

Hypothesis Testing

Estimation :- studying the samples & drawing the conclusion about population is known as estimation.

There are 2 types of estimations.

- 1) point estimation
- 2) Interval estimation.

Point estimation :- By studying the single observation and drawing the conclusion about population is known as point estimation.

Interval estimation :- By studying the set of observations & drawing the conclusion about the population is known as interval estimation.

Procedure for statistical testing

Steps for Hypothesis testing

- 1) Null hypothesis (H_0) :-

prepare the assumptions to the problem without any favoritism.

- 2) Alternative Hypothesis (H_1) :-

The converse of null hypothesis is known as alternative hypothesis

- 3) If the alternative hypothesis contains not equality (\neq) sign, then the name of the test is known as two tailed test.

- 4) If the alternative hypothesis contains inequality sign then the test is known as single tailed test.

5) If the inequality is less than ($<$) then it is known as left single tailed test.

6) If the inequality sign is greater than ($>$), then it is known as right single tailed test.

→ Degrees of Freedom (d.f.) :-

The no. of observations that are actually participating for testing the hypothesis is known as degrees of freedom.

Ex: If the no. of observations are n , its degrees of freedom is $n-1$.

→ Types of test :-

1) ~~Sample~~

1) small sample test

2) large sample test.

1) If no. of obs in given data are ≤ 30 then it is known as small sample test and is defined as $t = \frac{\bar{x} - E(x)}{\sqrt{\frac{s^2}{n}}} \sim t_{n-1}$

defined as $t = \frac{\bar{x} - E(x)}{S.E.(x)} \sim t_{n-1}$

\downarrow \downarrow

Calculated part tabulated part

S.E. - Standard Error.

2) If no. of samples (n) ≥ 30 then it is known as large sample test & is defined as

$$Z = \frac{x - E(x)}{S.d(x)} \sim N(0,1)$$

⇒ Levels of Significance (LOS):-

There are 4 diff accepting levels for testing the hypothesis. Those are :-

90% 95% 98% 99%

The rejection levels are

10% 5% 2% 1%

* levels of significance mathematically denoted by t value.

cut

ii) Inference :- / (conclusion)

Since the calculated value is less than tabulated value at diff LOS in diff d.f, our null hypothesis will be accepted, otherwise rejected.

i) Statistical errors :- (Types of errors)

Type I Error :-

Rejecting H_0 when H_0 is true.

Rej H_0 | H_0 is true

It is denoted by α

ii) Type II Error :-

Accepting H_0 when H_0 is false

Accep H_0 | H_0 is false

It is denoted by β

Notation

	Population	Sample
Size	N	n
Mean	μ	$\bar{x} = \frac{\sum x_i}{n}$
Variance	σ^2	$S^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$
Proportion	P	$p = \frac{x}{n}$

$S.d = \sqrt{\text{Population variance}}$

$S.E = \sqrt{\text{Sample variance}}$

Types of tests

chisquare test (χ^2 -test)

There are two types of chisquare tests

- 1) chisquare goodness of fit for independent attributes
- 2) chisquare goodness of fit for "rxc tables"

1) $H_0: P_1 = P_2 = \dots = P_n$

$H_1: P_1 \neq P_2 \neq \dots \neq P_n$

d.f : $n-1$

Los : at diff levels

Test statistics:-

$$\chi^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i} \sim \chi_{n-1}^2$$

O - observed frequencies

E - expected frequencies / estimated values

Sample table:-

O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$

The total of last column itself is calculated value

Inference

Since χ^2 calculated value $<$ χ^2 table value at diff los in diff d.f., our null hypothesis will be accepted otherwise rejected.

χ^2 -test ($r \times c$ tables)

$$H_0: p_1 = p_2 = p_3 = \dots = p_n$$

$$H_1: p_1 \neq p_2 \neq p_3 \neq \dots \neq p_n$$

$$\text{d.f.} : (r-1 \times c-1)$$

r = no. of rows

c = no. of columns

los: at different levels

Test statistics:-

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} \sim \chi^2_{(r-1 \times c-1)}$$

↓
follows

Inference:-

If χ^2 calculated value is less than χ^2 tabulated value, our null hypothesis will be accepted otherwise rejected.

2x2 contingency table:-

	α	β	R.T
I	a	b	a+b
II	c	d	c+d
C.T	a+c	b+d	$N = a+b+c+d$

$$E(a) = \frac{C.T \times R.T}{N} = \frac{(a+b)(a+c)}{N}$$

$$E(b) = \frac{(a+b)(b+d)}{N}$$

$$E(c) = \frac{(a+c)(c+d)}{N}$$

$$E(d) = \frac{(c+d)(b+d)}{N}$$

	O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
a				
b				
c				
d				

Test the χ^2 goodness of fit for following data
 Comment the data at 5% LOS

	α	β	R.T
A	18	26	44
B	45	52	97
C.T	63	48	N = 141

$$H_0 : P_1 = P_2 = P_3 = P_4$$

$$H_1 : P_1 \neq P_2 \neq P_3 \neq P_4$$

$$\text{d.f} : r = 2 : c = 2$$

$$\text{d.f} = (2-1)(2-1) = 1$$

$$\text{LOS} : 0.05$$

Test statistics:-

$$\chi^2 = \sum \frac{(O-E)^2}{E} \sim (r-1)(c-1)$$

$$E(a) = \frac{44 \times 63}{141} = 19.65$$

$$E(b) = \frac{44 \times 78}{141} = 24.34$$

$$E(c) = \frac{63 \times 97}{141} = 43.34$$

$$E(d) = \frac{97 \times 78}{141} = 53.65$$

i.e., There is no significant differences b/w the population proportion

O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
a	19.65	2.7225	0.1385
b	24.34	2.7556	0.1132
c	43.34	2.7556	0.0635
d	53.65	2.7225	0.0507
			<u>0.3659</u>

$$\chi^2_{\text{cal}} = \sum \frac{(O-E)^2}{E} = 0.3659 \quad \text{since } \chi^2_{\text{cal}} < \chi^2_{\text{table}} \text{ in 1 d.f at 0.05 LOS}$$

$\chi^2_{\text{table}} = 3.84$ at 5% LOS in 1 d.f
 \therefore our null hypothesis will be accepted

2) Test χ^2 goodness of fit for following data

	A	B	C	D
α	17	21	18	30
β	15	40	26	35
γ	11	25	35	43

$$d.f = (3-1)(4-1) \\ = 2 \times 3 = 6$$

$$H_0: p_\alpha = p_\beta = p_\gamma = p_A = p_B = p_C = p_D \\ H_1: p_\alpha \neq p_\beta \neq p_\gamma \neq p_A \neq p_B \neq p_C \neq p_D$$

$$LOS = 5\% = 0.05$$

Test statistics

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} \sim \chi^2_{(r-1)(c-1)}$$

~~$G(x)$~~

	A	B	C	D	
α	17	21	18	30	86
β	15	40	26	35	116
γ	11	25	35	43	114
	43	86	79	108	316

$$E(17) = \frac{43 \times 86}{316} = 11.702$$

$$E(15) = \frac{116 \times 43}{316} = 15.784$$

$$E(11) = \frac{114 \times 43}{316} = 15.512$$

$$E(21) = \frac{86 \times 86}{316} = 23.405$$

$$E(18) = \frac{86 \times 79}{316} = 21.5$$

$$E(30) = \frac{86 \times 108}{316} = 29.392$$

$$E(40) = \frac{116 \times 86}{316} = 31.56$$

$$E(26) = \frac{79 \times 116}{316} = 29$$

$$E(35) = \frac{79 \times 114}{316} = 29.645$$

$$E(25) = 31.025$$

$$E(35) = \frac{114 \times 79}{316} = 28.5$$

$$E(43) = \frac{114 \times 108}{316} = 38.962$$

O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
		28.068	2.398
17	11.702		
21	23.405	5.784	0.247
		12.25	0.569
18	21.5		
30	29.392	0.369	0.012
		0.614	0.038
15	15.784		
40	31.56	71.233	2.257
	29	9	0.310
26			
35	39.645	21.576	0.544
		20.358	1.312
11	15.512		
25	31.025	36.300	1.170
		42.25	1.482
35	28.5		
43	38.962	16.305	0.418
			<u>10.757</u>

$$\chi^2_{cal} = 10.757$$

$$\chi^2_{tab} = 12.59 \text{ at } 5\% \text{ LOS in } 6 \text{ d.f.}$$

Since $\chi^2_{cal} < \chi^2_{tab}$ in 6 d.f. at 0.05 LOS
our null hypothesis will be accepted

3) A die is rolled 120 times, check whether the die is unbiased or not for the following data.

face	1	2	3	4	5	6
freq	35	40	30	25	40	30

→ Avg of given data is expected frequency

H_0 : die is unbiased

H_1 : die is not unbiased

d.f : $n=6$

⇒ d.f : 5

LOS : 5% = 0.05

Test statistics:-

$$\chi^2 = \sum \frac{(O-E)^2}{E} \sim \chi^2_{n-1}$$

Calculation

$$\begin{aligned} \text{Exp} &= \frac{\sum f_i}{n} = \frac{\sum O}{n} \\ &= \frac{200}{6} = 33.3 \end{aligned}$$

for every obs value, exp value is same

O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
35	33.3	2.89	0.086
40	"	44.89	1.348
30	"	10.89	0.327
25	"	68.89	2.068
40	"	44.89	1.348
30	"	10.89	0.327
			<u>5.504</u>

$$\chi^2_{\text{cal}} = 5.504$$

$$\chi^2_{\text{tab}} = 11.07$$

Since $\chi^2_{\text{cal}} < \chi^2_{\text{tab}}$ at 5 d.f in 0.05 LOS

our null hypothesis will be accepted
i.e., the die is unbiased

Testing Single Sample mean (σ unknown)

[small sample test]

$$H_0 : \bar{x} = \mu$$

$$H_1 : \bar{x} \neq \mu$$

$$d.f : n-1$$

LOS : at different levels

Test statistics:

$$t = \frac{\bar{x} - \mu}{\sqrt{s^2/n}} \sim t_{n-1}$$

$$\text{Here } \bar{x} = \frac{\sum x_i}{n}$$

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

↓
Sum of squares
deviation from mean

Inference:-

Since t calculated value $< t$ tabulated value at different LOS in different d.f, our null hypothesis will be accepted, otherwise rejected.

Testing difference b/w two means (σ unknown)

$$H_0 : \mu_1 = \mu_2 ; \sigma_1^2 = \sigma_2^2$$

$$H_1 : \mu_1 \neq \mu_2 \quad \sigma_1^2 \neq \sigma_2^2$$

$$d.f : n_1 - 1 + n_2 - 1 \\ = n_1 + n_2 - 2$$

LOS : at different levels

Test Statistics:-

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \sim t_{n_1 + n_2 - 2}$$

$$\text{Here } \bar{x}_1 = \frac{\sum x_{1i}}{n_1}$$

$$\bar{x}_2 = \frac{\sum x_{2i}}{n_2}$$

$$s^2 \rightarrow \text{pooled sample variance} \\ s^2 = \frac{\sum (x_{1i} - \bar{x}_1)^2 + \sum (x_{2j} - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

Inference:- Since $t_{cal} < t_{tab}$ at diff LOS in diff d.f our null hypothesis will be accepted, otherwise rejected.

Equality of population Variances (F-test)

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$H_1: \sigma_1^2 \neq \sigma_2^2$$

$$d.f: F(n_1-1, n_2-1)$$

LOS: at different levels

Test statistics:-

$$F = \frac{S_1^2}{S_2^2} \sim F$$

$$S_1^2 = \frac{\sum (x_{1i} - \bar{x}_1)^2}{n_1 - 1}, \quad S_2^2 = \frac{\sum (x_{2j} - \bar{x}_2)^2}{n_2 - 1}$$

Inference:- Since $F_{cal} < F_{tab}$ at diff LOS in diff d.f, our Null hypothesis will be accepted, otherwise rejected.

1) The average life time of Computer is 1200 hours
A random sample of 10 computers are tested
whose averages are 1050, 1175, 1205, 1210, 1100,
1150, 1163, 1210, 1100, 1109

$$H_0: \mu = 1200$$

$$H_1: \mu \neq 1200$$

$$d.f: n = 10$$

$$n-1 = 9$$

$$LOS: 5\%$$

$$\begin{aligned}\bar{x} &= \frac{11472}{10} \\ &= 1147.2\end{aligned}$$

x_i	$(x_i - \bar{x})^2$	\bar{x}
1050	9147.84	
1175	772.84	
1205	3340.84	
1210	3943.84	
1100	2227.84	
1150	7.84	
1163	249.64	
1210	3943.84	
1100	2227.84	
1109	1459.24	
	27621.6	

$$S^2 = \frac{27621.6}{9} = 3069.066$$

$$t = \frac{\bar{x} - \mu}{\sqrt{s^2/n}} \sim t_{n-1}$$

$$= \frac{-52.8}{17.518}$$

$$s = 55.399$$

$$s/\sqrt{n} = 17.518$$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{-52.8}{17.518} = -3.014$$

$$t_{\text{tab}} = 2.262 \quad |t_{\text{cal}}| = 3.014$$

$$t_{\text{cal}} > t_{\text{tab}}$$

Inference:-

Since $t_{\text{cal}} < t_{\text{tab}}$ at 9 d.f our null hypothesis will be rejected

\therefore Average life of computer $\neq 1200$

2) The mean life expectancy is 59 years in the year 1980. A random sample of 12 people observed whose mean lives are 58, 57, 62, 61, 60, 59, 64, 55, 56, 56, 57, 61

$$H_0: \mu = 59$$

$$H_1: \mu \neq 59$$

$$d.f = 11$$

$$\bar{x} = \frac{706}{12} = 58.83$$

$$t = \frac{-0.17}{\frac{\sqrt{85.6668}}{12}} = \frac{-0.17}{2.67}$$

$$= -0.063$$

$$|t_{\text{cal}}| = 0.063$$

$$58 \quad 0.6889$$

$$57 \quad 3.3489$$

$$62 \quad 10.0489$$

$$61 \quad 4.7089$$

$$60 \quad 1.3689$$

$$59 \quad 0.0289$$

$$64 \quad 26.7289$$

$$55 \quad 14.6689$$

$$56 \quad 8.0089$$

$$56 \quad 8.0089$$

$$57 \quad 3.3489$$

$$61 \quad 4.7089$$

$$706 \quad 85.6668$$

$$t_{tab} = 2.201$$

$$t_{cal} < t_{tab}$$

Inference

- Q) Test the equality of means & equality of variances for the following observations & comment at 5% level.

x_1	1	7	9	11	19	26	35
x_2	15	22	33	40	42	51	58

$$H_0: \mu_1 = \mu_2 ; \sigma_1^2 = \sigma_2^2$$

$$H_1: \mu_1 \neq \mu_2 ; \sigma_1^2 \neq \sigma_2^2$$

$$df: n_1 = 7, n_2 = 7 \quad \left\{ \begin{array}{l} t_{n_1+n_2-2} = t_{12} \\ \text{df} = 12 \end{array} \right.$$

$$F(v_1, v_2) = F(n_1-1, n_2-1) = F(6, 6)$$

LOS : 5%

Test statistics :

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2(\frac{1}{n_1} + \frac{1}{n_2})}} \sim t_{n_1+n_2-2}$$

$$F = \frac{s_1^2}{s_2^2} \sim F(v_1, v_2)$$

Calculation

	x_{1i}	x_{2i}	$(x_{1i} - \bar{x}_1)^2$	$(x_{2i} - \bar{x}_2)^2$
	1	15	207.93	496.3984
	7	22	70.8964	233.4784
	9	33	41.2164	18.3184
	11	40	19.53	7.3984
	19	42	12.81	22.2784
	26	51	11.93	188.2384
	35	58	383.37	429.3184
	108	261	247.68	1395.4288
			707.407864	

$$\bar{x}_1 = 15.42$$

$$\bar{x}_2 = 37.28$$

$$F = \frac{S_1^2}{S_2^2} = \frac{847.68}{6} = 141.28$$

$$S_2^2 = \frac{1395.4288}{6} = 232.57$$

$$F = \frac{232.57}{141.28} = 1.646$$

$$F_{cal} = 1.646$$

$$F_{tab} = 4.28$$

$$S^2 = \frac{847.68 + 1395.4288}{12} = 186.92$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} = \frac{-21.86}{7.307} = -2.99$$

$$t_{cal} = -2.99$$

$$t_{tab} = 2.179$$

$$t_{cal} > t_{tab}$$

$$F_{cal} < F_{tab}$$

Inference for mean:-

Since $t_{cal} > t_{tab}$ at 12 d.f in 0.05 LOS
our null hypothesis will be rejected i.e., there is
significant differences b/w population mean

Inference for F:-

Since $F_{cal} < F_{tab}$ at (6,6) d.f in 0.05 LOS
our null hypothesis will be accepted, i.e., there
is no significant differences b/w population
Variances.

Q) Test the equality of means & equality of
Variances for the following observations

x_1	11	15	21	28	32	40	46	58	
x_2	21	18	26	3	11	17	19	21	28

$$H_0: \mu_1 = \mu_2 : \sigma_1^2 = \sigma_2^2$$

$$H_1: \mu_1 \neq \mu_2 : \sigma_1^2 \neq \sigma_2^2$$

$$d.f: \left. \begin{array}{l} n_1 = 8 \\ n_2 = 9 \end{array} \right\} t_{n_1+n_2-2} = t_{15}$$

$$F(v_1, v_2) = F(n_1 - 1, n_2 - 1) = F(8 - 1, 9 - 1) = F(7, 8)$$

LOS : 5%

Test statistics :-

x_{1i}	x_{2i}	$(x_{1i} - \bar{x}_1)^2$	$(x_{2i} - \bar{x}_2)^2$
11	21	415.14	7.72
15	18	268.14	0.048
21	26	107.64	60.52
28	3	11.39	231.648
32	11	0.39	52.128
40	17	74.39	1.488
46	19	213.89	0.608
58	21	708.89	7.728
<u>251</u>	<u>28</u>	<u>1799.87</u>	<u>95.64</u>
	164		457.52

$$\bar{x}_1 = \frac{251}{8} = 31.375$$

$$\bar{x}_2 = \frac{164}{9} = 18.22$$

$$S_1^2 = 257.124$$

$$S_2^2 = \frac{457.52}{8} = 57.19$$

$$F_{\text{Cal}} = \frac{257.124}{57.19} = 4.495$$

$$F_{\text{tab}}(7, 8) = 3.50$$

$$S^2 = \frac{1799.87 + 457.52}{15} = 150.492$$

$$t_{\text{Cal}} = \frac{13.155}{5.966} = 2.2042$$

$$t_{\text{tab}} = 2.131$$

Inference for mean:-

Since $t_{\text{cal}} > t_{\text{tab}}$ in 15 d.f at 5% LOS, our null hypothesis will be ~~accepted~~ ^{rejected}, i.e., there is ~~no~~ significant diff b/w population means.

Inference for variance:

Since $F_{cal} > F_{tab}$ at ^(7,8) d.f in 5% LOS, our null hypothesis will be rejected. i.e., there is a significant diff b/w population variances

ANOVA (Analysis of Variance)

one way classification

Testing the consistency between the observations among the heterogeneity groups is known as analysis of variance.

Treatment	T_1	a_{11}	a_{12}	a_{13}	a_{14}	
	T_2	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}
	T_3	a_{31}	a_{32}			
	T_4	a_{41}	a_{42}	a_{43}		

calculation

$$c.f = \frac{G^2}{n}$$

$$\text{Total sum of squares (TSS)} = \sum \sum y_{ij}^2 - c.f$$

$$\text{Treatment sum of squares (TrSS)} = \frac{(\sum y_i)^2}{r} - c.f$$

$$\text{Error sum of squares (ESS)} = TSS - TrSS$$

ANOVA table

S.V	df	S.S	MSS	F_{cal}	F_{tab}
Treatment	$k-1$	$TrSS$	$M_{Tr} = \frac{TrSS}{k-1}$	$F = \frac{M_{Tr}}{M_E}$	$F(k-1, n-k)$
Error	$n-k$	ESS	$M_E = \frac{ESS}{n-k}$		
Total	$n-1$	TSS			

Q2)

T_1	7	11	17	18	21		74
T_2	8	6	3	2	6	8	33
T_3	4	2	1	8			15
T_4	3	6	11	17	15		52
T_5	5	4	0	3			12
T_6	2	1					3
	29	30	32	48	42	8	<u>189</u>

$$C.f = \frac{G^2}{n} = \frac{(189)^2}{26} = 1373.88$$

$$T_{SS} = 2257 - 1373.88 = 883.12$$

$$T_{RSS} = \frac{9647}{6} - 1373.88 = 1607.833 - 1373.88 = 233.95$$

$$E_{SS} = 883.12 - 233.95 = 649.17$$

ANOVA table

S.V	d.f	SS	MSS	F _{cal}	F _{tab}
Treat	6-1=5	233.95	$\frac{233.95}{5} = 46.79$	$\frac{46.79}{32.45} = 1.44$	F(5,20)
Error	26-6=20	649.17	$\frac{649.17}{20} = 32.45$		= 2.75
Total	26-1=25	883.12			

Inference: $F_{cal} > F_{tab}$ at (5,20) d.f in 0.05 LOS, our null hypothesis will be rejected.

i.e., there is a significant diff b/w the treatments

Q)

T_1	3	7	9	-1	6	24
T_2	2	3	0	4		9
	5	4	2	8	6	25
T_3	2	1	-1	-2	0	0
T_4						4
T_5	2	1	0	1		
						<u>62</u>

$n = 23$

$$C.f. = \frac{(62)^2}{23} = 167.13$$

$$T_{SS} = 366 - 167.13 = 198.87$$

$$T_{RSS} = \frac{(24)^2}{5} = 115.2$$

$$T_{RSS} = \frac{9^2}{4} = 20.25$$

$$T_{RSS} = 125$$

$$T_{RSS} = 0$$

$$T_{RSS} = 4$$

$$T_{RSS} = 264.45 - 167.13 = 97.32$$

$$S_{SS} = 198.87 - 97.32 = 101.55$$

Annova table

S.v	d.f	SS	MSS	F _{cal}	F _{tab}
Treat	$5 - 1 = 4$	97.32	24.33	4.313	$F(4, 18)$ $= 2.93$
Error	$23 - 5 = 18$	5.64 101.55	5.64		
Total	22	198.87			

Inference: $F_{cal} > F_{tab}$ at $F(4, 18)$ a.f. in 0.05
LOS, our null hypothesis will be rejected.

i.e., there is a significant diff b/w the treatments.

Paired t-test (small sample test)

Assumptions :-

- 1) Population SD is unknown
- 2) observations need not be independent for both the samples.
- 3) The size of the observations of both the samples must be same.

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$

$$d.f : n-1$$

LOS : at diff levels

Test statistics :-

$$t = \frac{\bar{d}}{s/\sqrt{n}} \sim t_{n-1}$$

Here $d_i = x_i - y_i$

$$\bar{d} = \frac{\sum d_i}{n}$$

$$s = \sqrt{\frac{\sum (d_i - \bar{d})^2}{n-1}}$$

Inference :-

Since $t_{cal} < t_{tab}$ at diff LOS in diff d.f our null hypothesis will be ~~rejected~~, accepted otherwise rejected.

- 2) Apply paired t-test for the following set of observations and test the significance b/w the population.

x	7	9	17	26	35	42	51
y	9	8	16	24	39	45	60

$$n = 7$$

$$d.f = 6$$

x	y	d _i	(d _i - \bar{d}) ²
7	9	-2	4
9	8	1	9
17	16	1	9
26	24	2	16
35	39	-4	16
42	45	-3	9
51	60	-9	81
		-14	88

$$S = \sqrt{\frac{88}{6}} = 3.829$$

$$\bar{d} = \frac{-14}{7} = -2$$

$$t_{cal} = \frac{-2}{3.829/\sqrt{7}} = \frac{-5.29}{3.829} = -1.381$$

$$|t_{cal}| = 1.381$$

$$t_{tab} = 2.447$$

Inference :-

Since $t_{cal} < t_{tab}$ at ~~1%~~ 0.05 LOS in 6 d.f

our null hypothesis will be accepted.

Q) ~~x~~

x	y	d _i = x _i - y _i	(d _i - \bar{d}) ²
		-9	64
21	30	-7	36
35	42	-5	16
46	51	-1	4
57	56	3	1089
84	52	-7	36
93	100	-8	49
102	110	-5	16
110	115	-8	1310

$$n = 8$$

$$d.f = 7$$

$$\bar{d} = \frac{-8}{8} = -1$$

$$S = \sqrt{\frac{1310}{7}} = 13.68$$

$$t = \frac{-1 \times \sqrt{8}}{13.68} = -0.206$$

$$|t| = 0.206$$

Inference :-

$$t_{tab} = 2.365$$

Since $t_{cal} < t_{tab}$ at 5% LOS in 7 d.f, our null hyp will be accepted