Chapter 13

Representation and Description

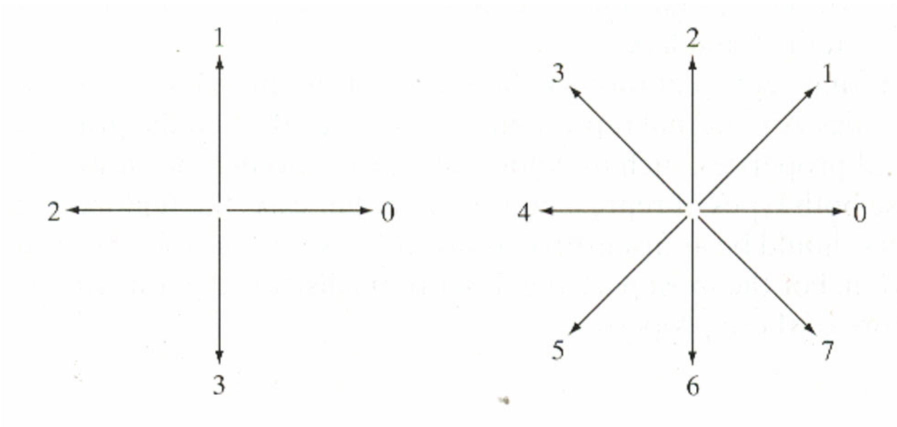
Preview

1. Representation
2. Boundary Descriptors
3. Regional Descriptors
4. Use of Principal Components for Description
5. Relational Descriptors

1. Representation

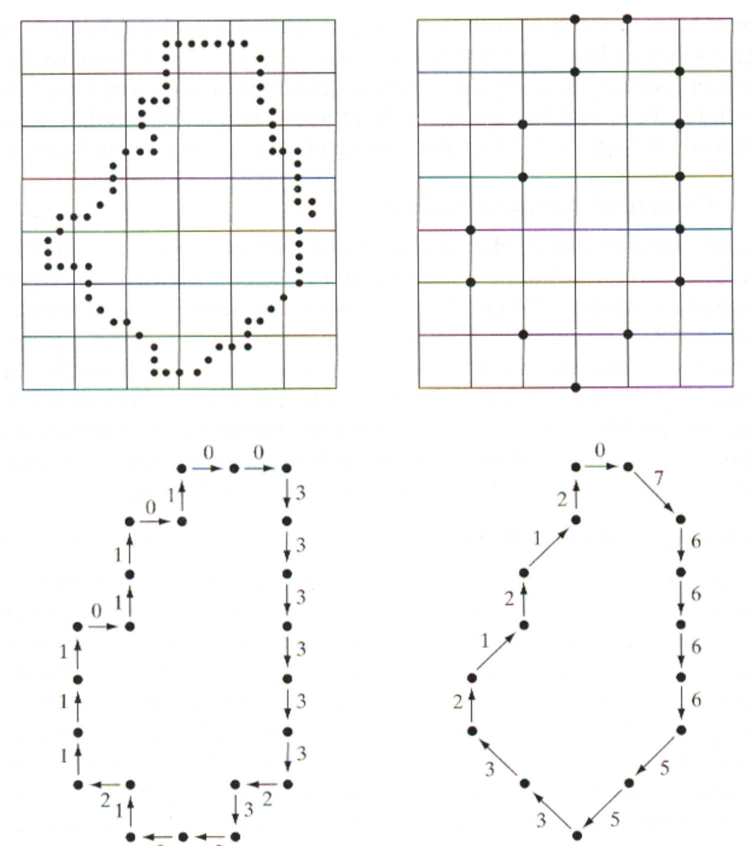
1.1 Chain Codes

- to represent a boundary by a connected sequence of line segments of specified length and direction.



a | b

Figure 13.1
Direction numbers for
(a) 4-directional chain code, and
(b) 8-directional chain code.



a	b
c	d

Figure 13.2
(a) Digital boundary with resampling grid superimposed.
(b) Result of resampling.
(c) 4-directional chain code.
(d) 8-directional chain code.

For Fig.13.2(c), starting at the top, left dot,

chain code is 0033......01.

For Fig.13.2(d), chain code is 0766......12.

Or using the first difference of the chain code, we get

030......1,

770......1, respectively.

1.2 Polygonal Approximations

Minimum perimeter polygons

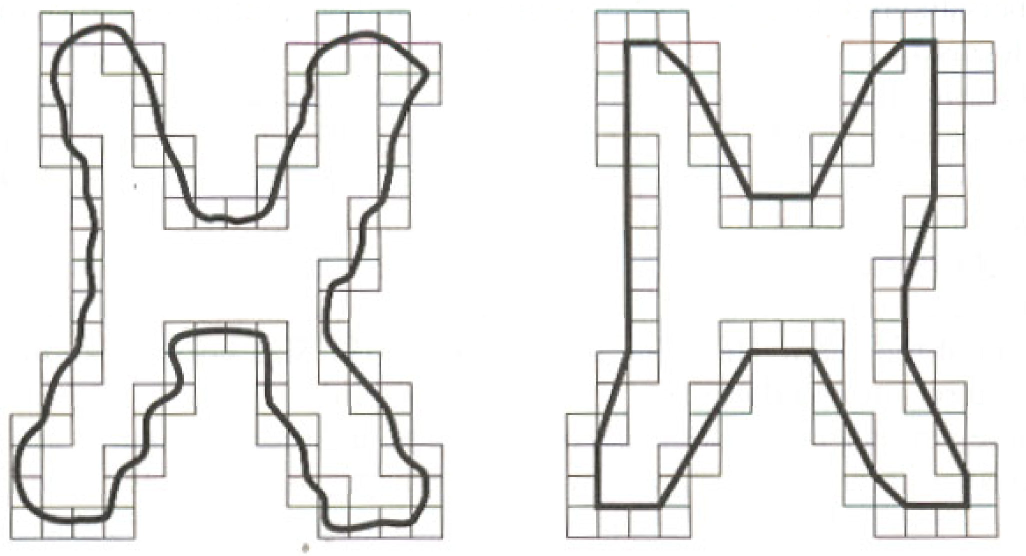
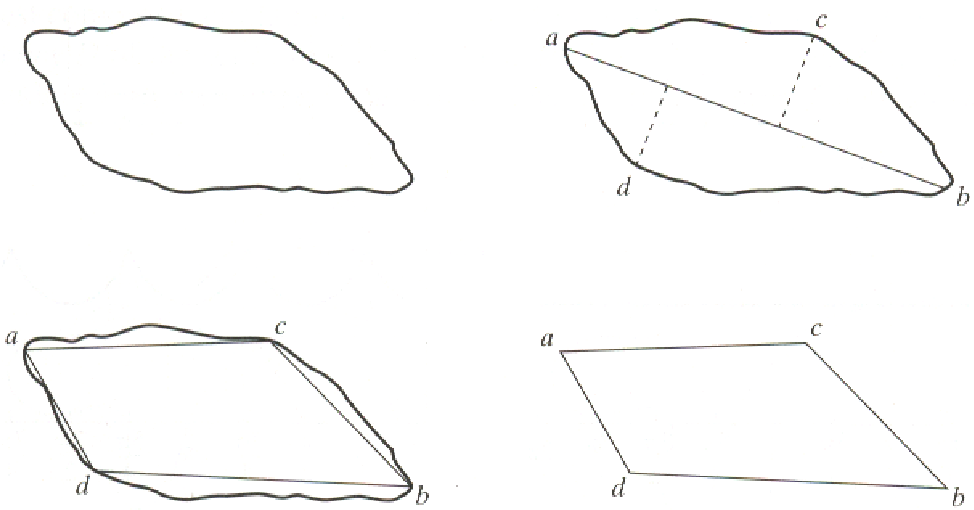


Figure 13.3

(a) Object boundary enclosed by cells.
(b) Minimum perimeter polygon.

Splitting and Merging techniques

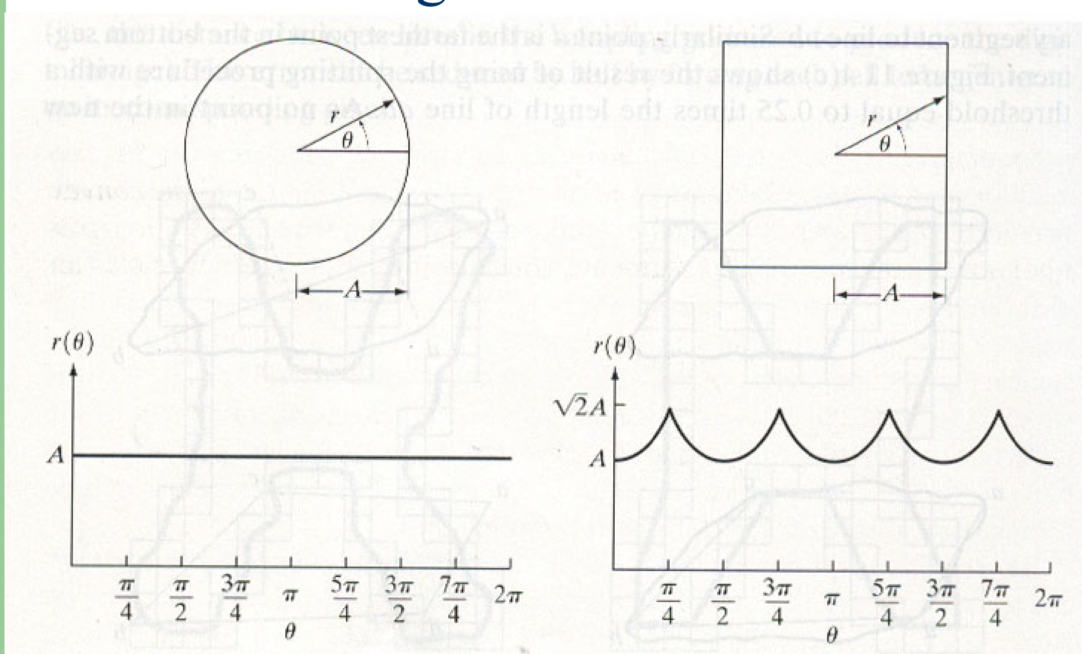


a	b
c	d

Figure 13.4
(a) Original boundary.
(b) Boundary divided into segments based on extreme points.
(c) Joining of vertices.
(d) Resulting polygon.

1.3 Signatures

plot the distance from the centroid to the boundary as the function of angle—translation invariant.



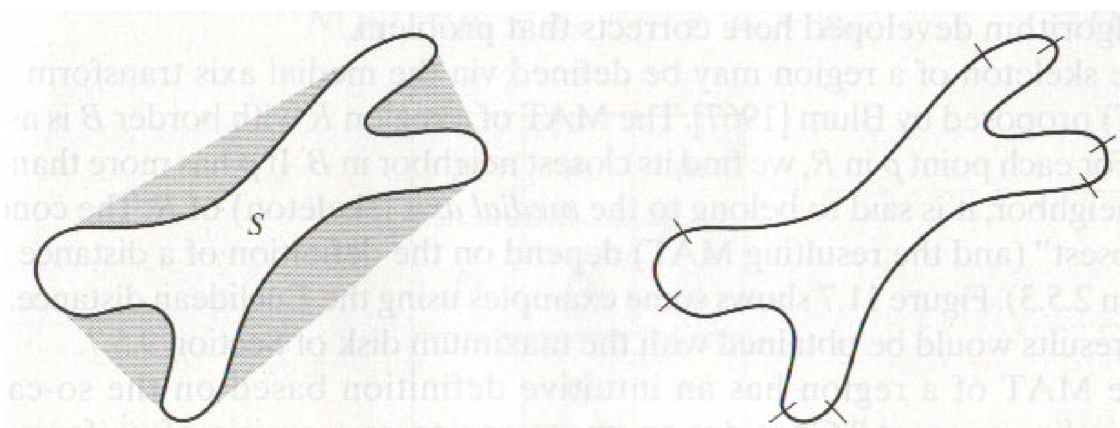
a	b

Figure 13.5
(a) Distance-versus-angle signatures.
In (a) $r(\theta)$ is constant. In (b), the signature consists of repetitions of the pattern

$r(\theta) = A\sec\theta$ for $0 \le \theta \le \pi/4$ and

$r(\theta) = A\csc\theta$ for $\pi/4 < \theta \le \pi/2$.

1.4 Boundary Segments



a	b

Figure 13.6
(a) A region, S, and its convex deficiency (shaded).
(b) Partitioned boundary.

1.5 Skeletons

Medial Axis Transformation (MAT, Blum 1967)

- for each point p in R, we find its closest neighbor in border B. If p has more than one such neighbor, it is said to belong to the medial axis (skeleton) of R.

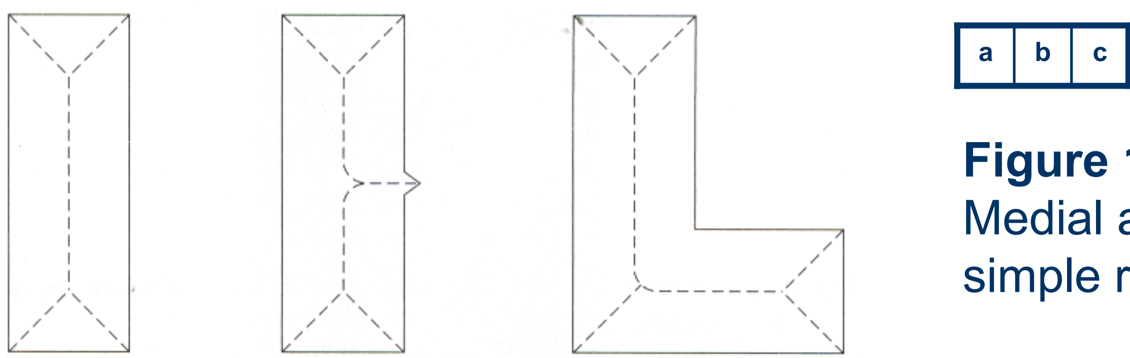


Figure 13.7
Medial axes (dashed) of three simple regions

Thinning Algorithm:

With reference to the 8-neighborhood notations shown in Fig. 13.8, step 1 flags a contour point p_1 for deletion, if the following conditions are satisfied:

(a) $2 \leq N(p_1) \leq 6$, where $N(p_1) = p_2 + p_3 + \ldots + p_8 + p_9$

(b) $T(p_1) = 1$, where $T(p_1)$=no. of 0-1 transitions of seq. $p_2, p_3, \ldots p_9, p_2.$

(c) $p_2 \bullet p_4 \bullet p_6 = 0$

(d) $p_4 \bullet p_6 \bullet p_8 = 0$

p_9	p_2	p_3
p_8	p_1	p_4
p_7	p_6	p_5

Figure 13.8 Neighborhood arrangement used by the thinning algorithm.

0	0	1
1	p_1	0
1	0	1

Figure 13.9
Illustrations of conditions (a) and (b) in equation on page 11. In this case $N(p_1)=4$ and $T(p_1)=3$.

Step 2: flags p_1 for deletion, if

(a) remains the same

(b) remains the same

(c) $p_2 \bullet p_4 \bullet p_8 = 0$

(d) $p_2 \bullet p_6 \bullet p_8 = 0$

Example:

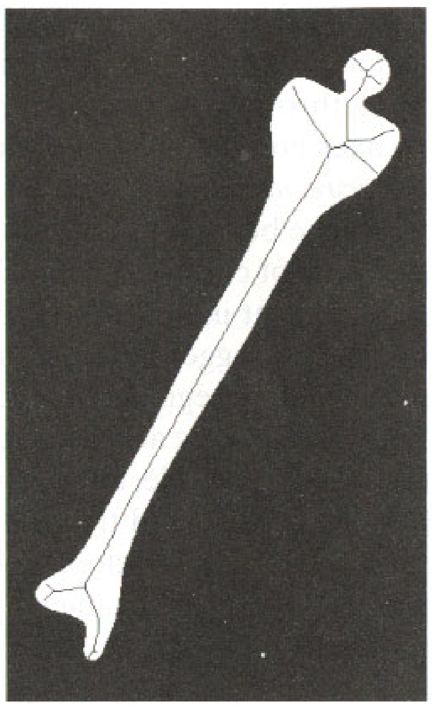


Figure 13.10
Human leg bone and skeleton of the region shown superimposed.

2. Boundary Descriptors

2.1 Some Simple Descriptors

- Length of a boundary
- Diameter of a boundary

  $$\text{Dian}(B) = \max_{i,j}\left[D\left(p_i, p_j\right)\right]$$

  where p_i and p_j are points on the boundary

  $\Rightarrow$ major axis

  minor axis

  $$\text{eccentricity} = \frac{\text{major axis}}{\text{minor axis}}$$

- Curvature: rate of change of slope.

  convex segment

  concave segment

  corner point: change $>90^0$

2.2 Shape Numbers

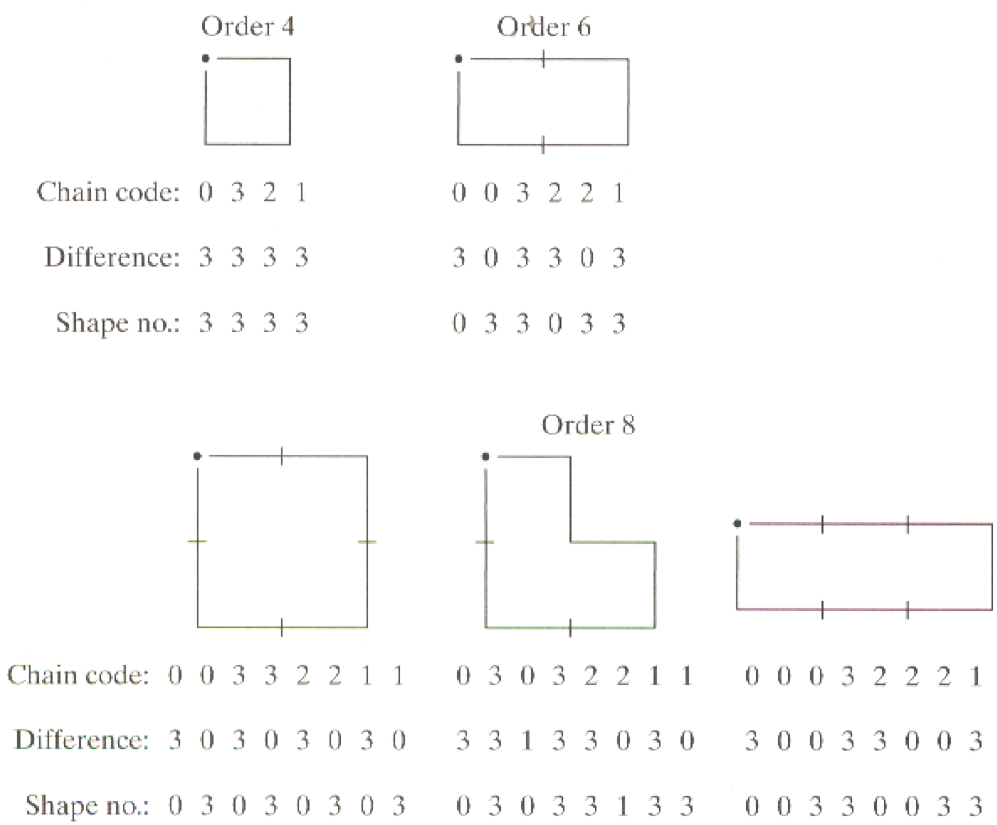
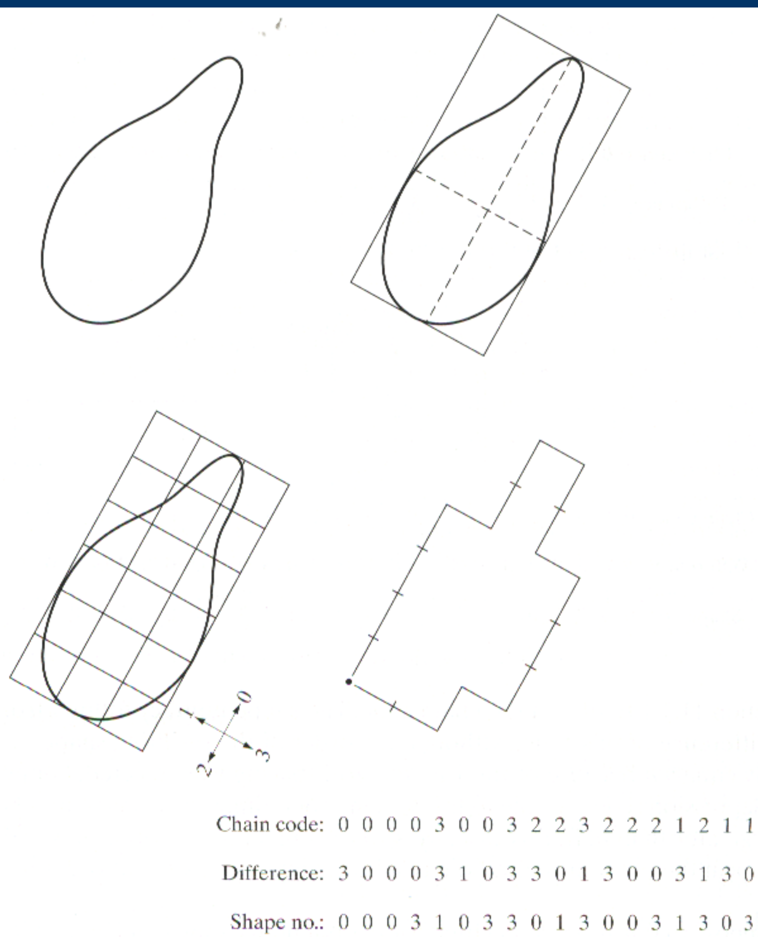


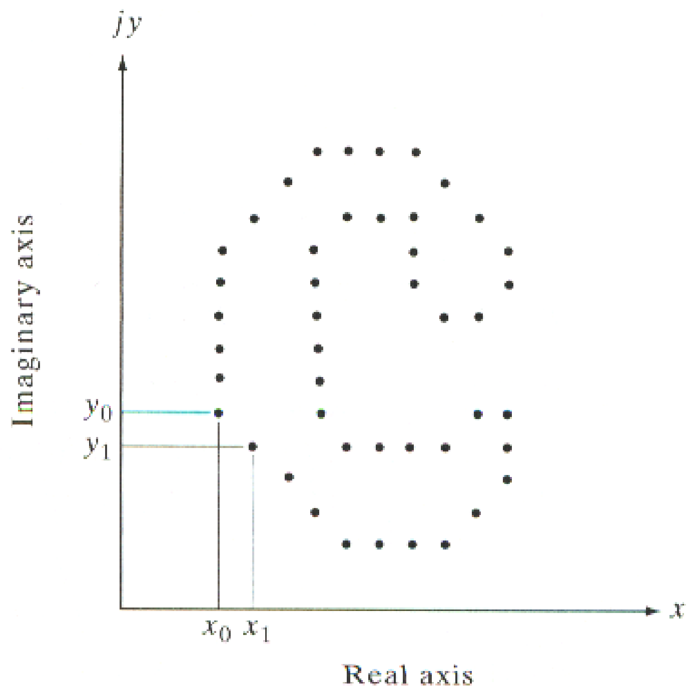
Figure 13.11
All shapes of order 4,6, and 8. The directions are from Fig.11.1(a) and the dot indicates the starting point.



a	b
c	d

Figure 13.12
Steps in the generation of a shape number.

2.3 Fourier Descriptors



$s(k)= x(k) + jy(k)$ for $k=0,1,2,...,K-1$

Figure13.13
A digital boundary and its representation as a complex sequence. The points (x_0,y_0) and (x_1,y_1) shown are (arbitrarily) the first two point in the sequence.

DFT of s(k) is

$$a(u) = \frac{1}{K}\sum_{k=0}^{K-1} s(k)\, e^{-j2\pi u k/K} \quad \text{for } u=0,1,2,\ldots,K-1$$

IDFT of a(u) restores s(k):

$$s(k) = \frac{1}{K}\sum_{u=0}^{K-1} a(u)\, e^{j2\pi u k/K} \quad \text{for } k=0,1,2,\ldots,K-1$$

If, only the first P coefficients are used,

$$\hat{s}(k) = \sum_{u=0}^{P-1} a(u)\, e^{j2\pi u k/K} \quad \text{for } k=0,1,2,\ldots,K-1$$

which approximates s(k).

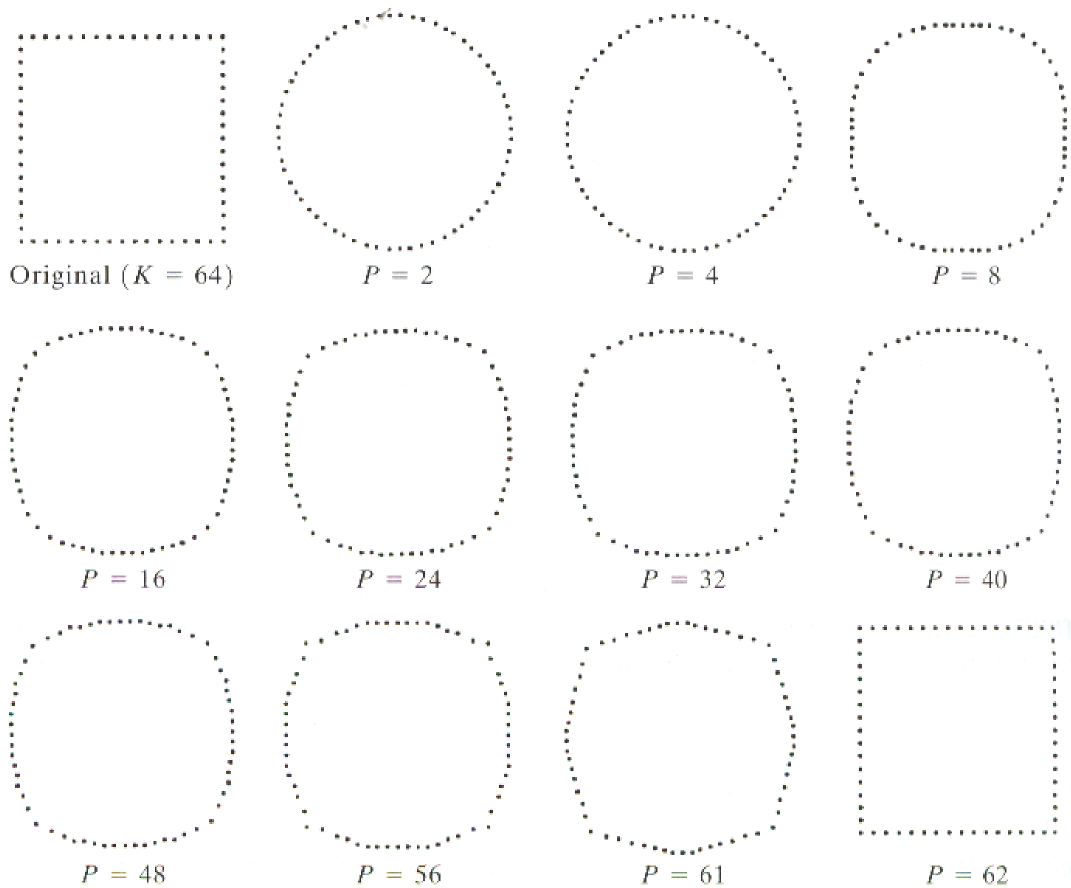
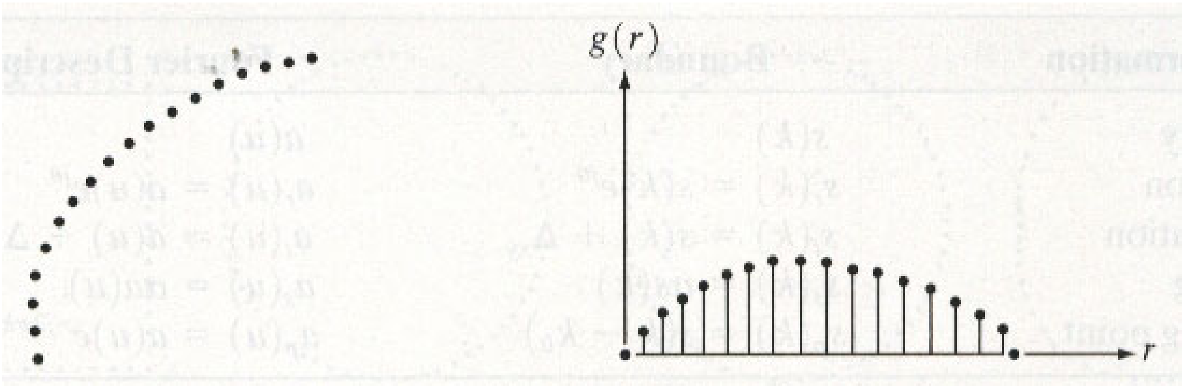


Figure 13.14 Examples of reconstruction from Fourier descriptors. P is the number of Fourier coefficients used in the reconstruction of the boundary.

Transformation	Boundary	Fourier Descriptor
Identity	$s(k)$	$a(u)$
Rotation	$s_r(k) = s(k)e^{j\theta}$	$a_r(u) = a(u)e^{j\theta}$
Translation	$s_t(k) = s(k) + \Delta_{xy}$	$a_t(u) = a(u) + \Delta_{xy}\delta(u)$
Scaling	$s_s(k) = \alpha s(k)$	$a_s(u) = \alpha a(u)$
Starting point	$s_p(k) = s(k - k_0)$	$a_p(u) = a(u)e^{-j2\pi k_0 u/K}$

Table 13.1 Some basic properties of Fourier descriptors.

2.4 Statistical Moments



$$\mu_n(r) = \sum_{i=0}^{K-1} (r_i - m)^n g(r_i) \qquad \text{where} \quad m = \sum_{i=0}^{K-1} r_i g(r_i)$$

Figure 13.15 (a) Boundary segment. (b) Representation as a 1-D function.

3. Regional Descriptors

3.1 Some Simple Descriptors

- area
- perimeter
- compactness=$(\text{perimeter})^2$/area
- mean and median of gray-level values
- minimum and maximum of gray-level values
- no. of pixels with values above and below the mean

3.2 Topological Descriptors

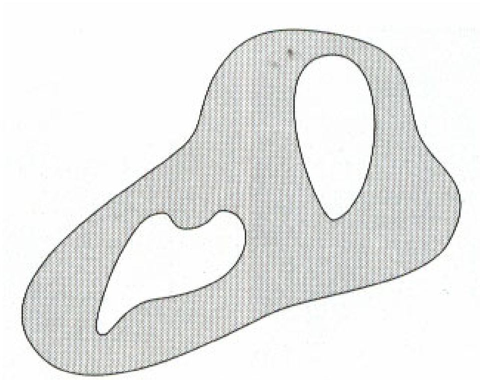


Figure 13.16 A region with two holes.

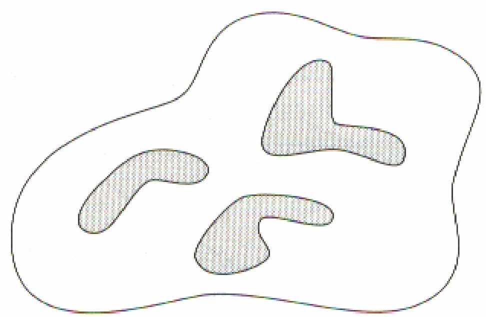


Figure 13.17 A region with three connected components

Euler number E:

E=C-H , where C is the no. of connected components,

H is the no. of holes.

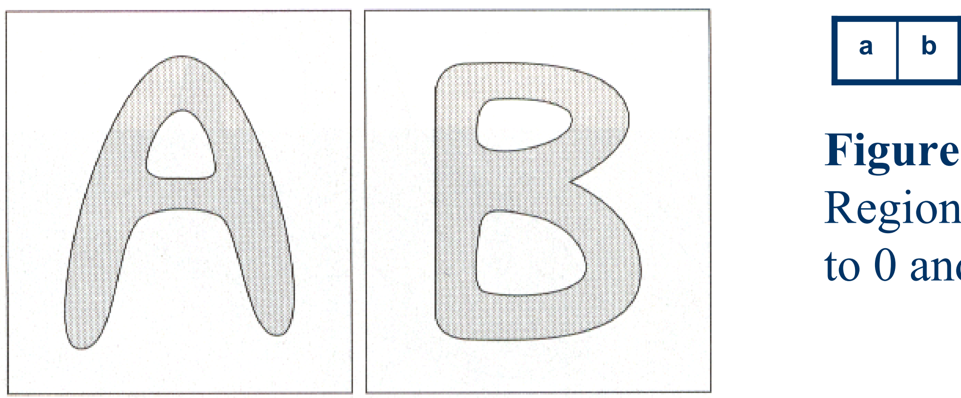


Figure 13.18

Regions with Euler number equal to 0 and -1, respectively.

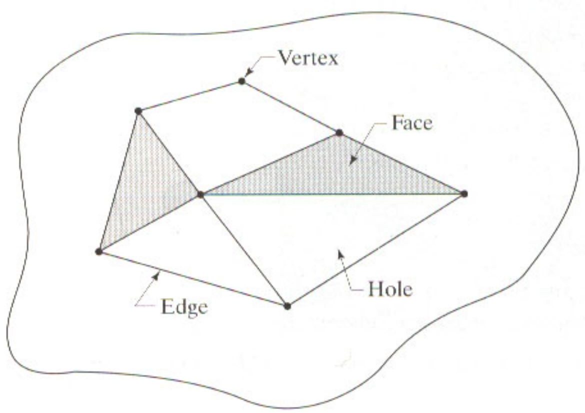
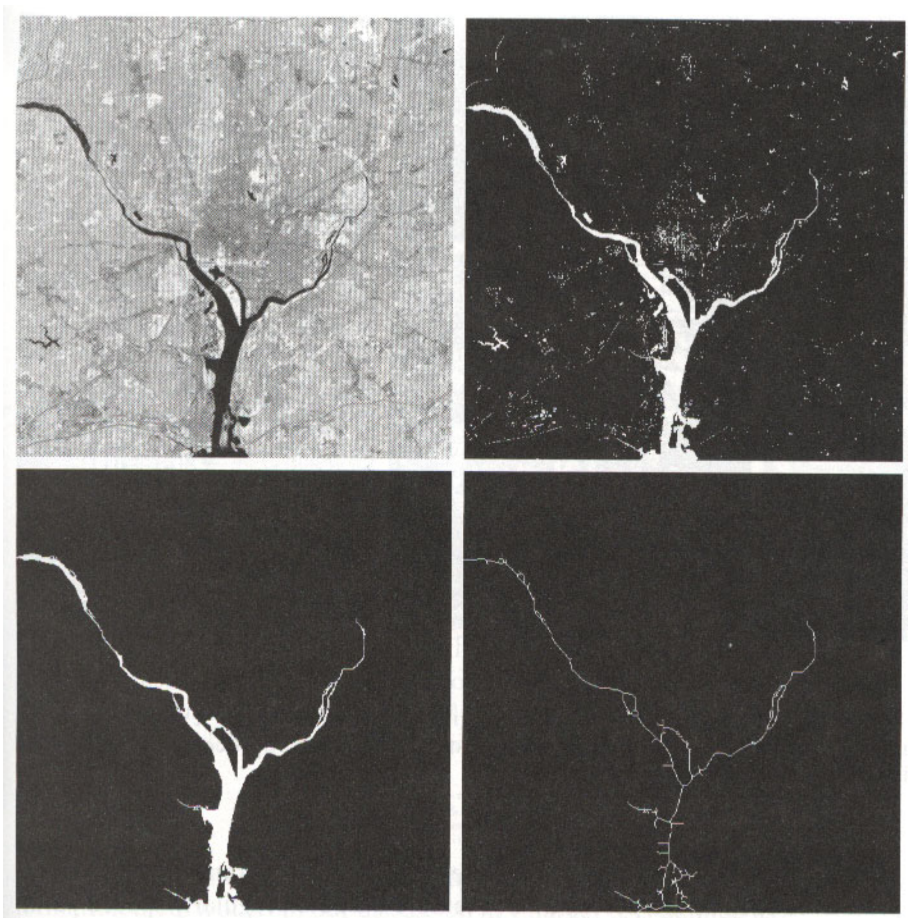


Figure13.19
A region containing a polygonal network.

Denoting the no. of vertices by V, the no. of edges by Q, and the no. of faces by F, we have the following relationship, called the Euler formula:

$$V - Q + F = C - H = E$$

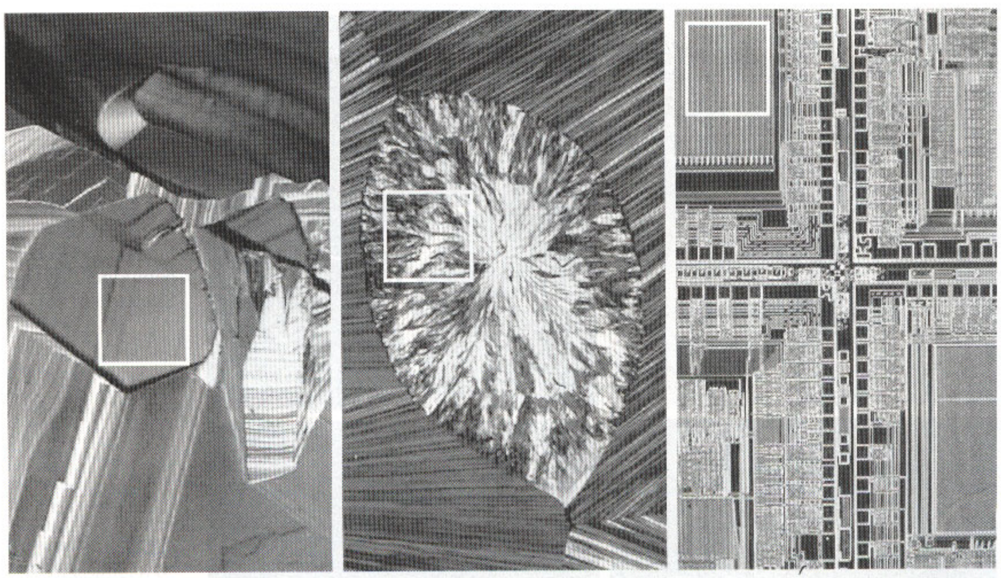
$$7 - 11 + 2 = 1 - 3 = -2$$



a	b
c	d

Figure 13.20
(a) Infrared image of the Washington, D.C. area.
(b) Thresholded image.
(c) The largest connected component of (b). Skeleton of (c).

3.3 Texture



a b c

Figure 13.21
The white squares mark, from left to right, smooth, coarse, and regular textures. These are optical microscope images of a superconductor, human cholesterol, and a microprocessor. (Courtesy of Dr. Michael W. Davidson, Florida State University.)

Statistical approaches:

Let z be a random variable denoting gray levels and let $p(z_i)$, i=1,2,...L-1, be the corresponding histogram. The n^{th} moment of z about the mean is

$$\mu_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n p(z_i) \ ,$$

$$\text{where} \quad m = \sum_{i=0}^{L-1} z_i p(z_i) \ .$$

Herein, $\mu_0 = 1$, $\mu_1 = 0$, and $\mu_2 = \sigma^2$.

The measure of gray-level contrast $R = 1 - \dfrac{1}{1+\sigma^2}$.

The third moment,

$$\mu_3(z)=\sum_{i=0}^{L-1}(z_i-m)^3 p(z_i)$$, is a measure of the skewness of the histogram.

The fourth moment is a measure of relative flatness.

Some additional texture measures based on histograms:

"uniformity" $$U=\sum_{i=0}^{L-1}p^2(z_i)$$

"average entropy" $$e=-\sum_{i=0}^{L-1}p(z_i)\log_2 p(z_i)$$

Texture	Mean	Standard deviation	R (normalized)	Third moment	Uniformity	Entropy
Smooth	82.64	11.79	0.002	−0.105	0.026	5.434
Coarse	143.56	74.63	0.079	−0.151	0.005	7.783
Regular	99.72	33.73	0.017	0.750	0.013	6.674

Table 13.2 Texture for the subimages shown in Fig.13.21.

Gray-level co-occurrence matrix:

Consider an image with three levels, $z_1=0$, $z_2=1$, and $z_3=2$, as follows:

$$
\begin{matrix}
0 & 0 & 0 & 1 & 2 \\
1 & 1 & 0 & 1 & 1 \\
2 & 2 & 1 & 0 & 0 \\
1 & 1 & 0 & 2 & 0 \\
0 & 0 & 1 & 0 & 1
\end{matrix}
$$

Defining a position operator P as "one pixel to the right and one pixel below" yields the following 3x3 matrix A:

$$A = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 3 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

Let n be the total number of point pairs in the image that satisfies P (n=16, the sum of all values in matrix A). Then the gray-level cooccurrence matrix C is formed by dividing every element of A by n.

c_{ij} is an estimate of the joint probability that a pair of points satisfying P will have values (z_i , z_j).

To categorize the texture of the region over which C was computed, a set of useful descriptors:

1. Maximum probability

$$\max_{i,j}\left(c_{ij}\right)$$

2. Element difference moment of order *k*

$$\sum_{i}\sum_{j}(i-j)^{k}c_{ij}$$

3. Inverse element difference moment of order k

$$\sum_i \sum_j c_{ij} / (i-j)^k \qquad i \neq j$$

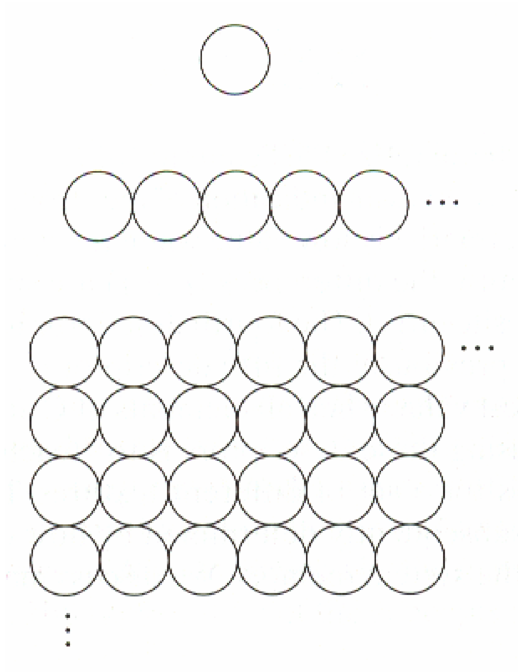
4. Uniformity

$$\sum_i \sum_j c_{ij}^{\ 2}$$

5. Entropy

$$-\sum_i \sum_j c_{ij} \log_2 c_{ij}$$

Structural approaches:



a
b
c

Figure 13.22
(a) Texture primitive.
(b) Pattern generated by the rule S→aS.
(c) 2-D texture pattern generated by this and other rules.

Spectral approaches:

Periodic or almost periodic 2-D patterns in an image

$\Rightarrow$ prominent peaks in the spectrum $S(r,\theta)$.

For each direction θ , $S(r,\theta) \Rightarrow S_\theta(r)$.

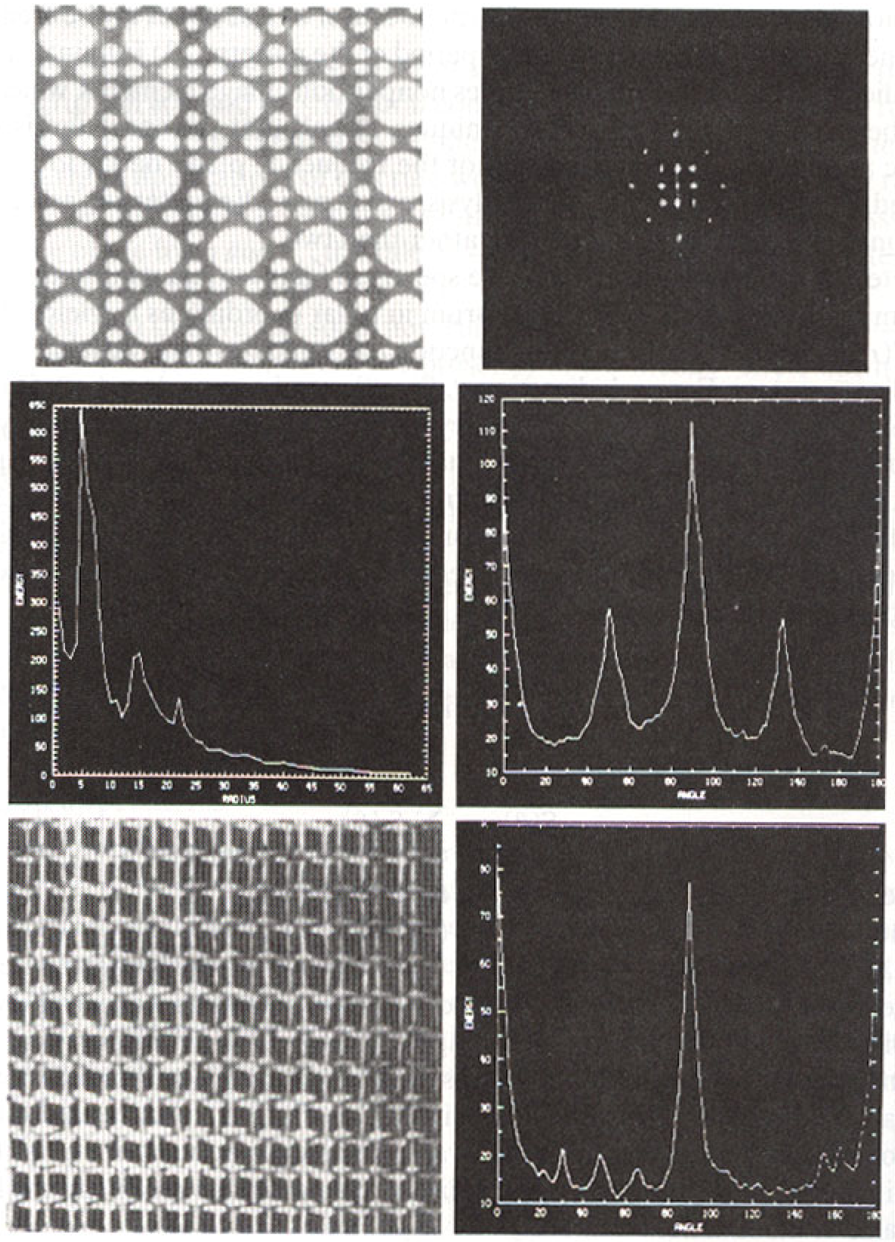
For each frequency r , $S(r,\theta) \Rightarrow S_r(\theta)$.

A more global description is obtained by integrating these functions:

$$S(r) = \sum_{\theta=0}^{\pi} S_\theta(r)$$

and

$$S(\theta) = \sum_{r=1}^{R_0} S_r(\theta)$$



a	b
c	d
e	f

Figure 13.23
(a) Image showing periodic texture.
(b) Spectrum.
(c) Plot of $S(r)$.
(d) Plot of $S(\theta)$.
(e) Another image with a different type of periodic texture.
(f) Plot of $S(\theta)$.
(Courtesy of Dr. Dragana Brzakovic, University of Tennessee.)

3.4 Moments of Two-Dimensional Functions

$$m_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q f(x,y)\,dxdy$$

central moments :

$$\mu_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(x-\bar{x}\right)^p \left(y-\bar{y}\right)^q f(x,y)\,dxdy$$

$$\text{where}\quad \bar{x} = \frac{m_{10}}{m_{00}} \quad \text{and} \quad \bar{y} = \frac{m_{01}}{m_{00}}$$

if f(x,y) is a digital image, then

$$\mu_{pq} = \sum_x \sum_y (x-\bar{x})^p (y-\bar{y})^q f(x,y).$$

$$\mu_{00} = \sum_x \sum_y \left(x-\bar{x}\right)^0 \left(y-\bar{y}\right)^0 f\left(x,y\right)$$

$$= \sum_x \sum_y f(x,y) = m_{00}$$

$$\mu_{10} = \sum_x \sum_y \left(x-\bar{x}\right)^1 \left(y-\bar{y}\right)^0 f\left(x,y\right) = m_{10} - \frac{m_{10}}{m_{00}}\left(m_{00}\right) = 0$$

$$\mu_{01} = \sum_x \sum_y \left(x-\bar{x}\right)^0 \left(y-\bar{y}\right)^1 f\left(x,y\right) = m_{01} - \frac{m_{01}}{m_{00}}\left(m_{00}\right) = 0$$

$$\mu_{11} = \sum_x \sum_y \left(x-\bar{x}\right)^1 \left(y-\bar{y}\right)^1 f\left(x,y\right) = m_{11} - \frac{m_{10}m_{01}}{m_{00}}$$

$$= m_{11} - \bar{x}m_{01} = m_{11} - \bar{y}m_{10}$$

$$\mu_{20} = \sum_x \sum_y \left(x-\bar{x}\right)^2 \left(y-\bar{y}\right)^0 f\left(x,y\right) = m_{20} - \frac{2m^2_{10}}{m_{00}} + \frac{m^2_{10}}{m_{00}}$$

$$= m_{20} - \frac{m^2_{10}}{m_{00}} = m_{20} - \bar{x}m_{10}$$

The normalized central moments, denoted η_{pq} , are defined as

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}{}^{\gamma}}$$

where $\gamma = \dfrac{p+q}{2} + 1$ for p+q=2,3,...

Invariant moments:

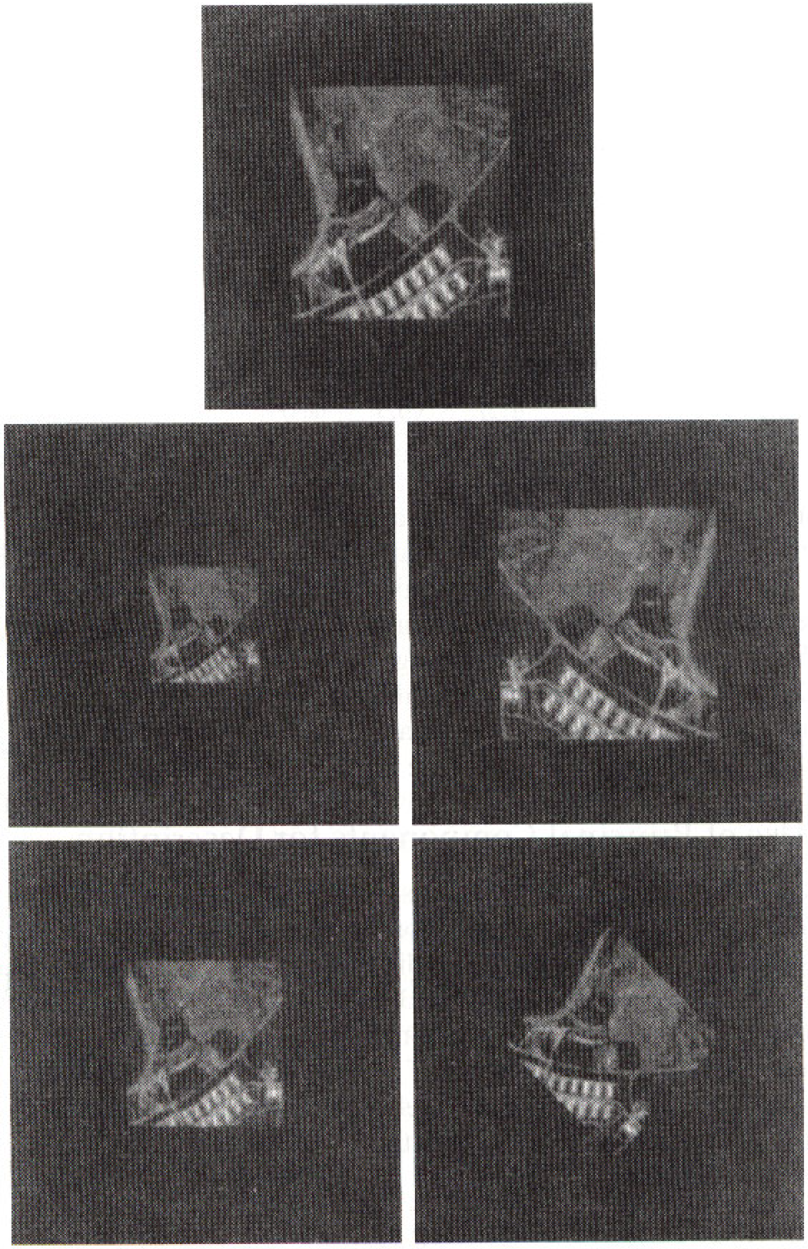
$$\phi_1 = \eta_{20} + \eta_{02}$$

$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}{}^2$$

$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2$$

$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2$$

$$\vdots$$



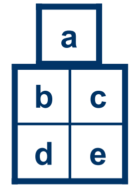


Figure 13.24
Images used to demonstrate properties of moment invariant (see Table 13.3).

Invariant (Log)	Original	Half Size	Mirrored	Rotated 2°	Rotated 45°
ϕ_1	6.249	6.226	6.919	6.253	6.318
ϕ_2	17.180	16.954	19.955	17.270	16.803
ϕ_3	22.655	23.531	26.689	22.836	19.724
ϕ_4	22.919	24.236	26.901	23.130	20.437
ϕ_5	45.749	48.349	53.724	46.136	40.525
ϕ_6	31.830	32.916	37.134	32.068	29.315
ϕ_7	45.589	48.343	53.590	46.017	40.470

Table 13.3 Moment invariants for the images in Figs.13.24(a)-(e).

4. Use of Principal Components for Description

Using one column vector to represent RGB images of size MxN:

$$x = \begin{bmatrix} x_{1r} \\ x_{1g} \\ x_{1b} \\ x_{2r} \\ x_{2g} \\ x_{2b} \\ \vdots \\ x_{MNr} \\ x_{MNg} \\ x_{MNb} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ x_n \end{bmatrix}$$

$n=3MN$

Treating x as random vector, its mean is

$$m_x = E\{x\},$$

and its covariance matrix

$$C_x = E\left\{(x - m_x)(x - m_x)^T\right\}.$$

Example

$$x_1 = (0,0,0)^T, \quad x_2 = (1,0,0)^T, \quad x_3 = (1,1,0)^T, \quad x_4 = (1,0,1)^T$$

$$m_x = \frac{1}{4}\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \quad C_x = \frac{1}{16}\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

Let e_i and λ_i , i=1,2,....,n, be the eigenvector and corresponding eigenvalues of C_x arranged in descending order so that $\lambda_j \geq \lambda_{j+1}$ for j=1,2,...,n-1.

Let **A** be the matrix such that its first row is the eigenvector corresponding to the largest eigenvalue, its second row is the eigenvector corresponding to the second largest eigenvalue, ... and so on, the last row is the eigenvector corresponding to the smallest eigenvalue.

Principal components transform (Hotelling transform):

$$y = A(x - m_x)$$

$$\Rightarrow \quad C_y = AC_x A^T$$

$$= \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix} \quad \text{with } m_y = 0.$$

and x can be recovered by

$$x = A^T y + m_x$$

Suppose, however, that instead of using all the eigenvectors of C_x we form matrix A_k from the k eigenvectors corresponding to the k largest eigenvalues, the vector reconstructed by A_k is

$$\hat{x} = A_k^T y + m_x \quad .$$

The mean square error between x and $\hat{x}$ is given by

$$e_{ms} = \sum_{j=1}^{n} \lambda_i - \sum_{i=1}^{k} \lambda_j = \sum_{j=k+1}^{n} \lambda_j \quad .$$

Example 1

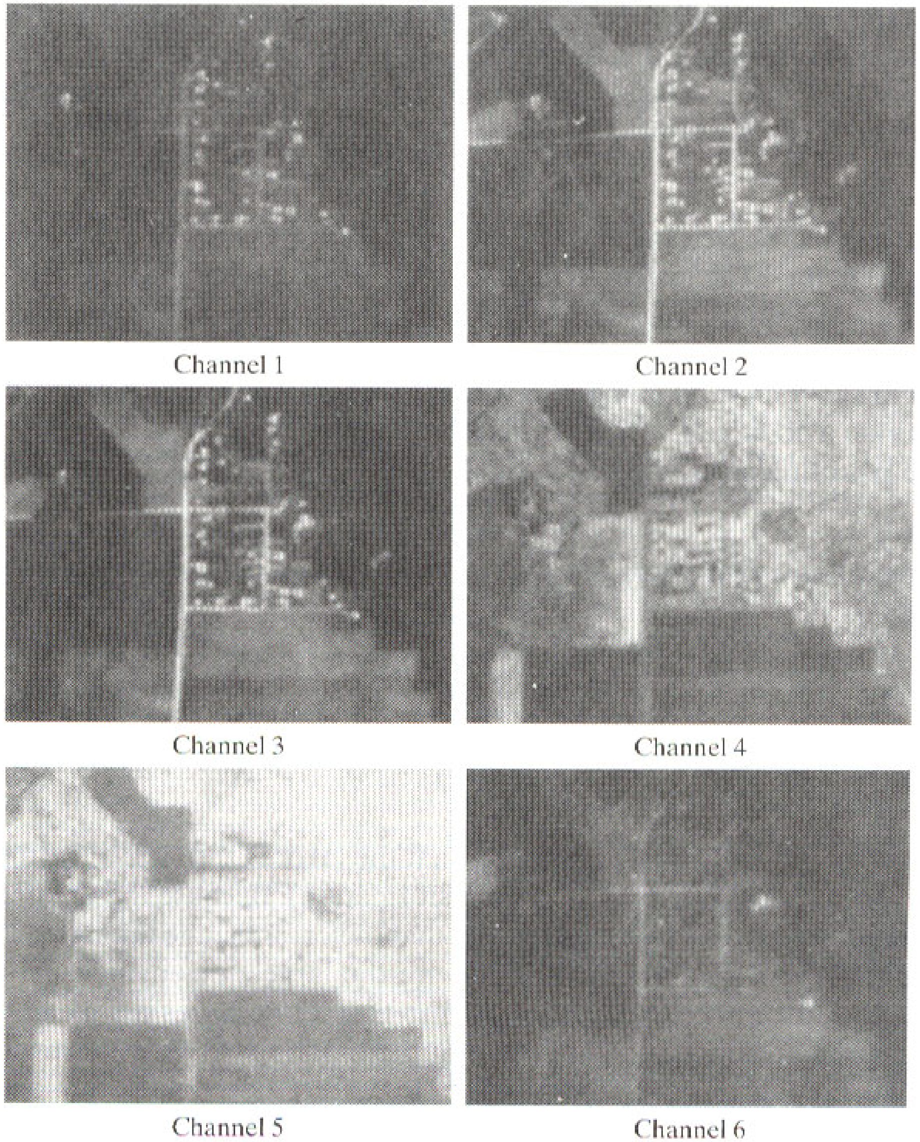


Figure 13.25
Six spectral images from an airborne scanner. (Courtesy of the Laboratory for Application of Remote Sensing, Purdue University)

Channel	Wavelength band (microns)
1	0.40–0.44
2	0.62–0.66
3	0.66–0.72
4	0.80–1.00
5	1.00–1.40
6	2.00–2.60

Table 13.4
Channel numbers and wavelengths.

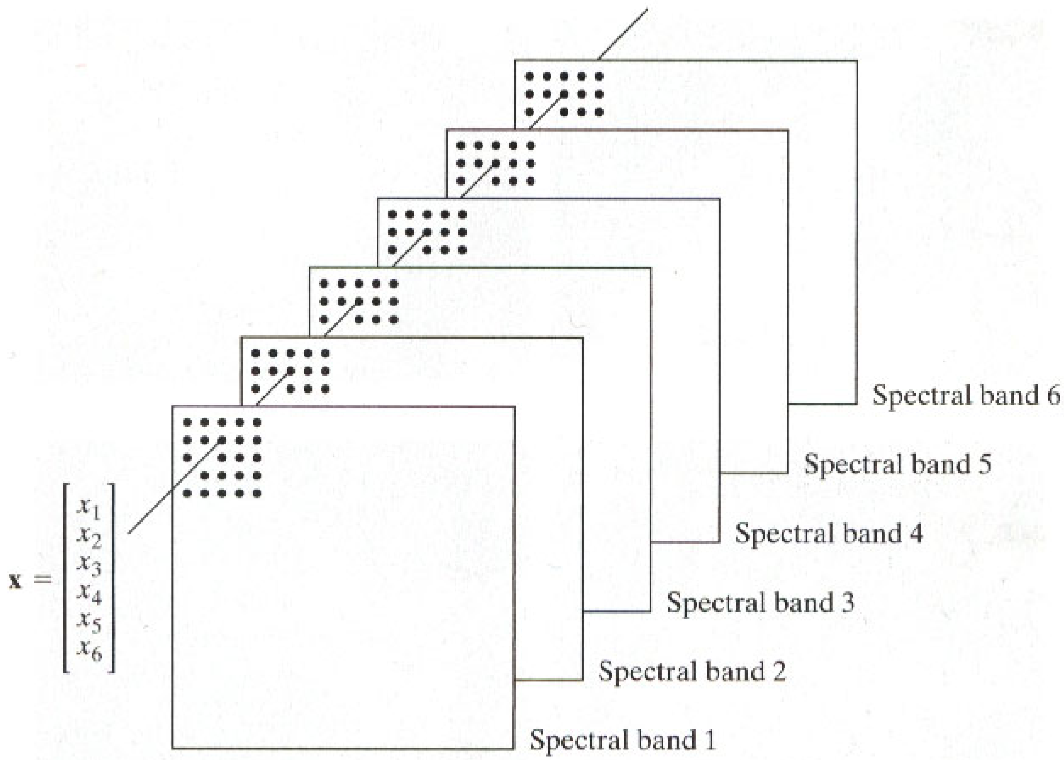
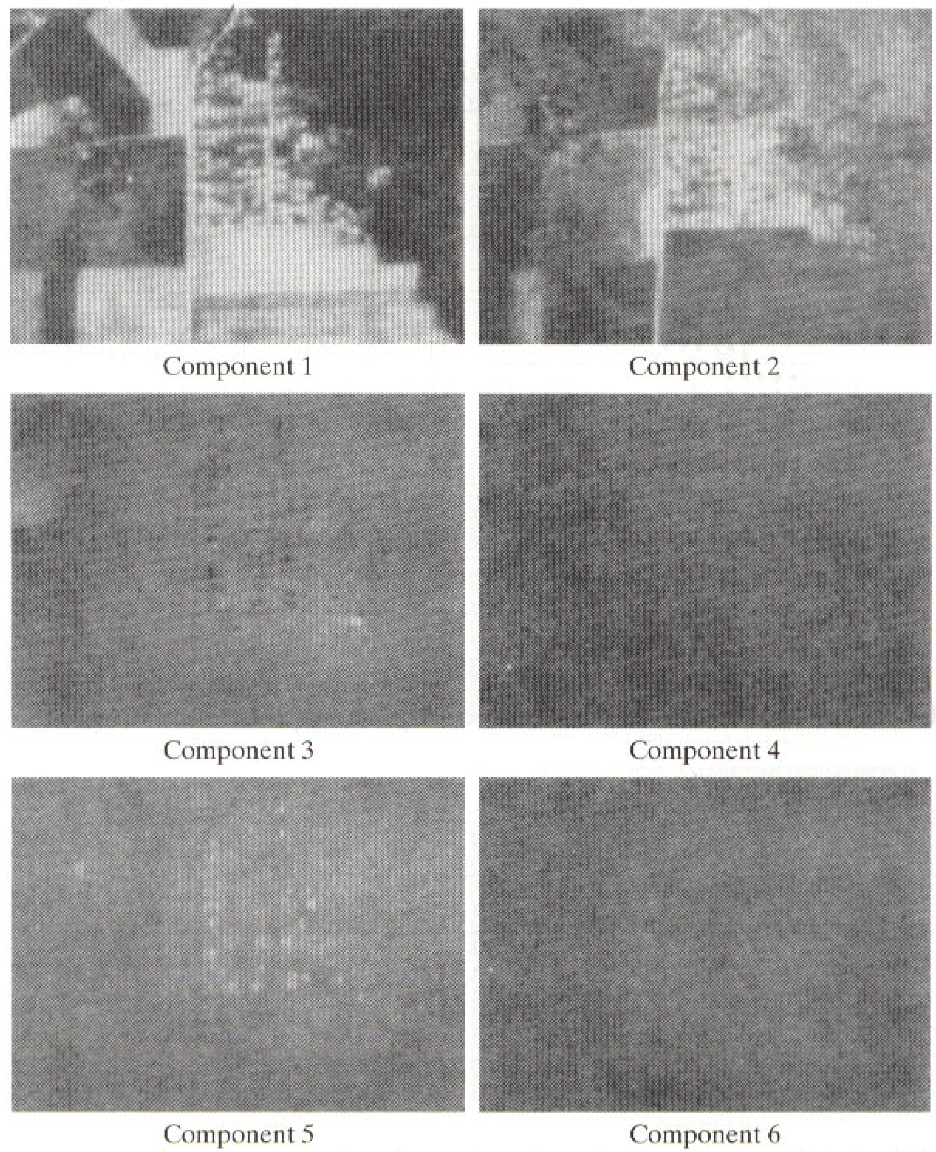


Figure 13.26
Formation of a vector from corresponding pixels in six images.

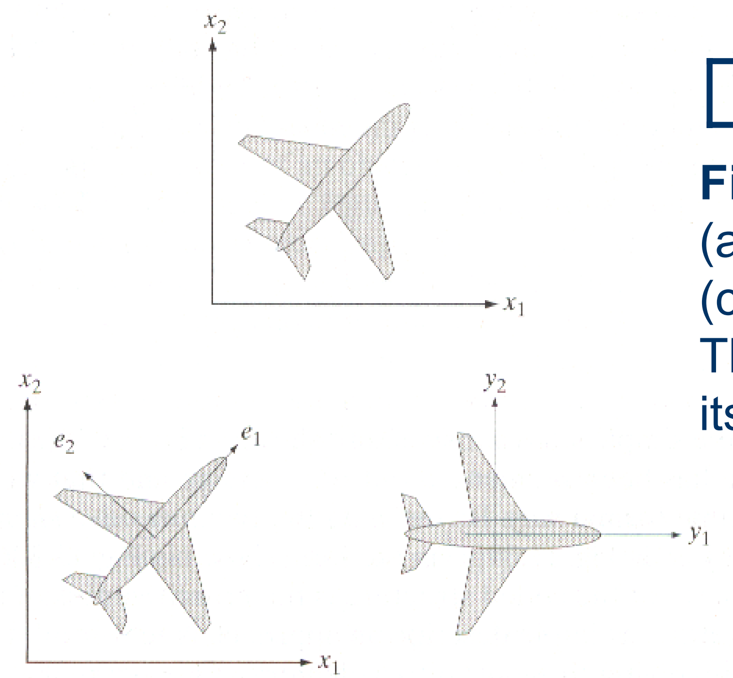
λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
3210	931.4	118.5	83.88	64.00	13.40

Table 13.5 Eigenvalues of the covariance matrix obtained from the images in Fig.13.25

Figure 13.27
Six principal-component images computed from the data in Fig. 13.25. (Courtesy of the Laboratory for Applications of Remote Sensing, Purdue University.)



Example 2



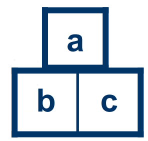


Figure 13.28
(a) An object. (b) Eigenvectors.
(c) Object rotated by using equation $y=A(x-m_x)$.
The net effect is to align the object along its eigen axes.