Conversion between Cartesian and Polar Co-ordinate Systems

The Cartesian image, *G(x, z) ,* containing pixels of dimensions (*dx)* x (*dz)* where *dx = dz =1,* is to be converted to polar co-ordinates by mapping the Cartesian grid, defined by the detector, onto a polar array with pixels of size (*dR)* x (*dΘ).* This pixel conversion process is adapted from a similar method used by Wrede and is described with respect to Figure 1. To retain a similar amount of information in polar co-ordinates, the polar pixel is chosen to be of a similar size as the Cartesian pixel: *dR dΘ ~-=* dx dz =1. As the differential area remains the same, the number of polar pixels required to define a given *R* will scale as *R,* as illustrated in Figure 1a. Prior to the conversion process, *G (x, z)* first undergoes a four-folding process, to generate a single symmetrized image quadrant, *G, (x, z), as follows:*

Image Centring and Four-way Folding

The center of the raw Cartesian image, (x_0, z_o), can be identified, by manually inspecting a four-way folded Cartesian quadrant of the original image, or fitting an ellipse or circle for symmetric data, or by implementing other methods such as the Bordas criterion. The symmetrized Cartesian image quadrant, G_q *(x, z) ,* generated from the folding process has the additional benefit of being a quarter of the size of the original raw image, which significantly reduces the computation time of the subsequent Legendre decomposition method, while still retaining all vital signal and angular information. When the center of the image has been identified, the pixel intensities in the folded Cartesian quadrant, ***I(x,z) ,*** are determined from the mean of four Cartesian pixels in the 2D image. The mean should ignore NaN values and zeros in order to weight properly the intensities. Additional corrections to detector sensitivity fluctuations can be made when there's a priori knowledge of isotropic intensity information (for example from random scattering). A calibration map per radius can be obtained using the median intensity as a reference, assuming the fluctuation from the median intensity per radius are small vs the total # of pixels.

For a symmetrized Cartesian quadrant, the number of polar pixels at each *R* equates to the integer part of π/2*(*R* + 1) . The origins of this relationship stem from the fact that one needs to define the circumference length of a 90° arc, defined as ¼(2πR)= πR/2. The factor of +1 originates from the fact that the smallest value of *R* is *R* = 0, and a single polar pixel is required to define this minimum *R.* For example, for the pixel column *R = 1,* the number of polar pixels equates to 3 using the above relationship.

Figure 1b illustrates how the signal in a polar pixel, *I (R, Θ) ,* is determined *via* its fractional overlap with the four surrounding Cartesian pixels. This process is best conceptually understood by rotating the polar pixel by *Θ* about its pivot. The absolute center of the polar pixel is first expressed in Cartesian co-ordinates, (x_p, z_p) , using: $x_p = R \sin\theta$, $z_p = \mathrm{R}\cos\theta$. From these decimal co-ordinates, the co-ordinates of the Cartesian pixel, (x_c, z_c) , that lies on

the center of the polar pixel are determined by rounding x_p and z_p to the nearest integers, respectively. The fractional intensity contribution from this Cartesian pixel, defined as I_1, to the total polar pixel intensity, *I (R, Θ)* , can be determined from $I_1 = I(x_c, Z_c)[x_{diff} Z_{diff}]$

With : $x_{diff} = 1 - |x_c - x_p|$ and $z_{diff} = 1 - |z_c - z_p|$

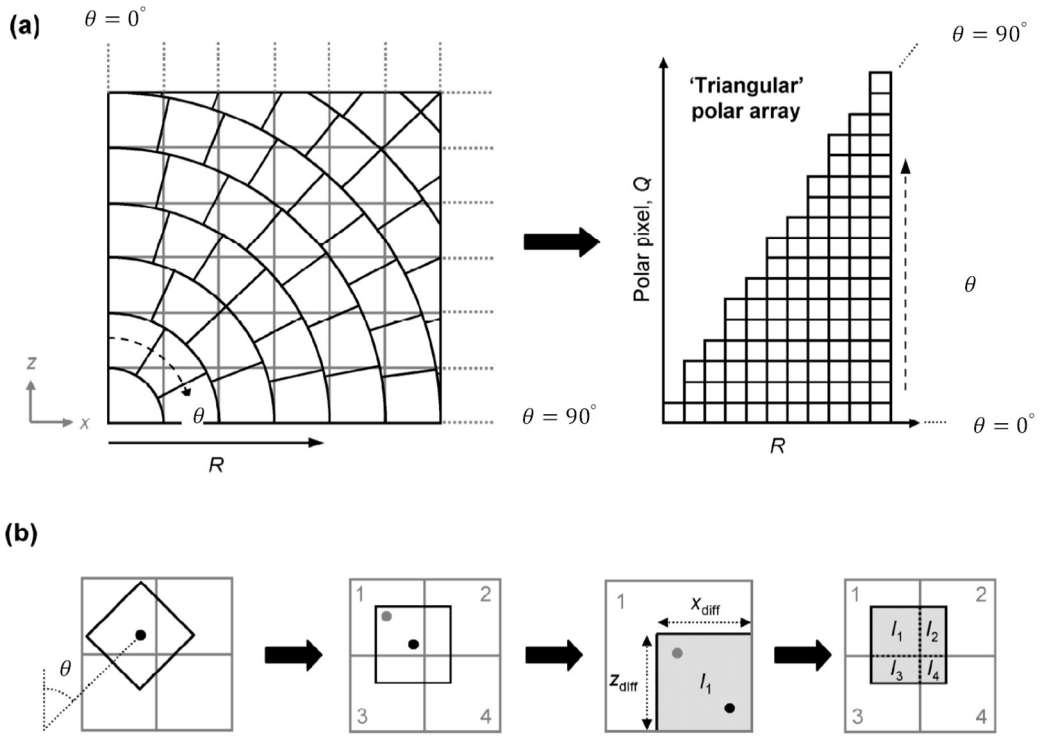

Figure 1 *(a) Schematic representation of the Cartesian pixel array, G(x, z) , shown in gray and the overlaid array of polar pixels, G(R, Θ) , shown in black. This polar array is then represented as a `triangular' array of polar pixels, Q, as show on the right. (b) Schematic of the Cartesian to polar conversion process. A polar pixel, (R, Θ) , is rotated about its central pivot (black dot) by Θ and the Cartesian pixel in which its center lies is determined (pixel 1). The intensity contribution of this pixel, I1 (grey shaded area), to the polar pixel intensity is then determined using x_{diff} and z_{diff}. Finally, the remaining intensity contributions from Cartesian pixels 2,3,4 (I_2 ,I_3 and I_4, respectively) are determined to calculate the final polar pixel intensity. With reference to the methodology described in this section, this example is shown for a translational move of x +1 and z — 1. In case of a NaN value in one of the 4 pixels, the remaining pixel are weighted to 1 according to their fractional area. In case all 4 pixels are NaN, the value of the polar pixel will be NaN.*

and $I(x_c,z_c)$ is the total signal of the Cartesian pixel at that coordinate. The next three Cartesian pixels that overlap with the polar pixel are identified using a series of simple comparisons between x_c and x_p, and z_c and z_p. With respect to x_c and x_p , if $x_c > x_p$ then the *x* coordinate translation will be *x* — 1, whereas if $x_c < x_p$ then the move will be *x* + 1. This process is then repeated for the z co-ordinate, and the remaining three fractional Cartesian intensities can then be calculated using:

$$I_2 = I(x_c \pm 1, z_c) \cdot [(1 - x_{diff}) z_{diff}]$$

$$I_3 = I(x_c, z_c \pm 1) \cdot [(1 - z_{diff}) x_{diff}]$$

$$I_4 = I(x_c \pm 1, z_c \pm 1) \cdot [(1 - x_{diff})(1 - z_{diff})]$$

where the total intensity of the polar pixel will be: $I(R, \theta) = \langle I_i \rangle$ excluding NaN and zeros.

When this process is completed for all polar pixels, the resulting raw polar image, $G(R, \Theta)$ is thus a triangular array because the number of angles, Θ, at which pixels can be defined scales linearly with ***R.*** Although not the traditional representation of a 2D image, it provides a convenient and intuitive display as one can simply read off the radial distribution along the ***R*** axis and the angular distribution by comparing the signal levels along the angular axis.

The main source of error in this polar to Cartesian conversion arises from the assumption that the polar pixel has the same dimensions as the Cartesian pixel. This becomes less pronounced as ***R*** increases. The errors are not cumulative and only present a problem at small ***R.***