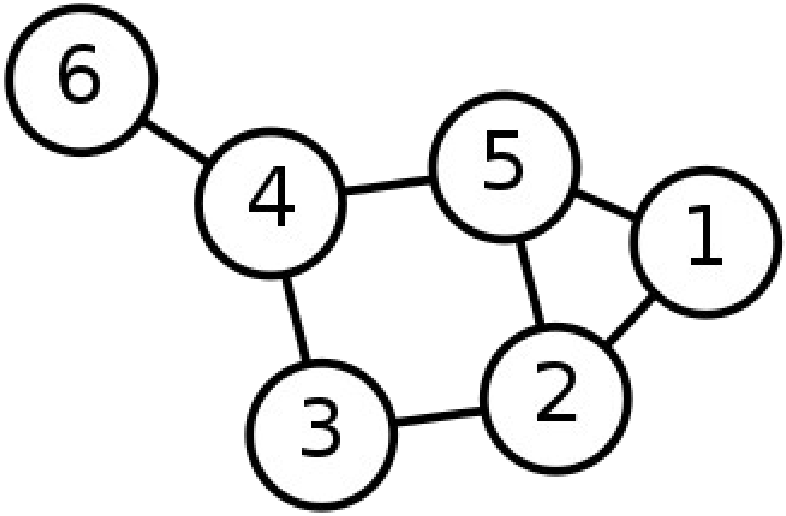
Dijkstra’s, Kruskals and Floyd-Warshall Algorithms

Single-Source Shortest Path Problem

<u>Single-Source Shortest Path Problem</u> - The problem of finding shortest paths from a source vertex *v* to all other vertices in the graph.



Dijkstra's algorithm

Dijkstra's algorithm - is a solution to the single-source shortest path problem in graph theory.

Works on both directed and undirected graphs. However, all edges must have nonnegative weights.

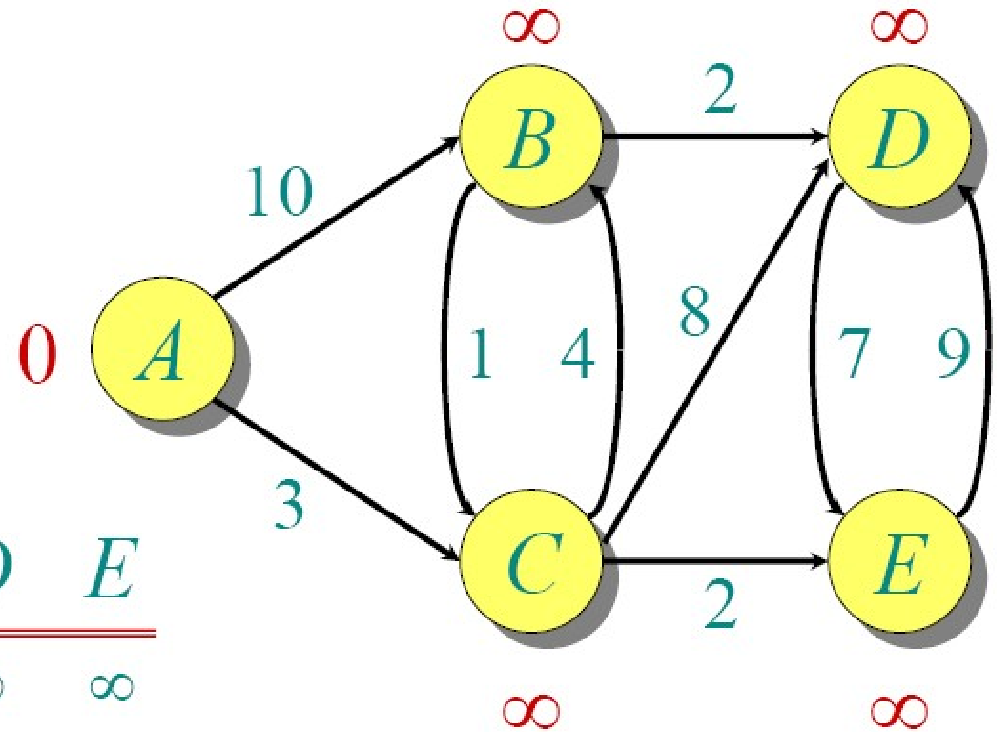
Approach: Greedy

Dijkstra's algorithm - Pseudocode

dist[s] ←0 (distance to source vertex is zero)
for all v ∈ V–{s}
do dist[v] ←∞ (set all other distances to infinity)
S←∅ (S, the set of visited vertices is initially empty)
Q←V (Q, the queue initially contains all vertices)
while Q ≠∅ (while the queue is not empty)
do u ← mindistance(Q,dist) (select the element of Q with the min. distance)
S←S∪{u} (add u to list of visited vertices)
for all v ∈ neighbors[u]
do if dist[v] > dist[u] + w(u, v) (if new shortest path found)
then d[v] ←d[u] + w(u, v) (set new value of shortest path)
(if desired, add traceback code)
return dist

Dijkstra Animated Example

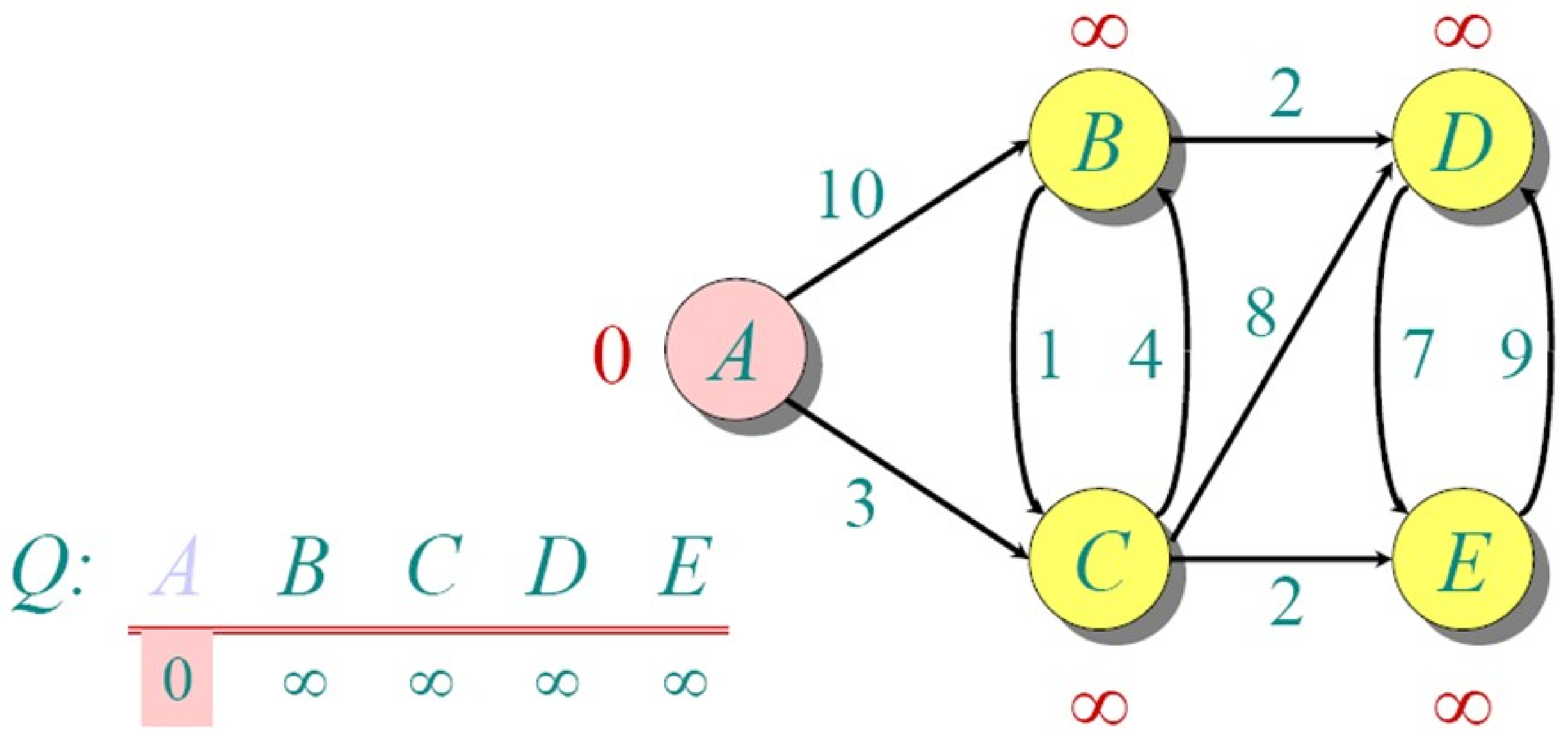
Initialize:



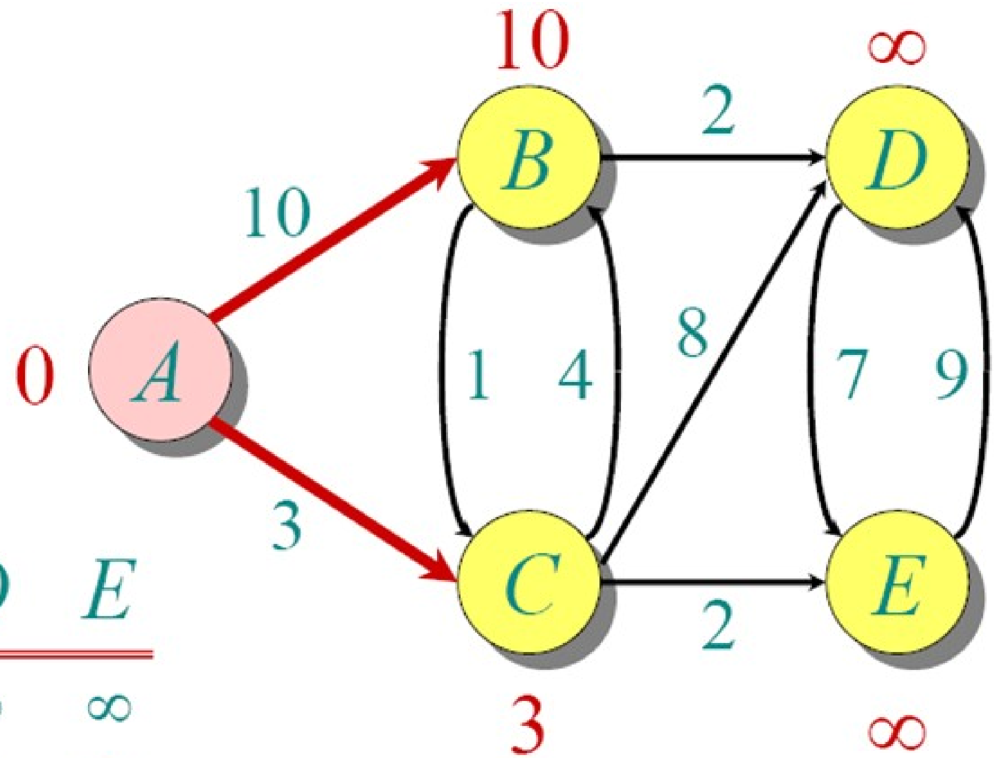
Q:	A	B	C	D	E
	0	∞	∞	∞	∞

S: {}

Dijkstra Animated Example



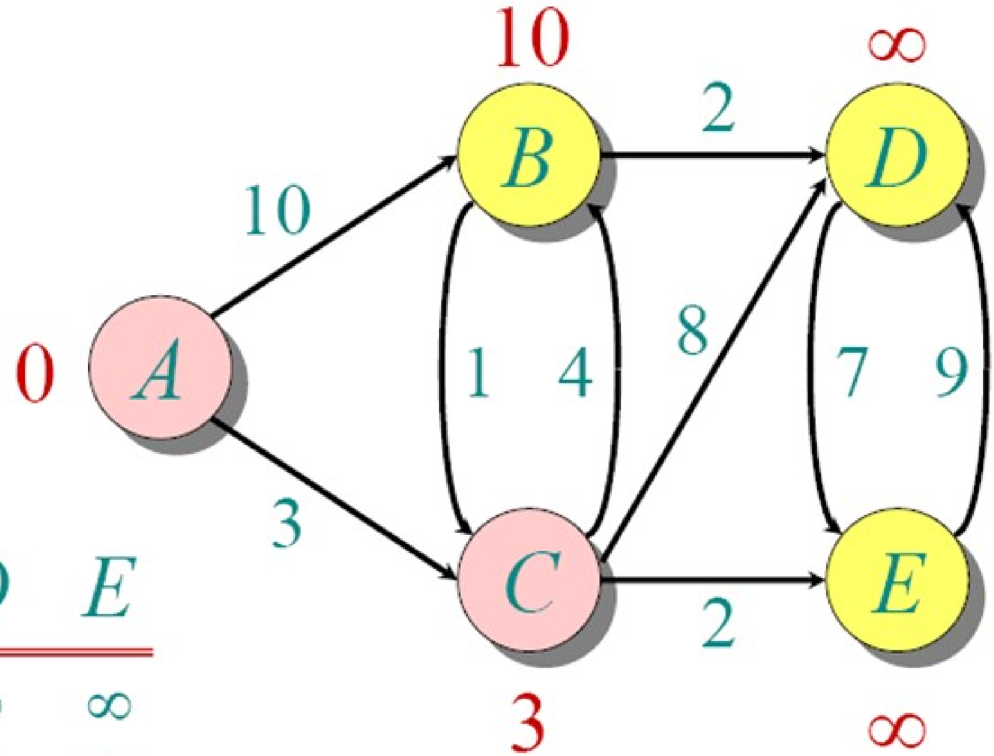
Dijkstra Animated Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞

S: { A }

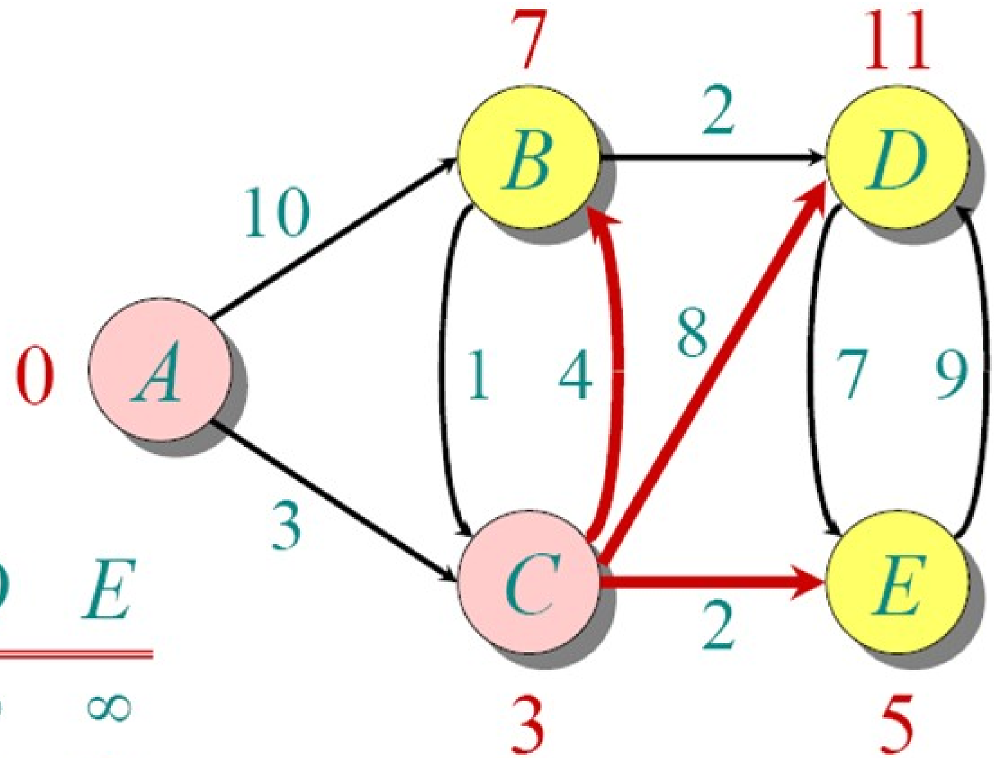
Dijkstra Animated Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞

S: { A, C }

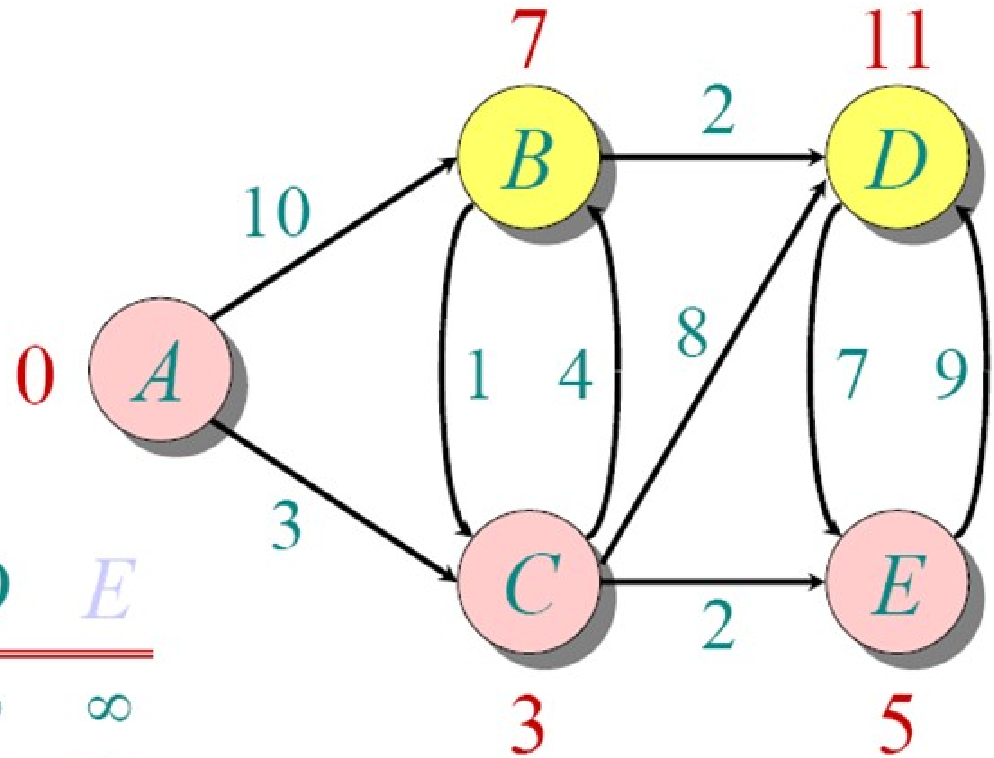
Dijkstra Animated Example



Q:	A	B	C	D	E
	$\boxed{0}$	∞	∞	∞	∞
		10	$\boxed{3}$	∞	∞
		7		11	5

S: { A, C }

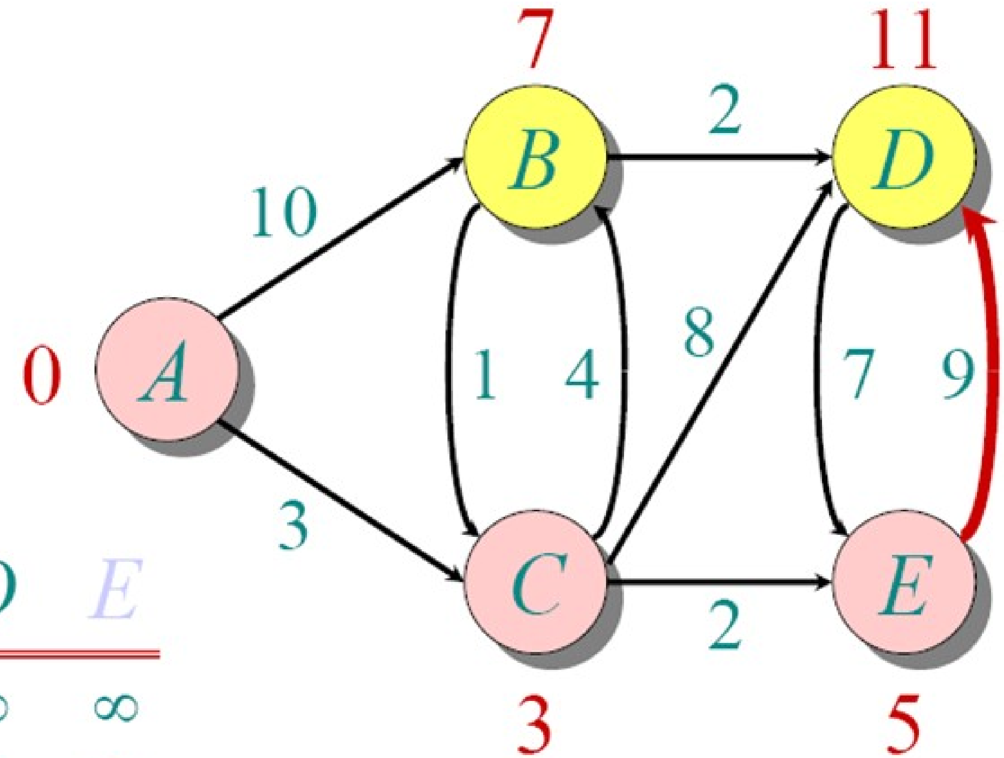
Dijkstra Animated Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5

S: { A, C, E }

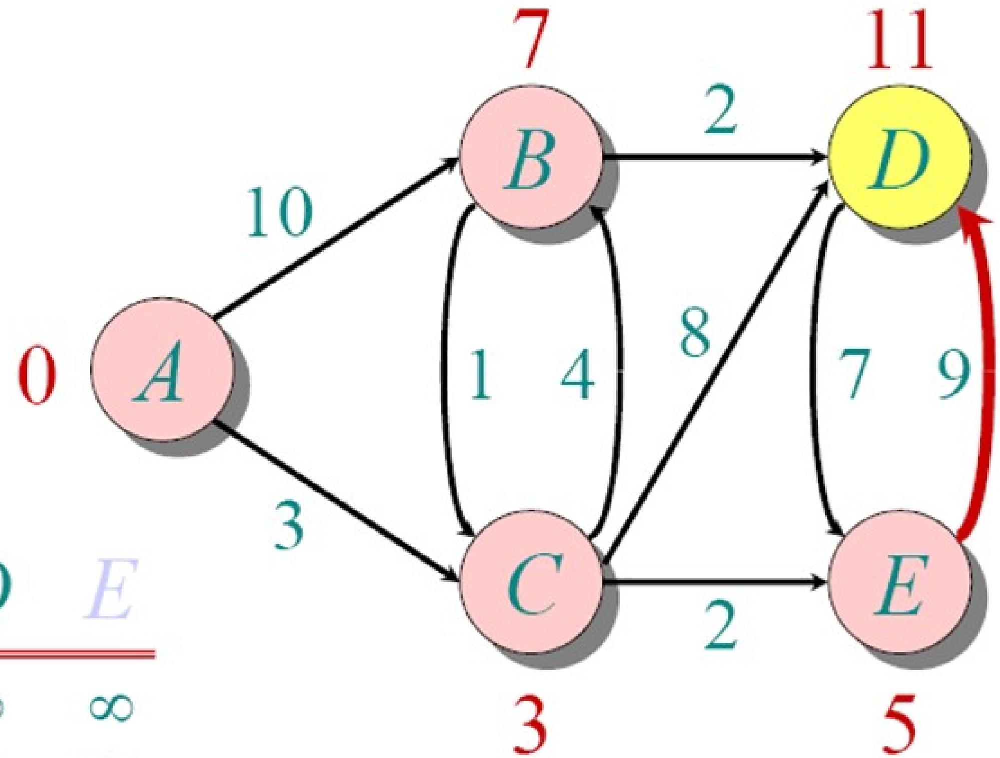
Dijkstra Animated Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	

S: { A, C, E }

Dijkstra Animated Example

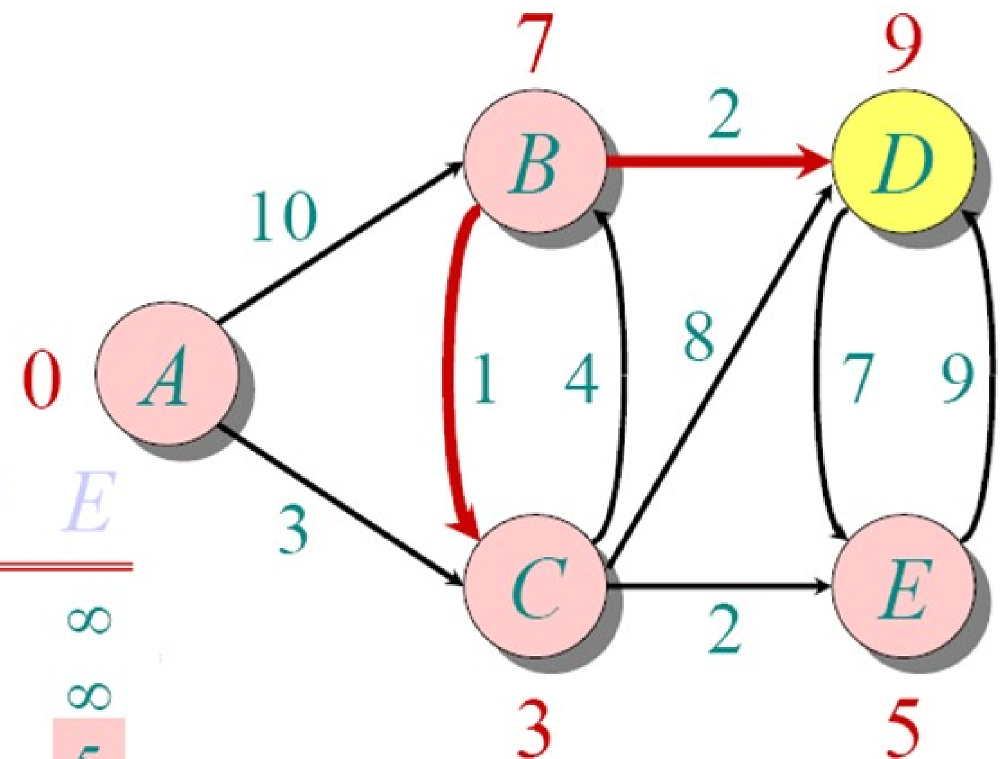


Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	

S: { A, C, E, B }

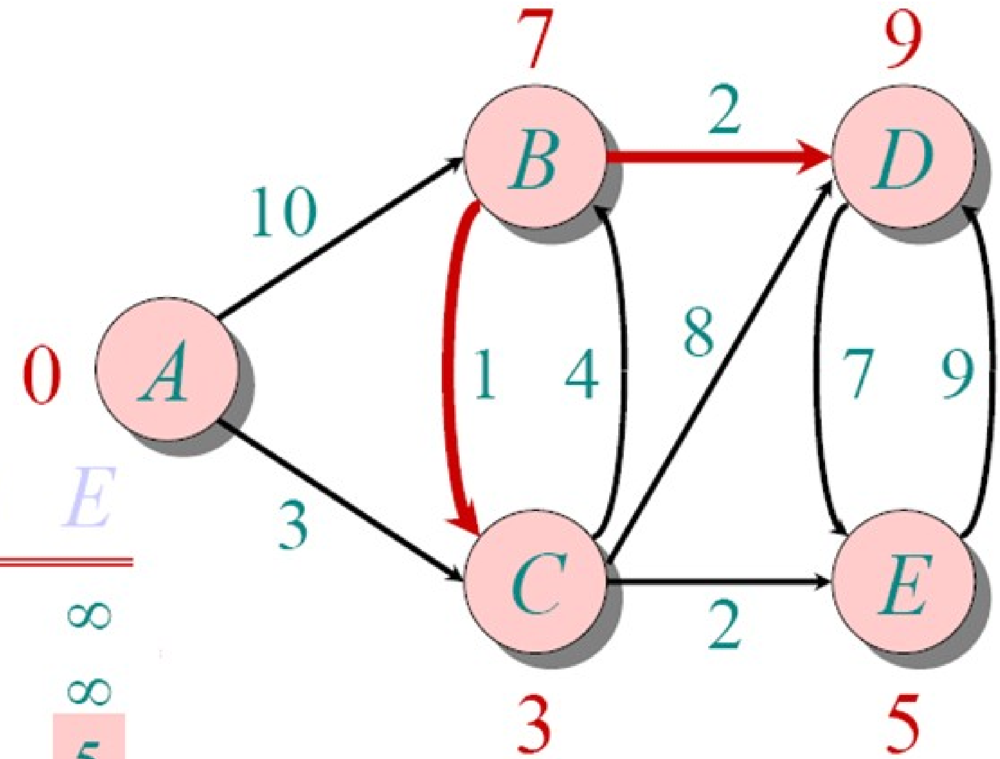
Dijkstra Animated Example

Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	
				9	



S: { A, C, E, B }

Dijkstra Animated Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	
				9	

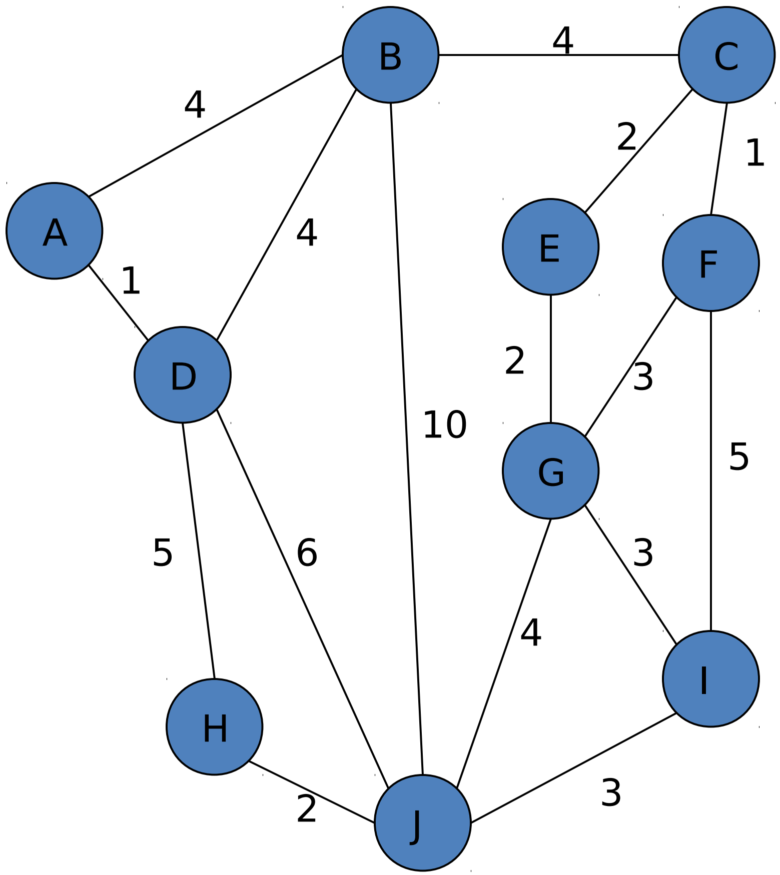
S: { A, C, E, B, D }

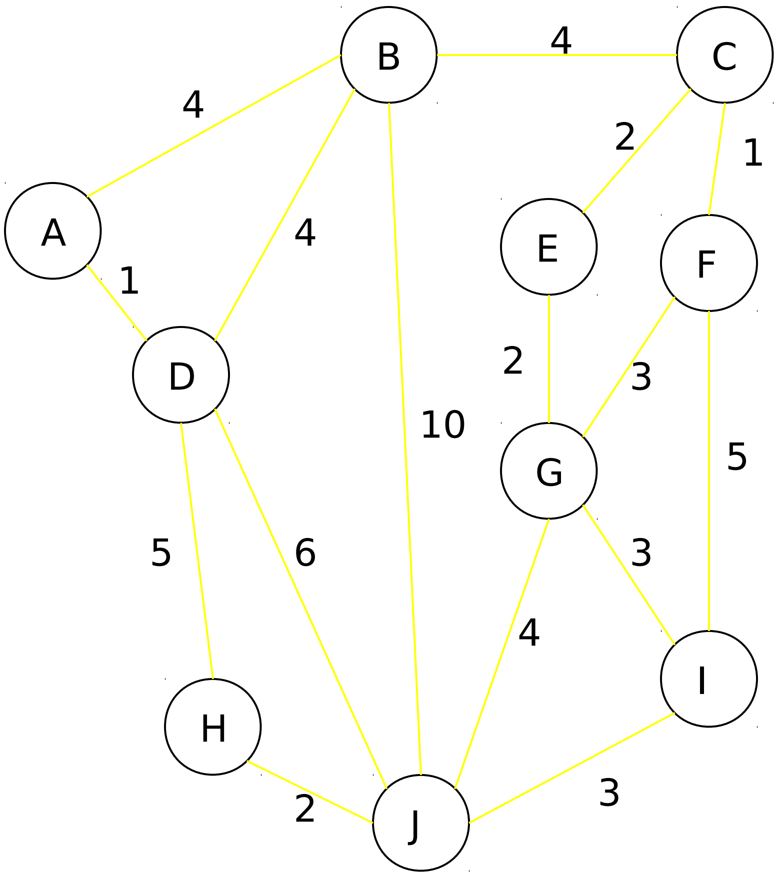
Kruskal’ Algorithm

Minimum Spanning Tree

Disjoint Sets

Complete Graph





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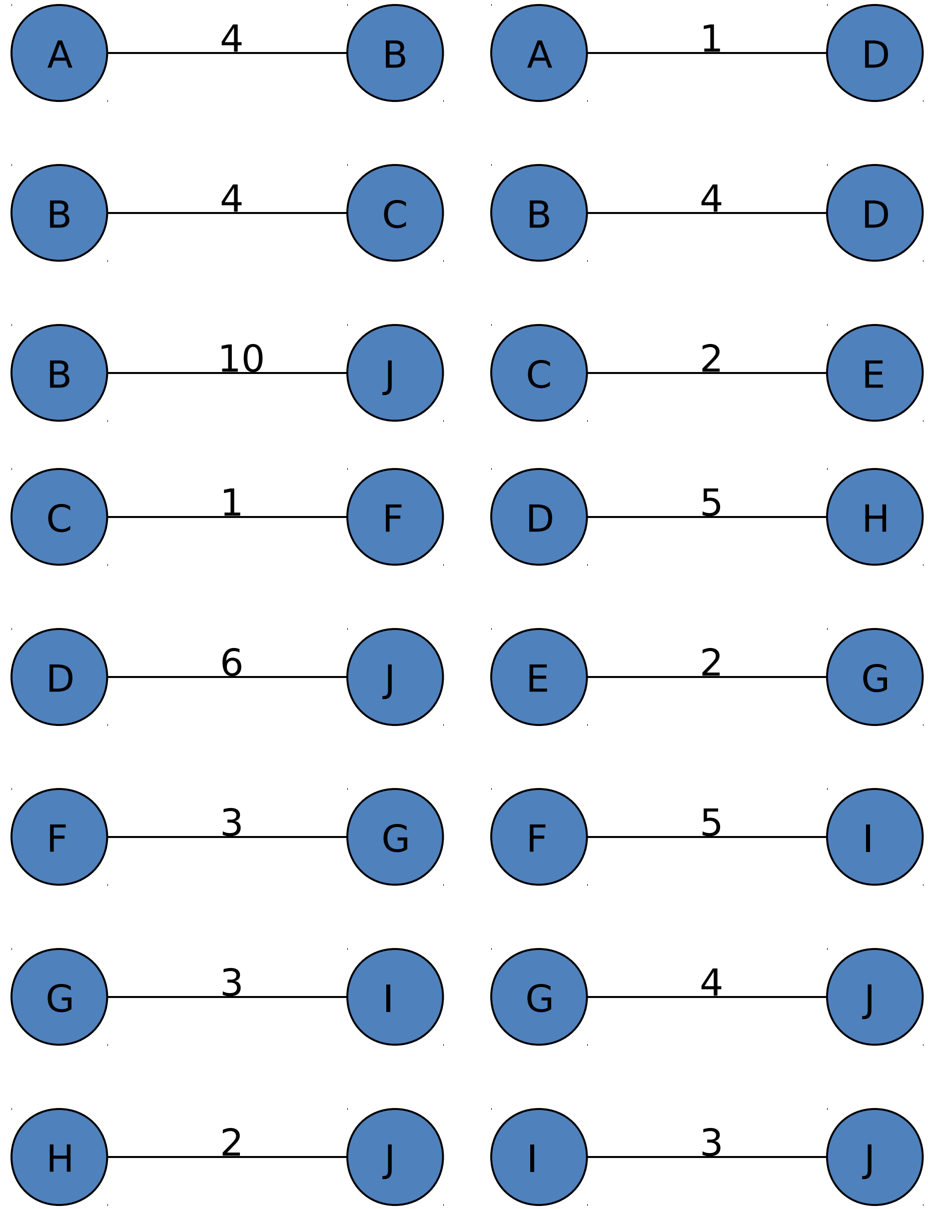

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C 2 E
C 1 F
D 5 H
D 6 J
E 2 G
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G 4 J
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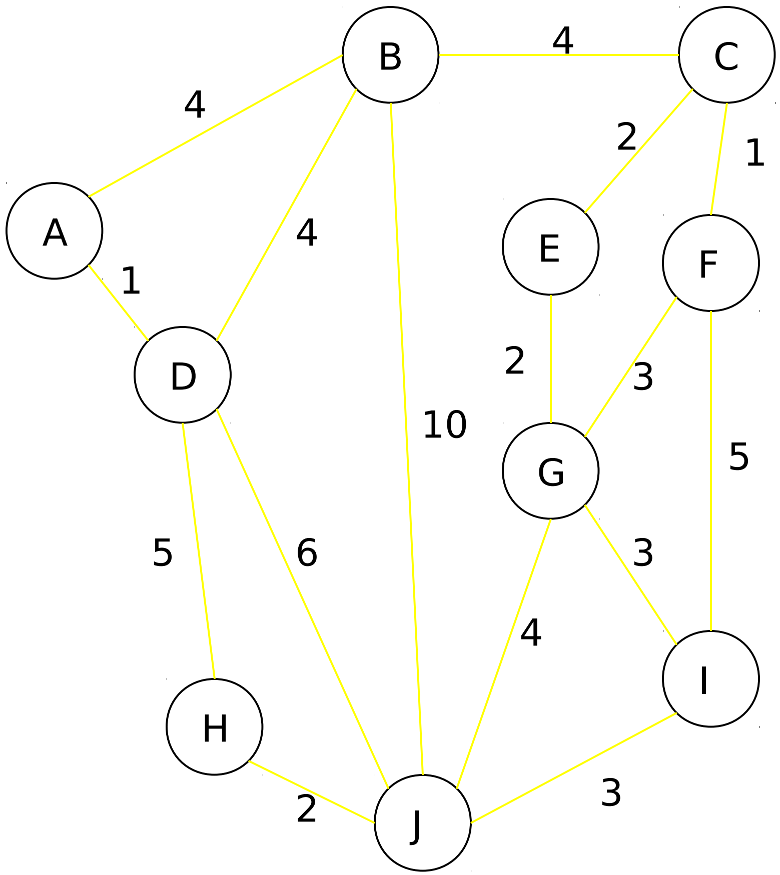
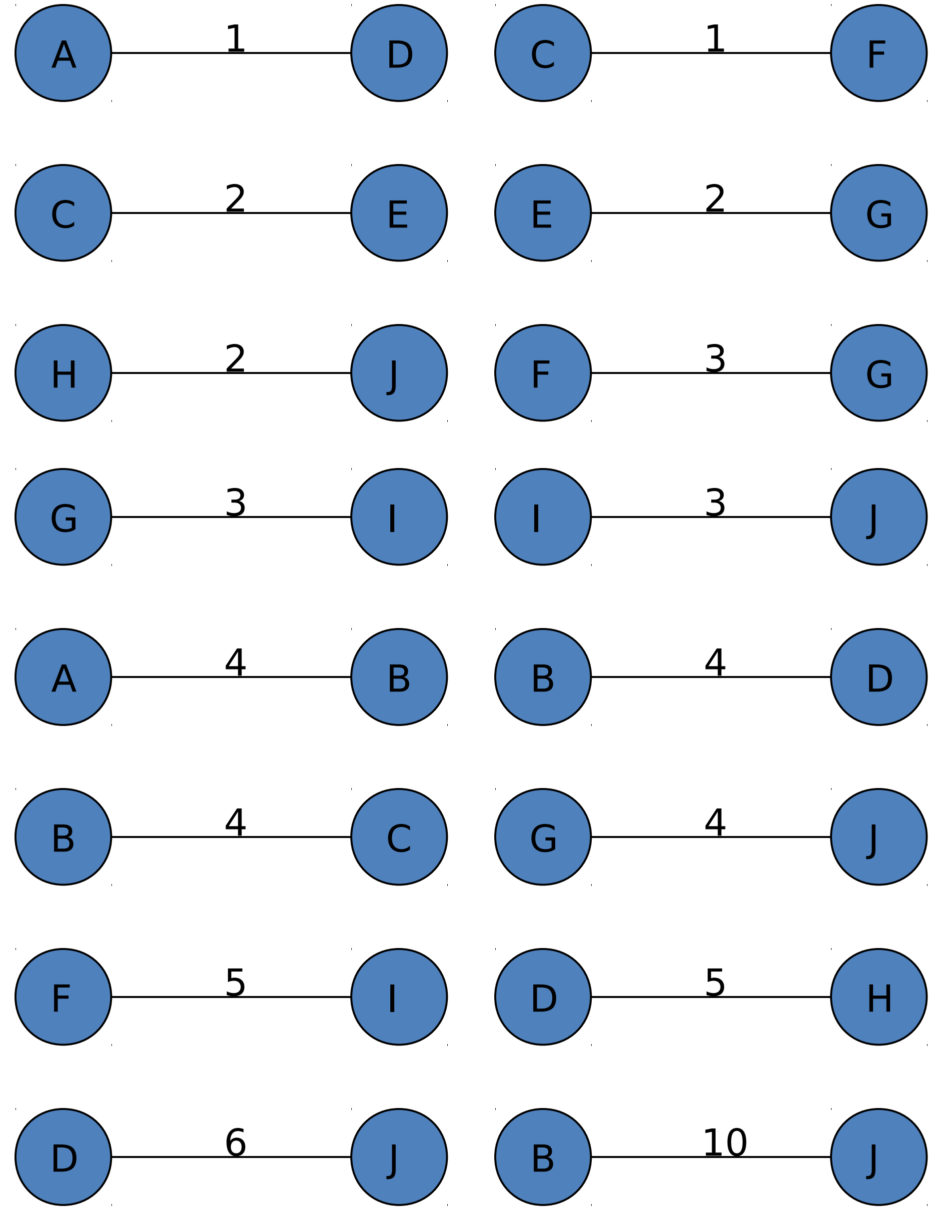
I 3 J

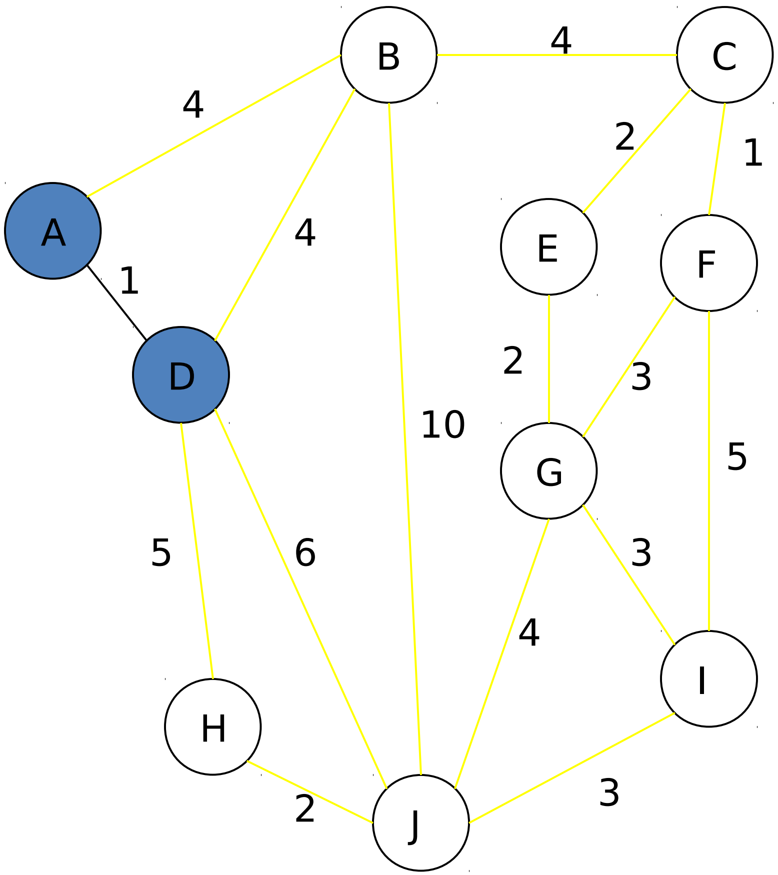
Sort Edges

(in reality they are placed in a priority queue - not sorted - but sorting them makes the algorithm easier to visualize)





Add Edge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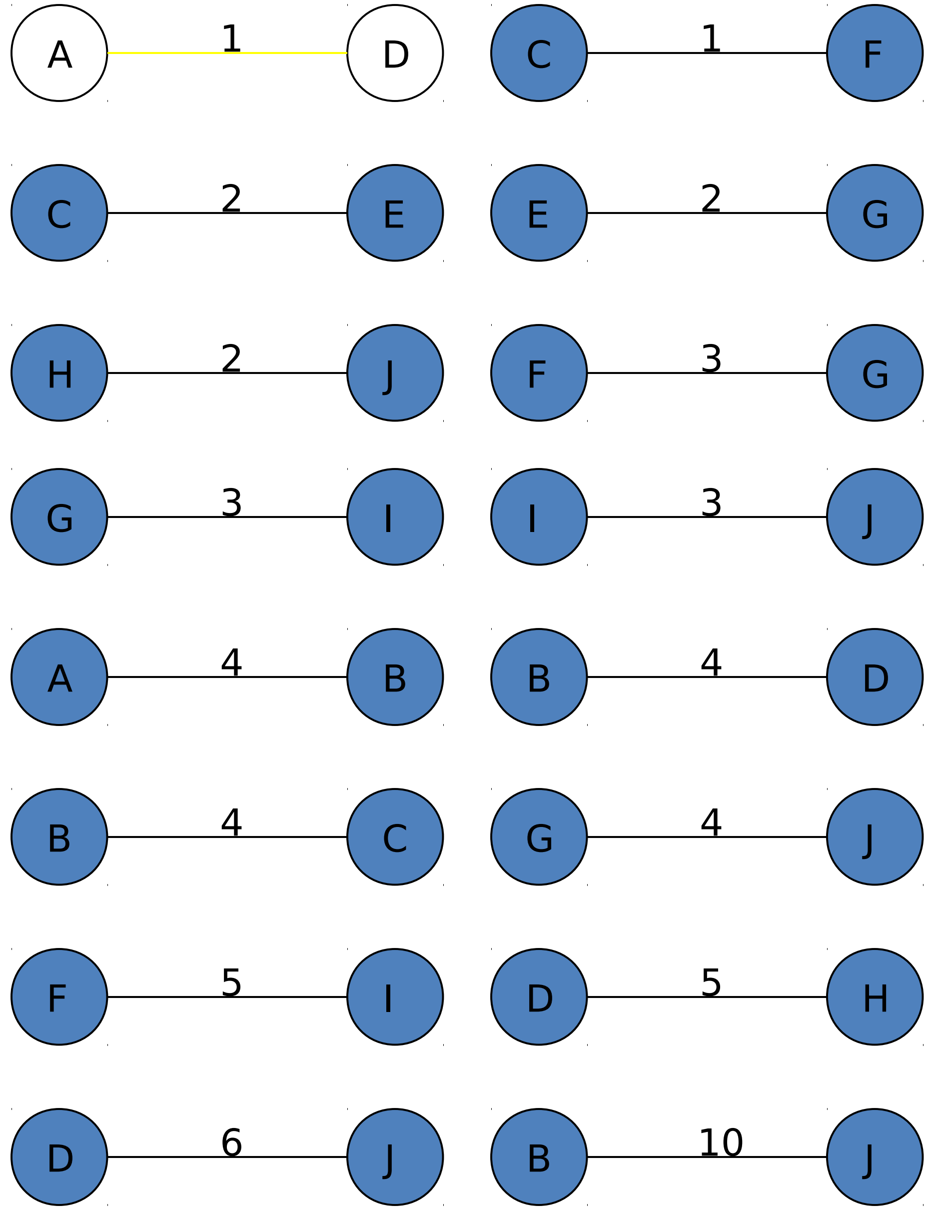
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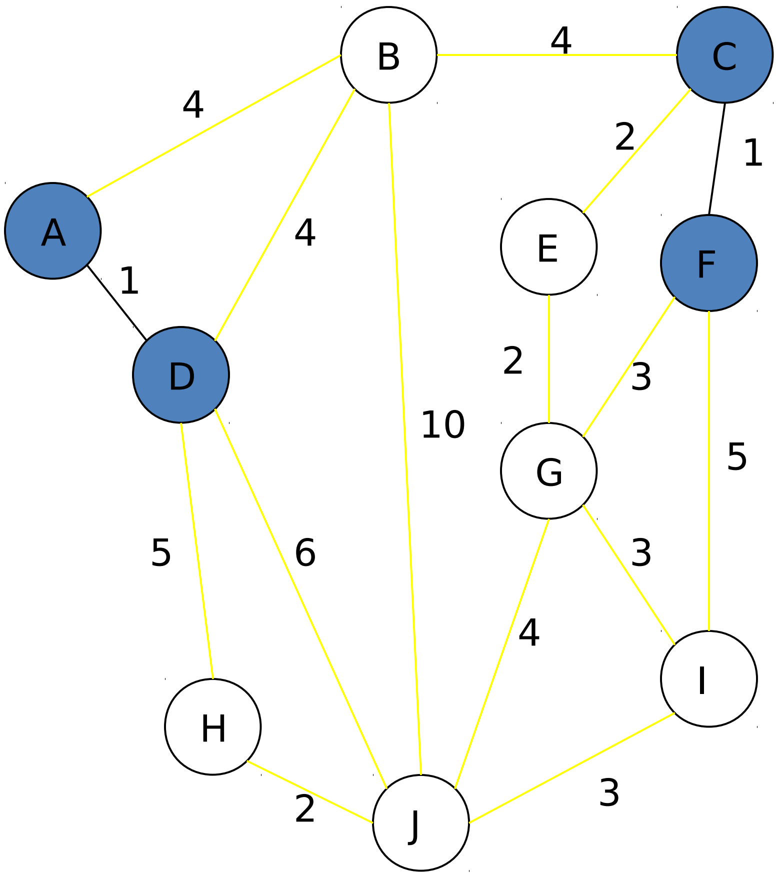
B 10 J

Add Edge



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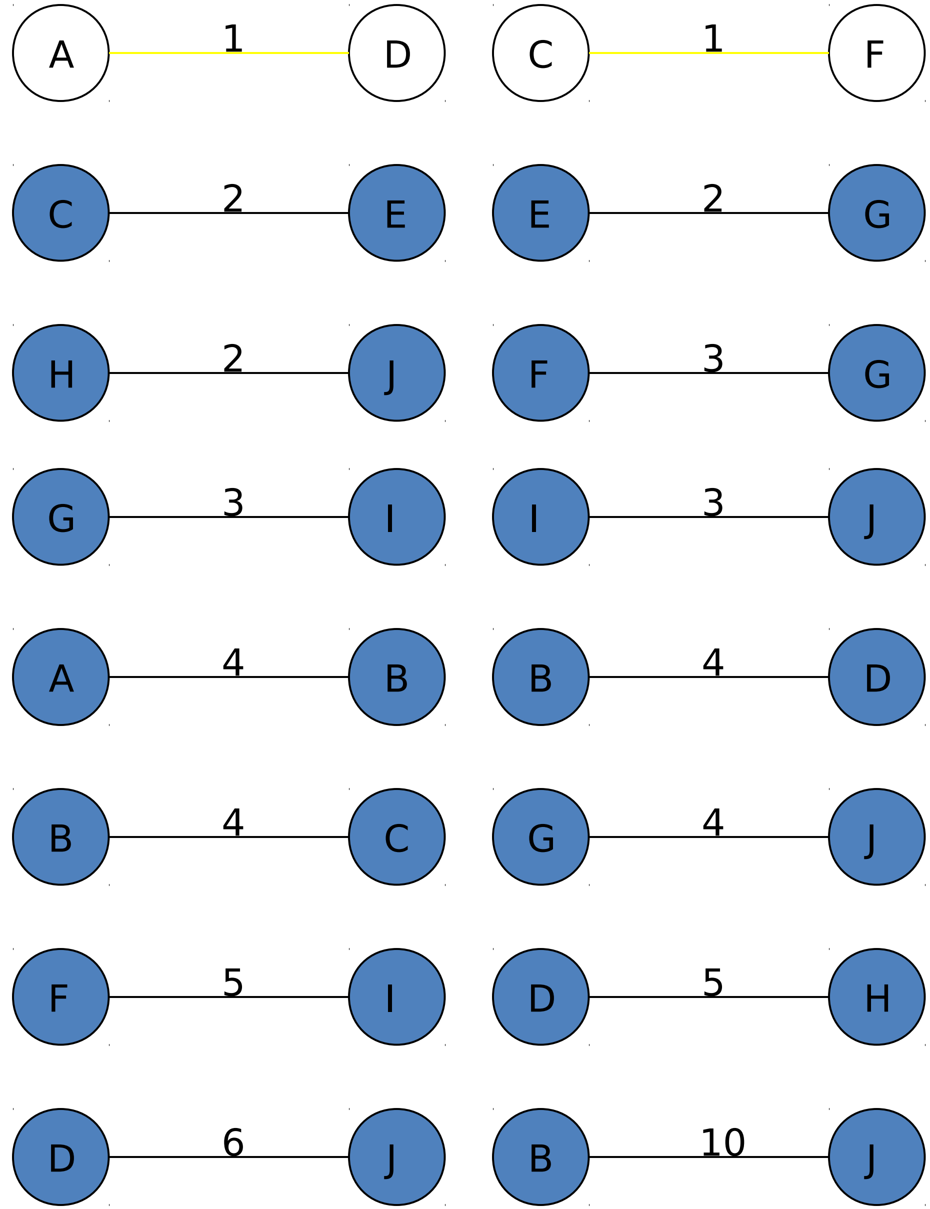

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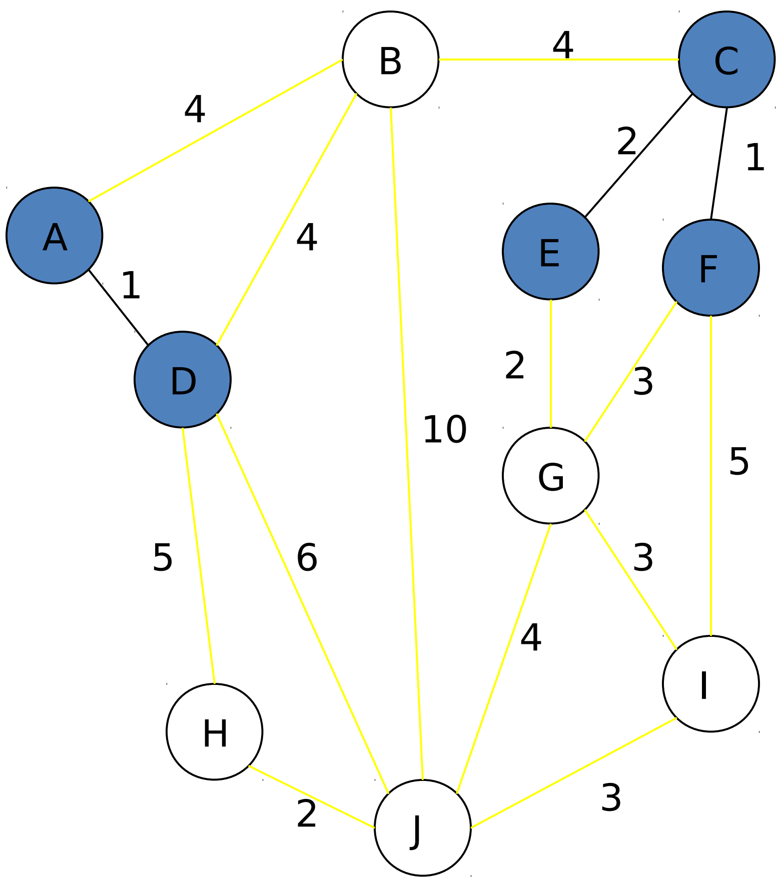

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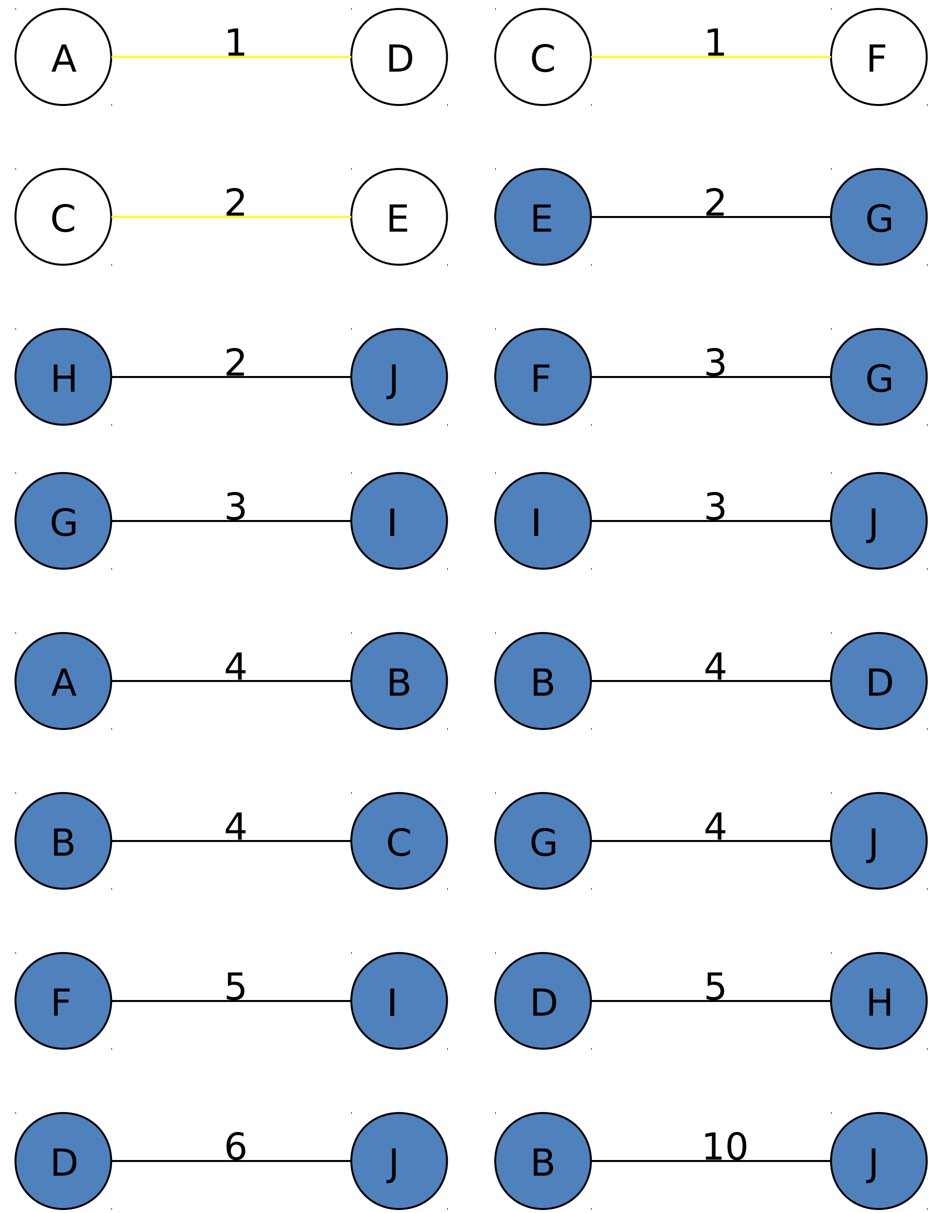

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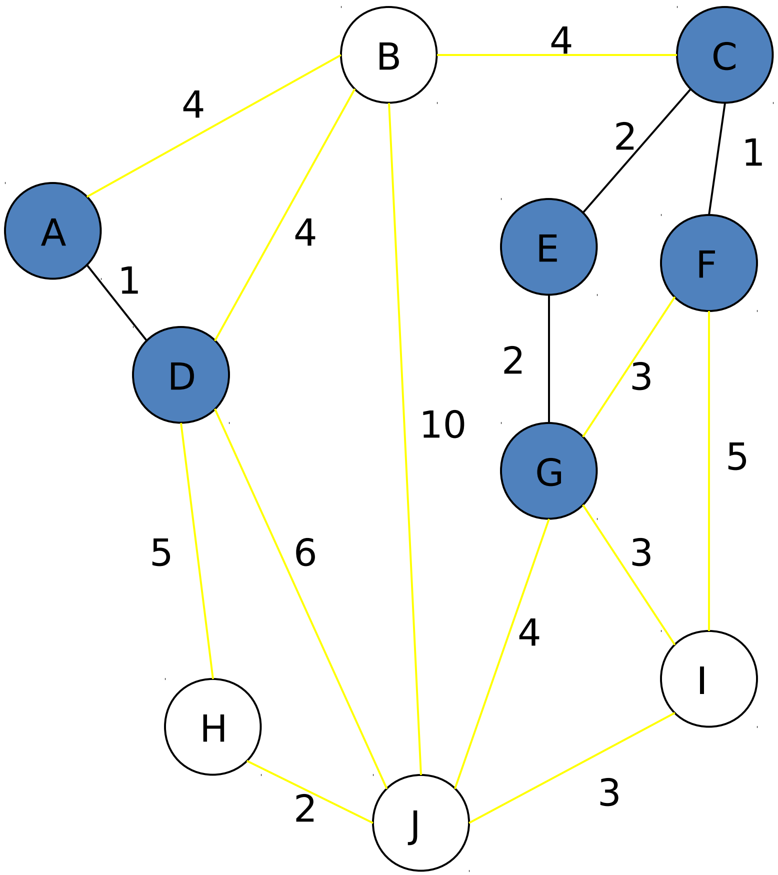

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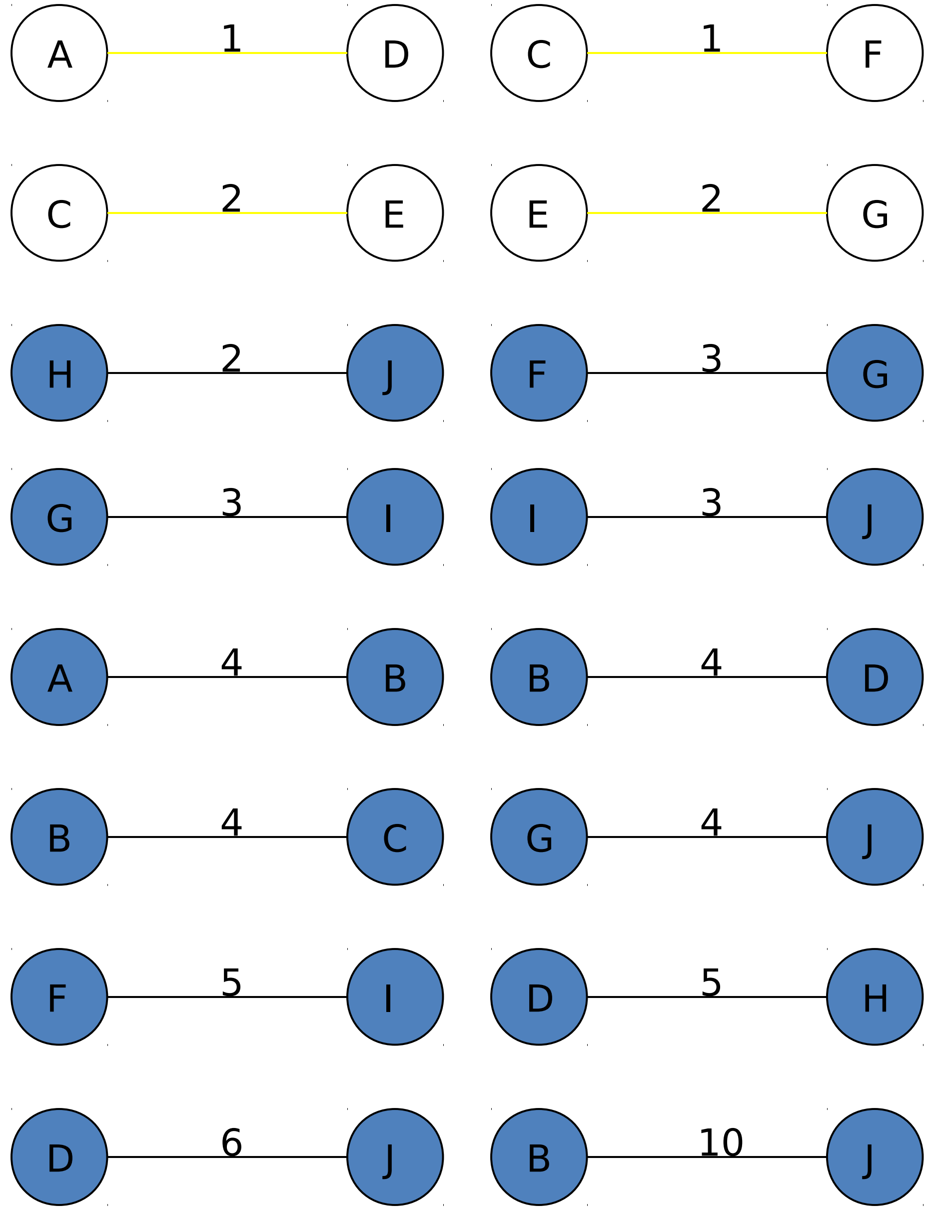

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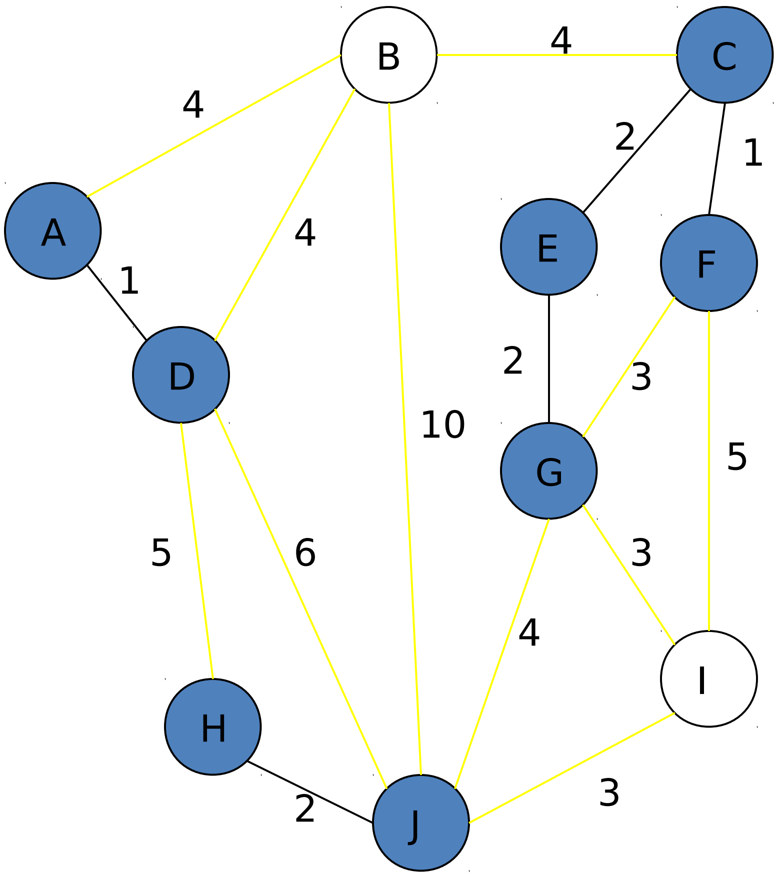

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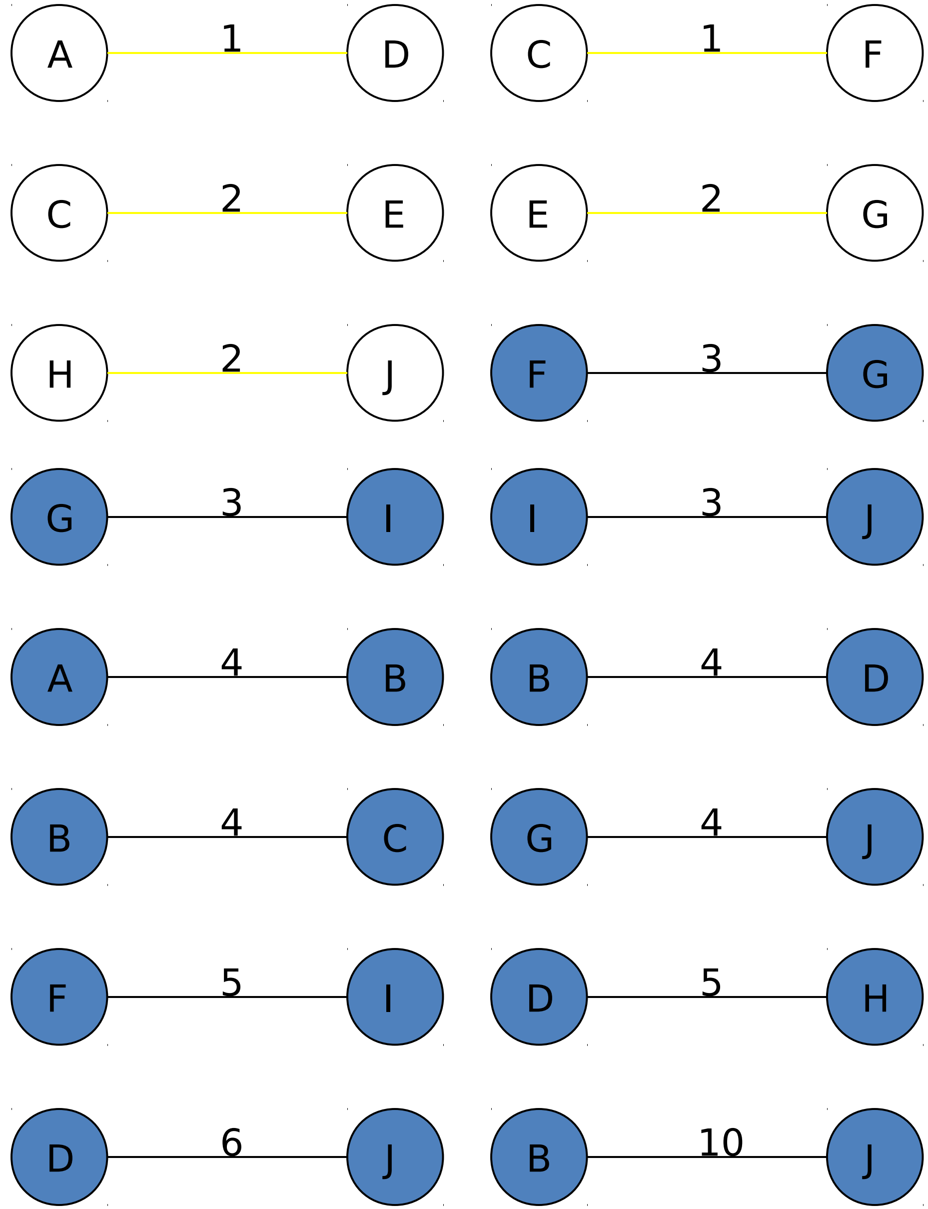

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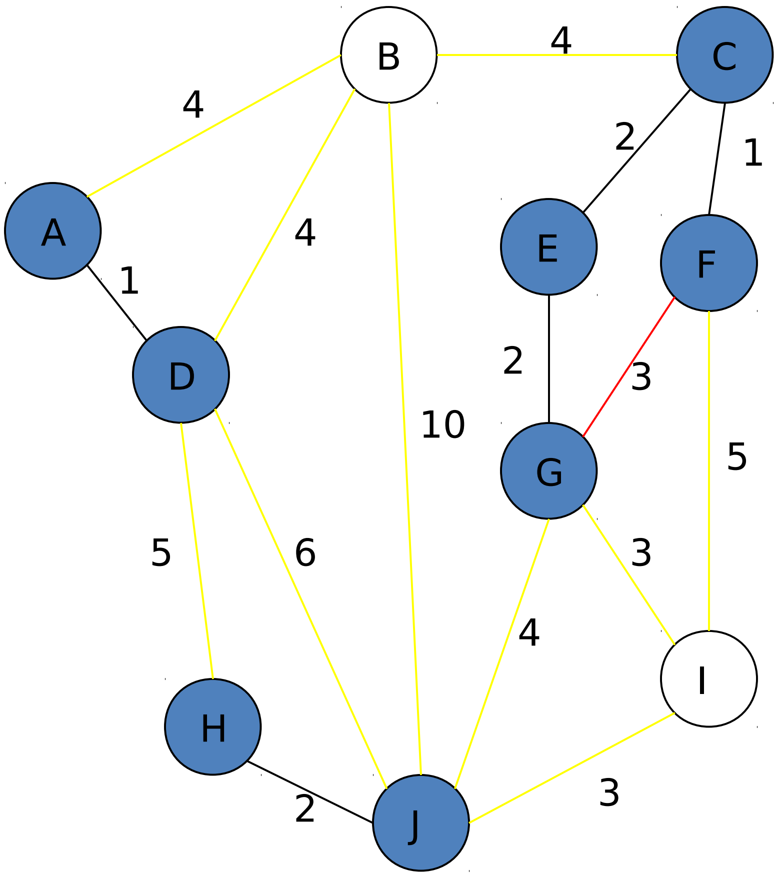
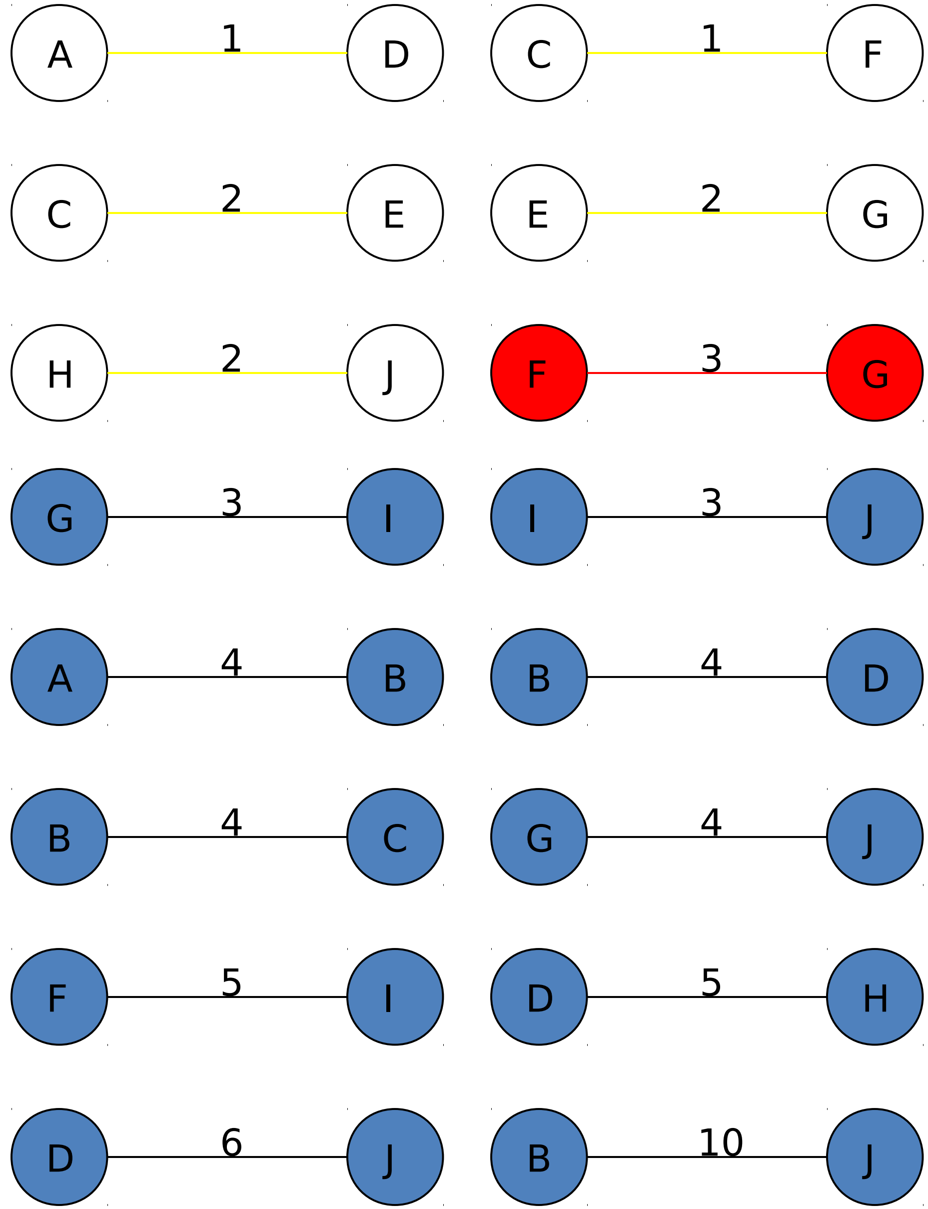
Cycle

Don’t Add Edge





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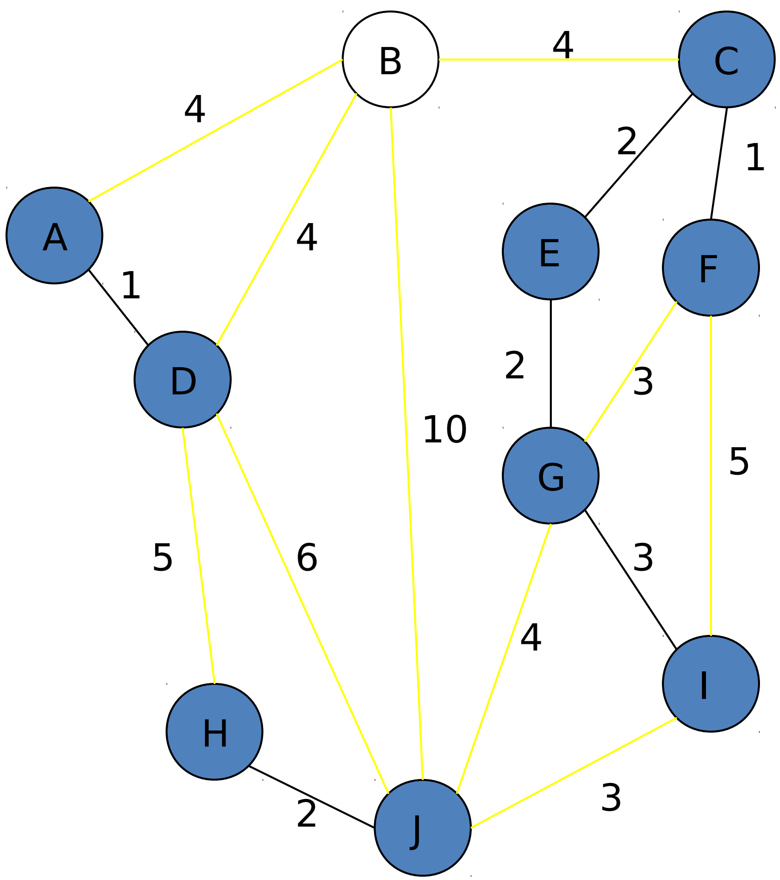
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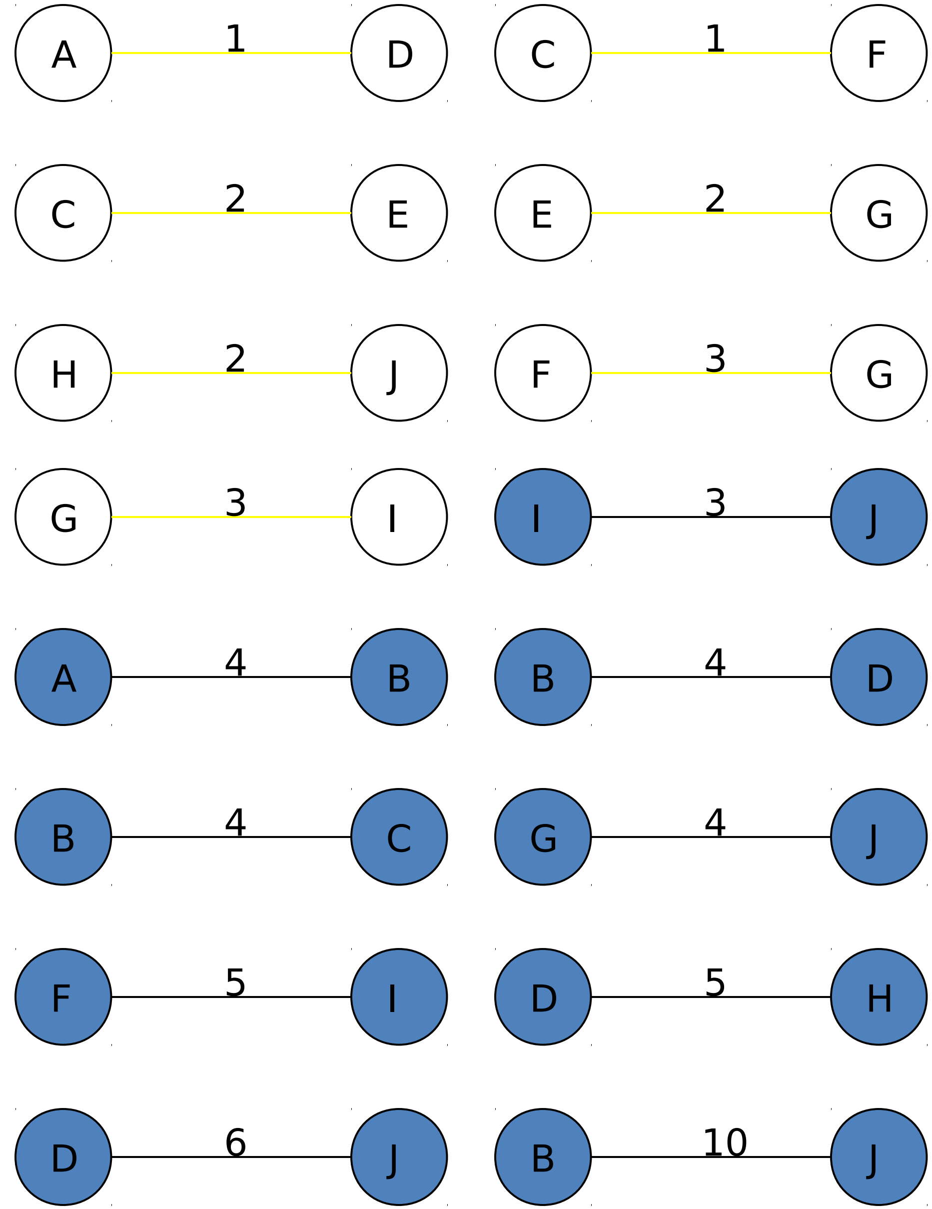
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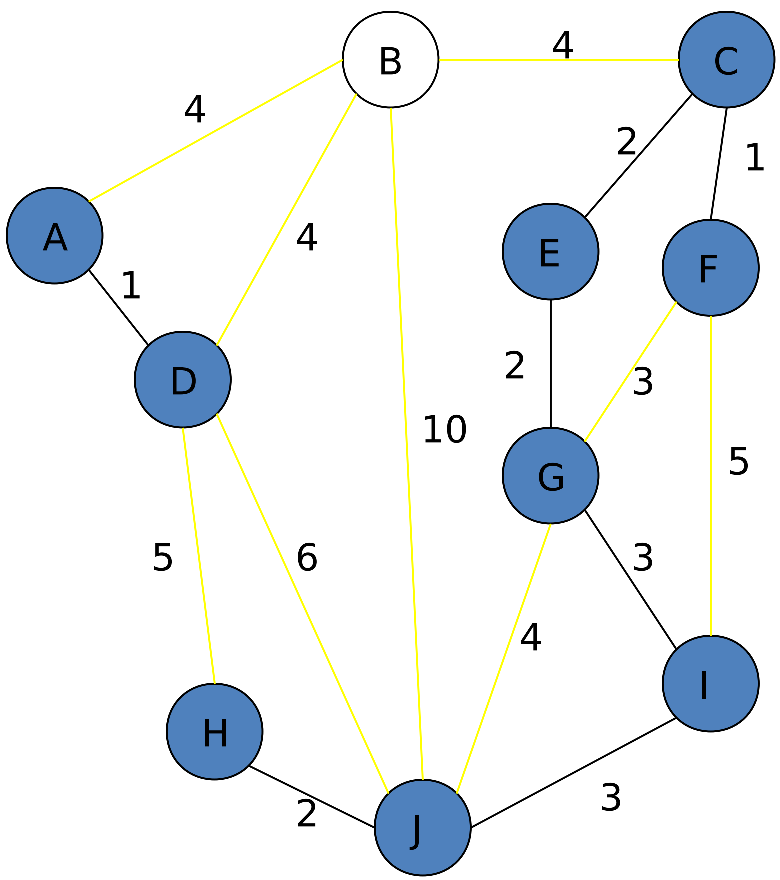
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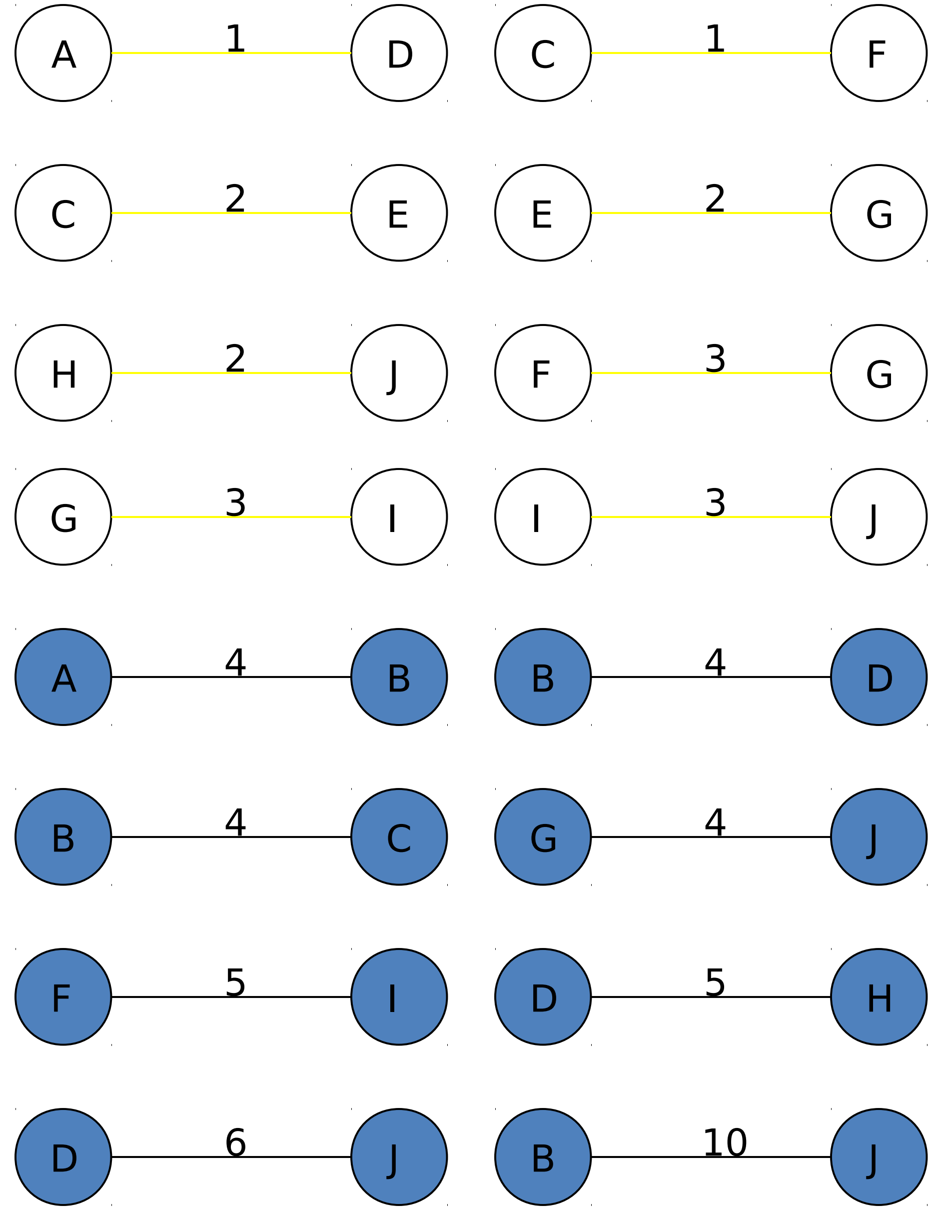
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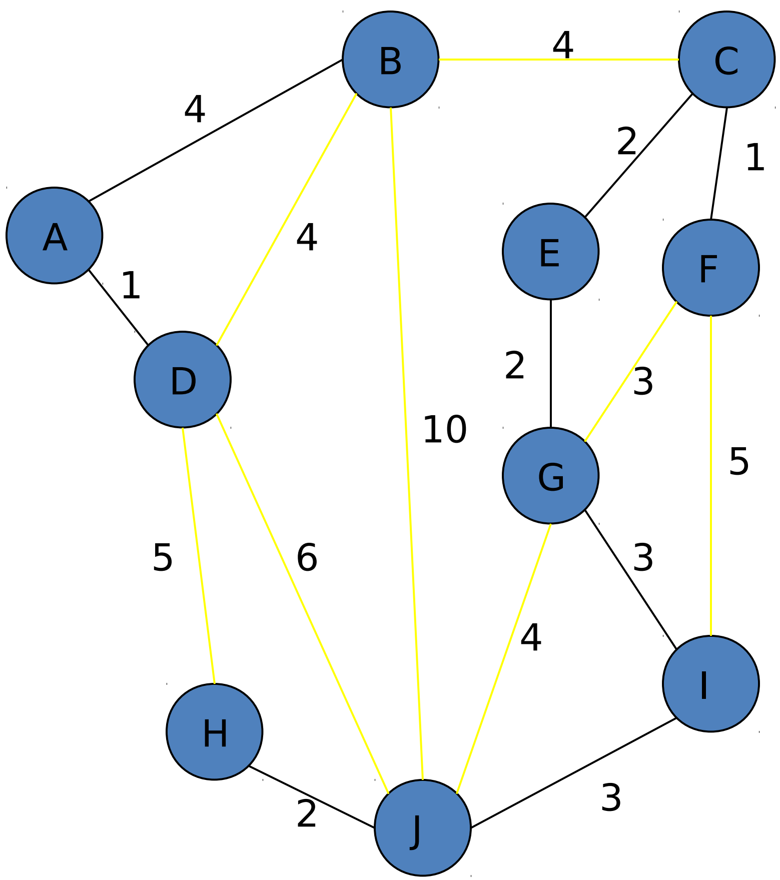
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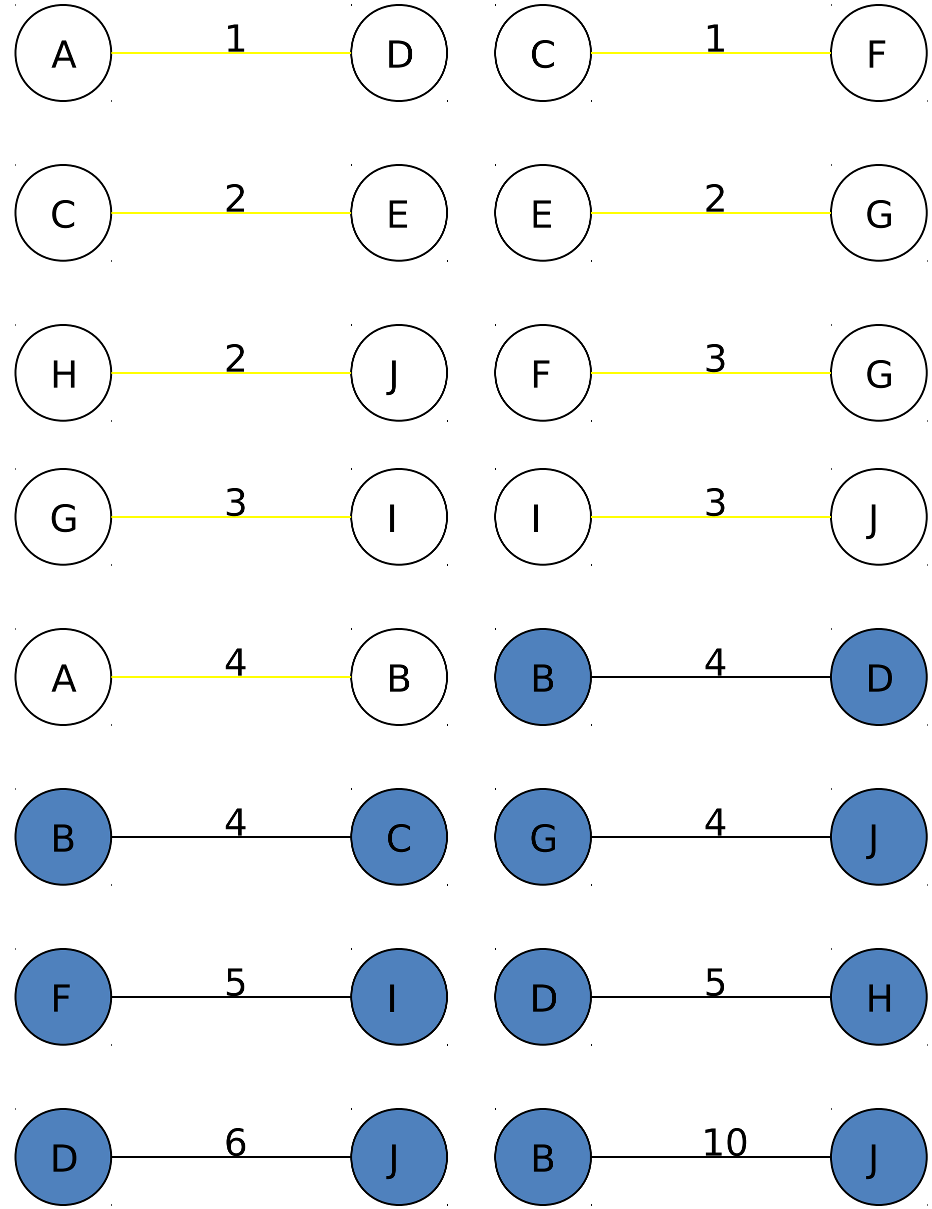
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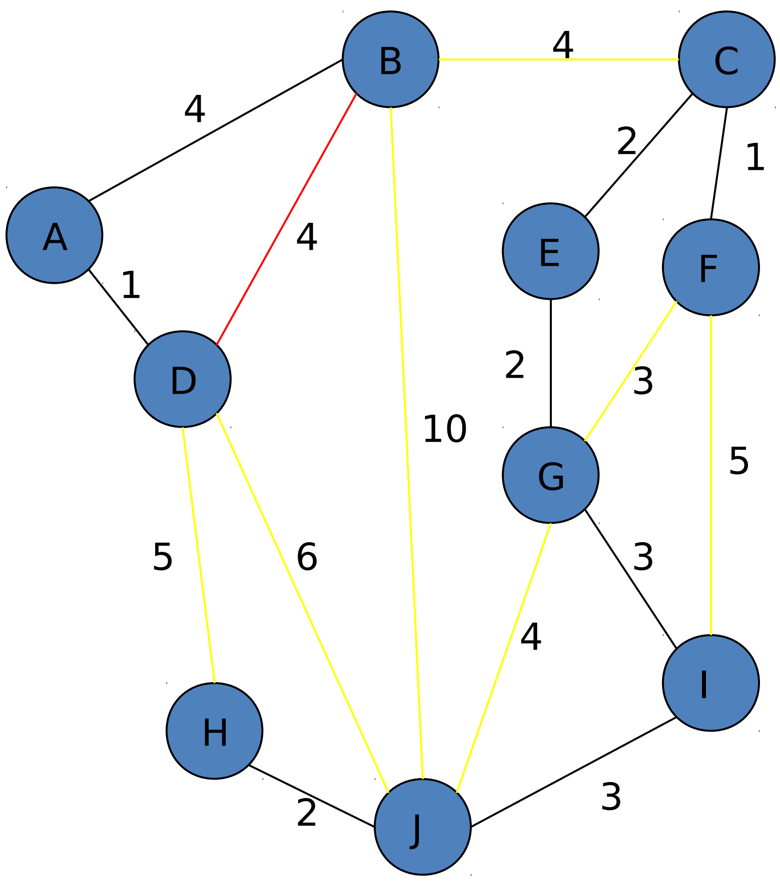
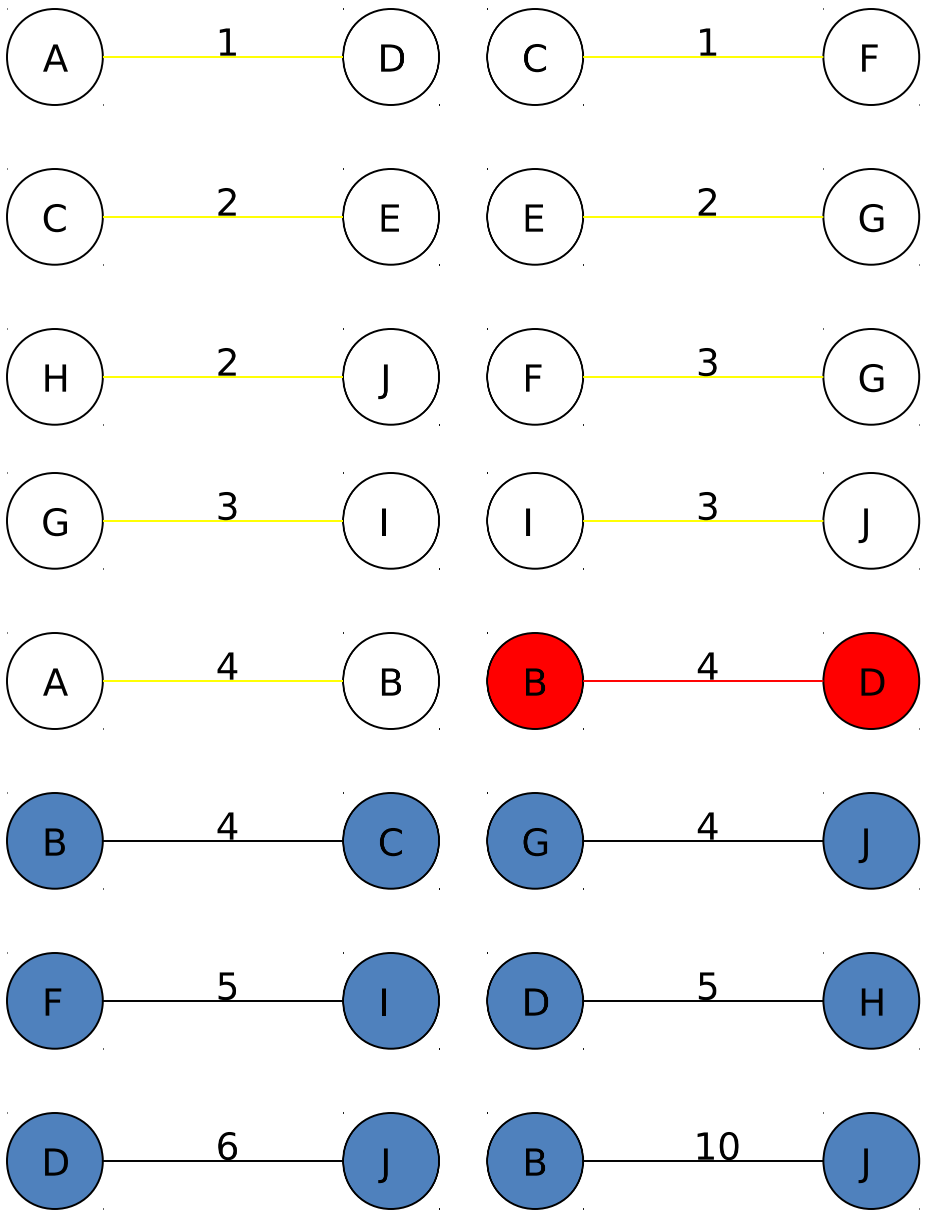
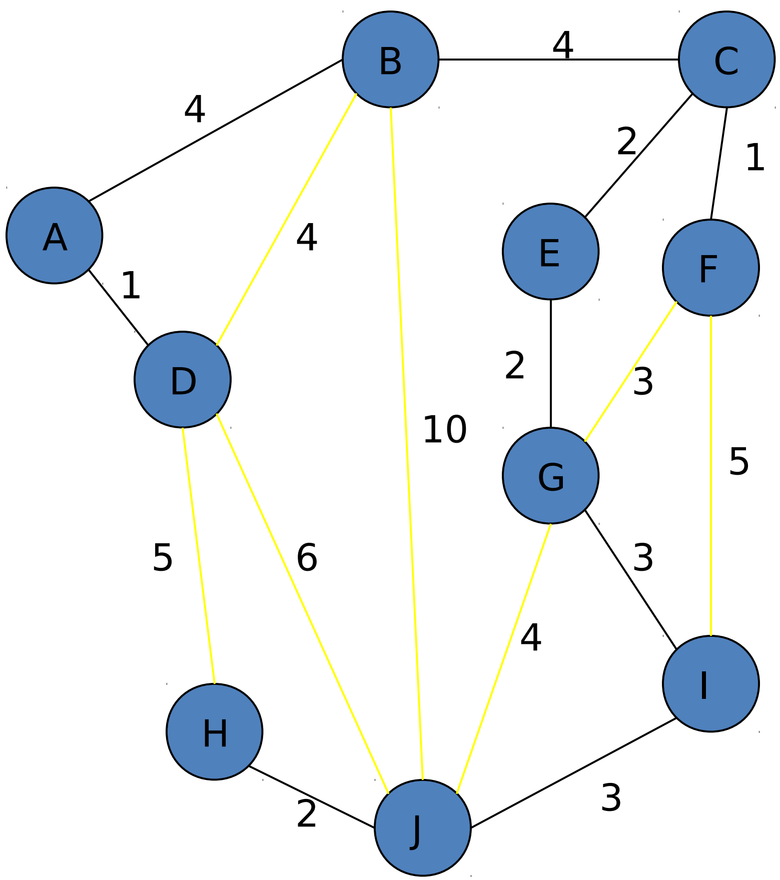
Cycle

Don’t Add Edge





Add Edge



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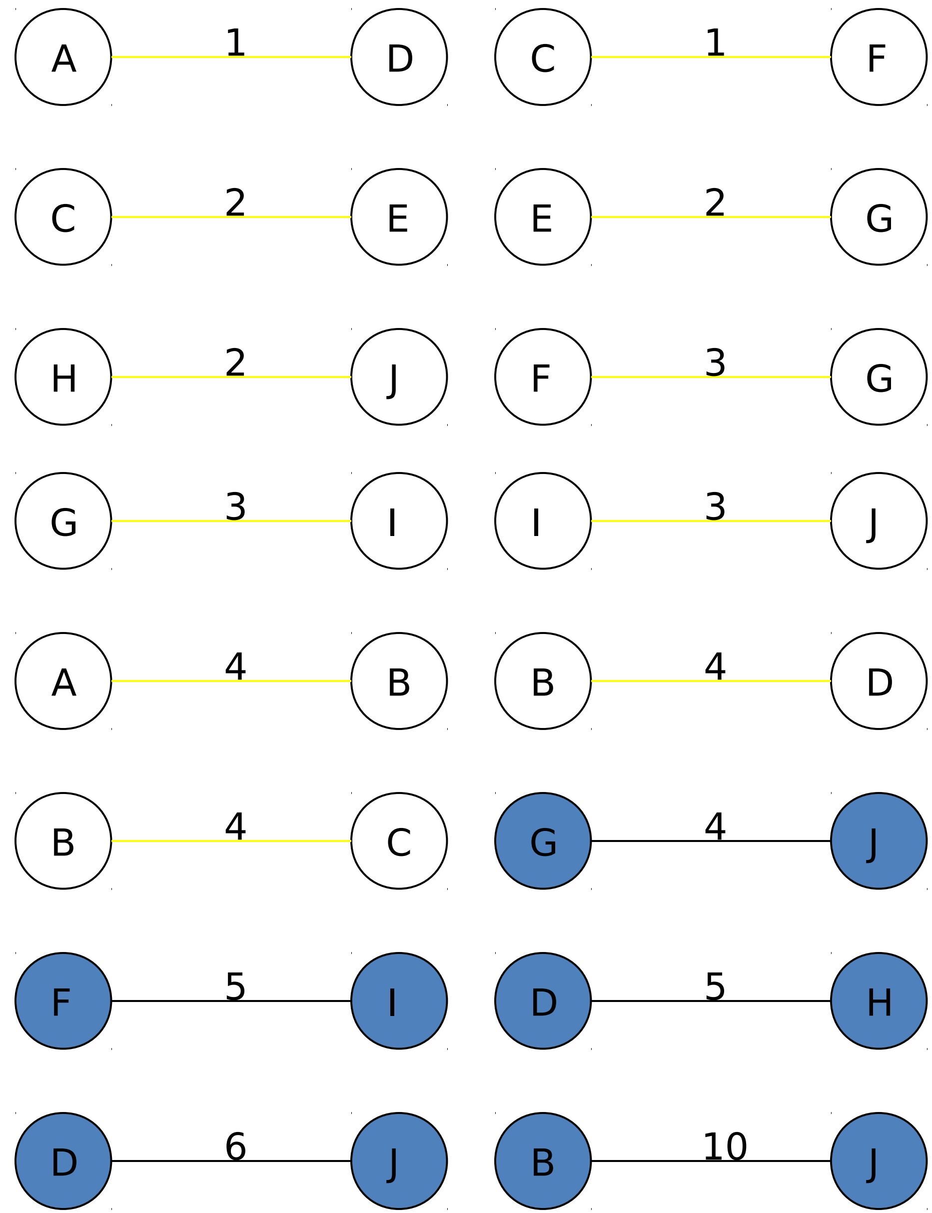

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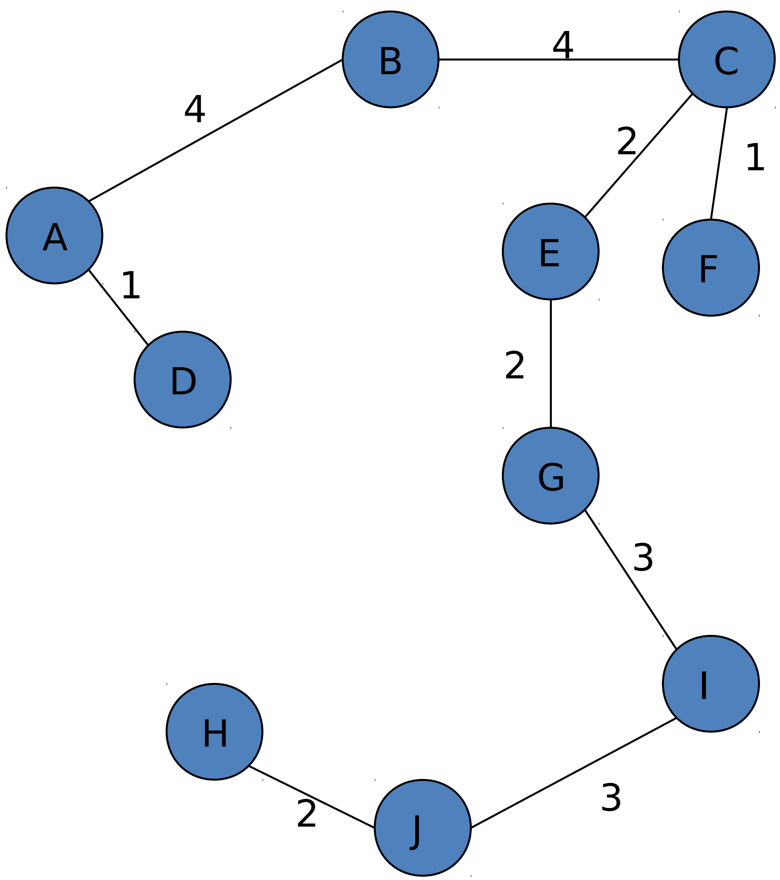
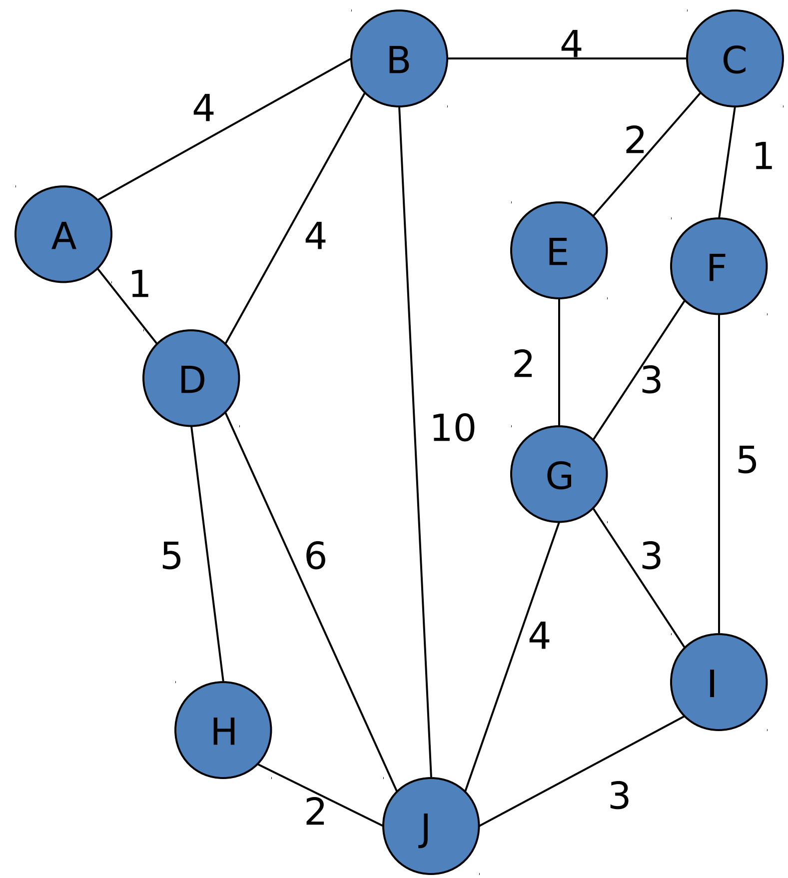
Minimum Spanning Tree



Complete Graph



```
MST-KRUSKAL(G, w)
A ← ∅
for each vertex v ∈ V[G]
      do MAKE-SET(v)
sort the edges of E into nondecreasing order by weight w
for each edge (u, v) ∈ E, taken in nondecreasing order by weight
      do if FIND-SET(u) ≠ FIND-SET(v)
            then A ← A ∪ {(u, v)}
                 UNION(u, v)
return A
```

MAKE-SET(x)

$p[x] \leftarrow x$

$rank[x] \leftarrow 0$

UNION(x, y)

LINK(FIND-SET(x), FIND-SET(y))

```
LINK(x, y)
    if rank[x] > rank[y]
        then p[y] ← x
        else p[x] ← y
             if rank[x] = rank[y]
                then rank[y] ← rank[y] + 1
```

FIND-SET(x)

    if $x \neq p[x]$

        then $p[x] \leftarrow$ FIND-SET($p[x]$)

    return $p[x]$

Floyd–Warshall algorithm

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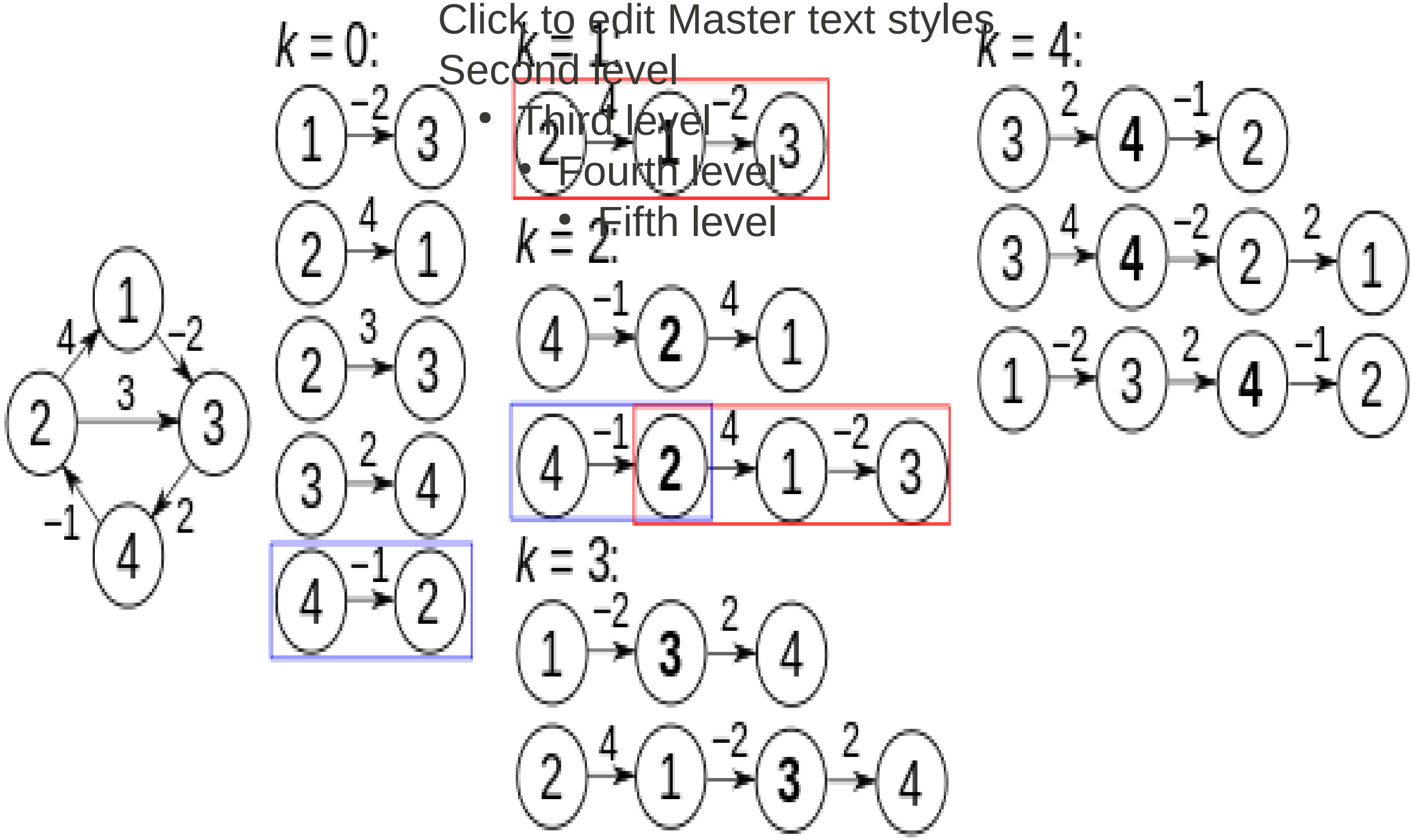
Second level

- Third level
  - Fourth level
    - Fifth level



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Second level

- Third level
  - Fourth level
    - Fifth level

```
let dist be a |V| × |V| array of minimum distances initialized to ∞ (infinity)
for each vertex v
   dist[v][v] ← 0
for each edge (u,v)
   dist[u][v] ← w(u,v)  // the weight of the edge (u,v)
for k from 1 to |V|
   for i from 1 to |V|
      for j from 1 to |V|
         if dist[i][k] + dist[k][j] < dist[i][j] then
            dist[i][j] ← dist[i][k] + dist[k][j]
```

Thank You