If the coefficient of friction is 0.3. Find the tension on the slack side of the belt.

Solution :

$T_1 = 3000$ N $\qquad \theta = 150°$

$\mu = 0.3 \qquad = \frac{150 \times \pi}{180}$ radians

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\therefore \quad \log \frac{T_1}{T_2} = \mu\theta \log e$$

$$= \frac{0.4343 \times 0.3 \times 150 \times \pi}{180}$$

$$= 0.341$$

Taking antilogarthms, $\quad \frac{T_1}{T_2} = 2.193$

ie.,

$$\frac{3000}{T_2} = 2.193$$

$$\therefore \quad T_2 = 1367.9 \text{ N}$$

Problem 9.7

In a crossed belt drive the difference in tensions between the tight and slack sides of the belt is 1000 N. Find the tensions on the slack and tight sides, if the angle of contact is 160° and the coefficient of friction is 0.3.

Solution :

$$T_1 - T_2 = 1000 \text{ N} \qquad \theta = 160° \qquad \mu = 0.3$$

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\therefore \quad \log \frac{T_1}{T_2} = 0.4343\ \mu\theta$$

$$= \frac{0.4343 \times 0.3 \times 160 \times \pi}{180}$$

Taking antilogarithms,

$$= 0.3638$$

$$\therefore \quad \frac{T_1}{T_2} = 2.31$$

But $\quad T_1 - T_2 = 1000$

$$\therefore \quad 2.31T_2 - T_2 = 1000$$

$$\therefore \quad T_2 = 763.35 \text{ N}$$

$$\therefore \quad T_1 = 1763.35 \text{ N}$$

Problem 9.8

A flat open belt drive consists of pulleys of diameter 1000 mm and 500 mm with the centre distance of 1500 mm. The coefficient of friction between the pulley and the belt is 0.3. When the maximum tension in the belt is 700 N, find the effective pull of the belt drive.

Solution :

$$D_1 = 1000 \text{ mm} \qquad D_2 = 500 \text{ mm} \qquad X = 1500 \text{ mm}$$

$$r_1 = 500 \text{ mm} \qquad r_2 = 250 \text{ mm} \qquad \mu = 0.3$$

$$T_1 = 700 \text{ N}$$

For an Open Belt Drive,

$$\therefore \quad \sin\phi = \frac{r_1 - r_2}{X}$$

$$\therefore \quad \phi = \sin^{-1}\frac{r_1 - r_2}{X}$$

Scanned by CamScanner

i.e.,

$$\phi = \sin^{-1}\frac{500 - 250}{1500}$$

$$= \sin^{-1}\frac{1}{6}$$

$$= 9.59°$$

The angle of lap is always considered for the smaller pulley,

$$\therefore \quad \theta = 180 - 2\phi$$

$$= 180 - 2 \times 9.59$$

$$= 160.82°$$

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\log\left(\frac{T_1}{T_2}\right) = 0.4343 \ \mu\theta$$

$$= \frac{0.4343 \times 0.3 \times 160.82 \times \pi}{180}$$

$$= 0.3657$$

Taking antilogarithms,

$$\frac{T_1}{T_2} = 2.321$$

$$T_2 = \frac{T_1}{2.321}$$

$$= \frac{700}{2.321}$$

$$= 301.6 \text{ N}$$

$\therefore$ Effective pull in the belt drive $= T_1 - T_2$

$= 700 - 301.6$

$= 398.4$ N

Problem 9.9

In a belt drive, the angle of lap on the driven pulley is 160° and the coefficient of fri between the pulley and the belt material is 0.28. If the width of the belt is 200 mm and the max tension in the belt is not to exceed 50 N per mm width, find the initial tension in the belt drive.

Solution :

$\theta = 160°$ $\mu = 0.28$ Belt Width = 200 mm

Maximum Tension = 50 N/ mm width

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\log\left(\frac{T_1}{T_2}\right) = 0.4343 \ \mu\theta$$

$$= \frac{0.4343 \times 0.28 \times 160 \times \pi}{180}$$

$$= 0.3395$$

Taking antilogarithms,

$$\frac{T_1}{T_2} = 2.187$$

Maximum Initial Tension, T_1 = 50 N/mm width of belt

$$= 50 \times 200$$

$$= 10000 \text{ N}$$

$$\therefore \quad T_2 = \frac{10000}{2.187}$$

$$= 4572.4 \text{ N}$$

$$= 4.5724 \text{ kN}$$

$\therefore$ Initial Tension $T_0 = \frac{10000 + 4572.4}{2}$

$$= 7586.2 \text{ N}$$

$$= 7.586 \text{ kN}$$

Problem 9.10

The driven pulley of 400 mm diameter of a belt drive runs at 200 rpm. The angle of lap is 165° and the coefficient of friction between the belt material and the pulley is 0.25. Find the power transmitted if the initial tension is not to exceed 10 kN.

Solution :

N_2 = 200 rpm θ = 165°

T_0 = 10 kN P = ?

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

Taking antilogarithms,

$$\log\left(\frac{T_1}{T_2}\right) = \mu\theta \ loge$$

$$\log\left(\frac{T_1}{T_2}\right) = 0.4343\ \mu\theta$$

$$= \frac{0.4343 \times 0.25 \times 165 \times \pi}{180}$$

$$= 0.3126$$

Taking antilogarithms,

$$\frac{T_1}{T_2} = 2.054$$

$$T_0 = \frac{T_1 + T_2}{2}$$

or $T_1 + T_2 = 2T_0$

$$= 2 \times 10000 \text{ N}$$

$$= 20000 \text{ N}$$

ie., $2.054\ T_2 + T_2 = 20000$

$$\therefore \quad T_2 = \frac{20000}{3.054}$$

$$= 6548.8 \text{ N}$$

$$\therefore \quad T_1 = 2.054 \times 6548.8$$

$$= 13451.2 \text{ N}$$

Belt Speed $v = \pi DN$

$$= \pi \times 0.4 \times 200$$

$$= 251.32 \text{ m/min}$$

Power Transmitted $= \dfrac{(T_1 - T_2)\ v}{60 \times 1000}$

$$= \frac{(13451.2 - 6548.8) \times 251.32}{60 \times 1000}$$

$$= 28.91 \text{ kW}$$

Problem: $\boxed{\text{R.S.Kharmi; Pg}>347}$

A shaft rotating at 200 rpm drives another shaft at 30 and transmits 6kw through a belt The belt is 100mm wid. and 10mm thick. The distance b/w the shafts is 4m, The Smaller pulley is 0.5m in diameter. Calculate the stress i the belt, if it is An open belt drive (Take: $\mu = 0.3$)

Sol: $N_1 = 200\,rpm$, $N_2 = 300\,rpm$, $P = 6\,kW$

$= 6 \times 10^3\,W$

$b = 100\,mm$, $t = 10\,mm$

$x = 4\,m$, $d_2 = 0.5\,m$

$\mu = 0.3$

Stress on Open belt :- ($\sigma_t = ?$)

We know that $\boxed{T_{max} = \sigma \cdot b \cdot t}$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \Rightarrow d_1 = \frac{300 \times 0.5}{200} = 0.75\,m$$

velocity of belt (V) $= \dfrac{\pi d_2 N_2}{60} = \dfrac{\pi \times 0.5 \times 300}{60} = 7.855\,m/s$

Now find the angle of Contact on the smaller pulley.

Open belt $\sin\alpha = \dfrac{r_1 - r_2}{x} = \dfrac{d_1 - d_2}{2x}$

$$\sin\alpha = \frac{0.75 - 0.5}{2 \times 4} = 0.03125$$

$$\alpha = 1.80$$

Angle of Contact $(\theta) = 180 - 2\alpha$

$= 180 - 2 \times 1.8 = 176.4^\circ$

$= 176.4 \times \dfrac{\pi}{180} = 3.08\,rad$

We know that:-

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu\theta$$

$= 0.3 \times 3.08$

$= 0.924.$

$$\frac{T_1}{T_2} = 2.52 \quad \text{--- (1)}$$

We know that $(P) = (T_1 - T_2)V \Rightarrow 6 \times 10^3 = (T_1 - T_2)\,7.855$

$T_1 - T_2 = 764\,N$ --- (2).

From eqn (1) & (2)

$T_1 = 1267\,N$ & $T_2 = 503\,N$

$\boxed{1\,MPa = 1\,\frac{MN}{m^2} = 1\,N/mm^2}$

$\boxed{T_1 = \sigma b t} \Rightarrow \boxed{1267 = \sigma \times 100 \times 10}$

$$\sigma = \frac{1267}{1000} = 1.267\,N/mm^2 = \boxed{1.267\,MPa}$$

R.S. khurmi Pg. 348

Problem: A leather belt is required to transmit 7.5 kW from a pulley 1.2 m in diameter, running at 250 rpm. The angle of contact is 165° and the co-efficient of friction b/w the belt and the pulley is 0.3. If the safe working stress [illegible] belt is 1.5 MPa. Density of leather 1 Mg/m3 and

1) Fluid Mechanics (Unit-IV)

Introduction:-

The difference between solid and fluid is the relative abilities to resists the external forces. A solid can assists tensile, compressive and shear forces upto certain limit. A fluid has no tensile strength and can assists compressive forces only, when it is kept in a container.

In fact as the fluid flows there exist shearing stresses (Tangential stresses) between adjacent fluid layer which results in opposing the movement of one layer over the other. An amount of shear stress in fluid depending on the magnitude of rate of deformation of the fluid element. However if a fluid is at rest ~~most~~ no shearing force existing there

Fluid:-

Fluid may be defined as a substance it is a capable of flowing (ability to flow)

→ It has no shape on its own, but conforms to the shape of containing vessel

→ Fluid is two types ① Liquid ② Gas.

Properties of fluid:-

① mass density (ρ):- "It is defined as the fluid mass which passes per the unit volume."

units kg/m^3 or gm/cc.

→ It is denoted by (ρ),

$$\boxed{\rho = \frac{m}{V}} ; (kg/m^3)$$

Note:- ② As Temp ↑ then ρ ↓

Note:- ① $\rho_{water} = 1000\ kg/m^3$

$\rho_{air} = 1.2\ kg/m^3$

② **Weight density (or) Specific weight :- [γ (or) w]**

"The weight (W) posses per a unit volume."

$$\boxed{w = \frac{W}{V}}$$ $\therefore W = mg$

$$= \frac{mg}{V} = \rho g.$$

$$\boxed{w = \rho g}$$ units :- N/m^3

$\boxed{1N = 1\ kg\text{-}}$

→ The specific weight of water is $\boxed{9810\ N/m^3}$.

w (or) $\gamma_{water} = 9810\ N/m^3$

w (or) $\gamma_{air} = 11.7\ N/m^3$

③ **Specific gravity (or) Relative density :- [S (or) G (or) RD]**

"It is the ratio of specific weight of a fluid to the specific weight of a standard fluid."

ie $$S = \frac{\gamma \text{ of a given substance}}{\gamma \text{ of a standard fluid(water)}}$$

Note:- Sp. gravity of water $= \dfrac{\text{sp. wt. of water}}{\text{sp. wt of standard fluid(water)}}$

$$= \frac{9810}{9810} = 1$$

ie $S_{water} = 1$

$S_{kerosen} = 0.8$

$S_{mercury} = 13.6$ etc

④ **Specific volume :- (ϑ)**

"The specific volume of a fluid is the volume of fluid to the unit weight."

$$\boxed{w = \frac{W}{V}}$$

ie, $\vartheta = \dfrac{V}{W}$

$$\boxed{\vartheta = \frac{1}{w}}$$

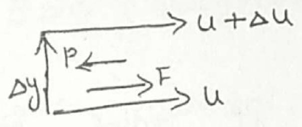
Problem ① If 5m3 of an oil weights 40kN. Calculate the a) mass density b) weight density c) sp. gravity d) sp. volume.

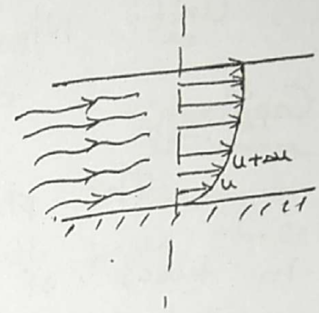
Problem ②

Viscosity:-

It is the property of the fluid by virtue of it offers the resistance to movement of one layer of fluid over adjacent layer.

→ It is primerly due to cohenion and molecular momentum exchange between the fluid layers





$F \propto A$

$F \propto \Delta u$

$F \propto \frac{1}{\Delta y}$

$$F \propto \frac{\Delta u}{\Delta y} \cdot A$$

$$\boxed{\tau = \frac{F}{A}}$$

$$\tau \propto \frac{\Delta u}{\Delta y}$$

$$\underset{\Delta y \to 0}{Lt} \frac{\Delta u}{\Delta y} = \boxed{\frac{du}{dy}}$$

$$\therefore \tau \propto \frac{du}{dy}$$

$$\boxed{\tau = \mu \frac{du}{dy}} \quad (or) \quad \boxed{\mu = \frac{\tau}{\left(\frac{du}{dy}\right)}}$$

where, μ = Co-efficient of viscosity.

A = Area of contact

du = relative velocity.

dy = Distance between layers

τ = Shear stress.

$\frac{du}{dy}$ ⇒ is called velocity potential.

Liquids:-

As $T\uparrow$ $\mu\downarrow$	cohesion$\downarrow$

Gases:-

As $T\uparrow$ $\mu_{gas}\uparrow$ $\rho\downarrow$ $\nu\uparrow\uparrow$
molecular momentum Exchange $\uparrow$

Units:- μ = Co-efficient of viscosity.

(or)

Dynamic viscosity.

$$\mu \to \frac{N/m^2}{1/sec} = \boxed{N \cdot sec/m^2}$$

1 poise = 0.1 N-s
1 Newton = 10^5 Dynes.

Kinematic viscosity (ν):-

It is defined as the ratio of Dynamic viscosity to de

ie $\boxed{\nu = \frac{\mu}{\rho}}$ units:- m^2/sec

Volume."

1N = 1 kg-f

Surface tension (σ):-

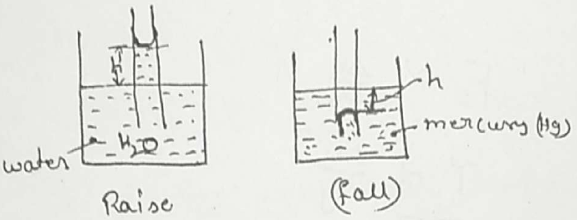
It is the surface energy per unit area.

$$\sigma = \frac{\text{surface energy}}{\text{Unit area.}}$$

Unit: N/m

Capillary:-

The phenomenon of raise or fall of liquids in tubes of a small diameter.



Raise (fall)

Note: ① Adhesion between water and glass is more, compare to Cohesion, thats why raise will takes place.

② Cohesion of mercury and glass is more, compare to adhesion.

③ wetting fluids will raise, non-wetting fluids will fall

Types of fluids:-

Acc to power law

$$\boxed{\tau = A\left(\frac{dv}{dy}\right)^n + B}$$

where, A = multiplying Constant
B = Additive Constant
n = power index.

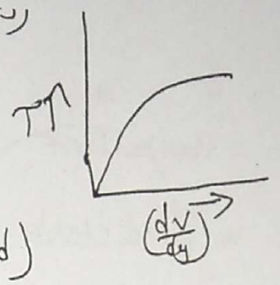
(i) If $A = \mu$; $n = 1$ & $B = 0$

$$\left.\tau = \mu \frac{dv}{dy}\right\}$$ Newton's law of viscosity.

Newtonian fluids: Fluids with satisfy newtons law of viscosity.

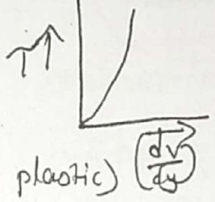
② if $A \neq 0$, $n<1$, $B=0$ (Pseudo plastic fluids)

Eg:- human blood

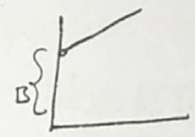


③ if $A \neq 0$, $n>1$, $B=0$ (Dilatant fluid)

Eg:- Butter



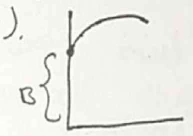
④ If $A \neq 0$, $n=1$, $B>0$ (Binghem plastic)



- It is a viscoplastic material.

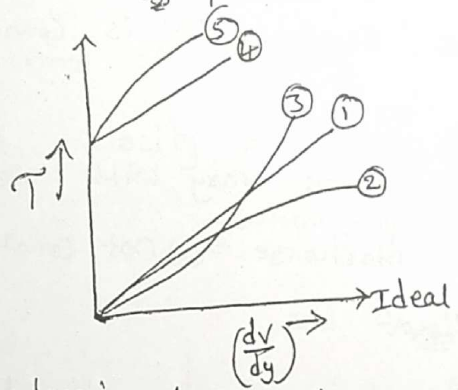
Eg:- clay, Suspensions, drilling mud, tooth paste, chocolate etc

⑤ If $A \neq 0$, $n<1$, $B>0$ (Thixotropic substance).



Eg:- printers Ink

⇒



Note:- ① X-axis represents ideal fluid

② Y-axis represents perfect rigid

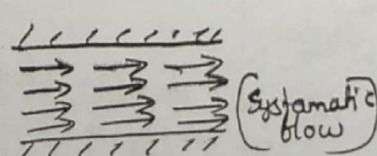
Rheology:- The study of non-newtanian fluids is Rheology.

Types of flows:-

① **Laminar and Turbulent flows**:-

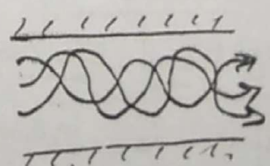
A laminar flow is characterised by a smooth flow of one lamina of fluid over another. fluid elements move in well defined paths and they retain the same relative position at succesive cross-sections of the flow passage

→ The laminar flow is also called the stream line or viscous flow



(Systamatic flow)

fig:- Laminar



(Zig-Zag flow)

fig:- Turbulent

In turbulent flow, the fluid elements move in erratic and unpredictable paths. Individual fluid particles are subject fluctuating ~~transfe~~ transverse velocities so that the motion is eddying, the random eddying motion is called turbulence.

Eg: rivers, canals and in the atmosphere.

② **Steady and un-steady flow:** Motion of a fluid is said to be steady when the fluid parameters at any point in the flow field remain Constant with respect to time.

→ In steady state flow, discharge "Q" is Constant. $\boxed{\text{ie } dQ = 0}$

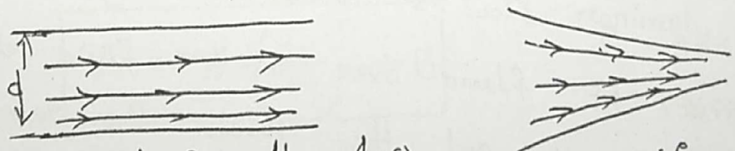
In turbulant

Flow is unsteady when conditions vary with respect to time.

- In unsteady flow, discharge is (Q) not constant $\boxed{\text{ie } dQ \neq 0}$

③ **Uniform and Non-Uniform flow:**

Flow is uniform in character, if the flow parameters like pressure, velocity, density, viscosity and temperature remain Constant through out the flow yield at any given time



Uniform flow ($d = c$)   non-uniform ($d \neq c$)

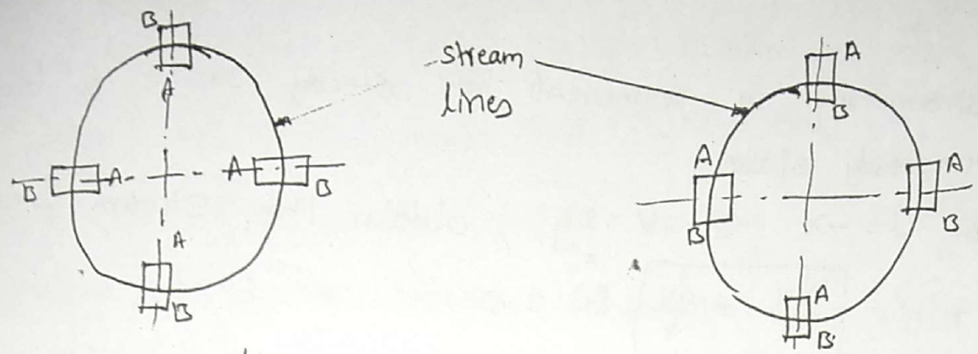
Flow is non-uniform if there is a change in the flow parameters from one section to another.

④ **Compressible and In Compressible flow:**

Flow is incompressible if the density changes due to pressure and temperature variations are insignificant in the flow field

When the density changes are appreciable, the flow is

Rotational and Irrotational Flow :-



A rotational flow exists, when the fluid particles rotate about their own mass centres whilst moving along a stream line

The flow is irrotational, when the fluid particles do not rotate about their mass centres whilst moving along a stream line.

Description of fluid flow :-

The motion of a fluid particle is expressed quantitatively in terms of velocity

the velocity of fluid particle is $\boxed{V = ui + vj + wk.}$

where u, v, w are the velocity components in the x, y and z direction respectively and i, j, k are the unit vectors in the direction of co-ordinate axes.

→ The magnitude of velocity is $\boxed{|V| = \sqrt{u^2+v^2+w^2}}$

$$u = \frac{dx}{dt}$$

$$v = \frac{dy}{dt} \quad \text{and} \quad w = \frac{dz}{dt}$$

→ The relative position of the fluid particles is thus not fixed but varies with time. At ~~a given instant~~

Stream line :-

It is an imaginary line, the tangent to which at any point represent the direction of velocity at that point.

$\boxed{\frac{dx}{u} = \frac{dy}{v}}$ for 2D.

eqn of stream line is $\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$ for 3D

Note: stream line is permanent for steady state & temperary for unsteady state

Problem: $U = -x$ & $V = 2y$, obtain the stream line function.

Sol:
$$\boxed{\frac{dx}{u} = \frac{dy}{V}} \text{ for 2-D.}$$

$$\frac{dx}{-x} = \frac{dy}{2y}$$

Intigrate on both side.

$$-\int \ln x = \frac{1}{2} \ln y$$

$$\ln(x^{-1}) = \ln y^{\frac{1}{2}}$$

$$x^{-1} = \sqrt{y}.$$

$$\frac{1}{x} = \sqrt{y} \Rightarrow \boxed{x\sqrt{y} = 1}$$

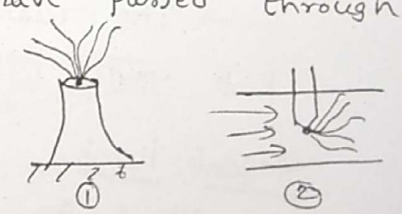
Stream tube:
It is an imaginary stream tube bounded by a no. of Stream lines.

⇒ No discharge normal to the Stream line. & Stream tube

→ stream tube is permanent for steady flow & temperary for un-steady flow.

Path line: Line traced by a single particle.

Streak line:
A streak line is the instantaneous ~~pitcu~~ picture of the Positions of all the fluid particles that have passed through a fixed point in the flow field.

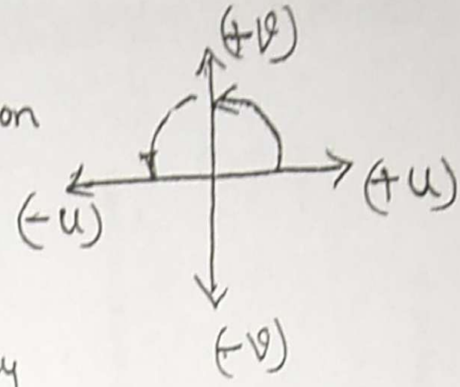


Velocity potential (or) potential function (ϕ):
It is defined as a scalar function of space and time that its negative derivative with respect to any direction gives the fluid velocity in that direction. $\phi = f(x, y, z)$ for steady state such that $u = -\frac{\partial \phi}{\partial x}$

$$V = -\frac{\partial \phi}{\partial y} \quad \text{and} \quad \omega = -\frac{\partial \phi}{\partial z}$$

Scanned by CamScanner

Stream Function. (ψ)



It is defined as the Scalar function of space and time, such that its partial derivative with respect to any direction gives the velocity Component at right angles to that direction.

- It is denoted by (ψ)
- Mathematically, for steady flow defined as:

$\psi = f(x, y)$ such that

$$\frac{\partial \psi}{\partial x} = v$$

$$\frac{\partial \psi}{\partial y} = -u$$

It is defined only for 2-D flow.

Scanned by CamScanner

$$u = -\frac{\partial \phi}{\partial x} = -\left[\frac{y^3}{3} - 2x + \frac{3x^2 y}{3}\right] = \frac{y^3}{3} + 2x - x^2 y$$

$\therefore$ $$u = \frac{y^2}{3} + 2x - x^2 y. \textbf{ Ans.}$$

$\therefore$ $$v = -\frac{\partial \phi}{\partial y} = -\left[-\frac{3xy^2}{3} + \frac{x^3}{3} + 2y\right] = \frac{3xy^2}{3} - \frac{x^3}{3} - 2y = xy^2 - \frac{x^3}{3}$$

(*ii*) The given value of ϕ, will represent a possible case of flow if it satisfies the Laplace equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

From equations (1) and (2), we have

Now $$\frac{\partial \phi}{\partial x} = -y^3/3 - 2x + x^2 y$$

$\therefore$ $$\frac{\partial^2 \phi}{\partial x^2} = -2 + 2xy$$

and $$\frac{\partial \phi}{\partial y} = -xy^2 + \frac{x^3}{3} + 2y$$

$\therefore$ $$\frac{\partial^2 \phi}{\partial y^2} = -2xy + 2$$

$\therefore$ $$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = (-2 + 2xy) + (-2xy + 2) = 0$$

$\therefore$ Laplace equation is satisfied and hence ϕ represent a possible case of flow. **Ans.**

Problem 5.11 *The velocity potential function is given by $\phi = 5(x^2 - y^2)$. Calculate the velocity components at the point (4, 5).*

Solution. $$\phi = 5(x^2 - y^2)$$

$\therefore$ $$\frac{\partial \phi}{\partial x} = 10x$$

$$\frac{\partial \phi}{\partial y} = -10y.$$

But velocity components u and v are given by equation (5.9) as

$$u = -\frac{\partial \phi}{\partial x} = -10x$$

$$v = -\frac{\partial \phi}{\partial y} = -(-10y) = 10y$$

The velocity components at the point (4, 5), *i.e.*, at $x = 4$, $y = 5$

$u = -10 \times 4 = -40$ units

$v = 10 \times 5 = 50$ **units. Ans.**

Problem 5.12 *A stream function is given by $\psi = 5x - 6y$. Calculate the velocity components and also magnitude and direction of the resultant velocity at any point.*

Solution. $$\psi = 5x - 6y$$

$$\frac{\partial \psi}{\partial x} = 5 \text{ and } \frac{\partial \psi}{\partial y} = -6.$$

The velocity components u and v in terms of stream function are given by equation (5.12) as

$$u = -\frac{\partial \psi}{\partial y} = -(-6) = 6 \textbf{ units/sec. Ans.}$$

$$v = \frac{\partial \psi}{\partial x} = 5 \textbf{ units/sec. Ans.}$$

Resultant velocity $$= \sqrt{u^2 + v^2} = \sqrt{6^2 + 5^2} = 36 + 25 = \sqrt{61} = 7.81 \textbf{ unit/sec}$$

Direction is given by, $$\tan \theta = \frac{v}{u} = \frac{5}{6} = 0.833$$

$$\theta = \tan^{-1} .833 = \mathbf{39° \; 48'}. \textbf{ Ans.}$$

Problem 5.13 *If for a two-dimensional potential flow, the velocity potential is given by*

$$\phi = x(2y - 1)$$

determine the velocity at the point P (4, 5). Determine also the value of stream function ψ at the point P.

(A.M.I.E., Winter, 1977)

Solution. Given : $$\phi = x(2y - 1)$$

The velocity components in the direction of x and y are

$$u = -\frac{\partial \phi}{\partial x} = -\frac{\partial}{\partial x}[x(2y - 1)] = -[2y - 1] = 1 - 2y$$

$$v = -\frac{\partial \phi}{\partial y} = -\frac{\partial}{\partial y}[x(2y - 1)] = -[2x] = -2x$$

At the point P (4, 5), *i.e.*, at $x = 4$, $y = 5$

$u = 1 - 2 \times 5 = -9$ units/sec

$v = -2 \times 4 = -8$ units/sec

$\therefore$ Velocity at $P = -9i - 8j$

Resultant velocity at $P = \sqrt{9^2 + 8^2} = \sqrt{81 + 64} = 12.04$ units/sec $=$ **12.04 units/sec. Ans.**

Value of Stream Function at P

We know that $$\frac{\partial \psi}{\partial y} = -u = -(1 - 2y) = 2y - 1 \quad ...(i)$$

$$\frac{\partial \psi}{\partial x} = v = -2x \quad ...(ii)$$

Integrating equation (*i*) w.r.t. 'y', we get

$$\int d\psi = \int (2y - 1)\, dy \text{ or } \psi = \frac{2y^2}{2} - y + \text{Constant of integration.}$$

The constant of integration is not a function of y but it can be a function of x. Let the valu[e] constant of integration is k. Then

$$\psi = y^2 - y + k.$$

Differentiating the above equation w.r.t. 'x', we get

$$\frac{\partial \psi}{\partial x} = \frac{\partial k}{\partial x}.$$

But $\frac{\partial \psi}{\partial x}$ from equation (*ii*) $= -2x$

Equating the value of $\frac{\partial \psi}{\partial x}$, we get $\frac{\partial k}{\partial x} = -2x$.

Integrating this equation, we get $k = \int -2x dx = -\frac{2x^2}{2} = -x^2$.

Substituting this value of k in equation (*iii*), we get $\psi = y^2 - y - x^2$. **Ans.**

$\therefore$ Stream function ψ at P (4, 5) $= 5^2 - 5 - 4^2 = 25 - 5 - 16 = 4$ units. **Ans.**

Problem 5.14 *The stream function for a two-dimensional flow is given by $\psi = 2xy$, calcula[te] velocity at the point P (2, 3). Find the velocity potential function ϕ.*

Solution. Given : $\psi = 2xy$

The velocity components u and v in terms of ψ are

$$u = -\frac{\partial \psi}{\partial y} = -\frac{\partial}{\partial y}(2xy) = -2x$$

$$v = \frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x}(2xy) = 2y.$$

At the point P (2, 3), we get $u = -2 \times 2 = -4$ units/sec

$$v = 2 \times 3 = 6 \text{ units/sec}$$

$\therefore$ Resultant velocity at $P = \sqrt{u^2 + v^2} = \sqrt{4^2 + 6^2} = \sqrt{16 + 36} = \sqrt{52} = 7.21$ units/sec.

Velocity Potential Function ϕ

We know
$$\frac{\partial \phi}{\partial x} = -u = -(-2x) = 2x$$

$$\frac{\partial \phi}{\partial y} = -v = -2y$$

Integrating equation (*i*), we get

$$\int d\theta = \int 2x dx$$

or
$$\phi = \frac{2x^2}{2} + C = x^2 + C$$

where C is a constant which is independent of x but can be a function of y.

Differentiating equation (*iii*) w.r.t. 'y', we get $\frac{\partial \phi}{\partial y} = \frac{\partial C}{\partial y}$

Differentiating equation (*vi*) w.r.t. y, we get $\frac{\partial \psi}{\partial y} = -x - 0 + \frac{\partial k}{\partial y}$

But from equation (*v*), we have $\frac{\partial \psi}{\partial y} = -x + 4y$

Equating the two values of $\frac{\partial \psi}{\partial y}$, we get $-x + \frac{\partial k}{\partial y} = -x + 4y$ or $\frac{\partial k}{\partial y} = 4y$

Integrating the above equation, we get $k = \frac{4y^2}{2} = 2y^2$

Substituting the value of k in equation (*vi*), we get

$$\psi = -yx - 2x^2 + 2y^2. \textbf{ Ans.}$$

Solution. Given : $u = y^3/3 + 2x - x^2y$

$v = xy^2 - 2y - x^3/3.$

The velocity components in terms of stream function are

$$\frac{\partial \psi}{\partial x} = v = xy^2 - 2y - x^3/3$$

$$\frac{\partial \psi}{\partial y} = -u = -y^3/3 - 2x + x^2y$$

Integrating (*i*) w.r.t. x, we get $\psi = \int (xy^2 - 2y - x^3/3)\,dx$

or
$$\psi = \frac{x^2y^2}{2} - 2xy - \frac{x^4}{4 \times 3} + k,$$

where k is a constant of integration which is independent of x but can be a function of y.

Differentiating equation (*iii*) w.r.t. to y, we get

$$\frac{\partial \psi}{\partial y} = \frac{2x^2y}{2} - 2x + \frac{\partial k}{\partial y} = x^2y - 2x + \frac{\partial k}{\partial y}$$

But from (*ii*),
$$\frac{\partial \psi}{\partial y} = -y^3/3 - 2x + x^2y$$

Comparing the value of $\frac{\partial \psi}{\partial y}$, we get $x^2y - 2x + \frac{\partial k}{\partial y} = -y^3/3 - 2x + x^2y$

$\therefore$
$$\frac{\partial k}{\partial y} = -y^3/3$$

Integrating, we get
$$k = \int (-y^3/3)\,dy = \frac{-y^4}{4 \times 3} = \frac{-y^4}{12}$$

Substituting this value in (*iii*), we get

$$\psi = \frac{x^2y^2}{2} - 2xy - \frac{x^4}{12} - \frac{y^4}{12}.\ \textbf{Ans.}$$

Problem 5.17 *In a two-dimensional incompressible flow, the fluid velocity components are*
$u = x - 4y$ and $v = -y - 4x$.
Show that velocity potential exists and determine its form. Find also the stream function.

(Delhi University, 1996 ; A.M.I.E., Winter

Solution. Given : $u = x - 4y$ and $v = -y - 4x$

$\therefore$ $\frac{\partial u}{\partial x} = 1$ and $\frac{\partial v}{\partial y} = -1$

$\therefore$
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 1 - 1 = 0$$

Hence flow is continuous and velocity potential exists.

Let ϕ = Velocity potential.

Let velocity components in terms of velocity potential is given by

$$\frac{\partial \phi}{\partial x} = -u = -(x - 4y) = -x + 4y \qquad ...(i)$$

and
$$\frac{\partial \phi}{\partial y} = -v = -(-y - 4x) = y + 4x \qquad ...(ii)$$

Integrating equation (*i*), we get $\phi = -\frac{x^2}{2} + 4xy + C$ ...(*iii*)

where C is a constant of integration, which is independent of x.

This constant can be a function of y.

Differentiating the above equation, *i.e.*, equation (*iii*) with respect to 'y', we get

$$\frac{\partial \phi}{\partial y} = 0 + 4x + \frac{\partial C}{\partial y}$$

But from equation (*iii*), we have $\frac{\partial \phi}{\partial y} = y + 4x$

Equating the two values of $\frac{\partial \phi}{\partial y}$, we get

$$4x + \frac{\partial C}{\partial y} = y + 4x \quad \text{or} \quad \frac{\partial C}{\partial y} = y$$

Integrating the above equation, we get

$$C = \frac{y^2}{2} + C_1$$

where C_1 is a constant of integration, which is independent of x and y.

Taking it equal to zero, we get $C = \frac{y^2}{2}$.

Substituting the value of C in equation (*iii*), we get

$$\phi = -\frac{x^2}{2} + 4xy + \frac{y^2}{2}.\ \textbf{Ans.}$$

Value of Stream functions

Let ψ = Stream function

The velocity components in terms of stream function are

$$\frac{\partial \psi}{\partial x} = v = -y - 4x \qquad ...(iv)$$

$$\frac{\partial \psi}{\partial y} = -u = -(x - 4y) = -x + 4y \qquad ...(v)$$

Integrating equation (*iv*) w.r.t. x, we get

$$\psi = -yx - \frac{4x^2}{2} + k \qquad ...(vi)$$

where k is a constant of integration which is independent of x but can be a function of y.