EC-433 Digital Image Processing

Lecture 8

Histogram Processing

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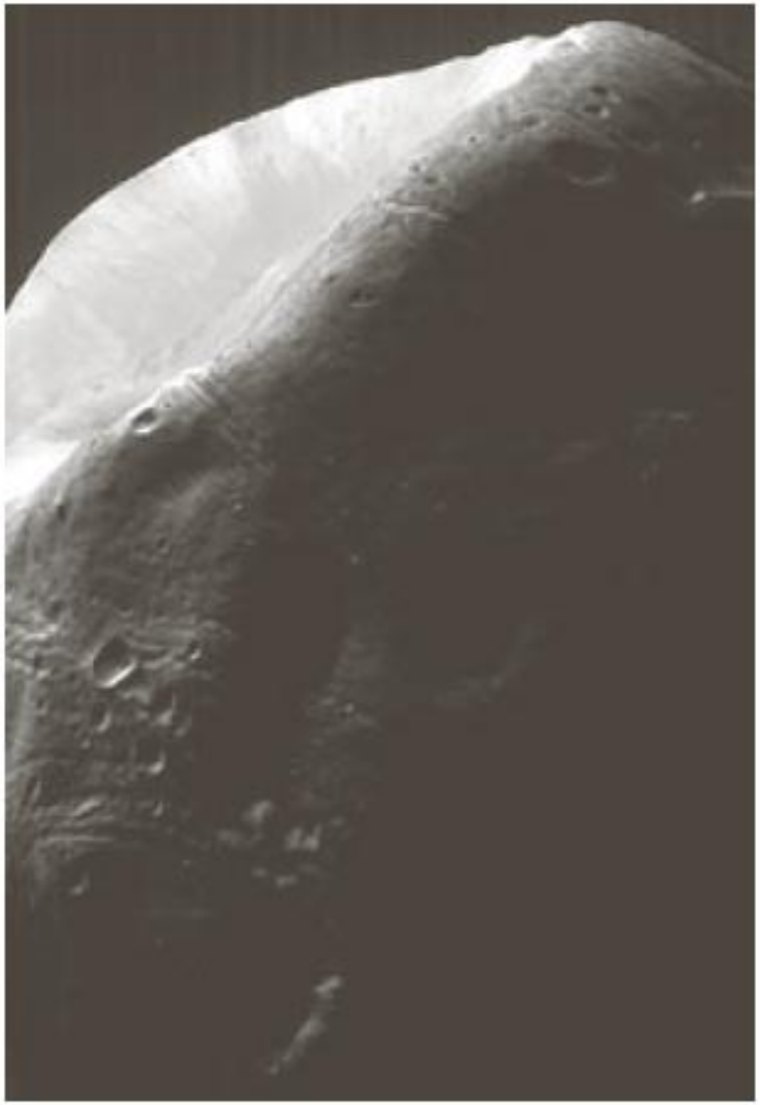
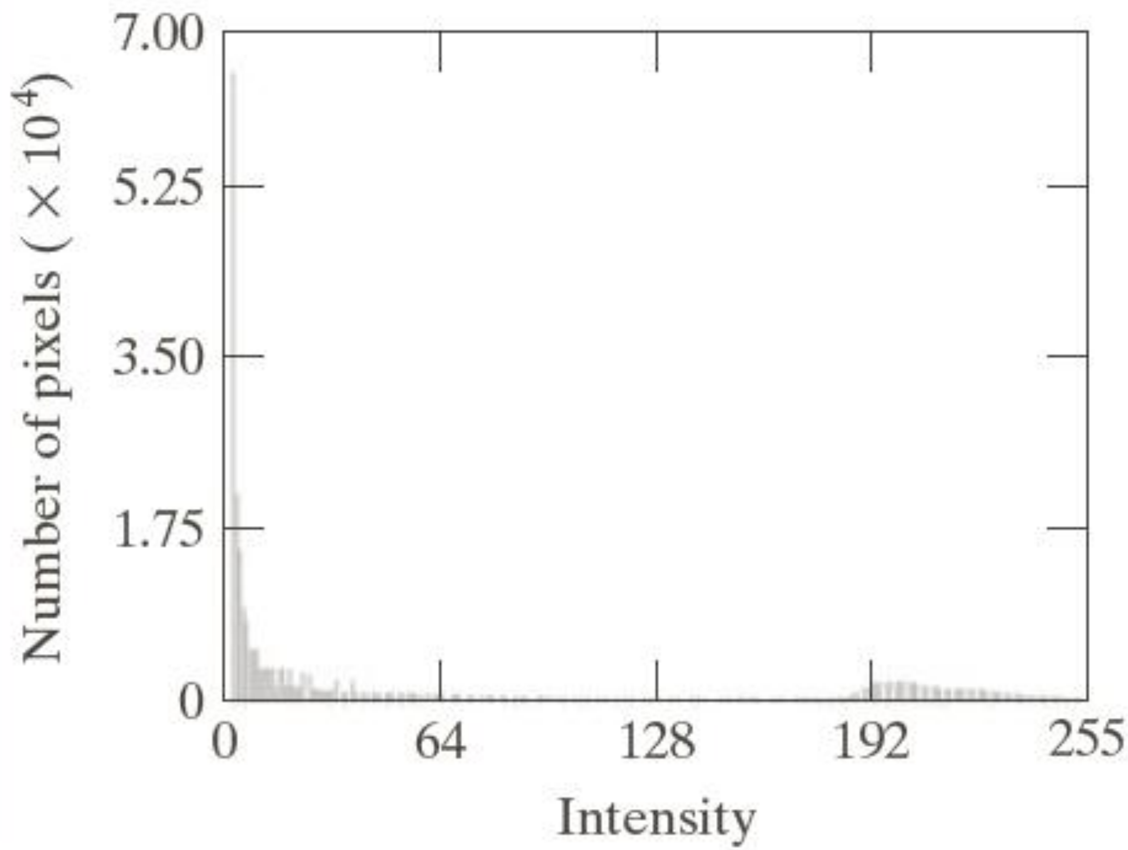
Dr. Rehan Hafiz, Gonzalez and Woods

Problems with Histogram Equalization

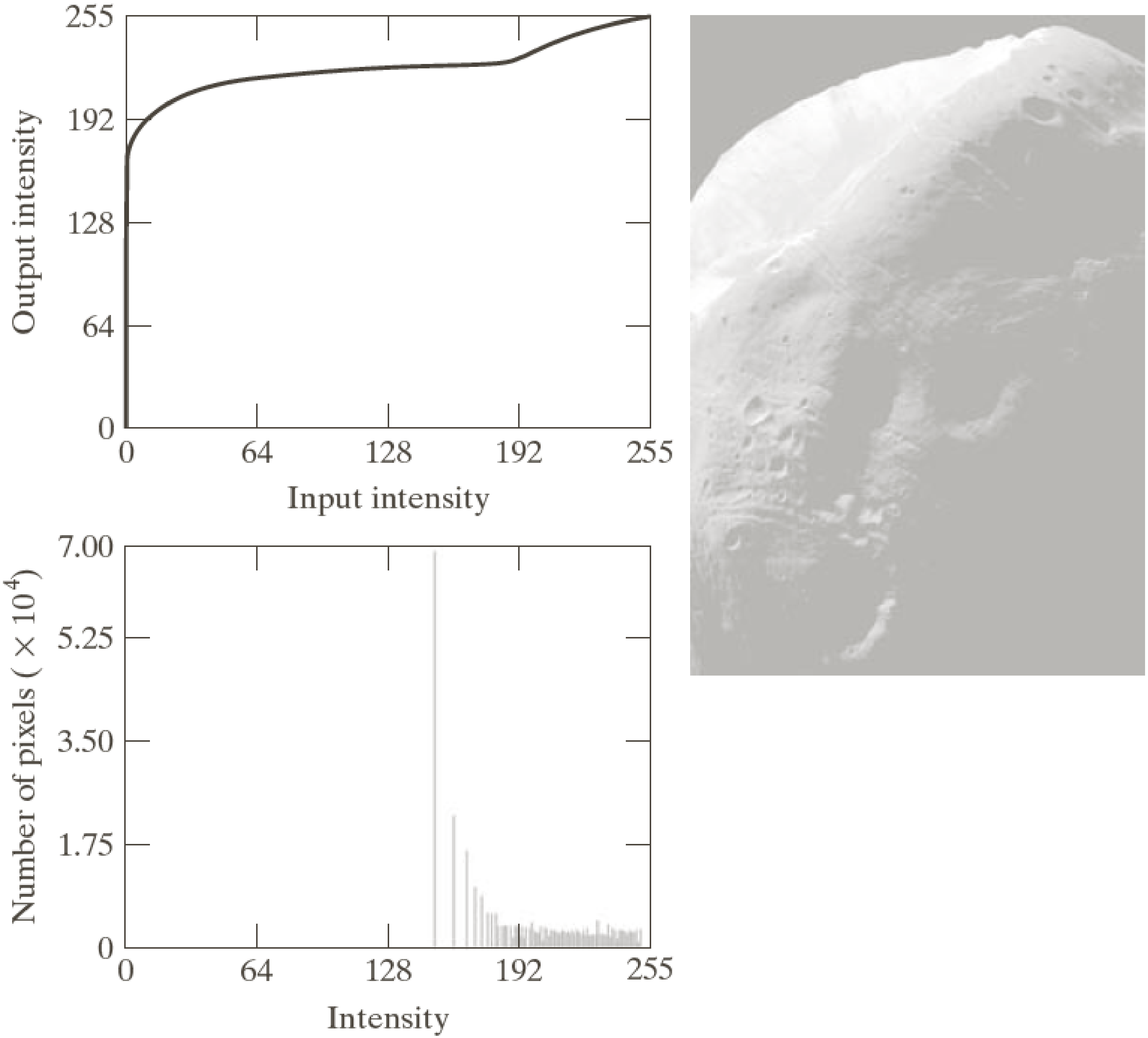
Image: Large concentration of dark gray-level pixels





a b

FIGURE 3.23
(a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)



a b
c

FIGURE 3.24
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).

Histogram Specification/Matching

- Histogram equalization method:
  - *Only generates one result: an image with approximately uniform histogram (without any flexibility)*
  - *Enhancement may not be achieved as desired*
- Histogram specification:
  - *Transform an image according to a specified gray-level histogram*
- Includes
  - *Specify particular histogram shapes ($p_z(z)$) capable of highlighting certain gray-level ranges*
  - *Obtain the transformation function for transformation of r to z*

Histogram Specification

$$r \xrightarrow[s=T(r)]{\textit{Transfer function } T= \textit{ cdf of } r} s$$

*Define a random variable **z** such that*

$$z \xrightarrow[S=G(z)]{\textit{Transfer function } G= \textit{ cdf of } z} s$$

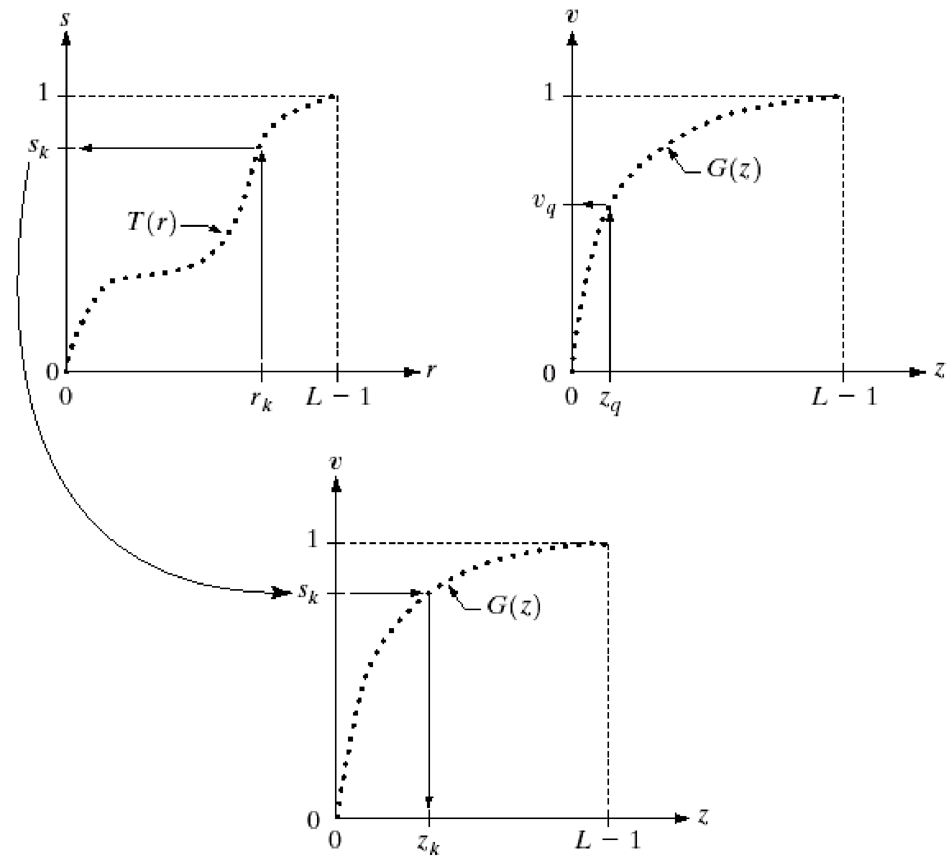
- $z=G^{-1}(s)$
- $z=G^{-1}(T(r))$
- Histogram Specification
  - *Apply HE on r to obtain s*
  - *Apply inverse of cdf of z on s*

Histogram Specification

- Step1: Equalize the levels of the original image
- Step 2: Specify the desired pdf and obtain the transformation function
- Step 3: Apply the inverse transformation function to the levels obtained in step 1

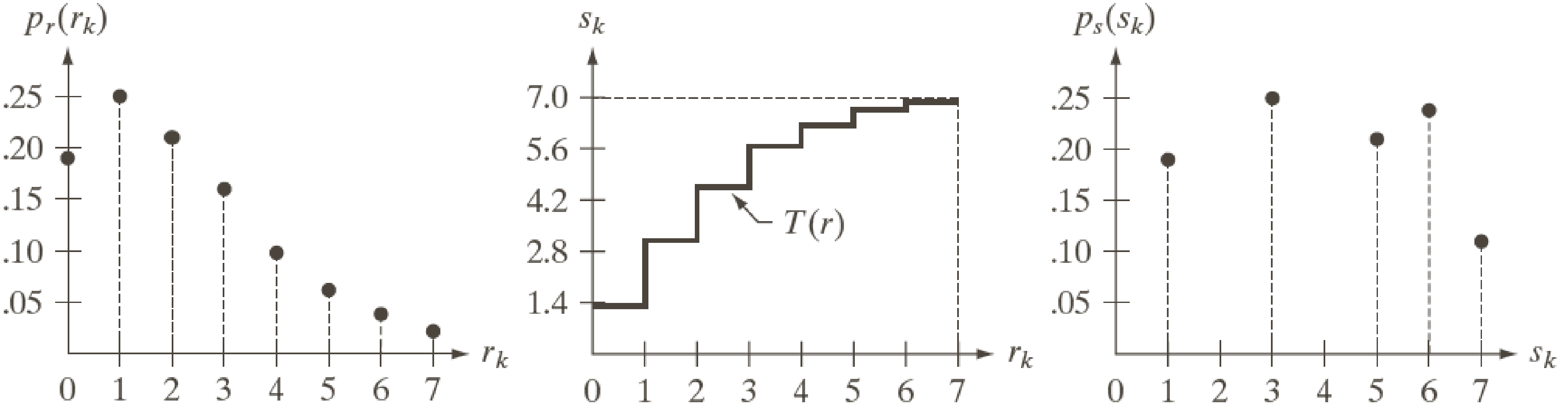
a b
c

FIGURE 3.19 (a) Graphical interpretation of mapping from r_k to s_k via $T(r)$. (b) Mapping of z_q to its corresponding value v_q via $G(z)$. (c) Inverse mapping from s_k to its corresponding value of z_k.

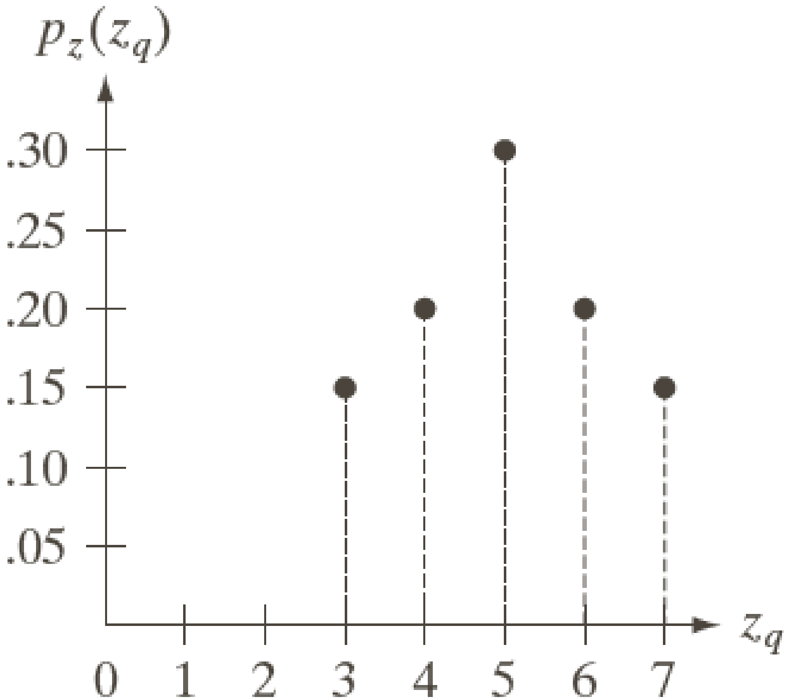


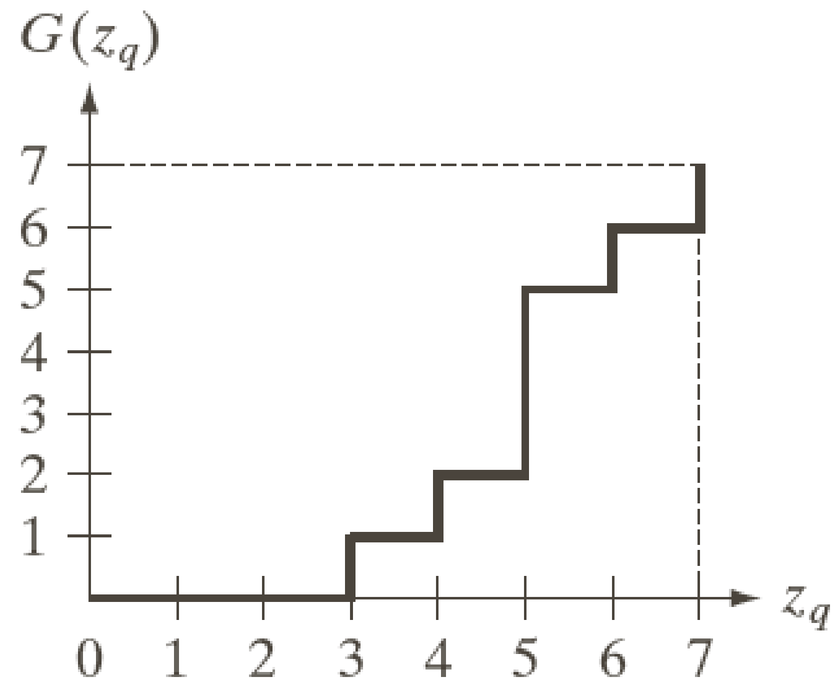
Step 1: Perform Histogram Equalization

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02



Step 2: Specify the Desired PDF and Get the CDF

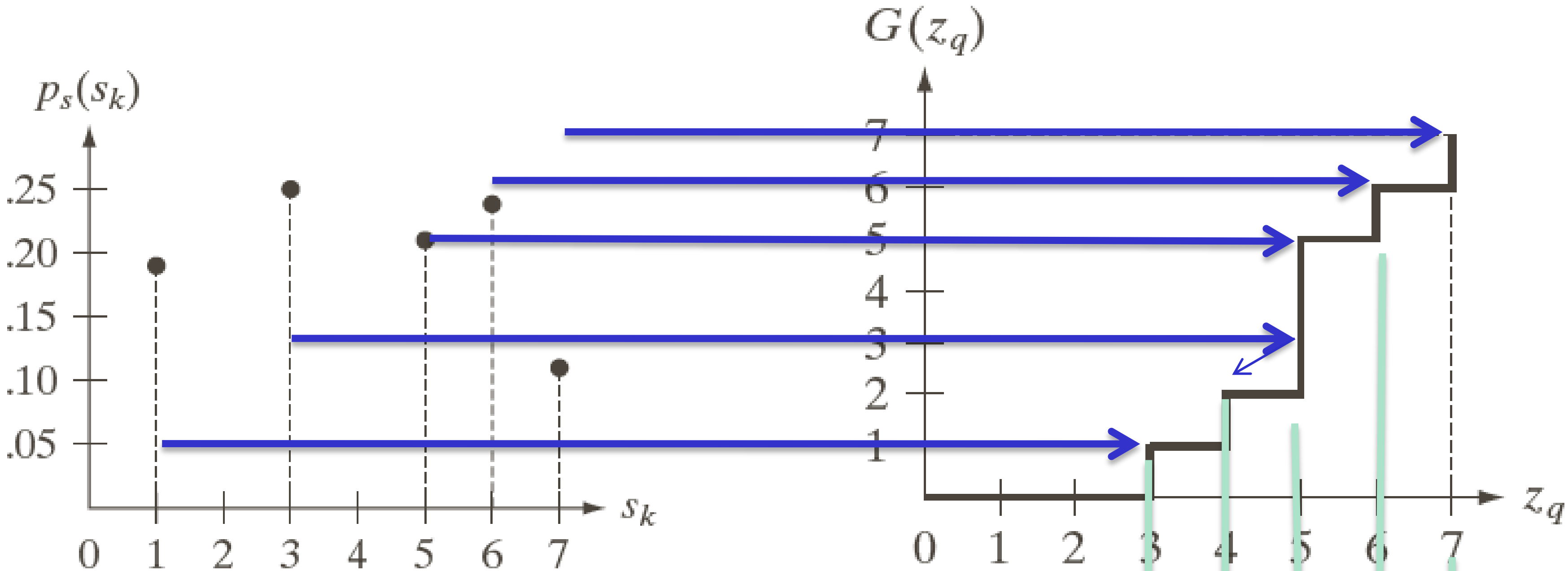




z_q	$G(z_q)$
$z_0 = 0$	0
$z_1 = 1$	0
$z_2 = 2$	0
$z_3 = 3$	1
$z_4 = 4$	2
$z_5 = 5$	5
$z_6 = 6$	6
$z_7 = 7$	7

TABLE 3.3 All possible values of the transformation function G scaled, rounded, and ordered with respect to z.

Step 3: Apply the Inverse PDF



Find the smallest value of z_q so that $G(z_q)$ is closest to s_k

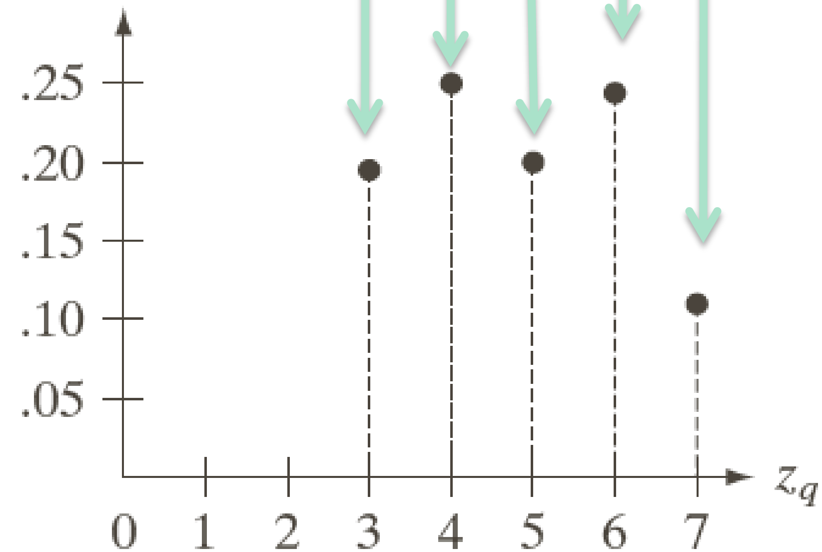
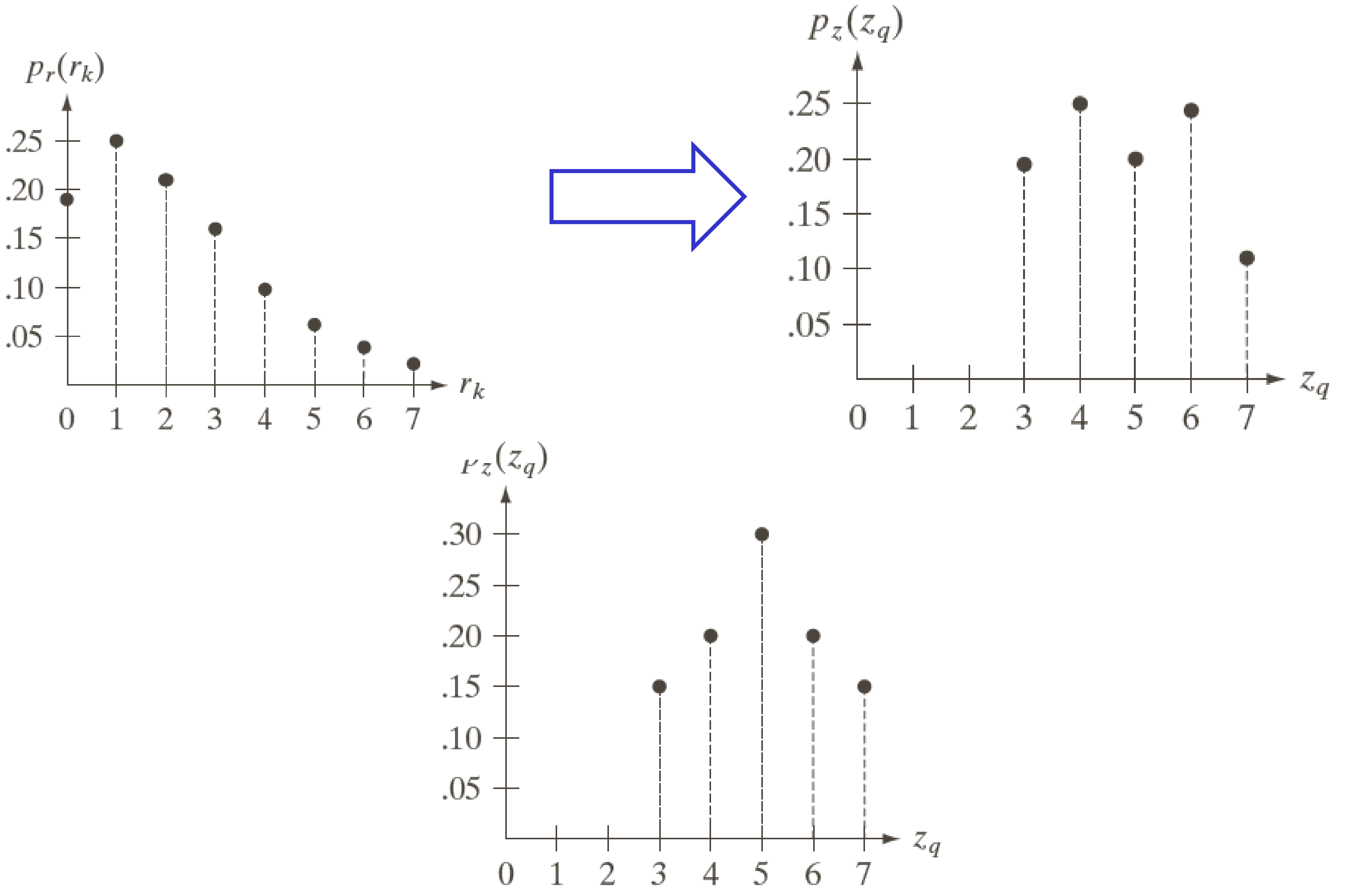


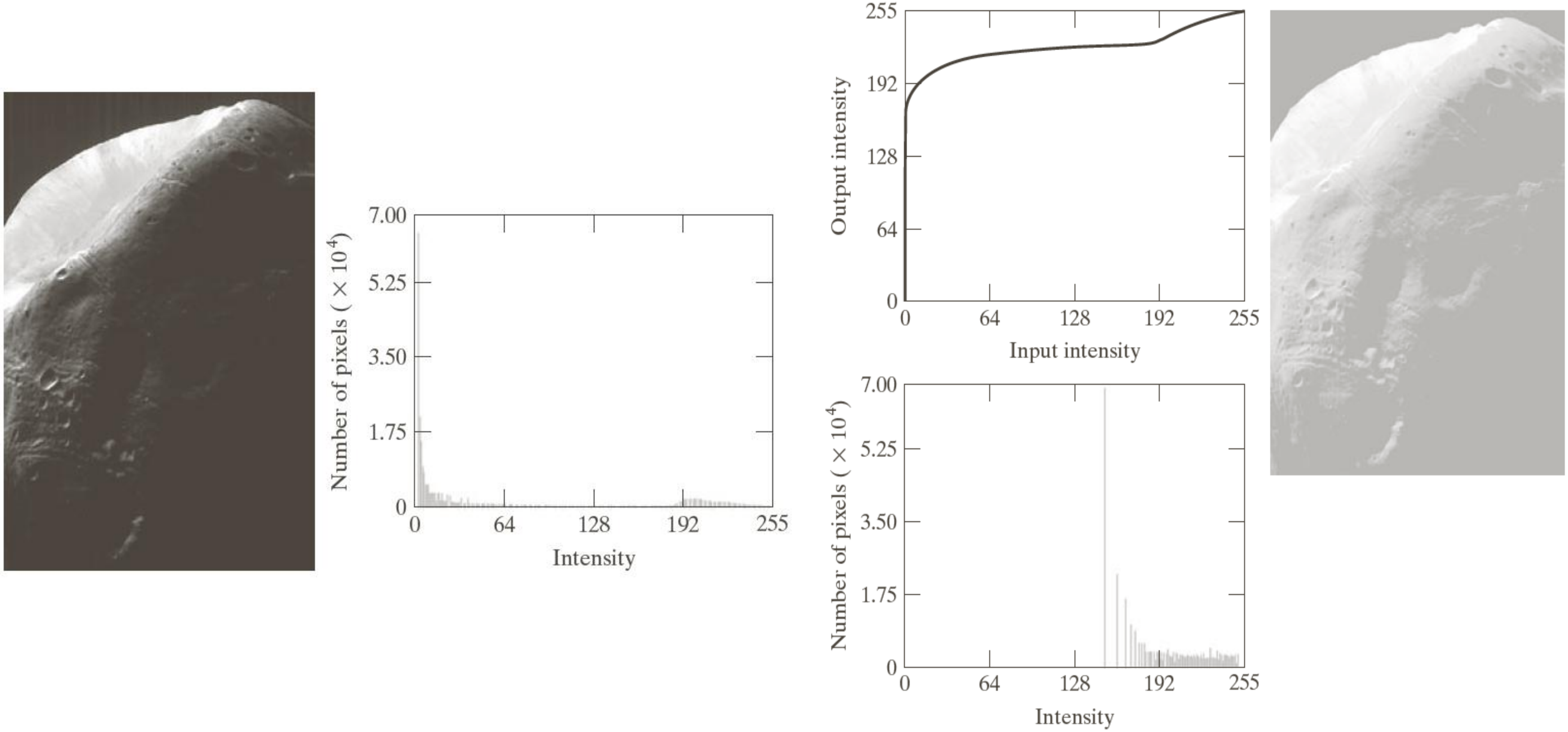
TABLE 3.4
Mappings of all the values of s_k into corresponding values of z_q.

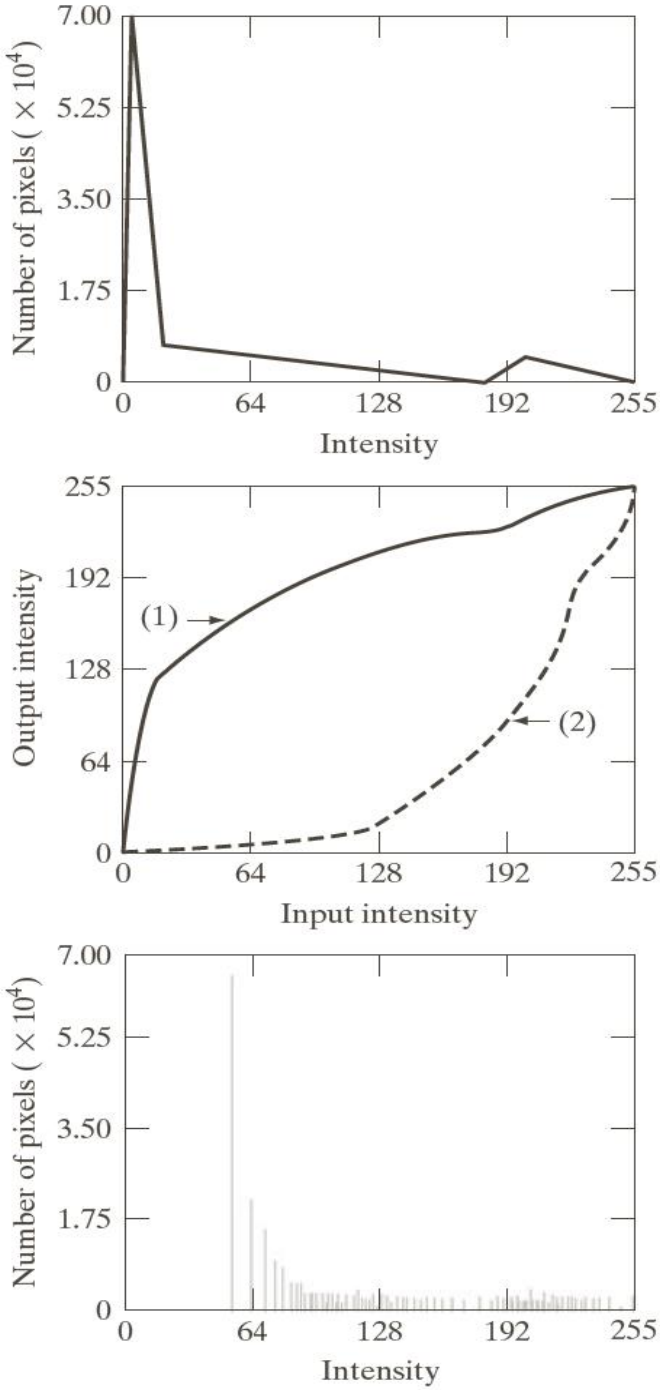
s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

Histogram Specification



Histogram Specification Example





Number of pixels (× 10^4)
7.00
5.25
3.50
1.75
0
0
64
128
192
255
Intensity
Output intensity
255
192
128
64
0
(1)
(2)
Input intensity

Histogram Specification

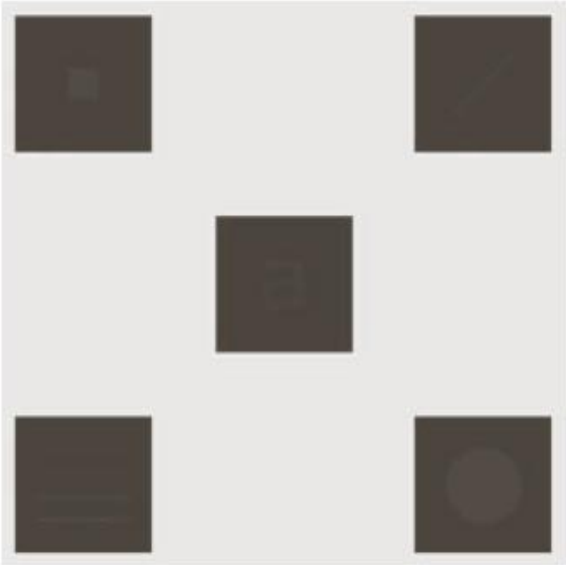
- Issues with Histogram specification/matching:
  - *No rule for specifying an optimal histogram*
  - *Each given enhancement task needs to be analyzed on a case-by-case basis*
  - *Histogram specification is somehow a trial-and-error process*

Local Histogram Processing

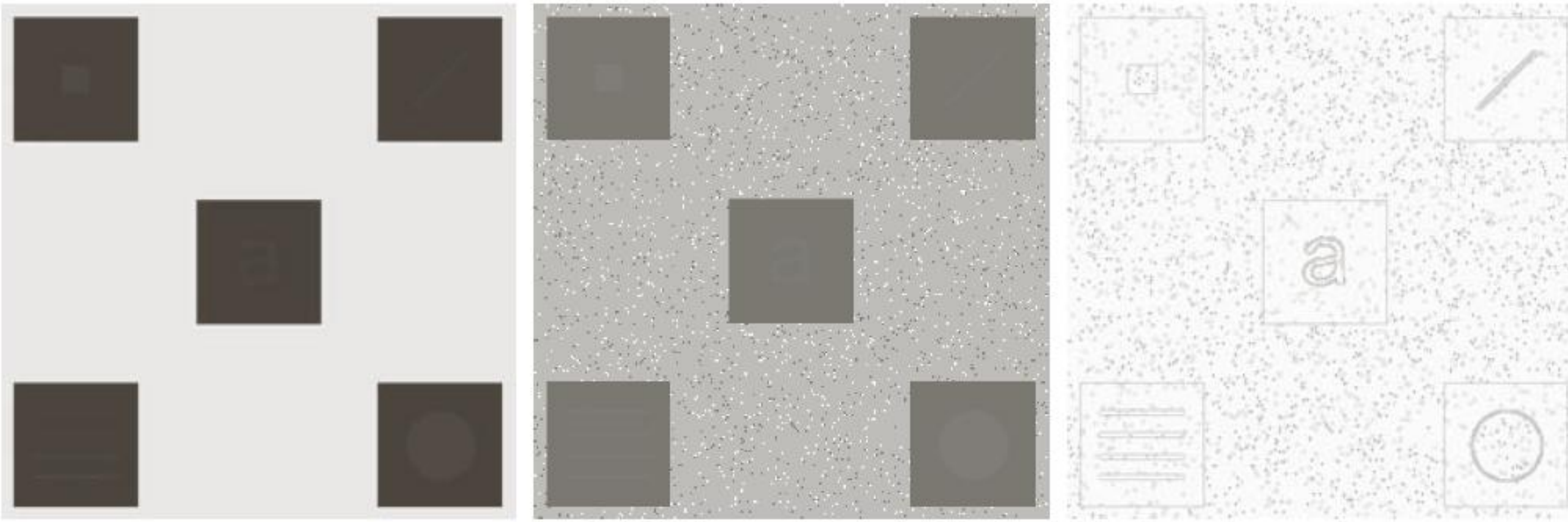
- The histogram processing methods mentioned up to now are global transformation where:
  - *Function is designed according to the gray-level distribution over an entire image*
  - *Global transformation methods may not be suitable for enhancing details over small areas*
  - *Where number of pixels in these small areas may have negligible influence on designing the global transformation function*

Local Histogram Processing

- To enhance details over small areas in an image
- Procedure
  - *Define a neighborhood (e.g. N8)*
  - *Move it from pixel to pixel.*
  - *For every pixel*
    - Histogram computed for the neighborhood
    - Transfer function computed for HE or H Spec
    - Applied on Centre Pixel



Local HE for 3x3 Neighborhood



a b c

FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3 × 3.

Using Histogram Statistics for Image Enhancement

- Mean gives the average brightness of the image
- Variance (σ^2) and its square root the standard deviation gives the deviation of intensities on average from the mean value (average contrast)
- Global statistics

$$m = \sum_{i=0}^{L-1} r_i \, p(r_i) \qquad m = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)$$

$$\mu_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 \, p(r_i) \qquad \sigma^2 = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[f(x, y) - m \right]^2$$

Using Histogram Statistics

- Local Statistics:
  - *S_{xy}: a neighborhood (subimage) of specific size centered at (x,y)*

$$m_{s_{xy}} = \sum_{i=0}^{L-1} r_i \, p_{s_{xy}}(r_i)$$

$$\sigma_{S_{xy}}^{\;2} = \sum_{i=0}^{L-1} \left[r_i - m_{s_{xy}} \right]^2 p_{s_{xy}}(r_i)$$

Local Enhancement using Histogram Statistics

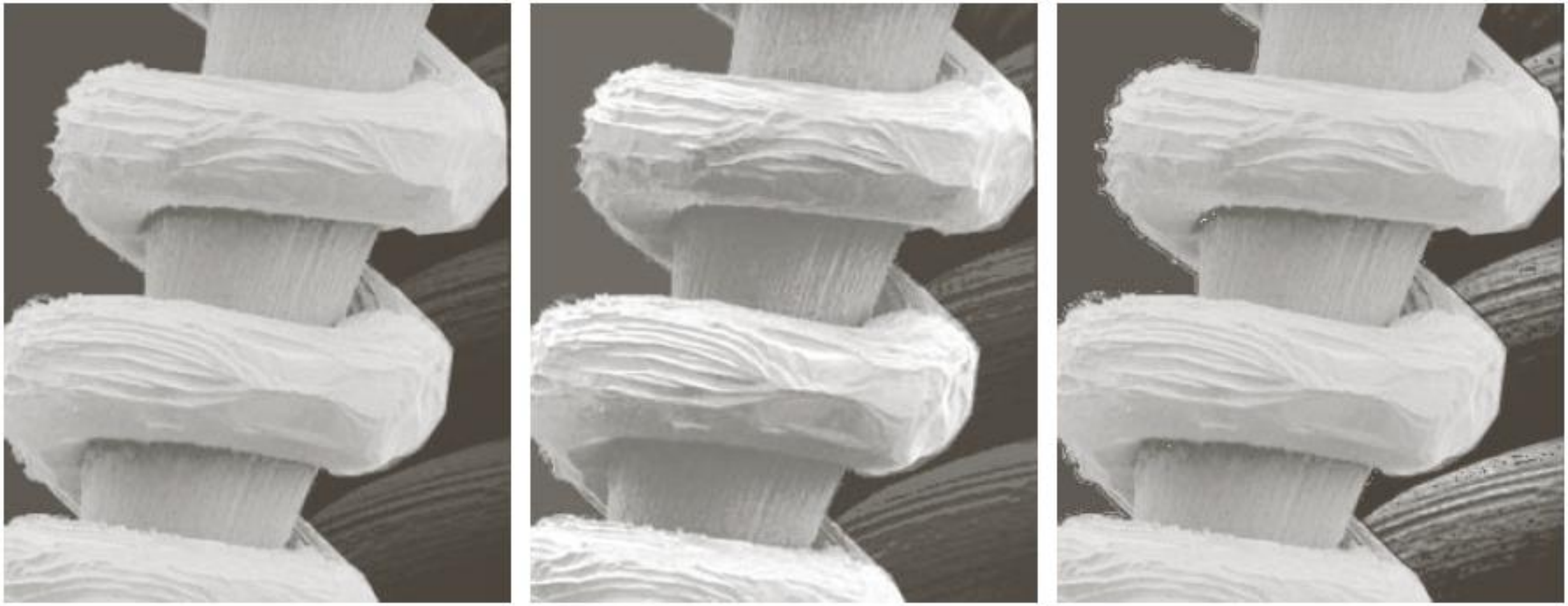
- The statistical parameters can be used in various ways
- Enhance the background filament
- Enhance details in dark areas while leaving light area unchanged.
- Define rules to chose the candidate pixels that need to be enhanced

Local Enhancement using Histogram Statistics

- A pixel at point (x,y) is considered if:
  - $m_{SXY} \leq k_0 M_G$, *where k0 is a positive constant less than 1.0, and* M_G *is global mean*
  - $\sigma_{sxy} \leq k_2 D_G$, *where* D_G *is the global standard deviation and* k_2 *is a positive constant*
  - *Also need to put a lower limit on SD to avoid distorting areas which don't have details, i.e.,* $k_1 D_G \leq \sigma_{sxy}$ *, with* $k1 < k2$
- A pixel that meets all above conditions is processed simply by multiplying it by a specified constant, *E*, to increase or decrease the value of its gray level relative to the rest of the image.
- The values of pixels that do not meet the enhancement conditions are left unchanged.

$$g(x,y) = \begin{cases} E.f(x,y) & \text{if } m_{S_{xy}} \leq k_0 M_G \text{ AND } k_1 D_D \leq \sigma_{S_{xy}} \leq k_2 D_G \\ f(x,y) & \text{otherwise} \end{cases}$$

Example



a b c

FIGURE 3.27 (a) SEM image of a tungsten filament magnified approximately 130×. (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)