Mathematics-III Unit Wise Important Questions

S. No	Question	Blooms Taxonomy Level	Course Outcome
UNIT-I **Linear ODE with variable coefficients and series solution (second order only)**			
1	Solve in series the equation $\frac{d^2y}{dx^2} - y = 0$ about x=0.	Evaluate	c
2	Solve in series the equation $y'' + y = 0$ about x=0.	Evaluate	c
3	Solve in series the equation $\frac{d^2y}{dx^2} + xy = 0$.	Evaluate	c
4	Solve in series the equation $y'' + x^2y = 0$ about x=0.	Evaluate	c
5	Solve in series the equation $2x^2 y'' + (x^2 - x) y' + y = 0$.	Evaluate	c
6	Solve in series the equation $x(1-x)y'' - (1+3x) y' - y = 0$.	Evaluate	c
7	Solve in series the equation $(x - x^2)y'' + (1-5x)y' - 4y = 0$.	Evaluate	c

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8	Solve $\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx} \ =\frac{12logx}{x^2}$.	Evaluate	c
9	Solve $\left(x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4\right)y=x^4$.	Evaluate	c
10	Find the power series solution of the equation $y''+\ (x-3\ )y'+y=0$ in powers of (x-2) (i.e,about x=2).	Analyse	c
	UNIT-II **Special functions**		
1	Show that $\int_0^x x^nJ_{n-1}\,(x)dx\ =\ x^nJ_n\,(x)$.	Analyse	d
2	Show that $\int_0^x x^{n+1}J_n\ \ x)dx\ =\ x^{n+1}J_{n+1}\,(x)$.	Analyse	d
3	Prove that $J_{\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\sin x$.	Analyse	d
4	Show that $J_0(x)=\frac{1}{\pi}\ {}_0^{\pi} cos(xsin\theta)d\theta$ satisfies Bessel's equation of order zero.	Analyse	d
5	Express $J_2(x)\ in\ terms\ of\ J_0(x)\ and\ J_1(x)$.	Apply	d
6	Show that $J_3(x)+3J_0'(x)+4J_0'''(x)=0$.	Analyse	d
7	Prove that $\frac{d}{dx}\ \ x^nJ_n(x)\ \ \ =x^nJ_{n-1}(x)$.	Analyse	d
8	Prove that ${}_0^r\,x\ \ J_0(ax)=\ \frac{r}{a}J_1(ar)$.	Analyse	d
9	Show that $J_n(x)$ is an even function if 'n' is even and odd function when 'n' is odd.	Remember	d
10	Prove that $\left[J_{\frac{1}{2}}\right]^2+\left[J_{-\frac{1}{2}}\right]^2=\frac{2}{\pi x}$.	Analyse	d
	UNIT-III **Complex functions-differentiation and integration**		
1	Let w = f(z) = z^2 find the values of w which correspond to (i) z = 2+i (ii) z = 1+3i	Analyse	e
2	Show that f(z) = $\|z\|^2$ is a function which is continuous at all z but not differentiable at any z $\neq$ 0.	Understand	e
3	Find all values of k such that f(x) = $e^x(cosky+isinky)$ is analytic.	Understand	e
4	Show that u = $e^{-x}(xsiny-ycosy)$ is harmonic.	Understand	e
5	Verify that u = x^2-y^2-y is harmonic in the whole complex plane and find a	Understand	e

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	conjugate harmonic function v of u .		
6	Find k such that f(x,y) = $x^3 + 3kxy^2$ may be harmonic and find its conjugate.	Analyse	f
7	Find the most general analytic function whose real part is u = $x^2 - y^2 - x$.	Analyse	f
8	Find an analytic function whose imaginary part is v = $e^x(xsiny + ycosy)$.	Understand	f
9	If f(z) is an analytic function of z and if u - v = $\frac{cosx + sinx - e^{-y}}{2cosx - e^y - e^{-y}}$,find f(z) subject to the condition f($\frac{\pi}{2}$) = 0.	Analyse	f
10	If f(z) is an analytic function of z and if u + v = $\frac{sin2x}{2cosh2y - cos2x}$ find f(z) in terms of z.	Remember	f
UNIT-IV Power series expansions of complex functions and contour integration			
1	Find the poles and residues of $\frac{1}{z^2-1}$.	Analyse	g
2	Find zeros and poles of $\left(\frac{z+1}{z^2+1}\right)^2$.	Analyse	g
3	Find the poles of the function $f(z) = \frac{1}{(z+1)(z+3)}$ and residues at these poles.	Analyse	g
4	Find the residue of the function $f(z) = \frac{z^3}{(z^2-1)}$ $at\ z = \infty$.	Evaluate	g
6	Find the residue of $\frac{z^2}{(z-a)(z-b)(z-c)}$ $at\ z = \infty$.	Evaluate	g

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7	Determine the poles and the residue of the function $f(z)=\dfrac{ze^z}{(z+2)^4(z-1)}$.	Remember	g
8	Find the region in the w-plane in which the rectangle bounded by the lines x= 0 ,y = 0 x=2 and y=1 is mapped under the transformation w = z+(2+3i).	Analyse	g
9	Obtain the Taylor series expansion of f(z) = $\dfrac{1}{z}$ about the point z = 1.	Analyse	k
10	Obtain the Taylor series expansion of f(z) = e^z about the point z = 1.	Evaluate	k
UNIT-V Conformal mapping			
1	Determine the bilinear transformation whose fixed points are 1,-1.	Remember	i
2	Determine the bilinear transformation whose fixed points are i,-i.	Analyse	i
3	Find the fixed points of the transformation $w=\frac{2i-6z}{iz-3}$.	Remember	i
4	Evaluate $\int_c \dfrac{2z-1}{z(2z+1)(z+2)}dz$ where c is the circle $\|z\|$ = 1.	Evaluate	i
5	Evaluate $\int_c \dfrac{12z-7}{(2z+3)(z-1)^2}dz$ where c is the circle $x^2+y^2=4$.	Understand	m
6	Evaluate $\int_c \dfrac{e^z}{(z-3)z}dz$ where c is the circle $\|z\|$ = 2 using Residue theorem.	Understand	m
7	Evaluate $\int_c \dfrac{3z-4}{(z-1)z}dz$ where c is the circle $\|z\|$ = 2 using Residue theorem.	Understand	m

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8	Show that $\int_0^{2\pi} \frac{d\theta}{4\cos^2\theta + \sin^2\theta} = \pi$.	Understand	m
9	Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$.	Understand	m
10	Evaluate $\int_0^{\infty} \frac{x\sin mx}{(x^4+16)} dx$.	Understand	m

1. Group - B (Long Answer Questions)

S. No	Question	Blooms Taxonomy Level	Course Outcome
	UNIT-I **Linear ODE with variable coefficients and series solution (second order only)**		
1	Solve$\left(x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y\right) = (1+x)^2$.	Evaluate	a
2	Solve$\left(x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + 8\right) y = 65cos(logx)$.	Understand	a
3	Solve$(x+1)^2 \frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx}$ = (2x+1)(2x+4).	Evaluate	a
4	$Solve(x+1)^2 \frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} + y = sin2(log(1+x))$.	Evaluate	a
5	Find the power series solution of the equation $y'' + (x-3)y' + y = 0$ in powers of (x-2) (i.e,about x=2).	Evaluate	c
6	Solve in series the equation $2x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + 3y = 0$.	Analyse	c
7	Solve in series the equation $(x-x^2)y'' + (1-x)y' - y = 0$.	Understand	c
8	Solve in series the equation $x(1-x)y'' - (1+3x)\ y' - y = 0$.	Evaluate	c
9	Solve $(x^2D^2 - 4xD + 6)y = (logx)^2$.	Analyse	b

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10	Solve $(x+a)^2\frac{d^2y}{dx^2} - 4(x+a)\frac{dy}{dx} + 6y = x.$	Evaluate	b
	UNIT-II **Special functions**		
1	State and prove Rodrigue's formula.	Evaluate	d
2	Show that $x^4 = \frac{8}{35}P_4(x) + \frac{4}{7}P_2(x) + \frac{1}{5}\ P_0(x).$	Understand	d
3	Express P(x) = $x^4 + 2x^3 + \ 2x^2 - x - 3$ in terms of Legendre Polynomials.	Evaluate	d
4	Using Rodrigue's formula prove that ${}_{-1}^{1} x^m\ p_n(x)dx = 0$ if m<n.	Evaluate	d
5	State and prove orthogonality of Legendre polynomials.	Analyse	d
6	If f(x) = 0 if -1 < x < 0 =1 if 0<x<1 then show that f(x) = $\frac{1}{2}P_0(x) + \frac{3}{4}P_1(x) - \frac{7}{16}\ P_3(x) + \cdots$	Evaluate	d
7	Show that P_n(x) is the coefficient of t^n in the expansion of $(1-2xt+t^2)^{\frac{-1}{2}}$.	Remember	d
8	Prove (2n+1) x$P_n(x) = (n+1)P_{n+1}(x) + \ nP_{n-1}(x).$	Understand	d
9	Prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = \begin{cases} 0, if \alpha \neq \beta \\ \frac{1}{2}\left[J_{n+1}(\alpha)\right]^2\ if\ \ \alpha = \beta \end{cases}.$	Evaluate	d
10	Show that a) $J_n(x) = \frac{1}{\pi}\ {}_0^{\pi} cos(n\theta - xsin\theta)d\theta$. b) $J_0(x) = \frac{1}{\pi}\ {}_0^{\pi} cos(xsin\theta)d\theta\ \ =\frac{1}{\pi}\ {}_0^{\pi} cos(xcos\theta)d\theta.$	Remember	d
	UNIT-III **Complex functions-differentiation and integration**		
1	Prove that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\ \ \ Realf(z)\vert^2 = 2\vert f'(z)\vert^2$ where w = f(z) is analytic.	Apply	e
2	Prove that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\ \ log\vert f'(z)\vert$ where w = f(z) is analytic.	Apply	e
3	If f(z) is a regular function of z prove that	Apply	e

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	$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\quad f(z)\|^2 = \|f'(z)\|^2.$		
4	Show that the function defined by f(z)= $\begin{cases} \frac{xy^2\,(x+iy)}{x^2+y^4}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is not analytic although Cauchy Riemann equations are satisfied at the origin.	Apply	e
5	Show that u = $x^3 - 3xy^2$ is harmonic and find its harmonic conjugate and the corresponding analytic function f(z) in terms of z.	Analyse	f
6	Evaluate $\int_0^{1+i}(x-y+ix^2)\,dz$ (i) along the straight from z = 0 to z = 1+i . (ii) along the real axis from z = 0 to z = 1 and then along a line parallel to real axis from z = 1 to z = 1+i along the imaginary axis from z = 0 to z = I and then along a line parallel to real axis z = i to z = 1+ i.	Apply	e
7	Verify Cauchy's theorem for the integral of z ^{3} taken over the boundary of the rectangle with vertices -1 ,1,1+i ,-1+i.	Apply	e
8	Evaluate $\int_c \frac{e^{2z}}{(z-1)(z-2)}$ dz where c is the circle $\|z\|$ =3.	Apply	e
9	Evaluate $\int_c \frac{z^3 e^{-z}}{(z-1)^3}$ dz where c is $\|z-1\| = \frac{1}{2}$ using Cauchy's integral formula.	evaluate	e
UNIT-IV **Power series expansions of complex functions and contour integration**			
1	Expand f(z) = $\frac{z-1}{z+1}$ in Taylor's series about the point (i) z = 0 (ii) z = 1 .	Apply	k
2	Expand f(z) = $\frac{z-1}{z^2}$ in Taylor's series in powers of z -1 and determine the region of convergence.	Evaluate	k
3	Obtain Laurent's series expansion of f(z) = $\frac{z^2-4}{z^2+5z+4}$ valid in 1< z < 2.	Analyse	k
4	Expand $f(z)=\frac{e^{2z}}{(z-1)^3}$ about z = 1 as Laurent's series and also find	Evaluate	k

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	the region of convergence.		
5	Expand f(z) = $\frac{7z-2}{z(z+1)(z-2)}$ about z=-1 in the region $1<\lvert z+1\rvert<3$ as Laurent's series .	Evaluate	k
6	Determine the poles and the residue of the function $f(z)=\frac{ze^z}{(z+2)^4(z-1)}$.	Evaluate	h
7	Evaluate $\oint_c \frac{4-3z}{(z-2)(z-1)z}$ dz where c is the circle $\lvert z\rvert$ = 1.5 using residue theorem.	Apply	e
8	Show that $\int_0^{2\pi}\frac{1+4\cos\theta}{17+8\cos\theta}d\theta=0$.	Apply	i
9	Evaluate $\int_0^{\infty}\frac{dx}{x^6+1}$.	Apply	i
10	Show that $\int_0^{2\pi}\frac{d\theta}{4\cos^2\theta+\sin^2\theta}=\pi$.	Analyse	i
	UNIT-V **Conformal mapping**		
1	Find the Bi-linear transformation which carries the points from $(0,1,\infty) to (-5,-1,3)$.	Evaluate	m
2	Find the image of the triangle with vertices 1,1+I,1-i in the z-plane under the transformation w=3z+4-2i.	Evaluate	m
3	Find the Bi-linear transformation which carries the points from (-i,0,i) to (-1,i,1).	Remember	m
4	Sketch the transformation $w=e^z$.	Understand	m
5	Sketch the transformation $w=\log z$.	Understand	m
6	Find the Bi-linear transformation which carries the points from $(1,i,-1) to (0,1,\infty)$	Apply	m
7	Show that transformation w = z^2 maps the circle $\lvert z-1\rvert=1$ into the cardioid r = 2(1+cosθ) where $w=re^{i\theta}$ in the w-plane.	Evaluate	m

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8	Determine the bilinear transformation that maps the points (1-2i,2+i,2+3i) into the points (2+i,1+3i,4).	Apply	m
9	Under the transformation $w = \dfrac{z-i}{1-iz}$, find the image of the circle (i) $w\| = 1$ $ii)$ $z\|=1$ in the w-plane.	Apply	m
10	Find the image of the region in the z-plane between the lines y = 0 and y = $\dfrac{\pi}{2}$ under the transformation w = e^z.	Evaluate	m

3. Group - III (Analytical Questions)

S. No	Questions	Blooms Taxonomy Level	Program Outcome
	UNIT-I **Linear ODE with variable coefficients and series solution (second order only)**		
1	Find the singular points and classify them (regular or irregular) $x^2 y'' + ax y' + by = 0$.	Analyse	b
2	Find the singular points and classify them (regular or irregular) $x^2 y'' + x y' + (x^2 - n^2)y = 0.$	Evaluate	b
3	Find the singular points and classify them (regular or irregular) $(1 - x^2)y'' - 2xy' + n(n+1)y = 0.$	Understand	b
4	Define Ordinary and Regular singular point.	Analyse	b
5	Explain Frobenius method about regular singular points.	Evaluate	b
6	Explain the method of solving Legendre's differential equation.	Analyse	a
7	Explain the method of solving Cauchy's differential equation.	Analyse	a
8	Find the singular points and classify them (regular or irregular) $x^2 y'' - 5y' + 3x^2 y = 0.$	Evaluate	b
9	Find the singular points and classify them (regular or irregular) $x^2 y'' + (x + x^2)y' - y = 0$.	Analyse	b
10	Find the singular points and classify them (regular or irregular) $x^3(x - 2) y'' + x^3 y' + 6y = 0.$	Understand	b
	UNIT-II **Special functions**		
1	Find the value of $J_{\frac{1}{2}}(x)$.	Evaluate	d
2	Find the generating function for $J_n(x)$.	Apply	
3	Write the integral form of Bessel's function .	Analyse	d
4	Find the value of $J_{\frac{5}{2}}(x)$.	Analyse	d

S. No	Questions	Blooms Taxonomy Level	Program Outcome
5	Express $J_5(x)$ *in terms of* $J_0(x)$ *and* $J_1(x)$.	Evaluate	d
6	Prove that $2J_0''(x) = J_2(x) - J_0(x)$.	Remember	d
7	Show that $x^3 = \frac{2}{5}P_3(x) + \frac{3}{5}P_1(x)$.	Analyse	d
8	Show that $J_0(x) = \frac{1}{\pi}\int_0^\pi cos(xsin\theta)d\theta$ satisfies Bessel's equation of order zero.	Analyse	d
9	Prove that $J_n(-x) = (-1)^n J_n(x)$ where n is a positive or negative integer .	Evaluate	d
10	Find the value of $\int_{-1}^{1} P_3(x)P_4(x)dx$.	Analyse	d
UNIT-III Complex functions-differentiation and integration			
1	Show that f(z) = z^3 is analytic for all z.	Understand	e
2	Find whether f(z) = sinxsiny – icosxcosy is analytic or not.	Understand	e
3	Show that both the real and imaginary parts of an analytic function are harmonic.	Analyse	e
4	Show that the function u = 2log$(x^2 + y^2)$ *is harmonic and find its* harmonic conjugate.	Analyse	e
5	Find an analytic function whose real part is u = $e^x[(x^2 - y^2)cosy - 2xysiny)]$.	Analyse	e
6	Find an analytic function whose real part is u = $\frac{sin2x}{cosh2y - cos2x}$.	Evaluate	e
7	If f(z) is an analytic function of z and if u - v = $\frac{cosx + sinx - e^{-y}}{2cosx - e^y - e^{-y}}$ find f(z) subject to the condition f($\frac{\pi}{2}$) = 0.	Analyse	e
8	Find whether f(z) = sinxsiny – icosxcosy is analytic or not.	Evaluate	e
9	Show that f(z) = x +iy is everywhere continous but is not analytic.	Understand	f
10	Show that u = $e^{-x}(xsiny - ycosy)$ is harmonic.	Analyse	f
UNIT-IV **Power series expansions of complex functions and contour integration**			
1	Find $\oint_c \frac{1}{(z^2+4)^2}$ dz where c is the circle $\lvert z - i \rvert$ =2.	Analyse	i

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2	Find the poles and residues at each pole of f(z) = $\frac{z\sin z}{(z-\pi)^3}$.	Remember	i
3	Find $\oint_c \frac{1}{(z^2+1)(z^2-4)}$ dz where c is the circle $\|z\|$ = 1.5	Understand	i
4	Find $\int_0^\infty \frac{dx}{x^6+1}$.	Understand	i
5	Find poles and residues of each pole of tanhz.	Understand	h
6	Write the poles of the function f(z) = $\frac{z^2}{(z-1)^2(z+2)}$.	Analyse	h
7	Find $\int_c \frac{2z-1}{z(2z+1)(z+2)}dz$ where c is the circle $\|z\|$ = 1.	Understand	i
8	Find the Laurent expansion of f(z) = $\frac{1}{z^2-4z+3}$ for 1<$\|z\|$ < 3 (ii) $z\|$ < 1 (iii) $\|z\|$ > 3.	Analyse	k
9	Expand f(z) = $\frac{(z-2)(z+2)}{(z+1)(z+4)}$ in the region where (i) $\|z\|$ < 1 (ii)1<$\|z\|$ < 4 .	Analyse	k
10	Find $\int_0^\infty \frac{\sin mx}{x}dx$.	Evaluate	i
UNIT-V Conformal mapping			
1	Show that the function $w = \frac{1}{z}$ transforms the straight line x=c in the z-plane into a circle in the w-plane.	Analyse	m
2	Find the fixed points of the transformation $w = \frac{2i-6z}{iz-3}$.	Analyse	m
3	Find the invariant points of the tranformation $w = \frac{z-1}{z+1}$.	Understand	m

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4	Find the critical points of $w = \dfrac{6z-9}{z}$.	Understand	m
5	Show that the transformation $w = \dfrac{2z+3}{z-4}$ changes the circle $x^2 + y^2 - 4x = 0$ into the straight line 4u+3=0.	Analyse	m
6	Find the image of the triangle with vertices at i,1+i,1-i in the z-plane under the transformation w = 3z+4-2i.	Understand	m
7	Find the image of the domain in the z-plane to the left of the line x=-3 under the transformation w = z^2.	Understand	m
8	Show that the function w = $\dfrac{4}{z}$ transforms the straight line x = c in the z-plane into a circle in the w- plane.	Understand	m
9	Define Translation ,Rotation and magnification of the transform.	Evaluate	m
10	Define and sketch Joukowski's transformation.	Apply	m