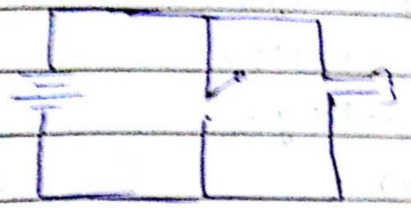
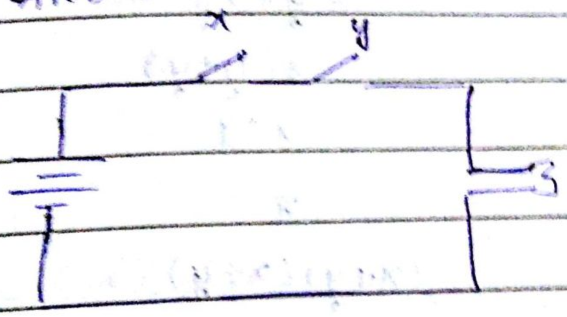
Boolean variable has only two values either 0 or 1.

Physical representation of 'x'

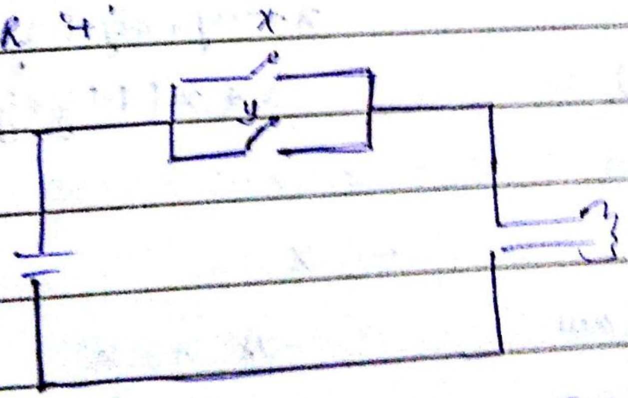
NOT x



AND '·'



OR '+'



$0 \cdot 0 = 0$

$0 \cdot 1 = 0$

$1 \cdot 0 = 0$

$1 \cdot 1 = 1$

$0 + 0 = 0$

$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 1$

$x \cdot 0 = 0$

$x + 0 = x$

$x \cdot x = x$

$x + x = 1$

$x + 1 = 1$

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$x\cdot\bar{x} = 0$

$x+\bar{x} = 1$

$x\cdot y = y\cdot x$

$x+y = y+x$

$x\cdot(y\cdot z) = (x\cdot y)\cdot z$ } Associative
$x+(y+z) = (x+y)+z$ }

$x\cdot(y+z) = xy+xz$ } Dis
$x+(yz) = (x+y)(x+z)$ }

RHS $= x+xy+xz+yz$

$= x(1+z+y)+yz$

$= \cancel{x1}\; x\cdot 1+yz$

$= x+(yz)$

$x+x\cdot y = x$

$x(1+y)$

$= x\cdot 1$

$= x$

$x\cdot y+x\bar{y} = x$

$x(\bar{y}+y)$

$= x\cdot 1$

$= x$

$x\cdot(x+y) = x$

LHS $= x+xy$

$= x(1+y)$

$= x\cdot 1$

$= x$

$(x+y)(x+\bar{y}) = x$

$x+xy+x\bar{y}+y\bar{y}$

$= \cancel{x+}\; x(1+y+\bar{y})$

$= x\cdot 1$

$= x$

$\cancel{x}\; \bar{\bar{x}} = x$

DeMorgan's law

$\overline{x+y} = \bar{x}\cdot\bar{y}$

$\overline{x\cdot y} = \bar{x}+\bar{y}$

$\overline{A+B+C} = \bar{A}\cdot\bar{B}\cdot\bar{C}$

$\overline{A+BC} = \bar{A}\cdot(\overline{B\cdot C})$

$\overline{A+B\cdot C}$

$= \bar{A}\cdot(\overline{B\cdot C})$

$= \bar{A}\cdot(\bar{B}+\bar{C})$

$x + \bar{x} y = x + y$

Proof: $\overline{\overline{x + \bar{x} y}} = \overline{\bar{x} \cdot (\overline{\bar{x} y})}$ $\qquad x(\bar{x} + \bar{y}) = xy$

$= \cancel{\overline{(x \cdot \bar{x} y)}}$

$= \bar{x} + \cancel{(\bar{x} y)}$

$=$

$\overline{x + \bar{x} y} = \bar{x} \cdot (\overline{\bar{x} \cdot y})$

$= \cancel{\bar{x} \cdot (x \cdot \bar{y})}$

$= \cancel{\bar{x}} \cdot$

$(\overline{\overline{x + \bar{x} y}}) = (\cancel{\overline{\bar{x} \cdot (\overline{\bar{x} y})}})$

$= (\cancel{\overline{\bar{x} \cdot (x \bar{y})}})$

$= \cancel{(\overline{\bar{x} \cdot (x}}$

$= (\overline{\bar{x} \cdot (\overline{\bar{x} y})}$

$= (\overline{\bar{x} \cdot (\bar{\bar{x}} + \bar{y})})$

$= (\overline{\bar{x} \cdot (x + \bar{y})})$

$= (\overline{\bar{x} \cdot x + \bar{x} \cdot \bar{y}})$

$= (\overline{0 + \bar{x} \cdot \bar{y}})$

$= (\overline{\bar{x} \cdot \bar{y}})$

$= \bar{\bar{x}} + \bar{\bar{y}}$

$= x + y$

$(x_1 + x_3)(\bar{x}_1 + \bar{x}_3) = x_1 \bar{x}_3 + \bar{x}_1 x_3$

LHS $= 0 + x_3 \bar{x}_1 + \bar{x}_3 x_1$

$= \bar{x}_1 x_3 + x_1 \bar{x}_3$

$x_1 \bar{x}_3 + \bar{x}_2 \bar{x}_3 + x_1 x_3 + \bar{x}_2 x_3 = \bar{x}_1 \bar{x}_2 + x_1 x_2 + x_1 \bar{x}_2$

LHS $= x_1(\bar{x}_3 + x_3) + \bar{x}_2(\bar{x}_3 + x_3)$

$= x_1 + \bar{x}_2$

RHS $= \bar{x}_2(\bar{x}_1 + x_1) + x_1 x_2$

$= \bar{x}_2 + x_1 x_2$

iv)

RHS $= (x_1+\bar{x}_2+x_3)(x_1+x_2+\bar{x}_3)(\bar{x}_1+\bar{x}_2+\bar{x}_3)$

$= (x_1+x_1\bar{x}_2+x_1x_3+x_1x_2+0+x_2x_3+x_1\bar{x}_3+\bar{x}_2\bar{x}_3+0)$

$(\bar{x}_1+x_2+\bar{x}_3)$

$= (x_1(1+\bar{x}_2+x_3+x_2+\bar{x}_3)+x_2x_3+\bar{x}_2\bar{x}_3)(\bar{x}_1+x_2+$

$= (x_1+x_2x_3+\bar{x}_2\bar{x}_3)(\bar{x}_1+x_2+\bar{x}_3)$

$= (0+\bar{x}_1x_2x_3+\bar{x}_1\bar{x}_2\bar{x}_3+x_1x_2+x_2x_3+\ldots$

$+x_1\bar{x}_3+0+\bar{x}_2\bar{x}_3$

$= x_2x_3(1+\bar{x}_1)+\bar{x}_2\bar{x}_3(1+\bar{x}_1)+x_1x_2+x_1\bar{x}_3$

$= x_2x_3+\bar{x}_2\bar{x}_3+x_1\bar{x}_3+x_1x_2$

Scanned by CamScanner

$= \overline{x_1 x_3} \cdot \overline{x_1 x_2 x_3} \cdot \overline{x_1 x_2} \cdot \overline{x_1 x_3}$

$= x_1 + \bar{x}_3 \cdot (x_1 + x_2 x_3) \cdot x_1 + x_3 \cdot x_1 + x_3$

$= (x_1 + x_2 x_3)(x_1 + x_2 + x_3)(x_1 + x_3)(x_1 + x_3)$

$= x_1 (x_1 \bar{x}_1 + x_1 \bar{x}_2 + x_1 x_3 + x_2 x_1 + x_2 x_2 + x_3 x_3)$

$[x_1 \bar{x}_1 + x_2 x_1 + x_2 x_1 + x_2 x_2]$

$= [x_1 x_2 + x_1 x_3 + x_3 x_1 + x_3 x_2 + x_3 x_3][x_1 x_1 + x_1 x_2]$

$= (x_1 x_2 + x_1 x_3 + x_3 x_1 + x_3 x_2)(x_1 x_2 + x_1 x_2)$

$= (x_1 x_2 x_3 + x_1 x_1 x_2 + x_1 x_2 x_3 + x_1 x_1 x_3 x_2 +$

$x_3 x_1 x_1 x_2 + x_3 x_2 x_1 x_2 + x_3 x_1 x_2)$

$= (x_1 x_2 x_3 + \bar{x}_1 x_2 \bar{x}_3) = (x_1 + x_2 + x_3) \cdot (x_1 + x_2 + x_3)$

RHS $= \overline{x_2 x_3 + x_1 \bar{x}_3 + x_2 x_3 + x_1 x_2 x_3}$

$= (\overline{x_2 x_3} \cdot (x_1 x_3) \cdot (x_2 x_3) \cdot (x_1 x_2 x_3))$

$= ((x_2 + \bar{x}_3)(\bar{x}_1 + x_3) \cdot (x_2 + x_3)(x_1 + \bar{x}_2 + x_3))'$

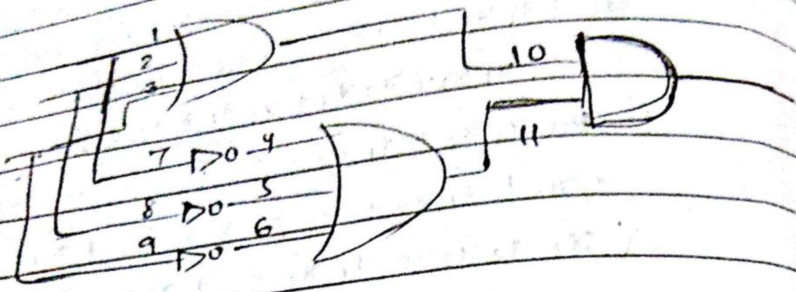
$= ((x_2 \bar{x}_1 + x_3 x_1 + x_2 x_3) \cdot (x_1 x_2 + x_1 x_3 + x_2 + x_2 x_3$
$+ x_2 x_1))'$

$= ((\bar{x}_1 x_2 + x_1 \bar{x}_3 + x_2 x_3) \cdot (x_2 (1 + x_3 + x_3 + x_1) + x_1 x_3))'$

$= ((x_1 x_2 + \bar{x}_1 x_3 + x_2 x_3) \cdot (x_2 + x_1 x_3))'$

The cost of circuits = The no. of gates it has + the no. of inputs to each gate.

= 6 + 11 = 17



4) LHS = $x_1\bar{x}_3 + x_2x_3 + \bar{x}_2\bar{x}_3$

= $\overline{\overline{x_1\bar{x}_3 + x_2x_3 + \bar{x}_2\bar{x}_3}}$

LH = $((\overline{x_1\bar{x}_3}).(\overline{x_2x_3}).(\overline{\bar{x}_2\bar{x}_3}))'$

= $((\bar{x}_1 + x_3)(\bar{x}_2 + \bar{x}_3).(x_2 + x_3))'$

= $((\bar{x}_1\bar{x}_2 + \bar{x}_2x_3 + \bar{x}_1\bar{x}_3).(x_2 + x_3))'$

= $(\bar{x}_1x_2\bar{x}_3 + \bar{x}_1\bar{x}_2x_3 + \bar{x}_2x_3)'$

= $(\bar{x}_1x_2\bar{x}_3 + \bar{x}_2x_3)'$

RHS = $(x_1 + \bar{x}_2 + x_3)(x_1 + x_2 + \bar{x}_3)(\bar{x}_1 + x_2 + \bar{x}_3)$

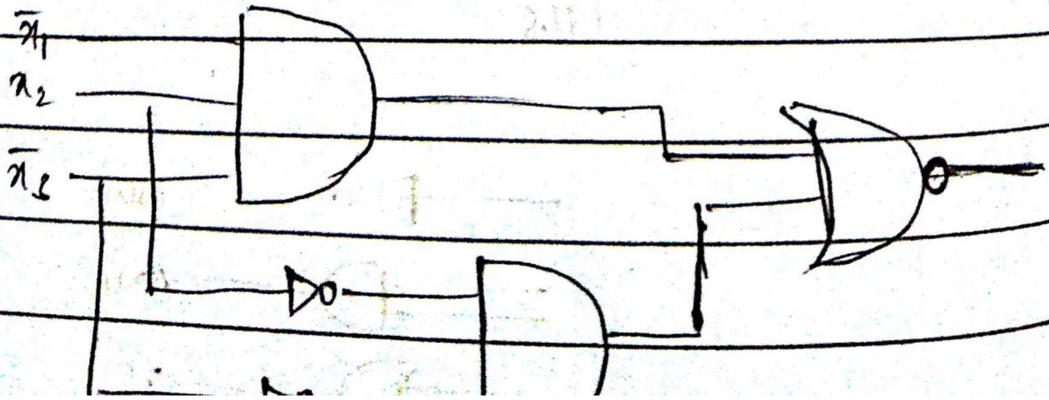
= $(\overline{x_1 + \bar{x}_2 + x_3} + \overline{x_1 + x_2 + \bar{x}_3} + \overline{\bar{x}_1 + x_2 + \bar{x}_3})'$

= $((\overline{x_1 + \bar{x}_2 + x_3}) + (\overline{x_1 + x_2 + \bar{x}_3}) + (\overline{\bar{x}_1 + x_2 + \bar{x}_3}))'$

= $((\bar{x}_1 \cdot x_2 \cdot \bar{x}_3) + (\bar{x}_1 \cdot \bar{x}_2 \cdot x_3) + (x_1 \cdot \bar{x}_2 \cdot x_3))'$

= $(\bar{x}_1 \cdot x_2 \cdot \bar{x}_3 + \bar{x}_2x_3)'$

= LHS



Minterm:

For a function of 'n' variables a product term in which each of the 'n' variables appear once is called a minterm.

Sum of Product forms:

A logical expression consisting of product (logical-AND) terms that are sum (logical OR) is said to be sum of product form.

If each product term is a min term then this expression is called canonical sop, for a function 'f'.

$$f = x_1\bar{x}_2 + \bar{x}_1 x_2$$

x_1	x_2	$\bar{x}_1$	$\bar{x}_2$	$x_1\bar{x}_2$	$\bar{x}_1 x_2$	f	minterm
0	0	1	1	0	0	0	$\bar{x}_1\bar{x}_2$
1	0	0	1	1	0	1	$x_1\bar{x}_2$
0	1	1	0	0	1	1	$\bar{x}_1 x_2$
1	1	0	0	0	0	0	$x_1 x_2$

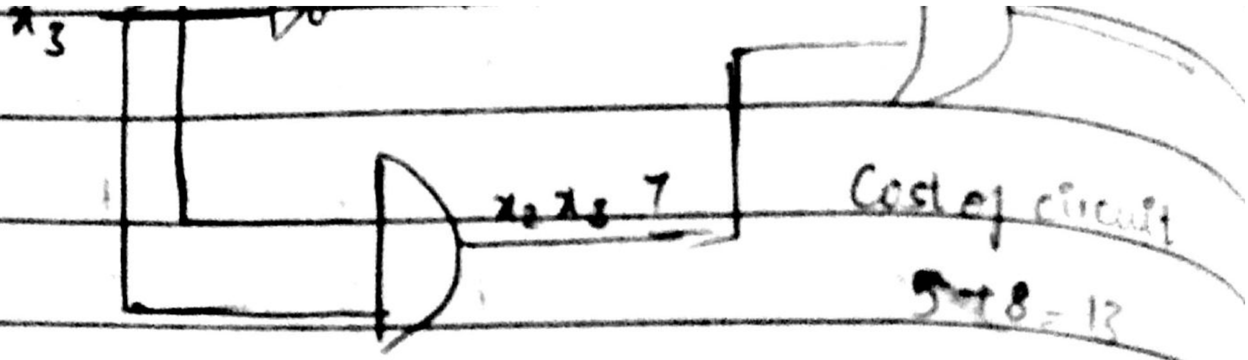
x_1	x_2	x_3	f	minterm
0	0	0	0	$\bar{x}_1\bar{x}_2\bar{x}_3$
0	0	1	1	$\bar{x}_1\bar{x}_2 x_3$
0	1	0	0	$\bar{x}_1 x_2\bar{x}_3$
0	1	1	0	$\bar{x}_1 x_2 x_3$
1	0	0	1	$x_1\bar{x}_2\bar{x}_3$
1	0	1	1	$x_1\bar{x}_2 x_3$
1	1	0	1	$x_1 x_2\bar{x}_3$
1	1	1	0	$x_1 x_2 x_2$

$$f = \bar{x}_1\bar{x}_2 x_3 + x_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_2 x_3 + x_1 x_2\bar{x}_3$$

an expression is a sum of those min-terms for which sum is 1.

Minterms are independent of f, same for any given 'f'.

Truth table depends on inputs.



Cost of circuit

5+8 = 13

Max·terms:

Max·terms are complement of minterms.

Product of sum forms:

A logical expression consisting of sums (logical OR) terms that are the factors of a logical product (logical AND) is said to be the product of sum form. If each term is a max-term then this expression is called canonical product of sum form. Taking max terms for each row in the truth table for which 'f' is 0 and forming a product of these max-terms, gives the expression for f

x_1	x_2	x_3	min-term		maxterm	f
0	0	0	$\bar{x}_1\bar{x}_2\bar{x}_3$	m_0	$x_1+x_2+x_3$	1
0	0	1	$\bar{x}_1\bar{x}_2 x_3$	m_1	$x_1+x_2+\bar{x}_3$	0
0	1	0	$\bar{x}_1 x_2 \bar{x}_3$	m_2	$x_1+\bar{x}_2+x_3$	1
0	1	1	$\bar{x}_1 x_2 x_3$	m_3	$x_1+\bar{x}_2+\bar{x}_3$	1
1	0	0	$x_1 \bar{x}_2 \bar{x}_3$	m_4	$\bar{x}_1+x_2+x_3$	0
1	0	1	$x_1 \bar{x}_2 x_3$	m_5	$\bar{x}_1+x_2+\bar{x}_3$	0
1	1	0	$x_1 x_2 \bar{x}_3$	m_6	$\bar{x}_1+\bar{x}_2+x_3$	0
1	1	1	$x_1 x_2 x_3$	m_7	$\bar{x}_1+\bar{x}_2+\bar{x}_3$	1

$f = \pi(1,4,5,6)$

$f = (x_1+x_2+\bar{x}_3)(\bar{x}_1+x_2+x_3)(\bar{x}_1+x_2+\bar{x}_3)(\bar{x}_1+\bar{x}_2+x_3)$

Prove the consensus property

a) $(x\cdot y)+(\bar{x}\cdot z)+(y\cdot z) = (x\cdot y)+(\bar{x}\cdot z)$

b) $(x+y)(y+z)(\bar{x}+z) = (x+y)(\bar{x}+z)$

Prove $(\bar{x}_1+x_2)(\bar{x}_1+x_3).(x_2+\bar{x}_3) = \bar{x}_1\bar{x}_3 + x_2x_3$

LHS $= (\bar{x}_1 + x_2\bar{x}_1 + \bar{x}_1x_3 + x_2x_3)(x_2+\bar{x}_3)$

$= (\bar{x}_1x_2 + x_2\bar{x}_1 + \bar{x}_1x_2x_3 + x_2x_3 + \bar{x}_1\bar{x}_3 + \bar{x}_1x_2\bar{x}_3)$

$= \bar{x}_1x_2 + x_2x_3 + \bar{x}_1\bar{x}_3$

$= \bar{x}_1x_2(x_3+\bar{x}_3) + \bar{x}_1\bar{x}_3 + x_2x_3$

$= \bar{x}_1x_2x_3 + \bar{x}_1x_2\bar{x}_3 + \bar{x}_1\bar{x}_3 + x_2x_3$

$= x_2x_3(1+\bar{x}_1) + \bar{x}_1\bar{x}_3(1+x_2)$

$= x_2x_3 + \bar{x}_1\bar{x}_3$

$\bar{f} = (x_1+x_2+\bar{x}_3)(\bar{x}_1+x_2+x_3)(\bar{x}_1+x_2+\bar{x}_3)(\bar{x}_1+\bar{x}_2+x_3)$

$= (0+\bar{x}_1x_2+\bar{x}_1\bar{x}_3+x_1x_2+x_2)(\bar{x}_1+\bar{x}_1x_2+\bar{x}_1\bar{x}_3+\bar{x}_1\bar{x}_2+\bar{x}_2\bar{x}_3$
$+\bar{x}_1x_3+x_2x_3)$

$= \cancel{\bar{x}_1(x_2+\bar{x}_3)}+(\bar{x}_1\bar{x}_3+x_2)(\bar{x}_1+\bar{x}_2\bar{x}_3+x_2x_3)$

$= \bar{x}_1\bar{x}_3+\bar{x}_1x_2+\bar{x}_1\bar{x}_2\bar{x}_3 + x_2x_3$ $= \bar{x}_1\bar{x}_3+\bar{x}_1x_2+x_2x_3$

$= \bar{x}_1\bar{x}_3+x_2x_3+\bar{x}_1x_2(x_3+\bar{x}_3)$

$x_1 x_2 \bar{x}_3 + x_1 x_2 \bar{x}_3$

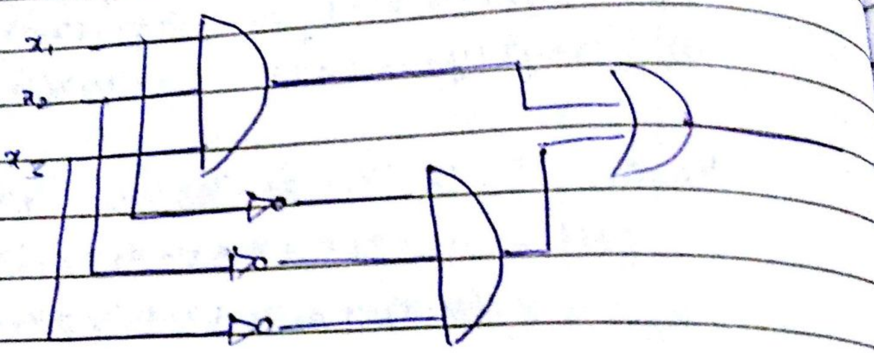
$= \bar{x}_1 \bar{x}_2 + x_2 x_3 + x_1 x_2 x_3 +$

$= \bar{x}_1 \bar{x}_3 + x_1 x_3$

x_1	x_2	x_3	f	minterms		Maxterms
0	0	0	1	$\bar{x}_1 \bar{x}_2 \bar{x}_3$	m_0	$x_1 + x_2 + x_3$
0	0	1	0	$\bar{x}_1 \bar{x}_2 x_3$	m_1	$x_1 + x_2 + \bar{x}_3$
0	1	0	0	$\bar{x}_1 x_2 \bar{x}_3$	m_2	$x_1 + \bar{x}_2 + x_3$
0	1	1	0	$\bar{x}_1 x_2 x_3$	m_3	$x_1 + \bar{x}_2 + \bar{x}_3$
1	0	0	0	$x_1 \bar{x}_2 \bar{x}_3$	m_4	$\bar{x}_1 + x_2 + x_3$
1	0	1	0	$x_1 \bar{x}_2 x_3$	m_5	$\bar{x}_1 + x_2 + \bar{x}_3$
1	1	0	0	$x_1 x_2 \bar{x}_3$	m_6	$\bar{x}_1 + \bar{x}_2 + x_3$
1	1	1	1	$x_1 x_2 x_3$	m_7	$\bar{x}_1 + \bar{x}_2 + \bar{x}_3$

$$f = \bar{x}_1 \bar{x}_2 \bar{x}_3 + x_1 x_2 x_3$$
$$= m_0 + m_7$$

$\bar{f} =$



$$f = \Pi(1,2,3,4,5,6)$$
$$= (x_1 + x_2 + \bar{x}_3)(x_1 + \bar{x}_2 + x_3)(x_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + x_2 + x_3)$$
$$(\bar{x}_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + \bar{x}_2 + x_3)$$

$$= (x_1 + \bar{x}_2 \bar{x}_3 + x_2 x_3)(0 + \bar{x}_1 \bar{x}_2 + \bar{x}_1 \bar{x}_3 + x_1 x_2 + \bar{x}_2 \bar{x}_3 + x_1 x_3$$
$$+ \bar{x}_2 x_3)$$
$$(\bar{x}_1 + \bar{x}_2 \bar{x}_3 + x_2 x_3)$$

$$= (x_1 + \bar{x}_2 \bar{x}_3 + x_2 x_3)(\bar{x}_1 \bar{x}_2 + \bar{x}_1 \bar{x}_3 + x_1 x_2 + x_2 \bar{x}_3 + x_1 x_3 + \bar{x}_2 x_3)$$
$$(\bar{x}_1 + \bar{x}_2 \bar{x}_3 + x_2 x_3)$$

$$= (\bar{x}_1 \bar{x}_2 \bar{x}_2 + 0 + \bar{x}_1 \bar{x}_2 \bar{x}_3 + x_1 x_2 + x_1 x_3 + 0 + x_1 x_2 x_3$$
$$+ x_1 \bar{x}_2 x_3 + 0)(\bar{x}_1 + \bar{x}_2 \bar{x}_3 + x_2 x_3)$$

Logic should be '1' when inputs are odd.

$= (\bar{x}_1 \bar{x}_2 x_3 + x_1 x_2 x_3 + x_1 \bar{x}_2 x_3)(\bar{x}_1 + \bar{x}_2 \bar{x}_3 + x_2 x_3)$

= ~~[illegible]~~

$= (x_1 x_2 \bar{x}_3 + \bar{x}_1 \bar{x}_2 \bar{x}_3 + 0 + x_1 x_2 x_3)$

$= \bar{x}_1 \bar{x}_2 \bar{x}_3 + x_1 x_2 x_3$

= SOP.

Karnov maps are used to find the minimum cost function circuits of a logical funcⁿ. It has cells which correspond to the rows in the truth table. Min term, is any two adjacent cells with either row / column can be combined. As few as possible and as large as possible groups of one's that cover all cases where the function has a value of 1 are found. Larger the group of one's the fewer the no. of variables in the corresponding product term. Diagonal ones cannot be grouped. Group can be of size of 1, 2, 4, 8, 16, 32.

$x_1 \backslash x_2$	0	1
0	1_{00}	1_{01}
1	0_{10}	1_{11}

$\rightarrow \bar{x}_1$, $\rightarrow x_2$

$\bar{x}_1 + x_2$

The term which doesnot vary is taken.

$x_1 \backslash x_2$	0	1
0	1_{00}	0_{01}
1	0_{10}	1_{11}

$\bar{x}_1 \bar{x}_2 + x_1 x_2$

$x_1 \backslash x_2$	0	1
0	1_{00}	1_{01}
1	1_{10}	1_{11}

$\rightarrow \bar{x}_1$, $\rightarrow x_1$

$f = 1$

0	1	0	1	m_2
0	1	1	1	m_3
1	0	0	0	m_4
1	0	1	1	m_5
1	1	0	0	m_6
1	1	1	1	m_7

$x_3 \backslash x_1x_2$	00	01 (010)	11	10
0	0	1 (2)	6	4
1	1	1 (3)	1 (7)	1 (5)
		011	111	101

$\rightarrow x_1x_3 + \bar{x}_1 x_2$

~~Correspond to the rows of the truth table~~

$$f_1 = \bar{x}_1\bar{x}_2\bar{x}_3 + \bar{x}_1 x_2 \bar{x}_3 + x_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_2 x_3 + x_1 x_2 \ldots$$

Find the min cost for a logical function

Bigger the group shorter the time lesser the cost.

$x_5 = 0$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	8	24	16
01	1 2	1 10	26	1 18
11	1 6	1 14	30	1 22
10	4	12	28	20

$x_5 = 1$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	1	9	25	17
01	1 3	1 11	27	19
11	1 7	1 15	31	23
10	5	13	29	21

$\bar{x}_1 x_4$

$\bar{x}_2 x_4 \bar{x}_5$

$\bar{x}_1 x_4 + \bar{x}_2 x_4 \bar{x}_5$

1) $t = \Sigma(0, 2, 4, 8, 10, 12, 14, 16, 18, 20, 22, 24, 25, 26, 28, 30, 31)$

2) $f = \Sigma(5, 6, 12, 14, 18, 28, 30)$

1)

$x_5 = 0$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	1 0	1 8	1 24	1 16
01	1 2	1 10	1 26	1 18
11	6	1 14	1 30	1 22
10	1 4	1 12	1 28	1 20

$x_5 = 1$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	1	9	1 25	17
01	3	11	27	19
11	7	15	1 31	23
10	5	13	29	21

$\bar{x}_3 \bar{x}_5$ $x_2\bar{x}_5$ $x_1\bar{x}_5$

$x_3\bar{x}_4\bar{x}_5$

$x_1\bar{x}_5 + x_2\bar{x}_5 + \bar{x}_3\bar{x}_5 + x_3\bar{x}_4\bar{x}_5 + x_1x_2\bar{x}_3\bar{x}_4 + x_1x_2x_3x_4$

$= x_1\bar{x}_5 + x_2\bar{x}_5 + \overline{x_3 x_5} + x_3\bar{x}_4\bar{x}_5 + x_1x_2$

$t = x_1\bar{x}_5 + x_2\bar{x}_5 + \bar{x}_3\bar{x}_5 + x_1x_2 + x_3\bar{x}_4\bar{x}_5$

Two four variable maps can be used to construct a five variable map. One map is directly beside the other, they are distinguished by $x_5=0$ and $x_5=1$.

Groups are formed if one appears at the same place in both the four variable maps.

Q) $f = \Sigma(5, 6, 12, 14, 18, 28, 30)$

$x_5 = 0$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	8	24	16
01	2	10	26	1 (18)
11	1 (6)	1 (14)	1 (30)	22
10	4	1 (12)	1 (28)	20

$x_1\bar{x}_2\bar{x}_3x_4\bar{x}_5$

$\bar{x}_1\bar{x}_2x_3x_4\bar{x}_5$

$x_2x_3\bar{x}_5$

$x_5 = 1$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	1	9	25	17
01	3	11	27	19
11	7	15	31	23
10	1 (5)	13	29	21

$\bar{x}_1\bar{x}_2x_3\bar{x}_4x_5 \quad x_5 = 1$

$$f = \bar{x}_1\bar{x}_2x_3x_4\bar{x}_5 + x_2x_3\bar{x}_5 + x_1\bar{x}_2\bar{x}_3x_4\bar{x}_5 + \bar{x}_1\bar{x}_2x_3\bar{x}_4x_5$$

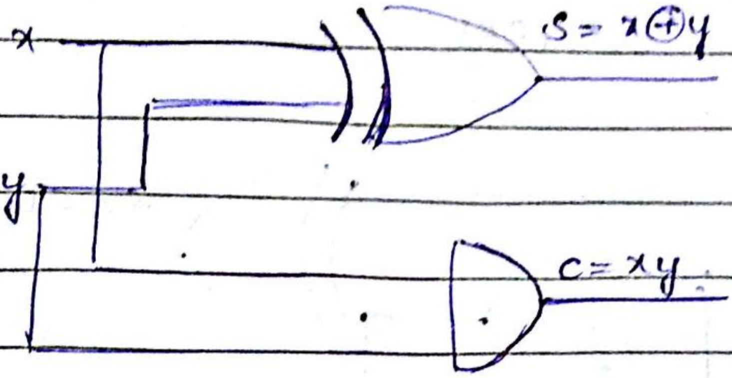
$$= \cancel{x_2x_3\bar{x}_5 + \bar{x}_1\bar{x}_2x_3}$$

x	y	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$S = \bar{x}y + x\bar{y}$

$= x \oplus y$

$C = xy$



HALF ~~ADDRESS~~ ADDER

z C_{i-1}	x	y	S	C
0	0	0	0	0
0	0	1	1	0

$S = \bar{x}y\bar{z} + \bar{z}x\bar{y}$

$+ z\bar{x}\bar{y} + zxy$

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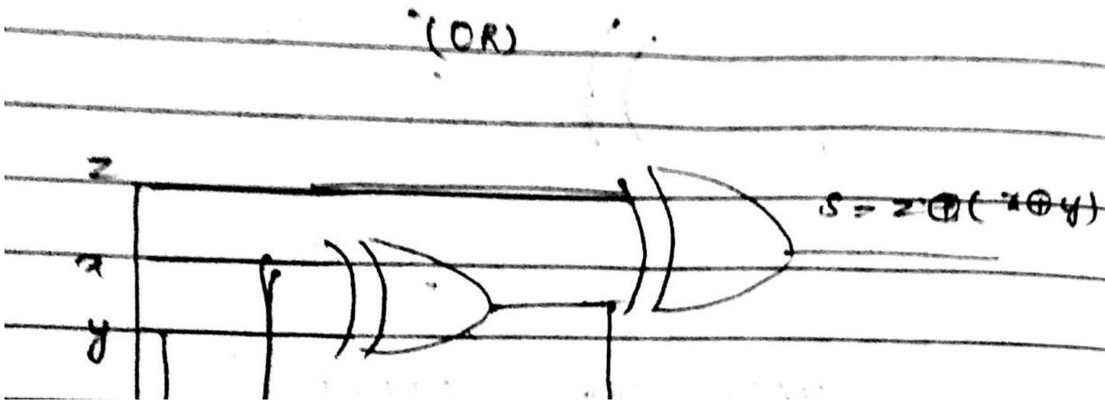

(OR)
z
x
y
S = z ⊕ (x ⊕ y)

1's compliment means every 0 is converted to 1 & every 1 to '0'.

Any given number, its 2' compliment is negative of the given number.

1) Make the number of bits equal.

2) Take 1's compliment of the subtrand

3) 1's compliment + 1 ~~to each~~ bit of subtrand gives 2's compliment.

4)

$C_{i+1} = x_i y_i + x_i c_i + y_i c_i - g_i + P_i c_i$

G.I is called the generatefunction. If G.I = 1

$\Longrightarrow x_i = 1, y_i = 1$

$\therefore C_{i+1} = 1$

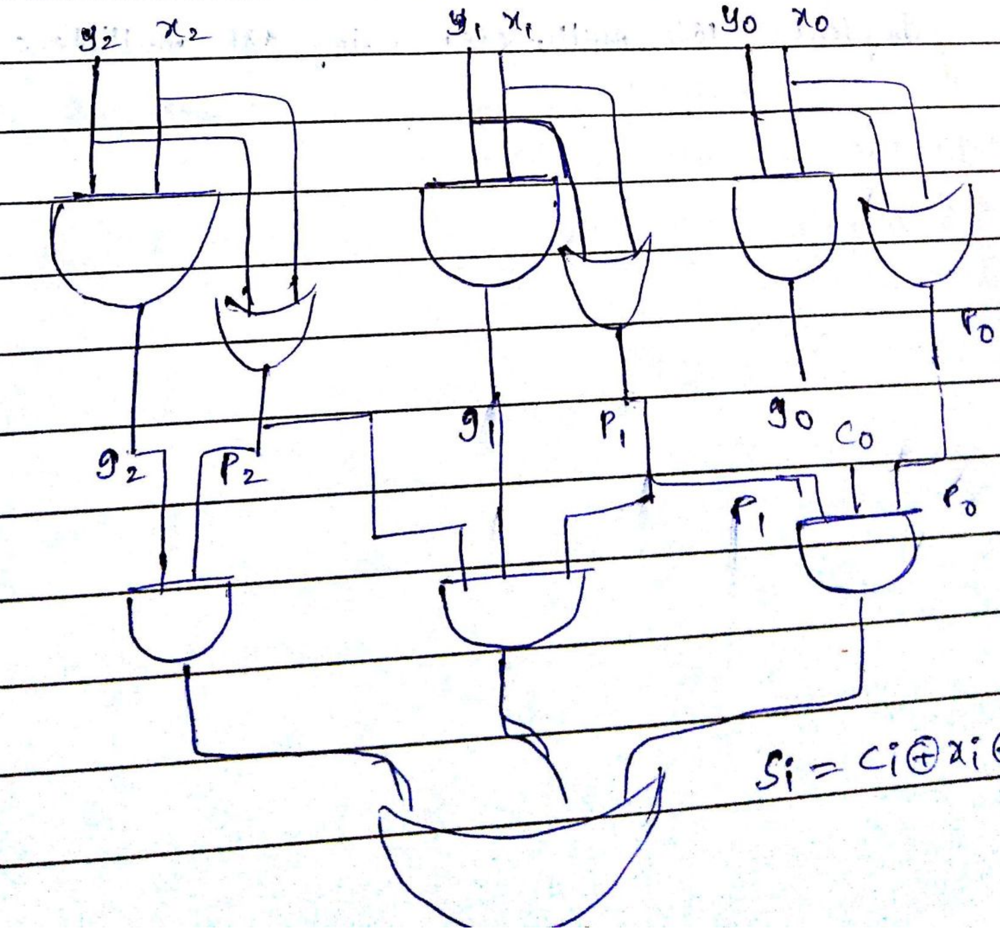
If PI = 1 then atleast $x_i = 1$ or $y_i = 1$ then ~~Ci~~ is cent out.

$$x = x_{n-1}\, x_{n-2} \,-----\, \overset{c_0}{x_1}\, x_0$$

$$y = y_{n-1}\, y_{n-2} \,----\, y_1\, y_0$$

$$s_1\; s_0$$

If $C_{i+1} = P_1 P_0 C_0 + P_2 P_1 g_1 + P_2 g_2$



$S_i = C_i \oplus x_i \oplus y_i$

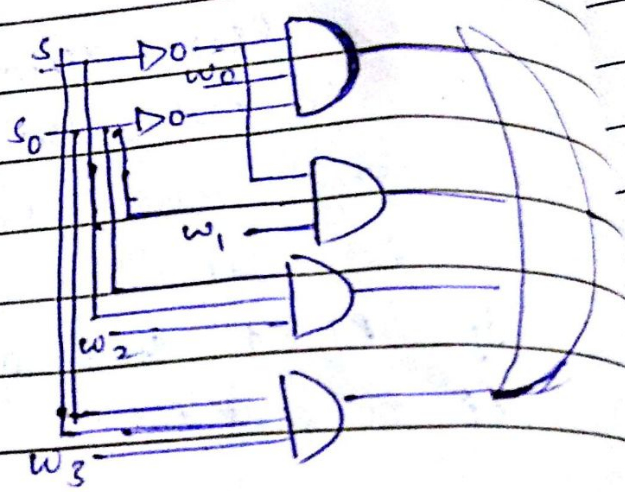
Multiplexer:

It has 'n' number of inputs but it gives only one output.

$\log_2 n \Rightarrow$ no of selection lines

4×1

s_1	s_0	f
0	0	w_0
0	1	w_1
1	0	w_2
1	1	w_3



Implent 16×1 multiplexer using 4×1 multiplexers.

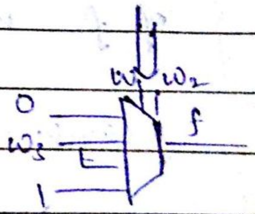
Synthesis of a logic function using Multiplexer:

Function can be implented by a multiplexer such that value of 'f' in each row of the truth table as constants to the multiplexes ~~signals~~ data inputs & selected inputs are driven by truth table input signals.

w_1	w_2	w_3	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$\Rightarrow$

w_1	w_2	f
0	0	0
0	1	w_3
1	0	w_3
1	1	1



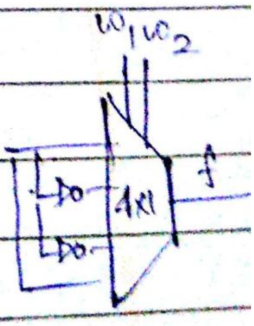
Implement a majority function

$f = w_1 \oplus w_2 \oplus w_3$ using a 4x1 multiplexer.

~~Extend~~ Redesign the same function using a 2X1 multiplexe

w_1	w_2	w_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

w_1	w_2	f
0	0	w_3
0	1	$\bar{w}_3$
1	0	$\bar{w}_3$
1	1	w_3



Shannon's ~~Channels~~ Expansion Theorem:

Any boolean function 'f' $(w_1, w_2, w_3 \ldots w_n)$ can be written in the form $\bar{w}_1 f(0, w_2, w_3 \ldots w_n) + w_1 f(1, w_2, w_3 \ldots w_n)$

Here $f(0, w_2, w_3 \ldots w_n)$ is called a cofactor of f with $\bar{w}_1$

$$f(w_1, w_2, w_3) = w_1 w_2 + w_1 w_3 + w_2 w_3$$
$$= w_1 w_2 + w_1 w_3 + (w_1 + \bar{w}_1) w_2 w_3$$
$$= \bar{w}_1 (w_2 w_3) + w_1 (w_2 + w_3 + w_2 w_3)$$
$$= \bar{w}_1 (w_2 w_3) + w_1 (w_2 + w_3)$$

$f(\omega_1, \omega_2 \ldots\ldots \omega_n)$

$= \overline{\omega_1}\,\overline{\omega_2}\, f(0,0,\omega_3,\omega_4, \ldots \omega_n)$

$+ \overline{\omega_1}\,\omega_2\, f(0,1,\omega_3 \ldots \omega_n)$

$+ \omega_1\overline{\omega_2}\, f(1,0,\omega_3 \ldots \omega_n)$

$+ \omega_1\omega_2\, f(1,1,\omega_3 \ldots \omega_n)$

$f(\omega_1, \omega_2, \omega_3) = \omega_1\omega_2 + \omega_1\omega_3 + \omega_2\omega_3$

$= \omega_1\omega_2 + \omega_1\omega_3(\omega_2 + \overline{\omega_2}) + (\omega_1 + \overline{\omega_1})\omega_2\omega_3$

$= \omega_1\omega_2 + \omega_1\omega_2\omega_3 + \omega_1\overline{\omega_2}\omega_3 + \omega_1\omega_2\omega_3 + \overline{\omega_1}\omega_2\omega_3$

$+ \overline{\omega_1}\,\overline{\omega_2}\cdot(0)$

$= \overline{\omega_1}\,\overline{\omega_2}(0) + \omega_1\omega_2 + \omega_1\overline{\omega_2}(\omega_3) + \omega_1\omega_2\omega_3$