17/09/18 :-

4. push down Automata :- (PDA)

Γ (Tow) Stack Alphabet $Z_0 \rightarrow$ initial symbol of stack

7 Tuples

$$M = (Q, \Sigma, \Gamma, q_0, Z_0, \delta, F)$$

The formal defination of PDA

A PDA involves 7 tuples

$$M = (Q, \Sigma, \Gamma, q_0, Z_0, \delta, F)$$

$Q \rightarrow$ Finite set of states

$\Sigma \rightarrow$ Finite set of alphabets

$\Gamma \rightarrow$ Finite stack alphabets.

$q_0 \rightarrow$ Start state

$Z_0 \rightarrow$ Start symbol in stack

$\delta \rightarrow$ Transtition f^n

$$\boxed{PDA \longrightarrow \delta : Q \times \Sigma \cup \{\epsilon\} \times \Gamma}$$

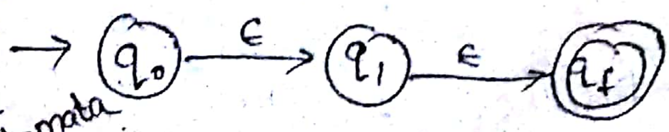
$F \rightarrow$ final states

A graphical notation of PDA :-

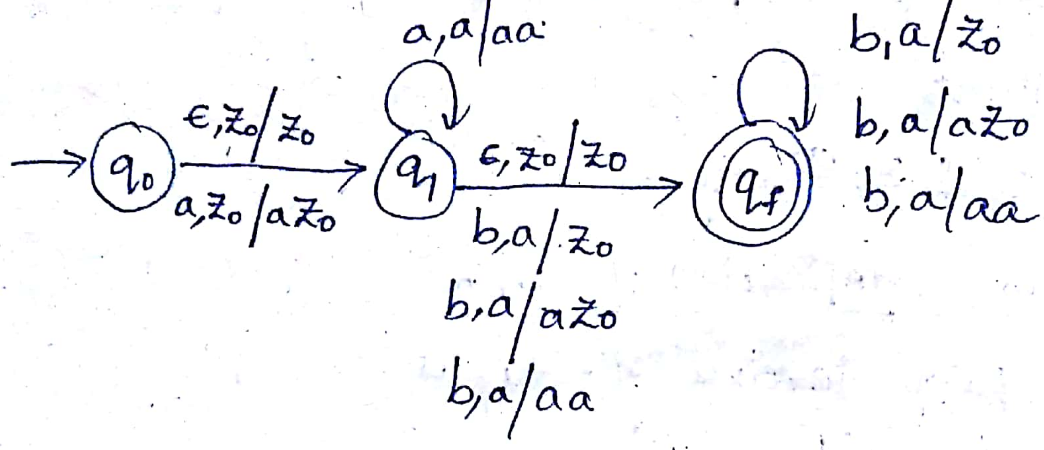
$$L = \{a^n b^n \mid n \geq 0\}$$

$$\{\epsilon, ab, aabb, aaabbb \cdots\}$$

For Finite Automata

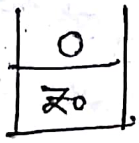


But in PDA here we are having Z_0 (extra tuples of stack



initially stack is empty

eg 01, 0011, 000111, ...



$\delta(q_0, 0, Z_0) = (q_1, 0Z_0)$

$\delta(q_1, 1, 0) = (q_2, Z_0)$ ✓ pop out '0'

when 1 input comes pop '0'

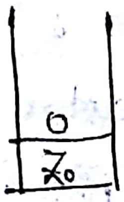
0011:-

$\delta(q_0, 0, Z_0) = (q_1, 0Z_0)$

$\delta(q_1, 0, 0) = (q_1, 00)$

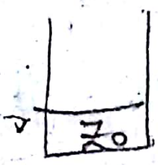
$\delta(q_1, 1, 0) = (q_2, 0Z_0)$

$\delta(q_2, 1, 0) = (q_2, Z_0)$



if same input comes then don't popout

000111:



$\delta(q_0, 0, Z_0) = (q_1, 0Z_0)$

$\delta(q_1, 0, 0) = (q_1, 00)$

$\delta(q_1, 0, 0) = (q_1, 00)$

$\delta(q_1, 1, 0) = (q_2, 00)$

$\delta(q_2, 1, 0) = (q_2, 0Z_0)$

$\delta(q_2, 1, 0) = (q_2, Z_0)$.

$\Rightarrow$ Suppose the PDA $P = (\{q_0, q_1\}, \{0, 1\}, \{X, Y, Z\}, \delta, q_0, Z, \{q_1\})$

has the following transition functions

1. $\delta(q_0, 0, Z) = \{(q_1, Z)\}$
2. $\delta(q_0, 1, Z) = \{(q_0, XZ)\}$
3. $\delta(q_0, 0, X) = \{(q_0, \epsilon)\}$
4. $\delta(q_0, 1, X) = \{(q_0, XX)\}$
5. $\delta(q_1, 0, Z) = \{(q_1, YZ)\}$
6. $\delta(q_1, 1, Z) = \{(q_0, Z)\}$
7. $\delta(q_1, 0, Y) = \{(q_1, YY)\}$
8. $\delta(q_1, 1, Y) = \{(q_1, \epsilon)\}$

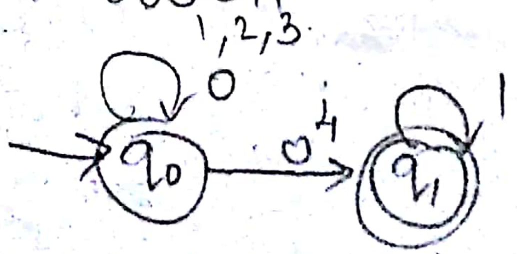
ID's

W = 000011

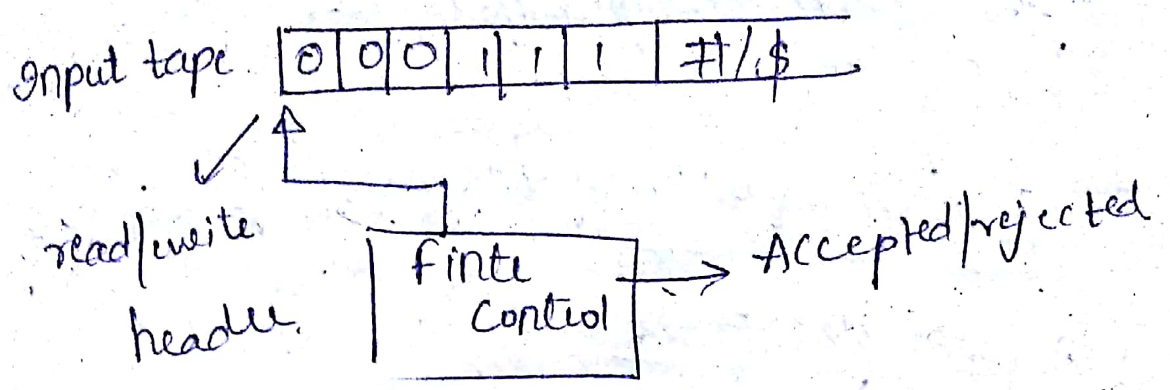
W = 011001

W = 0110110

W = 000011



19/9/18 :-



Stack

Instantaneous Description of PDA :-

The configuration of PDA by a tripple (q, ω, γ) input

where 'q' is the state 'ω' is the remaining i/p
and γ is the stack contents.

Conventionally we are representing the top of stack at the left end of γ and the bottom at the right end such tripple is called as Instantaneous Description.

left right

$(q_1, 1, \underline{11}) = (q_1, 1z)$

There are 3 imp principles about ID are:-

1) If a sequence of computation is legal for a PDA then computation formed by adding the same to the end of i/p in each id is also legal

2) If computation is legal for PDA adding same stack symbol into the stack in each id is also legal

3) If computation is legal for PDA same tail

of i/p is not consumed then we can remove from i/p and the result computation will still a legal

PDA $p = (\{q_0, q_1\}, \{0,1\}, \{X,Y,Z\}, \delta, q_0, Z, \{q_1\})$

(i) δ $\boxed{\text{it is in back}}$

Starting from the ID show all the reachable ID's when the i/p $|w| = 000011$; $= 01100$, $= 0110110$.

Q (no of states) $= \{q_0, q_1\}$

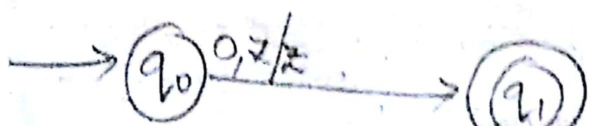
$\Sigma = \{0,1\}$ $\Gamma = \{X,Y,Z\}$, δ ;

$q_0 \rightarrow$ initial state

$Z \rightarrow$ start symbol of stack

$q_1 \rightarrow$ final state

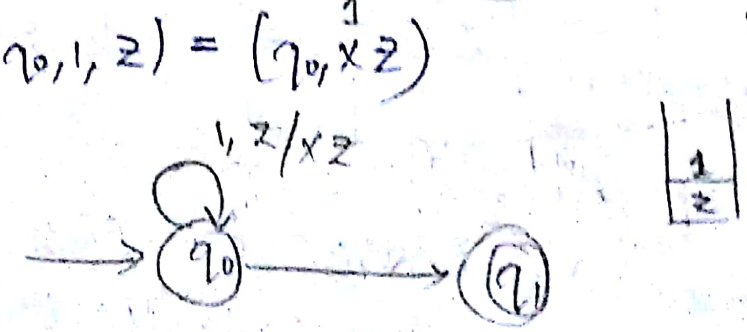
for 1st transition $\delta(q_0, 0, Z) = \{(q_1, Z)\}$



$0 \rightarrow Y$

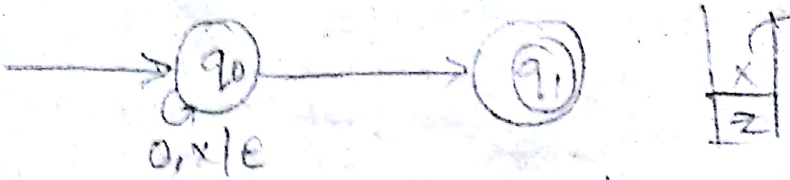
$1 \rightarrow X$

(ii) $\delta(q_0, 1, Z) = (q_0, \overset{1}{X}Z)$



iii) $\delta(q_0, 0, X) = (q_0, \epsilon)$ we are not inserting 0 because 1 is there ε → No transitions taken place in stack

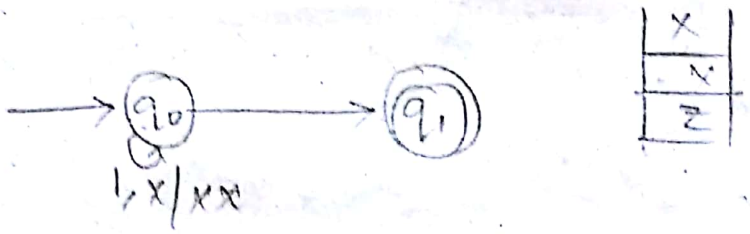
↳ all the stack elements poped / removed



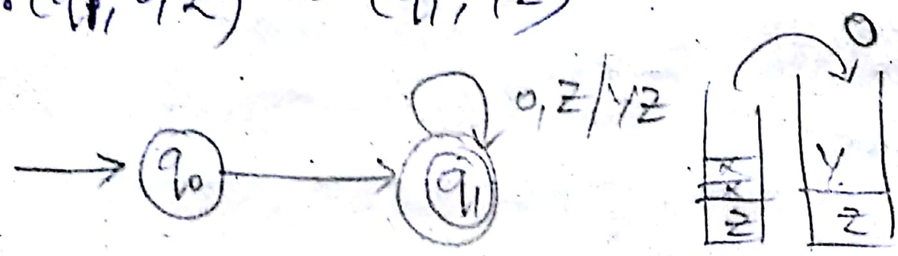
↳ No transition is inserted

↳ no insertion done because 1 is there '0' should be inserted

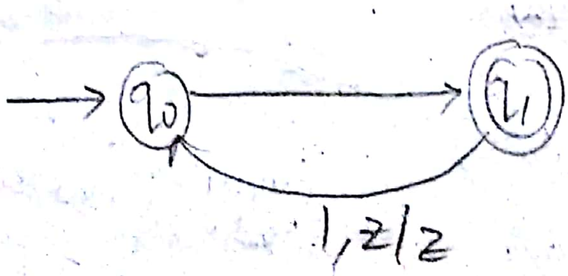
iv) $\delta(q_0, 1, x) = (q_0, xx)$



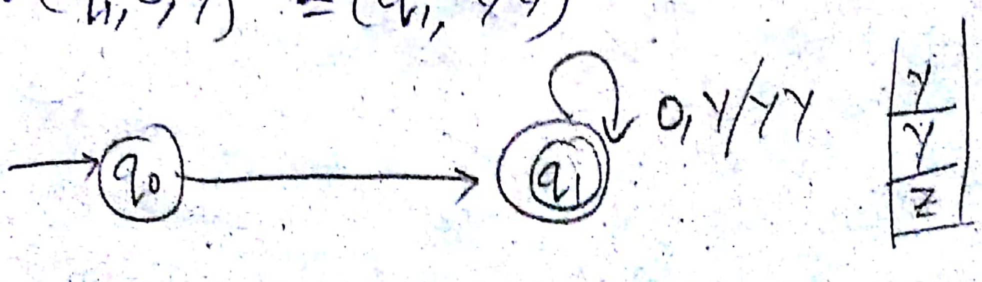
v) $\delta(q_1, 0, Z) = (q_1, YZ)$



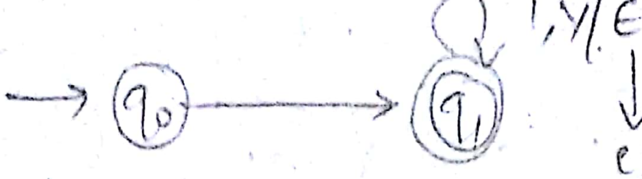
vi) $\delta(q_1, 1, Z) = \{q_0, Z\}$



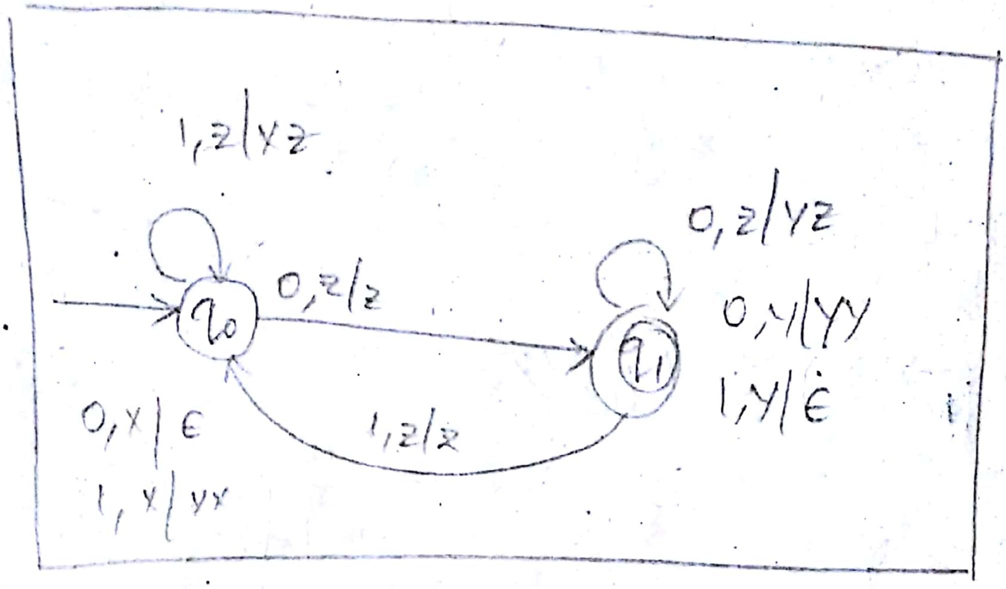
vii) $\delta(q_1, 0, Y) = (q_1, YY)$



viii) $\delta(q_1, 1, y) = (q_1, \epsilon)$



erasing all elements



i) w = 000011

$\delta(q_0, 0, z) = (q_1, z)$

$\delta(q_1, 0, z) = (q_1, yz)$ accepted

$\delta(q_1, 0, y) = (q_1, yy)$

$\delta(q_1, 0, y) = (q_1, yy)$ No transition (change)

$\delta(q_1, 1, y) = (q_1, \epsilon)$ → ~~poping all elements~~

$\delta(q_1, 1, y) = (q_1, \epsilon)$ → ~~No partition done at the stack~~

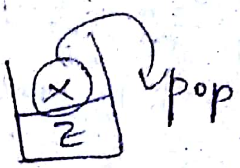
pop.

(ii) $w = 011001$

$\delta(q_0, 0, z) = (q_1, z) \rightarrow 0$

$\delta(q_1, 1, z) = (q_0, z) \rightarrow 1$

$\delta(q_0, 1, z) = (q_0, xz) \rightarrow 1$



$\delta(q_0, 0, x) = (q_0, \epsilon) \rightarrow 0 \rightarrow$ No change

$\cancel{\delta(q_0, 0, y) = \delta(q_1, yy)}$ $\delta(q_0, 0, x) = (q_0, \epsilon)$

$\cancel{\delta(q_1, 1, y) = (q_1, \epsilon)}$ $\delta(q_1, 1, x) = \cancel{(q_0, z)}$

No transition.

Not accepted.

(iii) $w = 0110110$

$\delta(q_0, 0, z) = (q_1, z)$ 0

$\delta(q_1, 1, z) = (q_0, z)$ 1

$\delta(q_0, 1, z) = (q_0, xz)$ 1 $\rightarrow$ Not accepted

$\delta(q_0, 0, x) = (q_0, \epsilon)$ 0

$\delta(q_0, 1,$

21/07/18 :-

PDA :- Question

$M = (\{q_0, q_1, q_2\}, \{a, b, c\}, \{a, b, z_0\}, \delta, q_0, z_0, \{q_2\})$

δ: $\delta(q_0, a, z_0) = \delta(q_0, az_0)$

$\delta(q_0, b, z_0) = \delta(q_0, bz_0)$

$\delta(q_0, a, a) = \delta(q_0, aa)$

$\delta(q_0, b, a) = \delta(q_0, ba)$ $\rightarrow$ $\delta(q_0, a, b)$

$\delta(q_0, b, b) = \delta(q_0, bb)$ $= \delta(q_0, ab)$

$\delta(q_0, c, z_0) = \delta(q_1, z_0)$

$\delta(q_0, c, a) = \delta(q_1, a)$

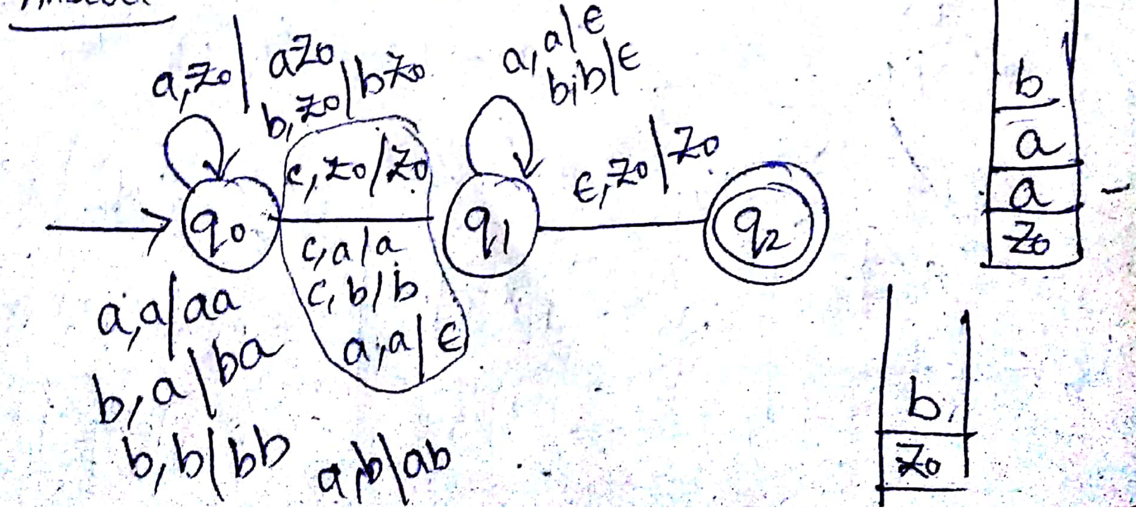
$\delta(q_0, c, b) = \delta(q_1, b)$

$\delta(q_1, a, a) = \delta(q_1, \epsilon)$

$\delta(q_1, b, b) = \delta(q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = \delta(q_2, z_0)$

Answer



Given string $w = bbacabb$.

$\delta(q_0, b, Z_0) = \delta(q_0, bZ_0) \longrightarrow b$

$\delta(q_0, b, b) = \delta(q_0, bb) \longrightarrow b$

$\delta(q_0, a, b) = \delta(q_0, ab). \longrightarrow a$

$\delta(q_0, c, a) = \delta(q_1, a). \longrightarrow c$

a
b
b
Z_0

$\delta(q_1, a, a) = \delta(q_1, \epsilon). \longrightarrow a$ (no change) operation)

$\delta(q_1, b, a) =$ no transition

$\delta(q_1, b, Z_0) \longrightarrow$ or pop the elements

↳ it also contain no transition

→ Not accepted

2) Design a PDA that accepts a string of well-formed paranthesis. consider the paranthesis is as:- (,) , [,] , { , }

→ left paranthesis should be inserted into stack.

→ when Right parenthesis comes which is equal to left then pop it

$\delta(q_0, (, Z_0) = (q_0, (Z_0).$

$\delta(q_0, [, Z_0) = (q_0, [Z_0)$

$\delta(q_0, \{, Z_0) = (q_0, \{Z_0)$

$\delta(q_0, (, () = (q_0, (()$

$\delta(q_0, [, [) = (q_0, [[)$

$\delta(q_0, \{, \{) = (q_0, \{\{)$

$\delta(q_0, (, \{) = (q_0, (\{)$

$\delta(q_0, (, [) = (q_0, ([)$

$\delta(q_0, [, \{) = (q_0, [\{)$ → $\delta(q_0, [, () = (q_0, [()$ ⇒

$\delta(q_0, \{, () = (q_0, [()$

$\delta(q_0, \{, [) = (q_0, \{[)$ ⇒

$\delta(q_0,), () = \delta(q_0, \epsilon)$

↳ deleting appropriate Symbol

$\delta(q_0, \}, \{) = \delta(q_0, \epsilon)$

$\delta(q_0,], [) = \delta(q_0, \epsilon)$

<u>String:-</u>

$w = \{[()\{\}]\}$

$\delta(q_0, \{, z_0) = \delta(q_0, \{z_0)$

$\delta(q_0, [, \{) = \delta(q_0, [\{)$

$\delta(q_0, (, [) = \delta(q_0, ([)$

$\delta(q_0,), ( ) = \delta(q_0, \epsilon) \rightarrow$ No change

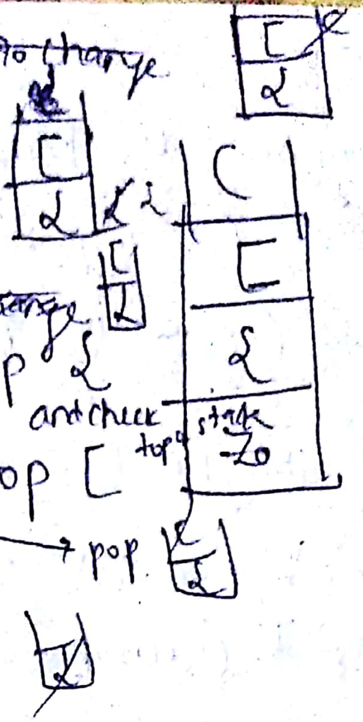
$\delta(q_0, \{, [\,) = \delta(q_0, \{[\,)$

$\delta(q_0, \}, \{) = \delta(q_0, \epsilon) \rightarrow$ ~~no change~~ pop $\{$ and check top stack Z_0

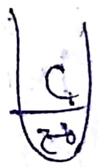
$\delta(q_0,], [\,) = \delta(q_0, \epsilon) \rightarrow$ pop $[$

$\rightarrow$ pop

$\delta(q_0, \}, \{) = \delta(q_0, \epsilon)$.



⇒ ( ) ( ) . (



⇒ Languages of PDA :-

equivalence of PDA & CFG's :-

- → all CFG are PDA's
- → PDA accepted by final state
- → PDA accepted by empty stack

26/09/18 :-

Grammer to pDA :-

$G = (V, T, P, S)$

$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

→ In this no final state will be present

$Q \to V$ (variable)

$\Sigma \to T$ (Terminal)

$\Gamma \to V \cup T$ (variable / Terminal)

Converting the production into transition

eg: $A \to \alpha \Rightarrow \delta(q, \epsilon, A) = (q, \alpha)$

$A \to a$

simple state

we don't know the i/p so we take into ϵ

left one in production

right one in production

① problem:-

$E \to I \mid E+E \mid E*E \mid (E)$

$I \to a \mid b \mid Ia \mid Ib \mid I0 \mid I1$

$Q = \{E, I\}$ $\Gamma = \{E, I, a, b, 0, 1, +, *, (,)\}$

$\Sigma = \{a, b, 0, 1, +, *, (,)\}$ $q_0 = E$ (left side of production) (will be stack symbol)

Z_0

δ : $E \to I$

$\delta(q, \epsilon, E) \Rightarrow (q, I)$

$E \to E+E$

$\delta(q, \epsilon, E) \Rightarrow (q, E+E)$

$E \to E*E$

$\delta(q, \epsilon, E) \Rightarrow (q, E*E)$

$E \to (E)$

$\delta(q, \epsilon, E) = (q, (E))$

$I \to a$

$\delta(q, \epsilon, I) = (q, a)$

$I \to b$

$\delta(q, \epsilon, I) = (q, b)$

$I \to Ia$

$\delta(q, \epsilon, I) = (q, Ia)$

$I \to Ib$

$\delta(q, \epsilon, I) = (q, Ib)$

$I \to I0$

$\delta(q, \epsilon, I) = (q, I0)$

$I \to I1$

$\delta(q, \epsilon, I) = (q, I1)$

$T = a, b, 0, 1, +, *, (,)$.

$\delta(q, a, a) = (q, \epsilon)$

$\delta(q, b, b) = (q, \epsilon)$

$\delta(q, 0, 0) = (q, \epsilon)$

$\delta(q, 1, 1) = (q, \epsilon)$

$\delta(q, +, +) = (q, \epsilon)$

$\delta(q, *, A) = (q, \epsilon)$

$\delta(q, (, () = (q, \epsilon)$

$\delta(q,),)) = (q, \epsilon)$

② $S \rightarrow AS / \epsilon$

$A \rightarrow 0A1 / A1 / 01$

$Q = \{S, A\}$

$\Sigma = \{0, 1\}$

$T = \{S, A, 0, 1\}$

$q_0 = S$

$\cancel{Z_0}$

$\delta : S \rightarrow AS$

$\delta(q, \epsilon, S) = (q, AS)$

$A \rightarrow 0A1$

$\delta(q, \epsilon, A) = (q, 0A1)$

$A \rightarrow A1$

$\delta(q, \epsilon, A) = (q, A1)$

$A \rightarrow 01$

$\delta(q, \epsilon, A) = (q, 01)$

$T = \{0, 1\}$

$\delta(q_1, 0, 0) = (q_1, \epsilon)$

$\delta(q_1, 1, 1) = \{q_1, \epsilon)$

converting (PDA to Grammer) :-

$M = \{\{q_0, q_1\}, \{a, b\}, \{a, Z_0\}, \delta, q_0, Z_0, \phi\}$ → stack element

δ :- $\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, Z_0) = (q_1, \epsilon)$

$G = (V, T, P, S)$

S → q_0 → start state

P → δ (Transitions tate)

T → i/p alphabet Σ

Variable is a 3 element component

$(\underset{Q}{\text{state}}, \underset{\text{stack symbol/alphabet}}{\Gamma}, \underset{Q}{\text{state}})$

Variable :- $[q_0, a, q_0]$ $[q_1, a, q_1]$

$[q_0, Z_0, q_0]$ $[q_1, Z_0, q_1]$

$[q_0, a, q_1]$ $[q_1, a, q_0]$

$[q_0, Z_0, q_1]$ $[q_1, Z_0, q_0]$

$S \rightarrow [q_0, a, q_0]$ ↙ Startsymbol production

$[q_0, a, q_1]$

$[q_0, Z_0, q_0]$ we should not Take all the

$[q_0, Z_0, q_1]$ Stack symbol only the top. of stack should be taken

---

$S: \; [q_0, Z_0, q_0]$ it should be in this form

$[q_0, Z_0, q_1]$ $V \rightarrow$ - -

↳ but variable has $(\; , \; , \;)$

(i) $\delta(q_0, a, Z_0) = (q_0, aZ_0)$ as their are 2 so consider 2 boxes for a & Z_0

$$[q_0, Z_0, q_0] \rightarrow a \underbrace{[q_0, a, q_0]}_{a} \; \underbrace{[q_0, Z_0, q_0]}_{Z_0} \rightarrow 00$$

State ↓ Stack Symbol 0 0

$\rightarrow 01$

$[q_0, Z_0, q_0] \rightarrow a \, [q_0, a, q_1] \, [q_1, Z_0, q_0]$ 1 0

$\rightarrow 10$

$[q_0, Z_0, q_1] \rightarrow a \, [q_0, a, q_0] \, [q_0, Z_0, q_1]$ 0 1

$\rightarrow 11$

$[q_0, Z_0, q_1] \rightarrow a \, [q_0, a, q_1] [q_1, Z_0, q_1]$

1 Same 1

Same

(iii) $\delta(q_0, b, a) = (q_1, e)$

$[q_0, a, q_1] \rightarrow b$

v) $\delta(q_1, \epsilon, Z_0) = (q_1, \epsilon)$

$[q_1, Z_0, q_1] \rightarrow \epsilon$

iv) $\delta(q_1, b, a) = (q_1, \epsilon)$

$[q_1, a, q_1] = b$

two inputs

o/p should be 4

ii) $\delta(q_0, a, a) = (q_0, aa)$

$[q_0, a, q_0] \rightarrow a [q_0, a, q_0][q_0, a, q_0]$ 00

$[q_0, a, q_0] \rightarrow a [q_0, a, q_1][q_1, a, q_0]$ 01

$[q_0, a, q_1] \rightarrow a [q_0, a, q_0][q_0, a, q_1]$ 10

$[q_0, a, q_1] \rightarrow a [q_0, a, q_1][q_1, a, q_1]$ 11

Acceptance
- → empty stack method
- → final state