⇒ Linear diff. eqn :-

* A diff. eqn is said to be a linear diff. eqn if the dependent variable & all its derivatives should be of 1st degree only and they should not occur as product.

i) $y, \left(\frac{dy}{dx}\right)^1, \left(\frac{d^2y}{dx^2}\right)^1, \left(\frac{d^3y}{dx^3}\right)^1, \ldots\ldots$

ii) $\left(y \cdot \frac{dy}{dx}\right), \; y \cdot \frac{d^2y}{dx^2}, \; \frac{dy}{dx} \cdot \frac{d^2y}{dx^2}, \; \frac{d^2y}{dx^2} \cdot \frac{d^3y}{dx^3} \ldots$ ✗

⇒ $D = \frac{d}{dx}$ $\qquad \frac{1}{D} = \int$

$D^2 = \frac{d^2}{dx^2}$ $\qquad \frac{1}{D^2} = \int\int$

$D^3 = \frac{d^3}{dx^3}$ $\qquad \frac{1}{D^3} = \int\int\int$

Higher order linear D.E's with constant co-efficients:-

⇒ Standard form of nth order linear equation

$$\frac{d^ny}{dx^n} + k_1 \frac{d^{n-1}y}{dx^{n-1}} + \ldots + k_{n-2}\frac{d^2y}{dx^2} + k_{n-1}\frac{dy}{dx} + k_n y = X$$

$$(D^n + k_1 D^{n-1} + \ldots + k_{n-2}D^2 + k_{n-1}D + k_n)\, y = X$$

$$\boxed{F(D)y = X}$$

$k_1, k_2, k_3, \ldots\ldots k_n$ are constants and X is a function of 'x' (or) constant.

⇒ Let the given diff. eqn be $F(D)y = X$

1) The complete sol^n of the given eqn is

$y = C.F + P.I$

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2) If $x=0$ then $f(D)y=0$ is called homogenous linear D.E

3) If $x \neq 0$ then $f(D)y=x$ is called non-homogenous linear D.E

4) The sol^n of homogenous linear D.E $f(D)y$ is called complimentary function (CF)

→ The no. of arbitrary constants in CF should be equal to the order of the given D.E

5) Particular integral (PI) of given eqn is

$$PI = \left[\frac{1}{f(D)}\right] x$$

6) If $x=0$ then the complete sol^n is only complimentary function.

7) By assuming T as an algebraic quantity or replacing D by m we get $f(D)=0$ or $f(m)=0$ will become an algebraic eqn and is called auxillary eqn.

⇒ Procedure to find C.F :-

→ By solving the auxillary eqn we get the roots. Based on nature of these roots we write c.f as follows :-

Roots	C.F
1) Real & distinct $D = m_1, m_2, m_3$	$CF = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}$
2) Real & repeated $D = m_1, m_1$	$CF = (c_1 + c_2 x) e^{m_1 x}$
3) Complex & distinct $D = a \pm ib$	$CF = e^{ax}[c_1 \cos bx + c_2 \sin bx]$
4) Complex & repeated $D = a \pm ib$ & $a \pm ib$	$CF = e^{ax}[(c_1 + c_2 x)\cos bx + (c_3 + c_4 x)\sin bx]$

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homogenous

non-homogenous

f(D)y is called

CF should be

.E

is

only

antity or

or f(m) = 0 will

called auxillary

get the

roots we

	$c_3 e^{m_3 x}$
	$+ c_2 \sin bx]$
	$) \cos bx + (c_3 + c_4 x) \sin bx]$

Roots	C.F
$D = 1, -\frac{1}{2}, -\frac{1}{2}$	$CF = c_1 e^{x} + (c_2 + c_3 x) e^{-x/2}$
$D = 2 \pm 3i, -1$	$C.F = e^{2x}[c_1 \cos 3x + c_2 \sin 3x] + c_3 e^{-x}$
$D = \pm 2i, \pm 4$	$C.F = c_1 \cos 2x + c_2 \sin 2x + Ae^{4x} + Be^{-4x}$

Q) Find the solution of $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$

$$(D^2 + 5D + 6)y = 0$$

Auxillary eqn is $M^2 + 5M + 6 = 0$

$$M^2 + 3M + 2M + 6 = 0$$

$$(M+3)(M+2) = 0$$

$$m = -2, -3$$

$$C.F = c_1 e^{-2x} + c_2 e^{-3x}$$

$$y = CF + P$$

Q) $\frac{d^3x}{dt^3} + \frac{d^2x}{dt^2} + \frac{dx}{dt} + x = 0$

Auxillary eqn is

$$m^3 + m^2 + m + 1 = 0$$

$$m = -1$$

	1	1	1	1
-1	0	-1	0	-1
	1	0	1	$\boxed{0}$

$$m^2 + 1 = 0$$

$$m^2 = -1$$

$$m = \pm i$$

$\therefore m = -1, \pm i$

$$x = c_1 e^{-t} + c_2 \cos t + c_3 \sin t$$

Q) $\frac{d^4y}{dx^4} + 81y = 0$

$$m^4 + 81 = 0$$

$$m^4 = -81$$

$$(m^2 + 9)^2 - 18m^2 = 0$$

$$(m^2 + 9)^2 - (\sqrt{18}\, m)^2 = 0$$

$$[(m^2 + 9) + (\sqrt{18}\, m)][(m^2 + 9) - (\sqrt{18}\, m)] = 0$$

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$m^2+9+\sqrt{18}\,m=0$ $\qquad$ $m^2+9-\sqrt{18}\,m=0$

$m=\dfrac{-\sqrt{18}\pm\sqrt{18-36}}{2}$, $\qquad$ $m=\dfrac{\sqrt{18}\pm\sqrt{18-36}}{2}$

$=\dfrac{-\sqrt{18}\pm\sqrt{18}\,i}{2}$ $\qquad$ $=\dfrac{\sqrt{18}\pm\sqrt{18}\,i}{2}$

$=\dfrac{-3\sqrt{2}\pm3\sqrt{2}\,i}{2}$ $\qquad$ $=\dfrac{3\sqrt{2}\pm3\sqrt{2}\,i}{2}$

$$y=e^{\frac{-3\sqrt{2}}{2}x}\left[c_1\cos\frac{3\sqrt{2}}{2}x+c_2\sin\frac{3\sqrt{2}}{2}x\right]+e^{\frac{3\sqrt{2}x}{2}}\left[c_3\cos\frac{3\sqrt{2}}{2}x+c_4\sin\frac{3\sqrt{2}x}{2}\right]$$

Q) $\dfrac{d^4y}{dx^4}-a^4y=0$

$m^4-a^4=0$

$m^4=a^4$

$(m^2)^2-(a^2)^2=0$

$(m^2+a^2)(m^2-a^2)=0$

$m^2=-a^2$ $\qquad$ $m^2=a^2$

$\Rightarrow m=\pm ia$ $\qquad$ $\Rightarrow m=\pm a$

$y=[c_1\cos ax+c_2\sin ax]+c_3e^{ax}$

<u>Procedure to find P.I</u>

$$P.I=\frac{1}{F(D)}X$$

Let the given eqn be $f(D)y=X$

$$\frac{1}{(D-\alpha)}X=e^{\alpha x}\int Xe^{-\alpha x}\,dx$$

Q) $\dfrac{dy}{dx}-3y=x^2$

$c.f=c_1e^{3x}$

<u>for P.I</u> $\qquad$ $F(D)=D-3$

$$\left(\frac{1}{D-3}\right)x^2=e^{3x}\int x^2e^{-3x}\,dx$$

$\downarrow$

by parts

$y=c.f+P.I$

for higher orders we use following cases :-

Case ① :- If $X = e^{ax}$ (or) constant

$$P.F = \left[\frac{1}{f(D)}\right] e^{ax} = \frac{1}{f(a)} e^{ax} \qquad [\because f(a) \neq 0]$$

Replace D by a in function f(D)

If $f(a) = 0$

Multiply the numerator with x

$$\frac{x}{f(D)} e^{ax} = \frac{x}{f'(a)} e^{ax} \qquad (\because f'(a) \neq 0]$$

Replace D by 'a' in $f'(D)$

$\sin \frac{3\sqrt{2}x}{2}]$

* P.I of $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} - 3y = e^{2x} + 7$

$$(D^2 + 6D - 3)y = e^{2x} + 7$$

$$P.I = \left[\frac{1}{(D^2 + 6D - 3)}\right] (e^{2x} + 7e^{0x})$$

$$= \frac{1}{[(2)^2 + 6(2) - 3]} [e^{2x}] + \frac{1}{[(0)^2 + 6(0) - 3]} [7e^{0x}]$$

$$= \frac{e^{2x}}{13} + \frac{7}{-3}$$

$$\boxed{P.I = \frac{e^{2x}}{13} - \frac{7}{3}}$$

* P.I Of $\frac{d^3y}{dx^3} - 9\frac{d^2y}{dx^2} + 27\frac{dy}{dx} - 27y = e^{3x}$

$$(D^3 - 9D^2 + 27D - 27)y = e^{3x}$$

$$P.I = \left[\frac{1}{D^3 - 9D^2 + 27D - 27}\right] e^{3x}$$

$$= \frac{1}{27 - 81 + 81 - 27} e^{3x} \qquad f(a) = 0$$

$$= \frac{x}{3D^2 - 18D + 27} e^{3x}$$

$$= \frac{x}{27 - 54 + 27} e^{3x} = \frac{(e^{3x})x^2}{6D - 18} = \frac{(e^{3x})x^3}{6}$$

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Case ② $x = \sin ax$ (or) $\cos bx$ (or) $\sin(ax+b)$ (or) $\cos(cx+d)$

$$P.I = \left[\frac{1}{F(D^2)}\right] \sin ax$$

$$= \left[\frac{1}{F(-a^2)}\right] \sin ax$$

Replace D^2 by $-a^2$ in function

If $F(-a^2) = 0$ then

$$P.I = \frac{x}{F'(D)} \sin ax = \frac{x \sin ax}{F'(-a^2)}$$

* $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = \cos 4x$

$$(D^2 + 2D + 3)y = \cos 4x$$

$$P.I = \left[\frac{1}{D^2+2D+3}\right] \cos 4x$$

$$\cancel{\frac{1}{(-4)^2+1}}$$

$$\cancel{P.I = \frac{1}{(-4)^2+2(}}$$

$$2D = \sqrt{4D^2}$$

$$= \sqrt{4(-a^2)}$$

$$= \frac{1}{(-4^2) + \sqrt{4(-16)} + 3} \cos 4x$$

$$= \frac{\cos 4x}{16 + \sqrt{64} + 3}$$

$$= \frac{\cos 4x}{16+8+3}$$

* P.I of $\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2} + 3y = \cos(2x+5)$

$$P.I = \left[\frac{1}{D^4+D^2+3}\right] \cos(2x+5)$$

Here $a = 2$

Replace D^2 by $-a^2$

$$= \left[\frac{1}{(-4)^2 + (-4) + 3}\right] \cos(2x+5)$$

$$= \frac{1}{15} \cos(2x+5)$$

$$P.I = \frac{1}{15} \cos(2x+5) \quad //$$

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* P.I of $\frac{d^2y}{dx^2} + 9y = e^{-3x} + \cos 3x$

$$P.I = \left[\frac{1}{D^2+9}\right]\left[e^{-3x} + \cos 3x\right]$$

$$= \left[\frac{1}{(-3^2)+9}\right]\left[e^{-3x} + \cos 3x\right]$$

$$F(-a^2) = 0$$

∴ Multiply numerator with x

& Diff $F(D)$

$$= \left[\frac{1}{(-3)^2+9}\right] e^{-3x} + x\left[\frac{1}{2D}\right]\cos 3x$$

$$P.I = \frac{e^{-3x}}{18} + \frac{x \sin 3x}{6}$$

* P.I of $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = \sin x$

$$P.I = \left[\frac{1}{D^3+D^2+2D+3}\right]\sin x \qquad a=1$$

$$= \left[\frac{1}{(-1)D+(-1)+2D+3}\right]\sin x \quad [\text{Replace } D^2 \text{ by } -a^2]$$

$$= \left[\frac{1}{D+2}\right]\sin x$$

$$= \left[\frac{1}{D+2}\cdot\frac{D-2}{D-2}\right]\sin x$$

$$= \left(\frac{D-2}{D^2-4}\right)\sin x$$

$$= \frac{D\sin x - 2\sin x}{-1-4} = \frac{\cos x - 2\sin x}{-5}$$

$$\boxed{P.I = \frac{2\sin x - \cos x}{5}}$$

Case (iii) : $X = x^m$ (or) poly in x (m is +ve integer)

$$P.I = \left[\frac{1}{f(D)}\right]x^m = \frac{1}{*[1+\phi(D)]}x^m = \frac{1}{*}[1+\phi(D)]^{-1}x^m$$

$$[1+\phi(D)]^{-1} = 1-\phi(D) + [\phi(D)]^2 - [\phi(D)]^3 + \dots$$

$$[1-\phi(D)]^{-1} = 1+\phi(D) + [\phi(D)]^2 + [\phi(D)]^3 + \dots$$

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* P.I of $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = x^2+2$

$$P.I = \left[\frac{1}{D^2+2D+2}\right](x^2+2)$$

$$= \frac{1}{2\left[1+\frac{D^2+2D}{2}\right]}(x^2+2)$$

$$= \frac{1}{2}\left[1+\left(\frac{D^2+2D}{2}\right)\right]^{-1}(x^2+2)$$

$$= \frac{1}{2}\left[1-\left(\frac{D^2+2D}{2}\right)+\left(\frac{D^2+2D}{2}\right)^2-\left(\frac{D^2+2D}{2}\right)^3+\cdots\right](x^2+2)$$

$$= \frac{1}{2}\left[(x^2+2)-(1+2x)+2\right]$$

$$= \frac{1}{2}(x^2+2-1-2x+2)$$

$$= \frac{1}{2}(x^2-2x+3)$$

* P.I of $\frac{d^3y}{dx^3} + 3\frac{dy}{dx} = x^3+x$

$$P.I = \left[\frac{1}{D^3+3D}\right](x^3+x)$$

$$= \frac{1}{3D\left[1+\frac{D^3}{3D}\right]}(x^3+x)$$

$$= \frac{1}{3D\left[1+\frac{D^2}{3}\right]}(x^3+x)$$

$$= \frac{1}{3D}\left[1+\frac{D^2}{3}\right]^{-1}(x^3+x)$$

$$= \frac{1}{3D}\left[1-\frac{D^2}{3}+\left(\frac{D^2}{3}\right)^2-\cdots\right](x^3+x)$$

$$= \frac{1}{3D}\left[x^3+x-\frac{1}{3}(6x+0)\right]$$

$$= \frac{1}{3D}\left[x^3+x-2x\right]$$

$$= \frac{1}{3D}(x^3-x)$$

$$P.I = \frac{1}{3}\left[\frac{x^4}{4}-\frac{x^2}{2}\right] \quad //$$

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Case (iv) :- $x = e^{ax} \cdot V$ $\quad \left[V = x^m \text{ (or) poly. in } x \right]$

$$P.I = \left[\frac{1}{F(D)}\right] e^{ax} \cdot V$$

$$= e^{ax} \left[\frac{1}{F(D+a)}\right] V$$

Replace ~~by~~ D by D+a in F(D)

$$\left[\frac{1}{F(D+a)}\right] V \begin{cases} \rightarrow \text{P.I of } V = \sin ax \text{ or } \cos bx \\ \rightarrow \text{P.I of } V = x^m \text{ or poly. in } x \end{cases}$$

* P.I of $\frac{d^2y}{dx^2} + y = e^x \cos x$.

$$P.I = \left[\frac{1}{D^2+1}\right] e^x \cos x$$

$$= e^x \left[\frac{1}{(D+1)^2+1}\right] \cos x$$

$$= e^x \left[\frac{1}{D^2+2D+2}\right] \cos x$$

$$= e^x \left[\frac{1}{(-1)+2D+2}\right] \cos x$$

$$= e^x \left[\frac{1}{2D+1} \cdot \frac{2D-1}{2D-1}\right] \cos x$$

$$= e^x \frac{(2D-1)}{4D^2-1} \cos x$$

$$= e^x \frac{2(-\sin x) - \cos x}{4(-1)-1}$$

$$= e^x \left[\frac{2\sin x + \cos x}{5}\right]$$

* P.I of $\frac{d^2y}{dx^2} - y = e^{2x} \cdot x^2$

$$P.I = \left[\frac{1}{D^2-1}\right] e^{2x} \cdot x^2$$

$$= e^{2x} \left[\frac{1}{D^2-1}\right] x^2$$

$$= \frac{e^{2x}}{-1} \left[1-D^2\right]^{-1} x^2 = -e^{2x} \left[1 + D^2 + D^4 + D^6 + \dots\right] x^2$$

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$$= e^{2x}\left[\frac{1}{(D+2)^2-1}\right]x^2$$

$$= e^{2x}\left[\frac{1}{D^2+4D+3}\right]x^2$$

$$= \frac{e^{2x}}{3}\left[\frac{1}{1+\frac{D^2+4D}{3}}\right]x^2$$

$$= \frac{e^{2x}}{3}\left[1+\frac{D^2+4D}{3}\right]^{-1}x^2$$

$$= \frac{e^{2x}}{3}\left[1-\left(\frac{D^2+4D}{3}\right)+\left(\frac{D^2+4D}{3}\right)^2-\cdots\right]x^2$$

$$= \frac{e^{2x}}{3}\left[x^2-\underbrace{\frac{4}{3}(2x)}_{-2/3}+\frac{16}{9}(2)\right]$$

$$= \frac{e^{2x}}{3}\left[x^2-\frac{2}{3}-\frac{8x}{3}+\frac{32}{9}\right]$$

<u>Case(v)</u> :- <u>$X = x^m \cdot V$</u> where $V = \sin ax$ (or) $\cos bx$

(or)

<u>$X = e^{ax} \cdot x^m \cdot V$</u>

$$P.I = \left[\frac{1}{F(D)}\right]x^m(\sin ax)$$

$$= \left[\frac{1}{F(D)}\right]\left[x^m\,(\text{I.P of } e^{iax})\right]$$

$$= \text{I.P of } \left\{\left[\frac{1}{F(D)}\right](x^m e^{iax})\right\}$$

using case (iv) & simplifying we get the answer

* $\sin ax$ = Imaginary part of e^{iax}
* $\cos bx$ = Real part of e^{ibx}

1) Find the solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$

Sol: $\cancel{P.I = \frac{1}{D^2-2D+1}}$

Auxillary eqn is $m^2-2m+1=0$

$\Rightarrow (m-1)^2=0$

$\Rightarrow m=1,1$

$C.F = (c_1+c_2x)e^x$

$$P.I = \left(\frac{1}{D^2-2D+1}\right)[xe^x \sin x]$$

$$\cancel{= e^x\left\{\frac{1}{D^2-2D+}\right.} = e^x\left[\frac{1}{(D+1)^2-2(D+1)+1}\right](x\sin x)$$

$$= e^x\left[\frac{1}{D^2+2D+1-2D-2+1}\right](x\sin x)$$

$$= e^x\left[\frac{1}{D^2}\right](x\sin x)$$

$$= e^x\left(\frac{1}{D^2}\right)[x \cdot \text{IP of } e^{ix}]$$

$$= \text{I.P of } \left\{e^x\left(\frac{1}{D^2}\right)(x \cdot e^{ix})\right\}$$

$$= \text{I.P of } \left\{e^x \cdot e^{ix}\left[\frac{1}{(D+i)^2}\right]x\right\}$$

$$= \text{I.P of } \left\{e^x \cdot e^{ix}\left(\frac{1}{D^2-1+2Di}\right)x\right\}$$

$$= \text{I.P of } \left\{-e^x e^{ix}\left(\frac{1}{-D^2-2Di+1}\right)x\right\}$$

$$= \text{I.P of } \left\{-e^x e^{ix}\left[\frac{1}{1-(D^2+2Di)}\right]x\right\}$$

$$= \text{I.P of } \left\{-e^{x+ix}\left[1-(D^2+2Di)\right]^{-1}x\right\}$$

$$= \text{I.P of } \left\{-e^x \cdot e^{ix}\left[1+(D^2+2Di)+(D^2+2Di)^2+\right]x\right\}$$

$$= \text{I.P of } \left\{-e^x e^{ix}\left[x+2i\right]\right\}$$

$$\cancel{= -2e^x e^{ix}} \qquad = \text{I.P of } \left\{-e^x(\cos x + i\sin x)(x+2i)\right\}$$

$$\cancel{= -2e^{x+ix}} \qquad = -xe^x\sin x - 2e^x\cos x$$

$$\cancel{= \text{I.P}}$$

2) $(D^4+2D^2+1)y = x^2\cos x$ find the soln

A.E is $m^4+2m^2+1=0$

$(m^2+1)^2=0$

$m^2 = -1$

$m = \pm i, \pm i$

$C.F = (c_1+c_2x)\cos x + (c_3+c_4x)\sin x$

$$P.I = \left[\frac{1}{D^4+2D^2+1}\right] x^2\cos x$$

$$= \left[\frac{1}{D^4+2D^2+1}\right] \left[x^2 (\text{R.P of } e^{ix})\right]$$

$$= \text{R.P of } \left[\frac{1}{D^4+2D^2+1}\right] (x^2 \cdot e^{ix})$$

$$= \text{R.P of } \left[e^{ix} \cdot \frac{1}{(D+i)^4+2(D+i)^2+1} \cdot x^2\right]$$

$$= \text{R.P. of } \left[e^{ix} \frac{1}{(D^2+2Di-1)^2+2(D^2+2Di-1)+1} \cdot x^2\right]$$

$$= \text{R.P of } \left\{e^{ix} \frac{1}{D^4 - 4D^2 + 1 + 4D^3 i - \cancel{4Di} + \cancel{2D^2} + \cancel{4Di} - \cancel{2D^2} + 1} \cdot x^2\right]$$

$$= \text{R.P of } \left[e^{ix} \frac{1}{D^4-4D^2+4D^3 i} \cdot x^2\right]$$

$$= \text{R.P of } \left\{\frac{-e^{ix}}{4D^2} \frac{1}{\left[1-\left(\frac{D^4+4D^3 i}{4D^2}\right)\right]} x^2\right]$$

$$= \text{R.P of } \left[\frac{-e^{ix}}{4D^2} \frac{1}{\left[1-\left(\frac{D^2}{4}+Di\right)\right]} x^2\right]$$

$$= \text{R.P of } \left\{\frac{-e^{ix}}{4D^2} \left[1-\left(\frac{D^2}{4}+Di\right)\right]^{-1} x^2\right\}$$

$$= \text{R.P of } \left\{\frac{-e^{ix}}{4D^2} \left[1+\left(\frac{D^2}{4}+Di\right)+\left(\frac{D^2}{4}+Di\right)^2+\dots\right] x^2\right\}$$

$$= \text{R.P of } \left\{\frac{-e^{ix}}{4D^2} \left[x^2+\frac{1}{2}+2ix-2\right]\right\}$$

$$= \text{R.P of } \left\{\frac{-e^{ix}}{4D^2} \left[x^2+2ix-\frac{3}{2}\right]\right\}$$

$$= \text{R.P of } \left\{-\frac{e^{ix}}{4} \left(\frac{x^4}{12}+\frac{ix^3}{3}-\frac{3x^2}{4}\right)\right\}$$

$$= \text{R.P of } \left\{-\frac{1}{4}(\cos x+i\sin x)\left(\frac{x^4}{12}+\frac{ix^3}{3}-\frac{3x^2}{4}\right)\right\}$$

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(08)

$$P.I = \left[\frac{1}{D^4+2D^2+1}\right] x^2 \cos x$$

$$= \text{R.P of } \left\{ \frac{1}{(D^2+1)^2} x^2 e^{ix} \right\}$$

$$= \text{R.P of } \left\{ e^{ix} \left(\left[\frac{1}{(D+i)^2+1}\right]^2 \right) x^2 \right\}$$

$$= \text{R.P of } \left\{ e^{ix} \left[\frac{1}{(D^2 \cancel{-1} + 2Di \cancel{+1})^2} \right] x^2 \right\}$$

$$= \text{R.P of } \left\{ e^{ix} \frac{1}{(2Di)^2 \left[1+\frac{D^2}{2Di}\right]^2} x^2 \right\}$$

$$= \text{R.P of } \left\{ e^{ix} \left(\frac{-1}{4D^2}\right) \left[1+\frac{D^2}{2Di}\right]^{-2} x^2 \right\}$$

Ans:- $\frac{-1}{48}\left[(x^4-9x^2)\cos x - (4x^3 \sin x) \right]$

$$= -\frac{x^4}{48}\cos x + \frac{3}{16}x^2 \cos x + \frac{x^3}{12}\sin x$$

$$= \frac{-1}{48}\left[(x^4-9x^2)\cos x - (4x^3 \sin x) \right]$$

Case (I)

$$X = e^{ax}$$

1) $(D^2+5D+6)y = e^x$

$$P.I = \frac{1}{F(D)} e^x = \frac{1}{D^2+5D+6} e^x$$

$$= \frac{1}{1+5+6} e^x = \frac{e^x}{12}$$

2) $(D+2)(D-1)^2 y = e^{-2x} + 2\sinh x$

$$P.I = \frac{1}{(D+2)(D-1)^2}\left[e^{-2x} + 2\sinh x\right]$$

$$= \frac{1}{(D+2)(D-1)^2}\left[e^{-2x} + e^x - e^{-x}\right]$$

$$\frac{1}{(D+2)(D-1)^2} e^{-2x} = \frac{1}{(D+2)(-2-1)^2} e^{-2x} \qquad [\because a = -2]$$

$F(D) = 0$

$$= \frac{1}{9}\left[\frac{1}{(D+2)} e^{-2x}\right]$$

$$= \frac{1}{9}\left[\frac{x}{1} e^{-2x}\right] = \frac{x}{9} e^{-2x}$$

$$\frac{1}{(D+2)(D-1)^2} e^x = \frac{1}{3} \cdot \frac{1}{(D-1)^2} e^x$$

$$= \frac{1}{3}\left[\frac{x}{2(D-1)} e^x\right]$$

$$= \frac{1}{3}\left[\frac{x^2}{2} e^x\right] = \frac{x^2}{6} e^x$$

$$\frac{1}{(D+2)(D-1)^2} e^{-x} = \frac{1}{(-1+2)(-1-1)^2} e^{-x} \qquad [\because a = -1]$$

$$= \frac{1}{4} e^{-x}$$

Hence $P.I = \frac{x}{9} e^{-2x} + \frac{x^2}{6} e^x + \frac{1}{4} e^{-x}$

Case-(II)

$$X = \sin(ax+b) \text{ or } \cos(ax+b)$$

1) $(D^3+1)y = \cos(2x-1)$

$$P.I = \frac{1}{D^3+1} \cos(2x-1) \qquad a = 2$$

Replace D^2 by $-a^2$

$$= \frac{1}{D(-4)+1} \cos(2x-1)$$

$$= \frac{1}{1-4D} \cos(2x-1) = \frac{1}{(1-4D)} \frac{(1+4D)}{(1+4D)} \cos(2x-1)$$

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$$= \frac{1+4D}{1-16D^2}\cos(2x-1)$$

$$= \frac{1+4D}{1-16(-4)}\cos(2x-1) = \frac{1+4D}{65}\cos(2x-1)$$

$$= \frac{1}{65}\left[\cos(2x-1) - 8\sin(2x-1)\right] //$$

2) $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = \sin 2x$

$$P.I = \frac{1}{D^3+4D}\sin 2x$$

$a = 2$

$$= \frac{1}{D(-4)+4D}\sin 2x$$

$$= \frac{1}{0}\sin 2x$$

$$\therefore P.I = \frac{x}{3D^2+4}\sin 2x$$

$$= \frac{x}{3(-4)+4}\sin 2x = \frac{x}{-12+4}\sin 2x = \frac{-x}{8}\sin 2x //$$

Case (III) $X = x^m$

1) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2+2x+4$

$$P.I = \frac{1}{D^2+D}(x^2+2x+4)$$

$$= \frac{1}{D(1+D)}(x^2+2x+4) = \frac{1}{D}(1+D)^{-1}(x^2+2x+4)$$

$$= \frac{1}{D}\left[1-D+D^2-D^3+\dots\right](x^2+2x+4)$$

$$= \frac{1}{D}\left(x^2+\cancel{2x}+4-\cancel{2x}-\cancel{2}+\cancel{2}\right)$$

$$= \frac{1}{D}(x^2+4) = \frac{x^3}{3}+4x //$$

Case (IV) $X = e^{ax}V$, V is a function of x

1) $(D^2-2D+4)y = e^x\cos x$

$$P.I = \frac{1}{D^2-2D+4}(e^x\cos x)$$

$$= e^x\left[\frac{1}{(D+1)^2-2(D+1)+4}\right](\cos x)$$

$$= e^x\left[\frac{1}{D^2+1+\cancel{2D}-\cancel{2D}-2+4}\right](\cos x)$$

$$= e^x\left[\frac{1}{D^2+3}\right](\cos x)$$

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$$= \frac{e^x}{-1+3}\cos x = \frac{e^x}{2}\cos x$$

~~Cos~~

EXAMPLES

1) Solve $y''+4y'+4y = 3\sin x+4\cos x$.

$$D^2+4D+4=0$$
$$(D+2)^2=0$$
$$D=-2,-2$$
$$C.F = (c_1+c_2x)e^{-2x}$$

$$P.I = \frac{1}{D^2+4D+4}(3\sin x+4\cos x) \qquad (a=1)$$

$$= \frac{1}{-1+4D+4}\cdot(3\sin x+4\cos x)$$

$$= \frac{1}{4D+3}(3\sin x+4\cos x)$$

$$= \frac{4D-3}{16D^2-9}(3\sin x+4\cos x)$$

$$= \frac{4D-3}{-25}(3\sin x+4\cos x)$$

$$= \frac{-1}{25}(\cancel{12\cos x}-16\sin x-9\sin x-\cancel{12\cos x})$$

$$= \frac{-1}{\cancel{25}}(-\cancel{25}\sin x) = \sin x$$

Complete solution is

$$y = (c_1+c_2x)e^{-2x}+\sin x$$

2) Solve $(D-2)^2 = 8(e^{2x}+\sin 2x+x^2)$

$$(D-2)^2=0$$
$$D=2,2$$
$$y = (c_1+c_2x)e^{2x}$$

$$P.I = \frac{1}{(D-2)^2}\left[8(e^{2x}+\sin 2x+x^2)\right]$$

$$\frac{1}{(D-2)^2}e^{2x} = \frac{x}{2(D-2)}e^{2x}$$

$$= \frac{x^2}{2}e^{2x}$$

$$\frac{1}{(D-2)^2}\sin 2x = \cancel{\frac{x}{2(D-2)}\sin 2x} = \frac{1}{D^2-4D+4}\sin 2x$$

$$= \frac{1}{-4-4D+4}\sin 2x$$

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$$= \frac{-1}{4D}\sin 2x$$

$$= -\frac{1}{4}\left(\frac{-\cos 2x}{2}\right) = \frac{\cos 2x}{8}$$

$$\frac{1}{(D-2)^2}x^2 = \frac{1}{(-2)^2\left[1-\frac{D}{2}\right]^2}x^2$$

$$= \frac{1}{4}\left(1-\frac{D}{2}\right)^{-2}x^2$$

$$= \frac{1}{4}\left[1+2\left(\frac{D}{2}\right)+3\left(\frac{D}{2}\right)^2+\cdots\right]x^2$$

$$= \frac{1}{4}\left[1+D+\frac{3D^2}{4}+\cdots\right]x^2$$

$$= \frac{1}{4}\left[x^2+2x+\frac{3}{2}\right]$$

$$\therefore \text{ P.I} = 8\left[\frac{x^2}{2}e^{2x}+\frac{1}{8}\cos 2x+\frac{1}{4}\left(x^2+2x+\frac{3}{2}\right)\right]$$

$$= 4x^2e^{2x}+\cos 2x+2x^2+4x+3$$

$$\therefore\ y = (c_1+c_2x)e^{2x}+4x^2e^{2x}+\cos 2x+2x^2+4x+3$$

3) $y''-2y'+2y = x+e^x\cos x$

$$D^2-2D+2=0$$

A.E is $m^2-2m+2=0$

$$m = \frac{2\pm\sqrt{4-8}}{2} = \frac{2\pm 2i}{2} = 1\pm i$$

$$\therefore\ \text{C.F} = e^x\left[c_1\cos x+c_2\sin x\right]$$

$$\text{P.I} = \frac{1}{D^2-2D+2}(x+e^x\cos x)$$

$$\frac{1}{D^2-2D+2}x = \frac{1}{2\left[1+\left(\frac{D^2}{2}-D\right)\right]}x$$

$$= \frac{1}{2}\left[1+\left(\frac{D^2}{2}-D\right)\right]^{-1}x$$

$$= \frac{1}{2}\left[1-\left(\frac{D^2}{2}-D\right)+\cdots\right]x$$

$$= \frac{1}{2}\left[x-0+1\right] = \frac{1}{2}(x+1)$$

$$\frac{1}{(D^2-2D+2)}e^x\cos x = e^x\cdot\frac{1}{(D+1)^2-2(D+1)+2}\cos x$$

$$= e^x\cdot\frac{1}{D^2+1+\cancel{2D}-\cancel{2D}-\cancel{2}+\cancel{2}}\cos x = e^x\cdot\frac{1}{D^2+1}\cos x$$

$$= e^x\cdot\frac{x}{2D}\cos x$$

$$= \frac{xe^x}{2}\sin x$$

$$\therefore\ y = e^x(c_1\cos x+c_2\sin x)+\frac{1}{2}(x+1)+\frac{xe^x}{2}\sin x$$

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4) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^{3x} + \sin 2x$

A.E is $m^2 - 3m + 2 = 0$

$m^2 - 2m - m + 2 = 0$

$m(m-2) - 1(m-2) = 0$

$(m-1)(m-2) = 0$

$m = 1, 2$

$C.F = c_1 e^{x} + c_2 e^{2x}$

$P.I = \frac{1}{D^2 - 3D + 2}\left[x e^{3x} + \sin 2x\right]$

$\frac{1}{D^2-3D+2}(x e^{3x}) = e^{3x} \cdot \frac{1}{(D+3)^2 - 3(D+3) + 2} x$

$= e^{3x} \frac{1}{D^2 + \cancel{9} + 6D - 3D - \cancel{9} + 2}(x)$

$= e^{3x} \frac{1}{D^2 + 3D + 2} \cdot x$

$= e^{3x} \frac{1}{\cancel{-1} + 3D + 2} \cancel{x}$ $= e^{3x} \cdot \frac{1}{2\left[1 + \left(\frac{D^2}{2} + \frac{3D}{2}\right)\right]} x$

$= e^{3x} \frac{1}{3D + 1} x$

$= e^{3x} \cdot \frac{3D - 1}{9D^2 - 1} x$ $= \frac{e^{3x}}{2}\left[1 + \left(\frac{D^2}{2} + \frac{3D}{2}\right)\right]^{-1} x$

$= e^{3x} \cdot \frac{3D - 1}{-10} x$ $= \frac{e^{3x}}{2}\left[1 - \left(\frac{D^2}{2} + \frac{3D}{2}\right) + \dots\right] x$

$= -\frac{e^{3x}}{10}[3 - x] =$

$= \frac{e^{3x}}{2}\left[x - 0 - \frac{3}{2}\right] = \frac{e^{3x}}{2}\left(x - \frac{3}{2}\right)$

$\frac{1}{D^2 - 3D + 2} \sin 2x = \frac{1}{-4 - 3D + 2} \sin 2x = \frac{1}{-2 - 3D} \sin 2x$

$= -\frac{1}{(3D + 2)} \sin 2x$

$= -\frac{(3D - 2)}{9D^2 - 4} \sin 2x$

$= -\frac{(3D - 2)}{-40} \sin 2x = \frac{3D - 2}{40} \sin 2x$

$= \frac{1}{40}(6\cos 2x - 2\sin 2x)$

$y = c_1 e^{x} + c_2 e^{2x} + e^{3x}\left(\frac{x}{2} - \frac{3}{4}\right) + \frac{1}{20}(3\cos 2x - \sin 2x)$ //

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5) $\frac{d^2y}{dx^2} - 4y = x \sinh x$

A.E is $m^2 - 4 = 0$

$m^2 = 4$

$m = 2, -2$

$y = c_1 e^{2x} + c_2 e^{-2x}$

$P.I = \frac{1}{D^2-4}(x \sinh x)$

$= \frac{1}{(D^2-4)}\left[x\left(\frac{e^x - e^{-x}}{2}\right)\right]$

$= \frac{1}{2(D^2-4)}\left[xe^x - xe^{-x}\right]$

$\frac{1}{(D^2-4)} x e^x = e^x \cdot \frac{1}{[(D+1)^2-4]} x$

$= e^x \cdot \frac{1}{(D^2+2D-3)} x$

$= \frac{e^x}{-3} \frac{1}{\left[1 - \left(\frac{D^2+2D}{3}\right)\right]} x$

$= \frac{e^x}{-3}\left[1 - \left(\frac{D^2+2D}{3}\right)\right]^{-1} x$

$= \frac{-e^x}{3}\left[1 + \left(\frac{D^2+2D}{3}\right) + \cdots\right] x$

$= -\frac{e^x}{3}\left[x + 0 + \frac{2}{3}\right] = -e^x\left[\frac{x}{3} + \frac{2}{9}\right]$

$\cancel{\frac{1}{(D^2-4)} \sinh x} = \frac{1}{(D^2-4)} x e^{-x} = e^{-x}\left[\frac{1}{(D^2-4)}\right] x$

$= e^{-x}\left[\frac{1}{(D-1)^2-4}\right] x$

$= e^{-x}\left[\frac{1}{D^2-2D-3}\right] x$

$= \frac{e^{-x}}{-3}\left[1 - \left(\frac{D^2-2D}{3}\right)\right]^{-1} x = -e^{-x}\left[\frac{x}{3} - \frac{2}{9}\right]$

$P.I = -\frac{x}{3}(e^x + e^{-x}) - \frac{2}{9}(e^x - e^{-x}) =$

$= -\frac{x}{3}\frac{\sinh x}{2} - \frac{2}{9}\frac{\cosh x}{2}$

$y = c_1 e^{2x} + c_2 e^{-2x} - \frac{x}{3}\sinh x - \frac{2}{9}\cosh x$ //

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6) $(D^2-1)y = x\sin 3x + \cos x$

A.E is $m^2-1=0$

$m^2=1$

$m=1,-1$

$y = c_1e^{x}+c_2e^{-x}$

$$P.I = \frac{1}{D^2-1}\left[x\sin 3x+\cos x\right]$$

$$\Rightarrow \frac{1}{D^2-1}\left[x\,(I.P \text{ of } e^{i3x}) + \cos x\right]$$

$$= \frac{1}{D^2-1}\,x\,(I.P \text{ of } e^{i3x}) + \frac{1}{D^2-1}\cos x$$

$$= I.P \text{ of } \frac{1}{D^2-1}\,e^{i3x}\,x + \frac{1}{D^2-1}\cos x$$

$$= I.P \text{ of } \left[e^{i3x}\,\frac{1}{(D+3i)^2-1}\,x\right] + \frac{1}{-1-1}\cos x$$

$$= I.P \text{ of } \left[e^{i3x}\,\frac{1}{D^2+6Di-10}\,x\right] - \frac{\cos x}{2}$$

$$= I.P \text{ of } \frac{e^{i3x}}{-10}\left[\frac{1}{1-\left(\frac{D^2+6Di}{10}\right)}\,x\right] - \frac{\cos x}{2}$$

$$= I.P \text{ of } \frac{e^{i3x}}{-10}\left[1-\left(\frac{D^2}{10}+\frac{3Di}{5}\right)^{-1}\right]x - \frac{\cos x}{2}$$

$$= I.P \text{ of } \frac{e^{i3x}}{-10}\left[1+\frac{D^2}{10}+\frac{3Di}{5}+\cdots\right]x - \frac{\cos x}{2}$$

$$= I.P \text{ of } \frac{e^{i3x}}{-10}\left[x+\frac{3i}{5}\right] - \frac{\cos x}{2}$$

$$= I.P \text{ of } \left(\frac{\cos 3x + i\sin 3x}{-10}\right)\left(x+\frac{3i}{5}\right) - \frac{\cos x}{2}$$

$$= \frac{-1}{10}\left(\frac{3\cos 3x}{5} + x\sin 3x\right) - \frac{\cos x}{2}$$

$$= \frac{1}{10}$$

$$y = c_1e^{x}+c_2e^{-x} - \frac{1}{50}\left(3\cos 3x + 5x\sin 3x + 25\cos x\right)$$

7) solve $\frac{d^2y}{dx^2}$

A.

C.F

P.I =

=

=

8) Sol

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7) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$

A.E is $m^2 - 2m + 1 = 0$

$(m-1)^2 = 0$

$m = 1, 1$

C.F $= (c_1 + c_2 x)e^x$

$$P.I = \frac{1}{D^2 - 2D + 1} xe^x \sin x$$

$$= e^x \frac{1}{(D+1)^2 - 2(D+1) + 1} x \sin x$$

$$= e^x \frac{1}{D^2 + \cancel{1} + \cancel{2D} - \cancel{2D} - 2 + 1} \cdot x \sin x$$

$$= e^x \cdot \frac{1}{D^2} x \sin x$$

$$= \frac{e^x}{D} \int x \sin x$$

$$= \frac{e^x}{D} \int \left[x(-\cos x) + \sin x \right] dx$$

$$= e^x \left[-x \sin x - \cos x - \cos x \right]$$

$$= -e^x \left[x \sin x + 2 \cos x \right]$$

$$y = (c_1 + c_2 x)e^x - e^x(x \sin x + 2 \cos x)$$ //

8) Solve $(D^4 + 2D^2 + 1)y = x^2 \cos x$

A.E is $m^4 + 2m^2 + 1 = 0$

$m = \pm i, \pm i$

C.F $= (c_1 + c_2 x)\cos x + (c_3 + c_4 x)\sin x$

$$P.I = \frac{1}{D^4 + 2D^2 + 1} x^2 \cos x = \frac{1}{D^4 + 2D^2 + 1} \left[x^2 (\text{R.P of } e^{ix}) \right]$$

Ans is $P.I = \frac{-1}{48} \left[(x^4 - 9x^2)\cos x - (4x^3 \sin x) \right]$

$$y = (c_1 + c_2 x)\cos x + (c_3 + c_4 x)\sin x + \frac{1}{48} \left[4x^3 \sin x - x^2(x^2 - 9)\cos x \right]$$ //

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9) Solve $(D^2-4D+4)y = 8x^2e^{2x}\sin 2x$

$$m^2-4m+4=0$$

$$(m-2)^2=0$$

$$m=2,2$$

$$C.F = (c_1+c_2x)e^{2x}$$

$$P.I = \frac{1}{D^2-4D+4}\, 8x^2e^{2x}\sin 2x$$

$$= 8e^{2x}\cdot\frac{1}{(D^2-4D+4)}(x^2\sin 2x)$$

$$= 8e^{2x}\cdot\frac{1}{(D-2)^2}(x^2\sin 2x)$$

$$= 8e^{2x}\,\frac{1}{(D+\cancel{2}-\cancel{2})^2}(x^2\sin 2x)$$

$$= 8e^{2x}\cdot\frac{1}{D^2}(x^2\sin 2x)$$

$$= 8e^{2x}\cdot\frac{1}{D}\int x^2\sin 2x$$

$$= 8e^{2x}\cdot\frac{1}{D}\left[x^2\left(\frac{-\cos 2x}{2}\right)-\int 2x\left(\frac{-\cos 2x}{2}\right)dx\right]$$

$$= 8e^{2x}\cdot\frac{1}{D}\left[\frac{-x^2}{2}\cos 2x + x\,\frac{\sin 2x}{2}-\int\frac{\sin 2x}{2}\,dx\right]$$

$$= 8e^{2x}\cdot\frac{1}{D}\left[\frac{-x^2}{2}\cos 2x+\frac{x}{2}\sin 2x+\frac{\cos 2x}{4}\right]dx$$

$$= 8e^{2x}\int\left\{\frac{-x^2}{2}\cos 2x+\frac{x}{2}\sin 2x+\frac{\cos 2x}{4}\right\}dx$$

$$= 8e^{2x}\left\{\left[\frac{-x^2}{2}\cdot\frac{\sin 2x}{2}-\int(-x)\frac{\sin 2x}{2}\,dx\right]+\left\{\int\frac{x}{2}\sin 2x\,dx\right\}+\frac{\sin 2x}{8}\right\}$$

$$= 8e^{2x}\left[\left(\frac{-x^2}{4}+\frac{1}{8}\right)\sin 2x+\int x\sin 2x\,dx\right]$$

$$= 8e^{2x}\left[\left(\frac{1}{8}-\frac{x^2}{4}\right)\sin 2x+x\left(\frac{-\cos 2x}{2}\right)-\int 1\left(\frac{-\cos 2x}{2}\right)dx\right]$$

$$= 8e^{2x}\left[\left(\frac{1}{8}-\frac{x^2}{4}\right)\sin 2x-\frac{x\cos 2x}{2}+\frac{\sin 2x}{4}\right]$$

$$= e^{2x}\left[(3-2x^2)\sin 2x-4x\cos 2x\right]$$

$$y = e^{2x}\left[(c_1+c_2x)+(3-2x^2)\sin 2x-4x\cos 2x\right]$$

10) Solve

A.

P.

Now

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[illegible]

$$\frac{dy}{dx} + [illegible]$$

$$I.F = [illegible]$$

$$= [illegible]$$

$$= [illegible]$$

$$I.F = \frac{1}{[illegible]} \sec x$$

$$= \frac{1}{[illegible]} \sec x$$

$$= \frac{1}{[illegible]} \left[\frac{1}{[illegible]} - \frac{1}{[illegible]} \right] \sec x$$

$$= \frac{1}{[illegible]} \left[\frac{1}{[illegible]} \sec x - \frac{1}{[illegible]} \sec x \right]$$

$$\text{Answer} = \frac{1}{[illegible]} \sec x = [illegible] \int \sec x \cdot e^{-[illegible]x} \, dx$$

$$= [illegible] \int \frac{[illegible]}{\cos x} \, dx$$

$$= [illegible] \int (1 - [illegible]) \, dx$$

$$= [illegible] \left[[illegible] \log \cos x \right]$$

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PROBLEMS

3) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = \cos x$

A.E is $m^2+1=0$

$m^2=-1$

$m=\pm i$

$y = C_1\cos x + C_2 \sin x$

$P.I = \frac{1}{D^2+D}\cos x$

$= \frac{1}{-1+D}\cos x = \frac{1}{D-1}\cos x = \frac{D+1}{D^2-1}\cos x$

$= \frac{D+1}{-2}\cos x$

$= \frac{-1}{2}[-\sin x + \cos x]$

$= \frac{\sin x - \cos x}{2}$

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4) $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 3x = \sin t$

A.E is $m^2+2m+3=0$

$$m = \frac{-2 \pm \sqrt{4-12}}{2}$$

$$= \frac{-2 \pm \sqrt{-8}}{2}$$

$$= \frac{-2 \pm \sqrt{8}\,i}{2}$$

$$= \frac{-2 \pm 2\sqrt{2}\,i}{2} = -1 \pm \sqrt{2}\,i$$

C.F $y = e^{-t}\left[c_1 \cos\sqrt{2}t + c_2 \sin\sqrt{2}t\right]$

$$P.I = \frac{1}{D^2+2D+3} \sin t$$

$$= \frac{1}{(D+1)^2+2(D+1)+3} \sin t = \frac{1}{D^2+1+2D+2D+2}$$

$$= \frac{1}{-1+2D+3} \sin t = \frac{1}{2D+2} \sin t$$

$$= \frac{1}{2} \frac{D-1}{D^2-1} \sin t$$

$$= \frac{1}{2} \frac{D-1}{-2} \sin t$$

$$= \frac{-1}{4}\left[\cos t - \sin t\right] = \frac{\sin t - \cos t}{4}$$

$$y = e^{-t}\left[c_1 \cos\sqrt{2}t + c_2 \sin\sqrt{2}t\right] + \frac{1}{4}(\sin t - \cos t)$$

5) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4\cos^2 x$

A.E is $m^2+3m+2=0$

$$(m+1)(m+2)=0$$

$$m = -1, -2$$

C.F $y = c_1 e^{-x} + c_2 e^{-2x}$

$$P.I = \frac{1}{D^2+3D+2}(4\cos^2 x) = \frac{1}{-1+3D+2} 4\cos^2 x$$

$$= \frac{1}{3D+1} \cdot 4\cos^2 x$$

$$= \frac{4}{9D^2-1}(3D-1)\cos^2 x$$

$$= 4\frac{(3D-1)}{-10}\cos^2 x = \frac{-2}{5}\left[-6\sin x - \cos^2 x\right]$$

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6) $(D^2-D+2)y = \sin 2x$

A.E is $m^2-m+2=0$

$m = \frac{1\pm\sqrt{1-8}}{2} = \frac{1\pm\sqrt{7}i}{2}$

7) $\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}+\frac{dy}{dx} = e^{-x}+\sin 2x$

A.E is $m^3+2m^2+m=0$

$m(m^2+2m+1)=0$

$m=0, \quad (m+1)^2=0$

$m=-1,-1$

$y = c_1+(c_2+c_3x)e^{-x}$

$P.I = \frac{1}{D^3+2D^2+D}(e^{-x}+\sin 2x)$

$= \frac{1}{\cancel{D}-2+\cancel{D}}e^{-x}+\frac{1}{D^3+2D^2+D}\sin 2x \qquad a=-1 \Rightarrow a^2=1$

$= -\frac{1}{2}e^{-x}+\frac{1}{D(1+(D^2+2D))}\sin 2x \qquad = -\frac{1}{2}e^{-x}+\frac{1}{\cancel{D}-2+\cancel{D}}\sin 2x$

$= -\frac{1}{2}e^{-x}+\frac{1}{D}[1+(D^2+2D)]^{-1}\sin 2x \qquad = -\frac{1}{2}e^{-x}-\frac{1}{2}\sin 2x$

$= \frac{1}{D^3+2D^2+D}[e^{-x}+\text{I.P of } e^{i2x}]$

$= \frac{1}{\cancel{D}-2+\cancel{D}}e^{-x}+\text{I.P of } \frac{e^{i2x}}{D^3+2D^2+D}$

$= -\frac{1}{2}e^{-x}+\text{I.P of } \frac{e^{i2x}}{-2}$

$= -\frac{1}{2}e^{-x}+\text{I.P of } \left[\frac{\cos 2x+i\sin 2x}{-2}\right]$

8) $\frac{d^2y}{dx^2}$

9) $(D^3-5$

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8) $\cancel{\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=e^{2x}\cos^2}$

9) $(D^3-5D^2+7D-3)y=e^{2x}\cosh x$

$$(D^3-5D^2+7D-3)y=e^{2x}\left(\frac{e^x+e^{-x}}{2}\right)$$

$$(D^3-5D^2+7D-3)y=\frac{e^{3x}+e^x}{2}$$

A.E is $m^3-5m^2+7m-3=0$

$$\begin{array}{c|cccc} & 1 & -5 & 7 & -3 \\ m=1 & 0 & 1 & -4 & 3 \\ \hline & 1 & -4 & 3 & \boxed{0} \end{array}$$

$m^2-4m+3=0$

$m^2-3m-m+3=0$

$m(m-3)-1(m-3)=0$

$m=1,3$

$\therefore m=1,1,3$

$$C.F=c_1e^{3x}+(c_2+c_3x)e^x$$

$$P.I=\frac{1}{D^3-5D^2+7D-3}\left(\frac{e^{3x}+e^x}{2}\right)$$

$$\frac{1}{2(D^3-5D^2+7D-3)}e^{3x}=\frac{1}{2[-9D+45+7D-3]}e^{3x}$$

$a=3\Rightarrow a^2=9$

$$=\frac{1}{2[42-2D]}e^{3x}$$

$$=\frac{1}{4[21-D]}e^{3x}=\frac{1}{4}\frac{(21+D)}{(441-D^2)}e^{3x}$$

$$=\frac{1}{4}\frac{(21+D)}{450}e^{3x}$$

$$=\frac{1}{1800}(21+D)e^{3x}$$

$$=\frac{1}{1800}\left[21e^{3x}+3e^{3x}\right]$$

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Problems

1) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = (1-e^x)^2$

Answers with $y(0)=1$ & $y'(0)=0$

2) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 3\sin x + 4\cos x$

3) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4\cos^2 x$

4) $\frac{d^2y}{dx^2} - 4y = x\sinh x$

5) $(D-2)^2 y = 8\,(e^{2x} + \sin x + x^2)$

6) $y'' - 2y' + 2y = x + e^x \cos x$

7) $y'' - 3y' + 2y = xe^{3x} + \sin 2x$

8) $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 5y = -2\cosh t$, then find 'y' when $y=0$ & $\frac{dy}{dt} = 1$ at $t=0$

1) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 1 + e^{2x} - 2e^x$

A.E is $m^2 + m + 1 = 0$

$$m = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$$

$$m_1 = \frac{-1+\sqrt{3}i}{2}, \quad m_2 = \frac{-1-\sqrt{3}i}{2} \qquad a = -\frac{1}{2} \quad b = \frac{\sqrt{3}}{2}$$

$$C.F = e^{-x/2}\left[c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x\right]$$

$$P.I = \frac{1}{D^2+D+1}(1 + e^{2x} - 2e^x)$$

$$\frac{1}{D^2+D+1}(1) = \frac{1}{0+0+1}(1) = 1 \quad [\because a = 0]$$

$$\frac{1}{D^2+D+1}e^{2x} = \frac{1}{4+2+1}e^{2x} = \frac{e^{2x}}{7}$$

$$\frac{1}{D^2+D+1}e^x = \frac{1}{1+1+1}e^x = \frac{e^x}{3}$$

$$\therefore P.I = 1 + \frac{1}{7}e^{2x} - \frac{2}{3}e^x \quad //$$

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1) $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$

$x = e^t \Rightarrow \log x = t$

$x^2 \frac{d^2y}{dx^2} = D_1(D_1 - 1)y, \quad x\frac{dy}{dx} = D_1 y \qquad \left[\because D_1 = \frac{d}{dt}\right]$

use this in the given eqn

$D_1(D_1-1)y - D_1 y + y = t$

$(D_1^2 - 2D_1 + 1)y = t$

$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = t$

A.E is $D_1^2 - 2D_1 + 1 = 0$

$\Rightarrow (D_1 - 1)^2 = 0 \Rightarrow D_1 = 1, 1$

$C.F = (c_1 + c_2 t)e^t$

$P.I = \left(\frac{1}{D_1^2 - 2D_1 + 1}\right) t$

$= [1 + (D_1^2 - 2D_1)]^{-1} t$

$= [1 - (D_1^2 - 2D_1) + (D_1^2 - 2D_1)^2 + \dots] t$

$= (t - 0 + 2) = t + 2$

$\therefore y = (c_1 + c_2 t)e^t + (t+2)$

$\Rightarrow y = (c_1 + c_2 \log x)x + (\log x + 2)$

2) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x \sin(\log x)$

$x = e^t \Rightarrow \log x = t$

$D_1(D_1 - 1)y + D_1 y + y = t\sin t$

$D_1^2 y - \cancel{D_1 y} + \cancel{D_1 y} + y = t\sin t$

$(D_1^2 + 1)y = t\sin t$

$\frac{d^2y}{dt^2} + y = t\sin t$

A.E is $D_1^2 + 1 = 0$

$D_1^2 = -1 \Rightarrow D_1 = \pm i$

$C.F = (c_1 \cos t + c_2 \sin t)$

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$$P.I = \left(\frac{1}{D_1^2+1}\right) t\sin t$$

$$= \frac{1}{D_1^2+1}\left[t\,(I.P \text{ of } e^{it})\right]$$

$$= I.P \text{ of } \frac{1}{D_1^2+1} e^{it}.t$$

$$= I.P \text{ of } \left[e^{it}.\frac{1}{D_1^2+1}.t\right]$$

$$= I.P \text{ of } e^{it}\left[(1+D_1^2)^{-1}\right]t$$

$$= I.P \text{ of } e^{it}\left[1-D_1^2+\cdots\right]t$$

$$= I.P \text{ of } e^{it}\left[t-0+\cdots\right]$$

$$= I.P \text{ of } e^{it}.t$$

$$= I.P \text{ of } \left\{\left[\cos t + i\sin t\right]t\right\}$$

$$= I.P \text{ of } \left\{t\cos t + it\sin t\right\}$$

$$= t\sin t$$

$$= \log x \sin \log x$$

$$\therefore\ y = c_1\cos\log x + c_2\sin\log x + \log x\sin(\log x) \quad //$$

3) $x^2y'' - 4xy' + 6y = x^2$

$$D_1(D_1-1)y - 4D_1y + 6y = e^{2t}$$

$$D_1^2y - D_1y - 4D_1y + 6y = e^{2t}$$

$$D_1^2y - 5D_1y + 6y = e^{2t}$$

$$(D_1^2-5D_1+6)y = e^{2t}$$

A.E is $D_1^2-5D_1+6=0$

$$\Rightarrow D_1^2-3D_1-2D_1+6=0$$

$$\Rightarrow D_1(D_1-3)-2(D_1-3)=0$$

$$\Rightarrow D_1 = 2, 3$$

$$C.F = c_1e^{2t} + c_2e^{3t}$$

$$P.I = \left(\frac{1}{D_1^2-5D_1+6}\right)e^{2t}$$

$$= \left(\frac{1}{4-10+6}\right)e^{2t}, \qquad f(a)=0$$

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$$= \left(\frac{t}{2D_1-5}\right) e^{2t}$$

$$= \left(\frac{t}{4-5}\right) e^{2t} = -te^{2t}$$

$$P.I = \cancel{x^2} - \log x \cdot x^2$$

$$y = c_1 e^{2t} + c_2 e^{3t} - te^{2t}$$

$$y = c_1 x^2 + c_2 x^3 - x^2 \log x \quad //$$

4) $x \frac{d^2y}{dx^2} - 2\frac{y}{x} = x + \frac{1}{x^2}$

$$x^2 \frac{d^2y}{dx^2} - 2y = x^2 + \frac{1}{x}$$

$$x = e^t \Rightarrow t = \log x$$

$$D_1(D_1-1)y - 2y = e^{2t} + e^{-t}$$

$$D_1^2 y - D_1 y - 2y = e^{2t} + e^{-t}$$

$$(D_1^2 - D_1 - 2)y = e^{2t} + e^{-t}$$

A.E is $D_1^2 - D_1 - 2 = 0$

$$D_1^2 - 2D_1 + D_1 - 2 = 0$$

$$D_1(D_1-2) + 1(D_1-2) = 0$$

$$D_1 = -1, 2$$

$$\cancel{y} \quad C.F = c_1 e^{-t} + c_2 e^{2t}$$

$$P.I = \left(\frac{1}{D_1^2 - D_1 - 2}\right)(e^{2t} + e^{-t})$$

$$\frac{1}{D_1^2 - D_1 - 2} e^{2t} = \frac{1}{4-2-2} e^{2t} \qquad [F(a) = 0]$$

$$= \frac{t}{2D_1 - 1} e^{2t}$$

$$= \frac{t}{4-1} e^{2t} = \frac{t}{3} e^{2t}$$

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UNIT-2

⇒ Laplace transformation

Definition:-

$f(t)$ be a function of t defined for all $t \geq 0$ the laplace transform of the function $f(t)$ is denoted by $L\{f(t)\}$ and is defined as

$$\boxed{L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt}$$

~~where~~ provided the above integral exists and s is a parameter which may be real or complex

$L\{f(t)\}$ is clearly function of s and is written as $\bar{f}(s)$

i.e., $L\{f(t)\} = \bar{f}(s)$

Linear property:-

If c_1, c_2 are two constants and f and g are two functions of t then

$$L\{c_1 f(t) + c_2 g(t)\} = c_1 L\{f(t)\} + c_2 L\{g(t)\}$$

$$L\{c_1 f(t) + c_2 g(t)\} = \int_0^\infty e^{-st}[c_1 f(t) + c_2 g(t)]\,dt$$

$$= \int_0^\infty [c_1 e^{-st} f(t) + c_2 e^{-st} g(t)]\,dt$$

$$L\{c_1 f(t) + c_2 g(t)\} = c_1 \int_0^\infty e^{-st} f(t)\,dt + c_2 \int_0^\infty e^{-st} g(t)\,dt$$

$$= c_1 L\{f(t)\} + c_2 L\{g(t)\}$$

⇒ Laplace transform of some simple function

1) $f(t) = k$ then find $L\{f(t)\}$

$$L\{f(t)\} = L\{k\}$$

$$= \int_0^\infty e^{-st} \cdot k\,dt$$

$$= k\left[\frac{e^{-st}}{-s}\right]_0^\infty$$

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$$= -k/s\left[e^{-\infty} - e^{0}\right]$$

$$L\{k\} = k/s$$

Note :-

$$L\{1\} = 1/s \;;\quad L\{10\} = 10/s$$

Find $L\{t^n\} = \int_0^\infty e^{-st} \cdot t^n \, dt$

γ function

$$\sqrt{n} = \int_0^\infty e^{-x} \cdot x^{n-1} \, dx$$

$$\sqrt{n+1} = n!$$

$$= \int_0^\infty e^{-st} \cdot t^n \, dt$$

$$= \int_0^\infty e^{-x} \left(\frac{x}{s}\right)^n \frac{dx}{s}$$

$$\left\{ \begin{array}{l} \text{Let } st = x \\ t = x/s \\ dt = dx/s \end{array} \right\}$$

$$= \frac{1}{s^{n+1}} \int_0^\infty e^{-x} x^n \, dx$$

$$= \frac{1}{s^{n+1}} \int_0^\infty e^{-x} (x^{n+1})^{-1} \, dx$$

$$= \frac{1}{s^{n+1}} \sqrt{n+1}$$

$$\boxed{L\{t^n\} = \frac{n!}{s^{n+1}}}$$

* $L\{e^{at}\} = \frac{1}{s-a} \; (s > a)$

$$L\{e^{at}\} = \int_0^\infty e^{-st} \cdot e^{at} \, dt$$

$$= \int_0^\infty e^{(a-s)t} \, dt$$

$$= \left[\frac{e^{(a-s)t}}{a-s}\right]_0^\infty$$

$$= \frac{1}{a-s}\left[e^{-\infty} - e^{0}\right]$$

$$= \frac{1}{a-s}(-1) = \frac{1}{s-a}$$

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* $L\{\sin at\} = \frac{a}{s^2+a^2} \quad (s>0)$

$$= \int_0^\infty e^{-st} \sin at \, dt$$

$$= \left[\frac{e^{-st}}{s^2+a^2} \left[-s \sin at - a\cos at \right] \right]_0^\infty \quad \left[\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2} (a \sin bx - b\cos bx) \right]$$

$$= 0 - \frac{1}{s^2+a^2} [0 - a] = \frac{a}{s^2+a^2}$$

* $L\{\sinh at\} = \frac{a}{s^2-a^2} \quad (|s|>|a|)$

$$L\{\sinh at\} = L\left\{ \frac{e^{at}-e^{-at}}{2} \right\}$$

$$= \frac{1}{2} L\{e^{at}\} - \frac{1}{2} L\{e^{-at}\}$$

$$= \frac{1}{2} \left(\frac{1}{s-a} \right) - \frac{1}{2} \left(\frac{1}{s+a} \right)$$

$$= \frac{1}{2} \left[\frac{2a}{s^2-a^2} \right] = \frac{a}{s^2-a^2} \quad //$$

* $L\{\cosh at\} = \frac{s}{s^2-a^2} \quad (s>|a|)$

$\sin A \sin B = \frac{1}{2} [\cos(A-B) - \cos(A+B)]$

* L.T of $\sin 6t \sin 4t$

$$L\{\sin 6t \sin 4t\}$$

$$= L\left\{ \frac{1}{2} (\cos 2t - \cos 10t) \right\}$$

$$= \frac{1}{2} [L\{\cos 2t\} - L\{\cos 10t\}]$$

$$= \frac{1}{2} \left[\frac{s}{s^2+2^2} - \frac{s}{s^2+10^2} \right]$$

$$= \frac{1}{2} \left[\frac{s}{s^2+4} - \frac{s}{s^2+100} \right]$$

$$= \frac{1}{2} \left[\frac{\cancel{s^3}+100s-\cancel{s^3}-4s}{(s^2+4)(s^2+100)} \right]$$

$$= \frac{48s}{(s^2+4)(s^2+100)} \quad //$$

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* $L\{\sin 2t \cos 3t\}$

$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$= \frac{2s^2-10}{(s^2+25)(s^2+1)}$

* change of scale property

If $L\{f(t)\} = \bar{f}(s)$ then $L\{f(at)\} = \frac{1}{a}\bar{f}(s/a)$

$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = \bar{f}(s)$

$L\{f(at)\} = \int_0^\infty e^{-st} f(at)\,dt$ Let $at = u$, $t = u/a$, $dt = du/a$

$= \int_0^\infty e^{-su/a} f(u)\,\frac{du}{a}$

$= \frac{1}{a}\int_0^\infty e^{-\frac{s}{a}u} f(u)\,du$

$= \frac{1}{a}\bar{f}(s/a)$

* First shifting theorem

$L\{f(t)\} = \bar{f}(s)$ then $L\{e^{at}f(t)\} = \bar{f}(s-a)$

$= \int_0^\infty e^{-st} f(t)\,dt$

$= \int_0^\infty e^{-st} e^{at} f(t)\,dt$

$= \int_0^\infty e^{(a-s)t} f(t)\,dt$

$= \int_0^\infty e^{-(s-a)t} f(t)\,dt$

$= \bar{f}(s-a)$

—ᴍᴍ— $V = IR$

—ᴍᴍ— $I = C\frac{dV}{dt}$

—||— $V = L\frac{dI}{dt}$

* $L\{e^{-at}f(t)\} = \bar{f}(s+a)$

* $L\{e^{at}\cdot t^n\} = \frac{n!}{(s-a)^{n+1}}$

* $L\{e^{at}\cos bt\} = \frac{s-a}{(s-a)^2+b^2}$

* $L\{e^{at}\sin bt\} = \frac{b}{(s-a)^2+b^2}$

* $L\{\sin^3 2t\} = \frac{1}{4}L\{3\sin 2t - \sin 6t\}$

$= \frac{3}{}$

* $L\{\sin^3 2t\} = L\{\frac{1}{4}(3\sin 2t - \sin 6t)\}$

$$= \frac{3}{4} L\{\sin 2t\} - \frac{1}{4} L\{\sin 6t\}$$

$$= \frac{48}{(s^2+4)(s^2+36)}$$

* find $L\{t^2 \sin at\}$

$$L\{t^2\} = \frac{2!}{s^3}$$

$$L\{e^{iat}; t^2\} = \frac{2!}{(s-ia)^3} \cdot \frac{(s+ia)^3}{(s+ia)^3}$$

$$= \frac{2(s+ia)^3}{[(s-ia)(s+ia)]^3}$$

$$= 2\frac{[s^3 - ia^3 + 3ias^2 - 3a^2 s]}{(s^2+a^2)^3}$$

$$= \frac{2s^3 - 6a^2 s}{(s^2+a^2)^3} + i\frac{(6s^2 a - 2a^3)}{(s^2+a^2)^3}$$

Eq, the imaginary part on b.s

$$L\{t^2 \sin at\} = \frac{6a^2 s^2 - 2a^3}{(s^2+a^2)^3}$$

1) * find $L\{t^3 e^{-3t}\} = \frac{3!}{(s+3)^4}$

2) * find L.T of $f(t)$ $\quad f(t) = \begin{cases} t & 0<t<4 \\ 5 & t>4 \end{cases}$

3) * $L\{f(t)\}$

$$f(t) = \begin{cases} t/T & 0<t<T \\ 1 & \text{when } t>T \end{cases}$$

4) * find $L\{e^{-3t}(2\cos 5t - 3\sin 5t)\}$

5) Find $L\{f(t)\}$ if

$$f(t) = \begin{cases} \cos t & 0 < t < 2\pi \\ 0 & t > 2\pi \end{cases}$$

2) $$L\{f(t)\} = \int_0^4 e^{-st} \cdot t\,dt + \int_4^\infty e^{-st} \cdot 5\,dt$$

$$= \left[t\left[\frac{e^{-st}}{-s}\right] - \left(\frac{e^{-st}}{s^2}\right)\right]_0^4 + 5\left(\frac{e^{-st}}{-s}\right)_4^\infty$$

$$= \frac{4e^{-4s}}{-s} - \frac{e^{-4s}}{s^2} - 0 + \frac{1}{s^2}(0) + \frac{5}{s}e^{-4s}$$

$$= \frac{e^{-4s}}{s^2} - \frac{1}{s^2}(e^{-4s} - 1)$$

3) $$L\{f(t)\} = \int_0^T \frac{t}{T} e^{-st}\,dt + \int_T^\infty e^{-st}(1)\,dt$$

$$= \frac{1}{T}\int_0^T t e^{-st}\,dt + \int_T^\infty e^{-st}(1)\,dt$$

$$= \frac{1}{T}\left[\frac{te^{-st}}{-s} + \int_0^T \frac{e^{-st}}{s}(1)\,dt\right]_0^T + \left[\frac{e^{-st}}{-s}\right]_T^\infty$$

$$= \frac{1}{T}\left[\frac{te^{-st}}{-s} + \frac{t}{s} + \left(\frac{e^{-st}}{s^2}\right)_0^T\right] + \left(\frac{e^{-st}}{-s}\right)_T^\infty$$

$$= \frac{t}{T}\frac{e^{-sT}}{s} + \frac{t}{Ts} + \frac{e^{-st}}{s^2} - \frac{1}{s^2} + \frac{e^{-sT}}{s}$$

4) $L\{e^{-3t}(2\cos 5t - 3\sin 5t)\}$

5) L

Invers

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5) $L\{f(t)\} = \int_0^{2\pi} e^{-st}\cos t\,dt + \int_{2\pi}^{\infty} e^{-st}\cdot 0\,dt$

$L\{\cos at\} = \frac{s}{s^2+a^2}$.

$L\{\cos t\} = \int_0^{\infty} e^{-st}\cos t\,dt = \frac{s}{s^2+1}$

<u>Inverse laplace transformal</u>

* If $L\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\,dt = \bar{f}(s)$

$L^{-1}\{\bar{f}(s)\} = f(t)$

* $L\{1\} = \frac{1}{s}$ then $L^{-1}\{1/s\} = 1$

If $L\{f(t)\} = \bar{f}(s)$ then $f(t)$ is called inverse laplace transform of $\bar{f}(s)$ and we write it as

$L^{-1}\{\bar{f}(s)\} = f(t)$

* $L\{e^{at}\} = \frac{1}{s-a}$

$L^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$

* $L\{t^n\} = \frac{n!}{s^{n+1}} \Rightarrow L^{-1}\left\{\frac{1}{s^{n+1}}\right\} = \frac{t^n}{n!}$

* $L\{\cos at\} = \frac{s}{s^2+a^2} \Rightarrow L^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at$

* $L\{\sin at\} = \frac{a}{s^2+a^2} \Rightarrow L^{-1}\left\{\frac{1}{s^2+a^2}\right\} = \frac{\sin at}{a}$

* $L\{e^{at}\cdot t^n\} = \frac{n!}{(s-a)^{n+1}} \Rightarrow L^{-1}\left\{\frac{1}{(s-a)^{n+1}}\right\} = \frac{e^{at}\cdot t^n}{n!}$

* $L\{\sinh at\} = \frac{a}{s^2-a^2} \Rightarrow L^{-1}\left\{\frac{1}{s^2-a^2}\right\} = \frac{\sinh at}{a}$

* $L\{\cosh at\} = \frac{s}{s^2-a^2} \Rightarrow L^{-1}\left\{\frac{s}{s^2-a^2}\right\} = \cosh at$

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* $L\{e^{at}\cosh bt\} = \dfrac{s-a}{(s-a)^2-b^2}$

$$L^{-1}\left\{\frac{s-a}{(s-a)^2-b^2}\right\} = e^{at}\cosh bt$$

Q) Inverse laplace transform of the following

i) $\dfrac{3(s^2-1)^2}{2s^5}$

$$L^{-1}\left\{\frac{3(s^2-1)^2}{2s^5}\right\}$$

$$\frac{3(s^2-1)^2}{2s^5} = \frac{3(s^4+1-2s^2)}{2s^5}$$

$$= \frac{3}{2s} + \frac{3}{2s^5} - \frac{3}{s^3}$$

$$L^{-1}\left\{\frac{3(s^2-1)^2}{2s^5}\right\} = L^{-1}\left\{\frac{3}{2}\left(\frac{1}{s}\right) + \frac{3}{2}\left(\frac{1}{s^5}\right) - 3\left(\frac{1}{s^3}\right)\right\}$$

$$= \frac{3}{2}(1) + \frac{3}{2}\left(\frac{t^4}{4!}\right) - 3\left(\frac{t^2}{2!}\right)$$

$$= \frac{3}{2} + \frac{t^4}{16} - \frac{3t^2}{2}$$

ii) $L^{-1}\left\{\dfrac{3s+5\sqrt{2}}{s^2+8}\right\}$

$$= \frac{3s}{s^2+8} + \frac{5\sqrt{2}}{s^2+8}$$

$$= 3\cos\sqrt{8}\,t + 5\sqrt{2}\,\frac{\sin\sqrt{8}\,t}{\sqrt{8}}$$

iii) $L^{-1}\left\{\dfrac{4s+15}{16s^2-25}\right\}$

$$\frac{4s+15}{16s^2-25} = \frac{4s}{(4s)^2-5^2} + \frac{15}{(4s)^2-5^2}$$

$$= \frac{s}{4\left[s^2-\left(\frac{5}{4}\right)^2\right]} + \frac{15}{(4)^2\left[s^2-\left(\frac{5}{4}\right)^2\right]}$$

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iv) $\frac{s+1}{s^2+s+1} = \frac{s+\frac{1}{2}+\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{3}{4}} = \frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+(\sqrt{\frac{3}{4}})^2} + \frac{\frac{1}{2}}{(s+\frac{1}{2})^2+(\sqrt{\frac{3}{4}})^2}$

$$= L^{-1}\left\{\frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+(\sqrt{\frac{3}{4}})^2}\right\} + L^{-1}\left\{\frac{\frac{1}{2}}{(s+\frac{1}{2})^2+(\sqrt{\frac{3}{4}})^2}\right\}$$

$$= e^{-t/2}\cos\frac{\sqrt{3}}{2}t + \frac{1}{2}\cdot\frac{e^{-t/2}\sin\frac{\sqrt{3}}{2}t}{\sqrt{3}/2}$$

$$= e^{-t/2}\cos\frac{\sqrt{3}}{2}t + \frac{1}{\sqrt{3}}e^{-t/2}\sin\frac{\sqrt{3}}{2}t \quad //$$

* $L^{-1}\left\{\frac{2s^2-4}{(s+1)(s-2)(s-3)}\right\}$

$$\frac{2s^2-4}{(s+1)(s-2)(s-3)} = \frac{A}{s+1} + \frac{B}{s-2} + \frac{C}{s-3}$$

$$= \frac{A(s-2)(s-3)+B(s+1)(s-3)+C(s+1)(s-2)}{(s+1)(s-2)(s-3)}$$

$$= \frac{-1}{6(s+1)} + \frac{4}{3(s-2)} + \frac{14}{4(s-3)}$$

$$= \frac{-1}{6(s+1)} - \frac{4}{3(s-2)} + \frac{7}{2(s-3)}$$

$$L^{-1}\left\{\frac{-1}{6(s+1)}\right\} - \frac{4}{3}L^{-1}\left\{\frac{1}{s-2}\right\} + \frac{7}{2}L^{-1}\left\{\frac{1}{s-3}\right\}$$

$$-\frac{1}{6}e^{-t} - \frac{4}{3}e^{2t} + \frac{7}{2}e^{3t}$$

* $L^{-1}\left\{\frac{2s^2-1}{(s^2+1)(s^2+4)}\right\}$ * $L^{-1}\left\{\frac{2s+1}{(s+2)^2(s^2-1)}\right\}$

put $s^2 = p$

* $L^{-1}\left\{\frac{4}{(s+1)(s+2)}\right\}$ * $L^{-1}\left\{\frac{s^2}{s^4-a^4}\right\}$

* $L^{-1}\left\{\frac{2p-1}{(p+1)(p+4)}\right\} = \frac{A}{p+1} + \frac{B}{p+4}$

$$\frac{-1}{p+1} + \frac{3}{p+4}$$

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$$\frac{-1}{s^2+1} + \frac{3}{s^2+4} = \frac{-1}{s^2+1^2} + \frac{3}{s^2+2^2}$$

$$= -1\,\frac{\sin t}{1} + 3\,\frac{\sin 2t}{2}$$

* $L^{-1}\left\{\frac{2s+1}{(s+2)^2(s^2-1)}\right\}$ Ans:- $\frac{-2}{3}e^{-2t} - te^{-2t} + \frac{2}{3}\cos t - \frac{1}{3}\sinh t$

$$= \frac{A}{(s+2)} + \frac{B}{(s+2)^2} + \frac{C}{s^2+1} + \frac{D}{s-1}$$

By solving $A = -2/3$, $D = 1/6$, $B = -1$, $C = 1/2$

$$= -\frac{2}{3}L^{-1}\left\{\frac{1}{s+2}\right\} - 1\,L^{-1}\left\{\frac{1}{(s+2)^2}\right\} + \frac{1}{2}L^{-1}\left\{\frac{1}{s+1}\right\} + \frac{1}{6}L^{-1}\left\{\frac{1}{s-1}\right\}$$

$$= -2/3\, e^{-2t} - 1\left[e^{-2t}\cdot t\right] + \frac{1}{2}e^{-t} + \frac{1}{6}e^{t}$$

* $L^{-1}\left\{\frac{s^2}{s^4-a^4}\right\}$

$$\frac{s^2}{(s^2)^2-(a^2)^2} = \frac{s^2}{(s^2+a^2)(s^2-a^2)}$$

Let $s^2 = k$

$$\frac{s^2}{(s^2+a^2)(s^2-a^2)} = \frac{A}{k+a^2} + \frac{B}{k-a^2}$$

$$= Ak - Aa^2 + Bk + Ba^2$$

$A+B=1$

$-A+B=0$

$2B=1$

$B = 1/2$ $A = 1/2$

$$= \frac{1}{2}L^{-1}\left\{\frac{1}{s^2+a^2}\right\} + \frac{1}{2}L^{-1}\left\{\frac{1}{s^2-a^2}\right\}$$

$$= \frac{1}{2}\,\frac{\sin at}{a} + \frac{1}{2}\cdot\frac{\sinh at}{a}$$

$$= \frac{1}{2a}\left[\sin at + \sinh at\right]$$

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$\frac{2}{3}\cos t - \frac{1}{3}\sin 2t$

$c = \frac{1}{2}$

$+ \frac{1}{2} L^{-1}\left\{\frac{1}{s+1}\right\}$

$L^{-1}\left\{\frac{1}{s-1}\right\}$

e^{t}

$s^2 = k$

$\}$

* $L^{-1}\left\{\frac{4}{(s+1)(s+2)}\right\}$

$$\frac{4}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$As + 2A + Bs + B$$

$$A + B = 0$$
$$2A + B = 4$$
$$-A = -4$$
$$A = 4$$
$$B = -4$$

$$= 4e^{-t} - 4e^{-2t}$$
$$= 4\left[e^{-t} - e^{-2t}\right]$$

→ Laplace transform of derivatives

→ If $f'(t)$ is continuous and $L\{f(t)\} = \bar{f}(s)$

then $L\{f'(t)\} = s\,\bar{f}(s) - f(0)$

→ If $f(t)$ and its first $(n-1)$ derivatives be continuous

then Laplace transform of $f^n(t)$

$$L\{f^n(t)\} = s^n \bar{f}(s) - s^{n-1} f(0) - s^{n-2} f'(0) - s^{n-3} f''(0) - \cdots - f^{n-1}(0)$$

Note

$$L\{f''(t)\} = s^2 \bar{f}(s) - s f(0) - f'(0)$$

$$L\{f'''(t)\} = s^3 \bar{f}(s) - s^2 f(0) - s f'(0) - f''(0)$$

$$L\{f^{IV}(t)\} = s^4 \bar{f}(s) - s^3 f(0) - s^2 f'(0) - s f''(0) - f'''(0)$$

→ Laplace transform of integrals

→ If $L f(t) = \bar{f}(s)$ then

$$L\left\{\int_0^t f(t)\,dt\right\} = \frac{1}{s}\bar{f}(s)$$

$$L^{-1}\left\{\frac{1}{s}\bar{f}(s)\right\} = \int_0^t f(t)\,dt$$

→ Multiplication by t^n

If $L\{f(t)\} = \bar{f}(s)$ then $L\{t^n f(t)\} = (-1)^n \dfrac{d^n}{ds^n}[\bar{f}(s)]$

→ Division by t

If $L\{f(t)\} = \bar{f}(s)$ then $L\left\{\frac{1}{t} f(t)\right\} = \int_s^\infty \bar{f}(s)\,ds$

* Find $L\{t \sin^2 3t\}$

$$\sin^2 3t = \frac{1-\cos 6t}{2}$$

$$L\{\sin^2 3t\} = L\left\{\frac{1-\cos 6t}{2}\right\}$$

$$= \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+36}\right]$$

$$= \frac{1}{2}\left[\frac{\cancel{s^2}+36-\cancel{s^2}}{s(s^2+36)}\right] = \frac{18}{s(s^2+36)}$$

$$L\{\sin^2 3t\} = \frac{18}{s(s^2+36)}$$

If $L\{$

2) Find

L

If $L\{f(t)\} = F(s)$ then $L\{t f(t)\} = (-1)\frac{d^n}{ds} F(s)$

$$L\{t \sin^2 3t\} = (-1)\frac{d}{ds}\left[\frac{18}{s^3+36s}\right]$$

$$= (-1)\, 18(-1)(s^3+36s)^{-2}(3s^2+36)$$

$$= \frac{18(3s^2+36)}{(s^3+36s)^2}$$

2) Find the L.T of $\frac{e^{-at}-e^{-bt}}{t}$

$$L\left\{\frac{e^{-at}-e^{-bt}}{t}\right\} = \frac{1}{s+a} - \frac{1}{s+b}$$

$$= \frac{s+b-s-a}{(s+a)(s+b)}$$

$$= \frac{b-a}{(s+a)(s+b)}$$

$$L\left\{\frac{e^{-at}-e^{-bt}}{t}\right\} = \int_s^\infty \frac{b-a}{(s+a)(s+b)}\, ds$$

$$= \int_s^\infty \frac{1}{s+a}\, ds - \int_s^\infty \frac{1}{s+b}\, ds$$

$$= \left[\log(s+a)\right]_s^\infty - \left[\log(s+b)\right]_s^\infty$$

$$= \left[\log\infty - \log(s+a)\right] - \left[\log\infty - \log(s+b)\right]$$

$$= \log(s+b) - \log(s+a)$$

$$= \log\left[\frac{s+b}{s+a}\right]$$

$$= \left[\log\left(\frac{s+a}{s+b}\right)\right]_s^\infty$$

$$= \left[\log \frac{1+a/s}{1+b/s}\right]_s^\infty$$

$$= \log 1 - \log\frac{(s+a)}{(s+b)}$$

$$= 0 - \log\left(\frac{s+a}{s+b}\right)$$

$$= \log\left(\frac{s+b}{s+a}\right)$$

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3) Find inverse L.T of $\log\left(\frac{s+1}{s-1}\right)$

$$L^{-1}\left\{\log\left(\frac{s+1}{s-1}\right)\right\} = f(t) \quad (\text{say})$$

$$\Rightarrow L\{f(t)\} = \log\left(\frac{s+1}{s-1}\right)$$

If $L\{f(t)\} = F(s)$ then $L\{f(t)\,t\} = (-1)\frac{d}{ds}[F(s)]$

$$L\{t\,f(t)\} = -1\,\frac{d}{ds}\left[\log\frac{s+1}{s-1}\right]$$

$$L\{t f(t)\} = (-1)\left(\frac{s-1}{s+1}\right)\left(\frac{-2}{(s-1)^2}\right)$$

$$L\{t f(t)\} = \frac{2}{(s+1)(s-1)}$$

$$L\{t\,f(t)\} = \frac{1}{s-1} - \frac{1}{s+1}$$

$$\cancel{= e^{t} - e^{-t}}$$

$$t\,f(t) = L^{-1}\left\{\frac{1}{s-1} - \frac{1}{s+1}\right\}$$

$$t f(t) = e^{t} - e^{-t}$$

$$f(t) = \frac{e^{t} - e^{-t}}{t}$$

$$f(t) = \frac{2\sinh t}{t}$$

4) $L\left\{\frac{\cos at - \cos bt}{t}\right\}$

$$L\{\cos at - \cos bt\} = L\{\cos at\} - L\{\cos bt\}$$

$$= \frac{s}{s^2+a^2} - \frac{s}{s^2+b^2}$$

$$= \frac{s^3 + sb^2 - s^3 - a^2 s}{(s^2+a^2)(s^2+}$$

$$L\left\{\frac{\cos at - \cos bt}{t}\right\} = \int_s^{\infty}\left(\frac{s}{s^2+a^2} - \frac{s}{s^2+b^2}\right) ds$$

$$= \left[\frac{\log(s^2+a^2)}{2} - \frac{\log(s^2+b^2)}{2}\right]_s^{\infty}$$

5) Find

6) Find

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$$= \frac{1}{2}\left[\log\left(\frac{s^2+a^2}{s^2+b^2}\right)\right]_s^\infty$$

$$= \frac{1}{2}\log\left(\frac{s^2+b^2}{s^2+a^2}\right) //$$

5) Find $L\{te^{-t}\sin 3t\}$

$$L\{\sin 3t\} = \frac{3}{s^2+9}$$

$$L\{t\sin 3t\} = (-1)\frac{d}{ds}\left[\frac{3}{s^2+9}\right]$$

$$= -3(s^2+9)^{-2}(-1)(2s)$$

$$= \frac{6s}{(s^2+9)^2}$$

$$L\{e^{-t}\, t\sin 3t\} = \frac{6(s+1)}{\left[(s+1)^2+9\right]^2}$$

$\Downarrow$

$\left[\because \text{Replace } s \text{ by } s-a \quad a=-1 \Rightarrow s-a = s+1\right]$

6) Find $L\left\{\frac{1-e^t}{t}\right\}$

$$L\{1-e^t\} = L\{1\} - L\{e^t\}$$

$$= \frac{1}{s} - \frac{1}{s-1}$$

$$L\left\{\frac{1-e^t}{t}\right\} = \int_s^\infty \left(\frac{1}{s} - \frac{1}{s-1}\right) ds$$

$$= \left[\log s - \log(s-1)\right]_s^\infty$$

$$= \left[\log\frac{s}{s-1}\right]_s^\infty$$

$$= \left[\log\frac{1}{1-\frac{1}{s}}\right]_s^\infty$$

$$= \log 1 - \log\frac{s}{s-1}$$

$$= \log\left(\frac{s-1}{s}\right) //$$

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7) $L\{t^2 \cos 2t\}$

$L\{\cos 2t\} = \frac{s}{s^2+4}$

$L\{t^2 \cos 2t\} = (-1)^2 \frac{d^2}{ds^2}\left[\frac{s}{s^2+4}\right]$

$= \frac{d}{ds}\left[\frac{d}{ds}\left(\frac{s}{s^2+4}\right)\right]$

$= \frac{d}{ds}\left[\frac{s^2+4-s(2s)}{(s^2+4)^2}\right]$

$= \frac{d}{ds}\left[\frac{-s^2+4}{(s^2+4)^2}\right]$

$= \frac{(s^2+4)^2(-2s)-(4-s^2)\left[2(s^2+4)(2s)\right]}{(s^2+4)^4}$

$= \frac{(s^4+16+8s^2)(-2s)-(4-s^2)(4s^3+16s)}{(s^2+4)^4}$

$= \frac{-2s^5-32s-16s^3-\cancel{16s^3}-64s+4s^5+\cancel{16s^3}}{(s^2+4)^4}$

$= \frac{2s^5-96s-16s^3}{(s^2+4)^4} = 2s(s^4+48-8s^2)$

8) $L\{te^{-2t}\cos 3t\}$

$L\{\cos 3t\} = \frac{s}{s^2+9}$

$L\{t\cos 3t\} = (-1)\frac{d}{ds}\left[\frac{s}{s^2+9}\right]$

$= (-1)\left[\frac{s^2+9-2s^2}{(s^2+9)^2}\right]$

$= \frac{s^2-9}{(s^2+9)^2}$

$L\{te^{-2t}\cos 3t\} = \frac{(s+2)^2-9}{\left[(s+2)^2+9\right]^2}$

9) $L\{$

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* Evalu

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9) $L\{t(3\sin 2t - 2\cos 2t)\}$

$$L\{3\sin 2t - 2\cos 2t\} = \frac{3\times 2}{s^2+4} - \frac{2\times s}{s^2+4}$$

$$= \frac{6}{s^2+4} - \frac{2s}{s^2+4}$$

$$L\{t(3\sin 2t - 2\cos 2t)\} = (-1)\cdot\frac{d}{ds}\left[\frac{6}{s^2+4} - \frac{2s}{s^2+4}\right]$$

$$= (-1)\left[\left(\frac{-12s}{(s^2+4)^2}\right) - \left(\frac{(s^2+4)2 - 2s(2s)}{(s^2+4)^2}\right)\right]$$

$$= (-1)\left[\left(\frac{-12s}{(s^2+4)^2}\right) - \left(\frac{2s^2+8-4s^2}{(s^2+4)^2}\right)\right]$$

$$= \frac{(-1)}{(s^2+4)^2}\left[-12s - 2s^2 - 8 + 4s^2\right]$$

$$= \frac{(-1)}{(s^2+4)^2}\left[2s^2 - 12s - 8\right]$$

$$= \frac{12s + 8 - 2s^2}{(s^2+4)^2}$$

* Evaluate $\int_0^\infty te^{-2t}\sin t\,dt$

$$\int_0^\infty e^{-st} f(t)\,dt = L\{f(t)\}$$

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$L\{f(t)\} = \bar{f}(s)$

$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \bar{f}(s)\,ds$

* Find $\int_0^\infty \frac{\sin mt}{t}\,ds$

$L\{\sin mt\} = \frac{m}{s^2+m^2}$

$L\left\{\frac{\sin mt}{t}\right\} = \int_s^\infty \frac{m}{s^2+m^2}\,ds$

$= \left[m \cdot \frac{1}{m} \tan^{-1}(s/m)\right]_s^\infty$

$= \pi/2 - \tan^{-1}(s/m) = \bar{f}(s)$

$L\left\{\frac{\sin mt}{t}\right\} = \pi/2 - \tan^{-1}(s/m) = \bar{f}(s)$

$\int_0^\infty e^{-st} \cdot \frac{\sin mt}{t}\,dt = \pi/2 - \tan^{-1}(s/m)$

$s \to 0$

$\int_0^\infty \frac{\sin mt}{t}\,dt = \pi/2 - 0 = \pi/2$ (if $m>0$)

$-\pi/2$ (if $m<0$)

$\left[\because \tan^{-1}(s/m) = \pi/2 \text{ if } m>0,\ -\pi/2 \text{ if } m<0\right]$

* Find $L\left\{\int_0^t \frac{e^t \sin t}{t}\,dt\right\}$

$L\{\sin t\} = \frac{1}{s^2+1}$

$L\left\{\frac{\sin t}{t}\,dt\right\} = \int_s^\infty \frac{1}{s^2+1}\,ds$

$= \left[\tan^{-1}(s)\right]_s^\infty$

$= \tan^{-1}(\infty) - \tan^{-1}(s)$

$= \pi/2 - \tan^{-1}(s)$

$= \cot^{-1}(s)$

$L\left\{\frac{e^t \sin t}{t}\,dt\right\} = \cot^{-1}(s-1)$

$\left[\because \text{If } L\{f(t)\} = \bar{f}(s),\ L\left\{\int_0^t f(t)\right\} = \frac{1}{s}\bar{f}(s)\right]$

$L\left\{\int_0^t \frac{e^t \sin t}{t}\,dt\right\} = \frac{1}{s}\cot^{-1}(s-1)$

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* $L\{f(t)\} = \bar{f}(s)$ and $f(t) = e^{-2t}\sin 4t$ then find $L\{f''(t)\}$

$$L\{\sin 4t\} = \frac{4}{s^2+16}$$

$$L\{e^{-2t}\sin 4t\} = \frac{4}{(s+2)^2+16} = \frac{4}{s^2+4s+20}$$

$$\bar{f}(s) = \frac{4}{s^2+4s+20}$$

$f(0) = 0$

$f'(0) = 4 - 0 = 4$

$$f(t) = e^{-2t}\sin 4t$$

$$f'(t) = 4e^{-2t}\cos 4t - 2e^{-2t}\sin 4t$$

$$L\{f''(t)\} = s^2\bar{f}(s) - s f(0) - f'(0)$$

$$= s^2 \cdot \frac{4}{s^2+4s+20} - 4$$

$$= \frac{4s^2}{s^2+4s+20} - 4$$

$$= \frac{-16s-80}{s^2+4s+20} \quad //$$

* $L\left\{\int_0^t e^{-t}\cos t \, dt\right\}$

$$L\{\cos t\} = \frac{s}{s^2+1}$$

$$L\{e^{-t}\cos t\} = \frac{(s+1)}{(s+1)^2+1} = \frac{s+1}{s^2+2s+2}$$

$$L\left\{\int_0^t e^{-t}\cos t \, dt\right\} = \frac{s+1}{s(s^2+2s+2)} \quad //$$

* <u>Convolution theorem</u>

If $L^{-1}\{\bar{f}(s)\} = f(t)$ and $L^{-1}\{\bar{g}(s)\} = g(t)$ then

$$L^{-1}\{\bar{f}(s)\,\bar{g}(s)\} = \int_0^t f(u)\, g(t-u)\, du$$

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Proof :-

We have to show

$$L^{-1}\{\bar{f}(s).\bar{g}(s)\} = \int_0^t f(u)\, g(t-u)\, du = \phi(t) \quad \text{(say)}$$

(or) $L\{\phi(t)\} = \bar{f}(s).\bar{g}(s)$

$$L\{\phi(t)\} = \int_0^\infty e^{-st}\phi(t)\, dt = \int_0^\infty e^{-st}\left\{\int_0^t f(u)\, g(t-u)\, du\right\} dt$$

$$L\{\phi(t)\} = \int_{u=0}^{\infty}\int_{t=u}^{\infty}\left\{e^{-st} f(u)\, g(t-u)\, dt\right\} du$$

$$= \int_{u=0}^{\infty} f(u)\, e^{-su}\int_{t=u}^{\infty}\left\{e^{-s(t-u)} g(t-u)\, dt\right\} du$$

$$= \int_0^\infty e^{-su} f(u)\left\{\int_0^\infty e^{-sv} g(v)\, dv\right\} du$$

$$L\{\phi(t)\} = \bar{g}(s)\int_0^\infty e^{-su} f(u)\, du = \bar{f}(s).\bar{g}(s)$$

$$L\left\{\int_0^t f(u)\, g(t-u)\, du\right\} = \bar{f}(s).\bar{g}(s) \quad \text{(or)}$$

$$L^{-1}\{\bar{f}(s).\bar{g}(s)\} = \int_0^t f(u)\, g(t-u)\, du$$

* Find $L^{-1}\left\{\frac{1}{s(s^2+4)}\right\}$.

$$L^{-1}\left\{\frac{1}{s(s^2+2^2)}\right\}$$

We have $L^{-1}\{1/s\} = 1$ $\quad L^{-1}\left\{\frac{1}{s^2+4}\right\} = \frac{\sin 2u}{2}$

By convolution theorem

$$L^{-1}\left\{\frac{1}{s}\cdot\frac{1}{s^2+4}\right\} = \int_0^t \frac{\sin 2u}{2}\cdot 1\, du$$

$$= \left[\frac{-\cos 2u}{4}\right]_0^t$$

$$= \frac{-1}{4}\left[\cos 2t - 1\right]$$

$$= \frac{1}{4}\left[1-\cos 2t\right]$$

* Find

* Find

1) L^{-1}

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* Find $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$

$$= L^{-1}\{\cos au \cos bt\}$$

$$= \int_0^t \cos au \cos b(t-u)\, du$$

$$= \int_0^t \frac{1}{2}\left[\cos(au + bt - bu) + \cos(au - bt + bu)\right] du$$

$$= \frac{1}{2}\int_0^t \cos(au + bt - bu)\, du + \frac{1}{2}\int_0^t \cos(au - bt + bu)\, du$$

$$= \frac{1}{2}\int_0^t \cos(bt + (a-b)u)\, du + \frac{1}{2}\int_0^t \cos[(a+b)u - bt]\, du$$

$$= \frac{1}{2}\left[\frac{\sin[bt+(a-b)u]}{(a-b)} + \frac{\sin(a+b)u - bt}{(a+b)}\right]_0^t$$

$$= \frac{1}{2}\left[\frac{\sin at}{a-b} + \frac{\sin at}{a+b} - \frac{\sin bt}{a-b} + \frac{\sin bt}{a+b}\right]$$

$$= \frac{1}{2}\left[\frac{(a+b)(\sin at - \sin bt) + (a-b)(\sin at - \sin bt)}{(a-b)(a+b)}\right]$$

$$= \frac{1}{2}\left[\frac{2(a\sin at - b\sin bt)}{a^2-b^2}\right]$$

$$= \frac{a\sin at - b\sin bt}{a^2-b^2}$$

* Find inverse $[L^{-1}]$

1) $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\} = L^{-1}\left\{\frac{s}{(s^2+a^2)} \cdot \frac{1}{(s^2+a^2)}\right\}$

$$L^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos au$$

$$L^{-1}\left\{\frac{1}{s^2+a^2}\right\} = \frac{\sin au}{a}$$

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* Laplace transform can be used to solve ordinary as well as partial diff eqn's. We shall apply this method to solve ordinary linear D.E with constant co-efficients. The advantage of this method is that it gives the particular solution directly without the necessity of first finding general sol^n & then evaluating arbitrary constants.

Procedure:-

1) Take laplace transform on both sides of the given D.E using initial conditions.

2) This gives the algebraic eqn

2) solve the algebraic eqn to get $\bar{y}$ in terms of s

3) Take inverse laplace transform on both sides. This gives y as a function of t which is the required sol^n

1) solve the D.E $\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = 0$

where $y=1$, $\frac{dy}{dt} = 2$ and $\frac{d^2y}{dt^2} = 2$ at $t=0$

$$L\left\{\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y\right\} = 0$$

$$\Rightarrow L\left\{\frac{d^3y}{dt^3}\right\} + 2L\left\{\frac{d^2y}{dt^2}\right\} - 2\left\{\frac{dy}{dt}\right\} - 2L\{y\} = 0$$

$$\left[s^3\bar{y} - s^2y(0) - sy'(0) - y''(0)\right] + 2\left[s^2\bar{y} - sy(0) - y'(0)\right] - \left[s\bar{y} - y(0)\right] - 2\bar{y} = 0$$

$$(s^3+2s^2-s-2)\bar{y} - s^2 - 2s - 2 - 2s - 4 + 1 = 0.$$

$$\Rightarrow (s^3+2s^2-s-2)\bar{y} = s^2+4s+5$$

$$\Rightarrow \bar{y} = \frac{s^2+4s+5}{s^3+2s^2-s-2}$$

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$s^3+2s^2-s-2 = (s-1)(s+1)(s+2)$

	1	2	-1	-2
$s=1$	0	1	3	2
	1	3	2	0

$(s-1)(s^2+3s+2)$

$$\frac{s^2+4s+5}{s^3+2s^2-s-2} = \frac{s^2+4s+5}{(s-1)(s+1)(s+2)}$$

$$= \frac{A}{s-1} + \frac{B}{s+1} + \frac{C}{s+2} = \frac{A(s+1)(s+2)+B(s-1)(s+2)+C(s-1)(s+1)}{(s-1)(s+1)(s+2)}$$

$\cancel{= A(s^2+}$

$s=1 \Rightarrow 10 = A(3)(2)$

$\Rightarrow A = 5/3$

$s=-1 \Rightarrow -2B = 2 \Rightarrow B = -1$

$s=-2 \Rightarrow 3C = 1 \Rightarrow C = 1/3$

$$\bar{y} = \frac{5}{3}\cdot\frac{1}{(s-1)} - \frac{1}{s+1} + \frac{1}{3}\left(\frac{1}{s+2}\right)$$

$$L^{-1}\{\bar{y}\} = \frac{5}{3}L^{-1}\left\{\frac{1}{s-1}\right\} - L^{-1}\left\{\frac{1}{s+1}\right\} + \frac{1}{3}L^{-1}\left\{\frac{1}{s+2}\right\}$$

$$\boxed{y(t) = \frac{5}{3}e^{t} - e^{-t} + \frac{1}{3}e^{-2t}}$$

2) $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-t}$ $\quad y(0)=0$ & $y'(0)=0$

$$L\left\{\frac{d^2y}{dt^2}\right\} + 4L\left\{\frac{dy}{dt}\right\} + 4\{y\} = L\{e^{-t}\}$$

$$\Rightarrow \left[s^2\bar{y} - sy(0) - y'(0)\right] + 4\left[s\bar{y} - y(0)\right] + 4\bar{y} = \frac{1}{s+1}$$

$$\Rightarrow \bar{y}(s^2+4s+4) - 0 - 0 - 0 = \frac{1}{s+1}$$

$$\Rightarrow \bar{y} = \frac{1}{(s+1)(s^2+4s+4)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+4s+4}$$

$$= \frac{A(s^2+4s+4) + (Bs+C)(s+1)}{(s+1)(s^2+4s+4)}$$

$s=-1 \Rightarrow A(1-4+4) = 1 \Rightarrow A = 1$

$$\Rightarrow \bar{y} = \frac{1}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

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3) $L\left\{\frac{d^2x}{dt^2}+2\frac{dx}{dt}+5x\right\} = L\{e^{-t}\sin t\}$ with $x(0)=0$, $\frac{dx}{dt}(0)=1$

$$L\left\{\frac{d^2x}{dt^2}+2\frac{dx}{dt}+5x\right\} = L\{e^{-t}\sin t\}$$

$$s^2\bar{x} - sx(0) - x'(0) + 2[s\bar{x} - x(0)] + 5\bar{x} = \frac{1}{1+(s+1)^2}$$

$$(s^2+2s+5)\bar{x} - 0 - 1 - 0 = \frac{1}{s^2+2s+2}$$

$$(s^2+2s+5)\bar{x} = \frac{1}{(s^2+2s+2)} + 1$$

$$\bar{x} = \frac{1}{(s^2+2s+2)(s^2+2s+5)} + \frac{1}{(s^2+2s+5)}$$

$$= \frac{1}{3}\left[\frac{1}{s^2+2s+2} - \frac{1}{s^2+2s+5}\right] + \frac{1}{s^2+2s+5}$$

$$= \frac{1}{3}\left[\frac{1}{(s+1)^2+1}\right] - \frac{1}{3}\left[\frac{1}{(s+1)^2+2^2}\right] + \frac{1}{(s+1)^2+2^2}$$

$$L^{-1}\{\bar{x}\} = \frac{1}{3}L^{-1}\left\{\frac{1}{(s+1)^2+1^2}\right\} - \frac{1}{3}L^{-1}\left\{\frac{1}{(s+1)^2+2^2}\right\} + L^{-1}\left\{\frac{1}{(s+1)^2+2^2}\right\}$$

$$= \frac{1}{3}e^{-t}\sin t - \frac{1}{3}\frac{e^{-t}\sin 2t}{2} + \frac{e^{-t}\sin 2t}{2}$$

$$= \frac{1}{3}e^{-t}\sin t + \frac{2}{6}e^{-t}\sin 2t$$

* unit impulse function (Dirac- delta function)

It is defined as

$$\delta_\varepsilon(t-a) = \begin{cases} 0 & \text{for } t < a \\ 1/\varepsilon & \text{for } a \le t \le a+\varepsilon \\ 0 & t > a \end{cases}$$

$$L\{\delta_\varepsilon(t-a)\} = \int_0^\infty \delta_\varepsilon(t-a)\, e^{-st}\, dt$$

$$= \int_0^a e^{-st}(0)\,dt + \int_a^{a+\varepsilon} e^{-st}\cdot\frac{1}{\varepsilon}\,dt + \int_{a+\varepsilon}^\infty e^{-st}\cdot(0)\,dt$$

$$= 0 + \left(\frac{1}{\varepsilon}\frac{e^{-st}}{-s}\right)_a^{a+\varepsilon} + 0$$

$$= \frac{-1}{s\varepsilon}\left[e^{-s(a+\delta)} - e^{-sa}\right]$$

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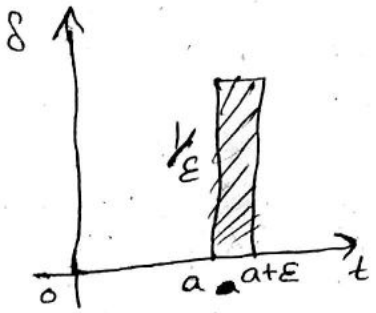
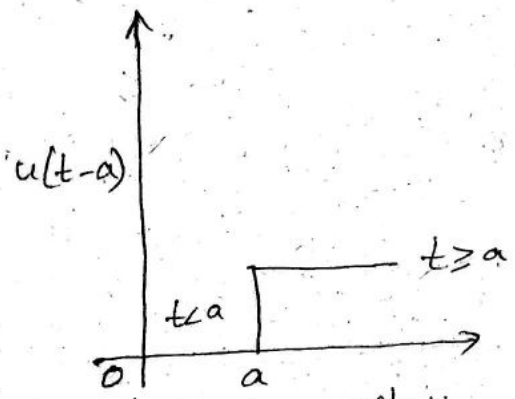
$$= \frac{e^{-as}}{s\varepsilon}\left[1-e^{-s\varepsilon}\right]$$



UNIT-STEP FUNCTION

→ USF is defined as

$$u(t-a) = \begin{cases} 0 & \text{for } t<a \\ 1 & \text{for } t\geq a \end{cases}$$



$$L\{u(t-a)\} = \int_0^\infty u_{t-a} \cdot e^{-st}\,dt$$

$$= \int_0^a 0 \cdot e^{-st}\,dt + \int_a^\infty 1 \cdot e^{-st}\,dt$$

$$= 0 + \left[\frac{e^{-st}}{-s}\right]_a^\infty$$

$$= \frac{-1}{s}\left[e^{-\infty} - e^{-as}\right] = \frac{e^{-as}}{s}$$

PERIODIC FUNCTION

A function $f(t)$ is said to be a P.F with period 'T' $f(t+T) = f(t)\ \forall t$

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* Find the laplace transform of periodic function $f(t)$ with period T

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

$$= \int_0^T e^{-st} f(t)\,dt + \int_T^{2T} e^{-st} f(t)\,dt + \int_{2T}^{3T} e^{-st} f(t)\,dt \ldots$$

let sub $t = u$, $t = u+T$, $t = u+2T$ .....

$$L\{f(t)\} = \int_0^T e^{-su} f(u)\,du + \int_0^T e^{-s(u+T)}\,du\, f(u+T) + \int_0^T e^{-s(u+2T)} f(u+2T)\,du$$

$$f(t) = f(t+T) = f(t+2T) = \ldots \quad \forall t$$

$$L\{f(t)\} = \int_0^T e^{-su} f(u)\,du + e^{-sT}\int_0^T e^{-su} f(u)\,du + e^{-2sT}\int_0^T e^{-su} f(u)\,du + \ldots$$

$$L\{f(t)\} = (1 + e^{-sT} + e^{-2sT} + \ldots)\int_0^T e^{-su} f(u)\,du$$

$$L\{f(t)\} = \frac{1}{(1-e^{-sT})}\int_0^T e^{-su} f(u)\,du$$

* Find the L.T of $f(t) = \begin{cases} t & 0 < t < c \\ 2c - t & c < t < 2c \end{cases}$ of period $2c$

$$L\{f(t)\} = \frac{1}{1-e^{-s2c}}\int_0^{2c} e^{-st} f(t)\,dt$$

$$= \frac{1}{1-e^{-2cs}}\left[\int_0^c e^{-st}\, t\,dt + \int_c^{2c} e^{-st}(2c-t)\,dt\right]$$

$$= \frac{1}{1-e^{-2cs}}\left[\left[t\left(\frac{e^{-st}}{-s}\right) - (1)\left(\frac{e^{-st}}{s^2}\right)\right]_0^c + \left[(2c-t)\left(\frac{e^{-st}}{-s}\right) - (-1)\left(\frac{e^{-st}}{s^2}\right)\right]_c^{2c}\right]$$

$$= \frac{1}{1-e^{-2cs}}\left[\cancel{-\frac{ce^{-cs}}{s}} - \frac{e^{-sc}}{s^2} + \frac{1}{s^2} + \frac{e^{-2cs}}{s^2} + \cancel{\frac{e^{-cs}c}{s}} - \frac{e^{-cs}}{s^2}\right]$$

$$= \frac{(1-2e^{-cs}+e^{-2cs})}{(1-e^{-2cs})\,s^2} = \frac{1}{s^2}\,\frac{(1+e^{-cs})^2}{(1-e^{-cs})(1+e^{-cs})} = \frac{(1-e^{-cs})}{s^2(1+e^{-cs})}$$

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* <u>Gamma functions:</u>

It is denoted by Γn and is defined as

$$\int_0^\infty e^{-x} \cdot x^{n-1}\, dx$$

$$\Gamma n = \int_0^\infty e^{-x} \cdot x^{n-1}\, dx$$

→ This integral is also known as Euler's integral of second kind. & it defines a function of 'n' for all +ve values of 'n'

<u>Note:</u> 1) $\Gamma 1 = 1$

2) $\Gamma n+1 = n\Gamma n$ (when $n>0$)

3) $\Gamma n = (n-1)!$ (n is an integer)

$$\Gamma n = \Gamma(n-1)+1 = (n-1)\Gamma n-1 = (n-1)\,\Gamma(n-2)+1$$

$$= (n-1)(n-2)\Gamma n-2 \quad = (n-1)!$$

4) $\Gamma \frac{1}{2} = \sqrt{\pi}$

5) $\Gamma \frac{3}{2} = \Gamma \frac{1}{2}+1 = \frac{1}{2}\Gamma \frac{1}{2} = \frac{\sqrt{\pi}}{2}$

6) $\Gamma \frac{5}{2} = \Gamma \frac{3}{2}+1 = \frac{3}{2}\Gamma \frac{3}{2} = \frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}$

7) $\int_0^\infty e^{-ky} \cdot y^{n-1}\, dy = \frac{\Gamma n}{k^n}$

$ky = x$

$y = x/k$

$dy = dx/k$

$$= \frac{1}{k^n}\int_0^\infty e^{-x} \cdot x^{n-1}\, dx = \frac{\Gamma n}{k^n}$$

1) Find $\int_0^1 (\log$

$\int_0^1 (\log$

Let

$= \int_\infty^0$

$= -$

2) S.T $\int_0^\infty e^{-y^{1/n}}$

$\int_0^\infty e$

Let

$\int_0^\infty e^{-}$

3) Find

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1) Find $\int_0^1 (\log 1/y)^{n-1}\, dy$

$$\int_0^1 \left(\log \tfrac{1}{y}\right)^{n-1} dy$$

Let $\log 1/y = x \Rightarrow \frac{1}{y} = e^x$

$\Rightarrow y = e^{-x} \Rightarrow dy = -e^{-x}\, dx$

$$= \int_{\infty}^{0} x^{n-1}(-e^{-x}\, dx)$$

$$= -\int_{0}^{\infty} x^{n-1}(-e^{-x}\, dx) = \int_0^\infty e^{-x} \cdot x^{n-1}\, dx = \Gamma n$$

2) S.T $\int_0^\infty e^{-y^{1/n}}\, dy = n\Gamma n$ Hence calculate $\int_0^\infty e^{-y^2}\, dy$

$$\int_0^\infty e^{-y^{1/n}}\, dy$$

Let $y^{1/n} = x$

$\Rightarrow y = x^n$

$\Rightarrow dy = n x^{n-1}\, dx$

$$= \int_0^\infty e^{-x} \cdot n x^{n-1}\, dx$$

$$= n \int_0^\infty e^{-x} \cdot x^{n-1}\, dx = n\Gamma n$$

$\int_0^\infty e^{-y^2}\, dy$ $\quad 1/n = 2 \Rightarrow n = 1/2$

$$= \tfrac{1}{2}\Gamma\tfrac{1}{2} = \tfrac{1}{2}\sqrt{\pi}$$

3) Find $\int_{-\infty}^{\infty} e^{-k^2x^2}\, dx$

$k^2x^2 = y \Rightarrow x^2 = y/k^2$

$\Rightarrow x = \sqrt{y}/k$

$dx = \frac{1}{2\sqrt{y}\,k}\, dy.$

$$\int_{-\infty}^{\infty} e^{-k^2x^2}\, dx = 2\int_0^\infty e^{-k^2x^2}\, dx = 2\int_0^\infty e^{-y} \cdot \frac{1}{2\sqrt{y}\,k}\, dy$$

$$= \frac{1}{k}\int_0^\infty e^{-y} \cdot y^{-1/2}\, dy = \frac{1}{k}\int_0^\infty e^{-y} \cdot y^{\frac{1}{2}-1}\, dy$$

$$= \frac{1}{k}\Gamma\tfrac{1}{2} = \sqrt{\pi}/k \;//$$

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5) find $\int_0^\infty x^4 e^{-x}\,dx = \Gamma 5 = 4! = 24$

6) find $\int_0^\infty e^{-x^2} x^2\,dx$

$x^2 = y \Rightarrow x = \sqrt{y}$

$\Rightarrow dx = \frac{1}{2\sqrt{y}}\,dy$

$= \frac{1}{2}\int_0^\infty e^{-y} y\, y^{-1/2}\,dy$

$= \frac{1}{2}\int_0^\infty e^{-y} \cdot y^{1/2}\,dy$

$= \frac{1}{2}\int_0^\infty e^{-y} \cdot y^{\frac{3}{2}-1}\,dy$

$= \frac{1}{2}\Gamma\frac{3}{2} = \sqrt{\pi}/4$

7) find $\int_0^\infty \sqrt{x} \cdot e^{-x^3}\,dx$ $\qquad x^3 = t$

$x = t^{1/3}$

$= \int_0^\infty t^{1/6} \cdot e^{-t} \cdot \frac{1}{3} t^{-2/3}\,dt$ $\qquad dx = \frac{1}{3} t^{-2/3}\,dt$

$= \frac{1}{3}\int_0^\infty e^{-t} \cdot t^{-1/2}\,dt$

$= \frac{1}{3}\int_0^\infty e^{-t} \cdot t^{\frac{1}{2}-1}\,dt$

$= \frac{1}{3} \cdot \Gamma\frac{1}{2} = \sqrt{\pi}/3$

* S.T $\int_0^\infty \frac{x^c}{c^x}\,dx = \frac{\Gamma(c+1)}{(\log c)^{c+1}}$ $\quad (c > 0)$

$= \cancel{\int_0^\infty t^{1/c}}$ $\qquad x^c = t \Rightarrow x = \cancel{t^{1/c}}$

$dx = \cancel{\frac{1}{c} t^{\frac{1}{c}-1}}\,dt$

$= \int_0^\infty \left(\frac{y}{\log c}\right)^c e^{-y} \cdot \frac{1}{\log c}\,dy$ $\qquad \Rightarrow \cancel{c \log x = \log t}$

$c^x = e^y \Rightarrow x = \frac{y}{\log c}$

$= \frac{1}{(\log c)^{c+1}} \int_0^\infty e^{-y}\, y^{(c+1)-1}\,dy$ $\qquad c^x \log c\, dx = e^y\, dy$

$= \frac{\Gamma(c+1)}{(\log c)^{c+1}}$ $\qquad \Rightarrow dx = \frac{e^y}{c^x \log c}\,dy = \frac{1}{\log c}$

* $\Gamma\frac{1}{2} = \sqrt{\pi}$

$\Gamma\frac{1}{2} = \int_0^\infty e$

Let

$= \infty$

$\Gamma\frac{1}{2}$

$(\Gamma$

chang

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* $\Gamma\frac{1}{2} = \sqrt{\pi}$

$$\Gamma\frac{1}{2} = \int_0^\infty e^{-x} \cdot x^{\frac{1}{2}-1} dx = \int_0^\infty e^{-x} \cdot x^{-\frac{1}{2}} dx$$

Let $x = y^2 \Rightarrow dx = 2y\,dy$

$$= \int_0^\infty e^{-y^2} \cdot \frac{1}{y} 2y\,dy = 2\int_0^\infty e^{-y^2} dy$$

$$\Gamma\frac{1}{2} = 2\int_0^\infty e^{-x^2} dx$$

$$\left(\Gamma\frac{1}{2}\right)^2 = \left(2\int_0^\infty e^{-y^2} dy\right)\left(2\int_0^\infty e^{-x^2} dx\right)$$

$$= 4\int_0^\infty\int_0^\infty e^{-(x^2+y^2)} dx\,dy$$

changing into polar form

$x = r\cos\theta$, $y = r\sin\theta$ and $dx\,dy = r\,dr\,d\theta$

$$= 4\int_{\theta=0}^{\pi/2}\int_{r=0}^{\infty} e^{-r^2} r\,dr\,d\theta$$

$$= 4\int_0^\infty \left\{\int_0^{\pi/2} r e^{-r^2} d\theta\right\} dr$$

$$= 4\int_0^\infty r e^{-r^2} (\theta)_0^{\pi/2} dr$$

$$= 4\frac{\pi}{2}\int_0^\infty r e^{-r^2} dr$$

$r^2 = t \Rightarrow 2r\,dr = dt$
$\Rightarrow r\,dr = \frac{dt}{2}$

$$= 2\pi\int_0^\infty e^{-t} \frac{dt}{2}$$

$$= \pi\left[\frac{e^{-t}}{-1}\right]_0^\infty$$

$$= -\pi(0-1) = \pi$$

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→ Find laplace transform of periodic function which as defined in one period as follows

1) $f(t) = kt$ — $0 < t < 2\pi$

2) $f(t) = t^2$ — $0 < t < 2\pi$

$$L\{f(t)\} = \frac{1}{1-e^{-2\pi s}} \int_0^{2\pi} e^{-st} f(t)\, dt$$

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→ Simple problems of electric circuits

→ Current is rate of flow of electricity ($i = dq/dt$)

→ Voltage drop across resistor $V = IR$, ~~Across to~~

$$V = R\left(\frac{dq}{dt}\right)$$

→ Across inductor $V = L\frac{di}{dt} = L\frac{d^2q}{dt^2}$

→ " Capacitor $V = q/c$

→ The algebraic sum of voltage drop around any closed circuit = resultant emf in circuit.

Q→ An uncharged condenser of capacity c is charged by applying an emf $E\sin(t/\sqrt{Lc})$ through leads of self-inductance L & negligible resistance. P.T at any time t, the charge on one of the plates is $\frac{Ec}{2}\left[\sin\frac{t}{\sqrt{Lc}} - \frac{t}{\sqrt{Lc}}\cos\frac{t}{\sqrt{Lc}}\right]$

Sol: If q is the charge on ~~L~~ condenser then D.E of circuit is $L\frac{d^2q}{dt^2} + \frac{q}{c} = E\sin\frac{t}{\sqrt{Lc}}$

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