III . 3

C- $q(x_k | x_{k-1}, z_k)$

$q(x_k|x_{k-1}, z_k, H_0)$ $=$ $\lambda_0 \underbrace{q_0(x_k|z_k)}_{H_0} + \lambda_1 \underbrace{p(x_k|x_{k-1})}_{H_1} + \lambda_2 \underbrace{u_{\mathcal{D}}(x_k)}_{H_2}$

$q(x_k|x_{k-1}, z_k, H_1)$ $\quad$ $q(x_k|x_{k-1}, z_k, H_2)$

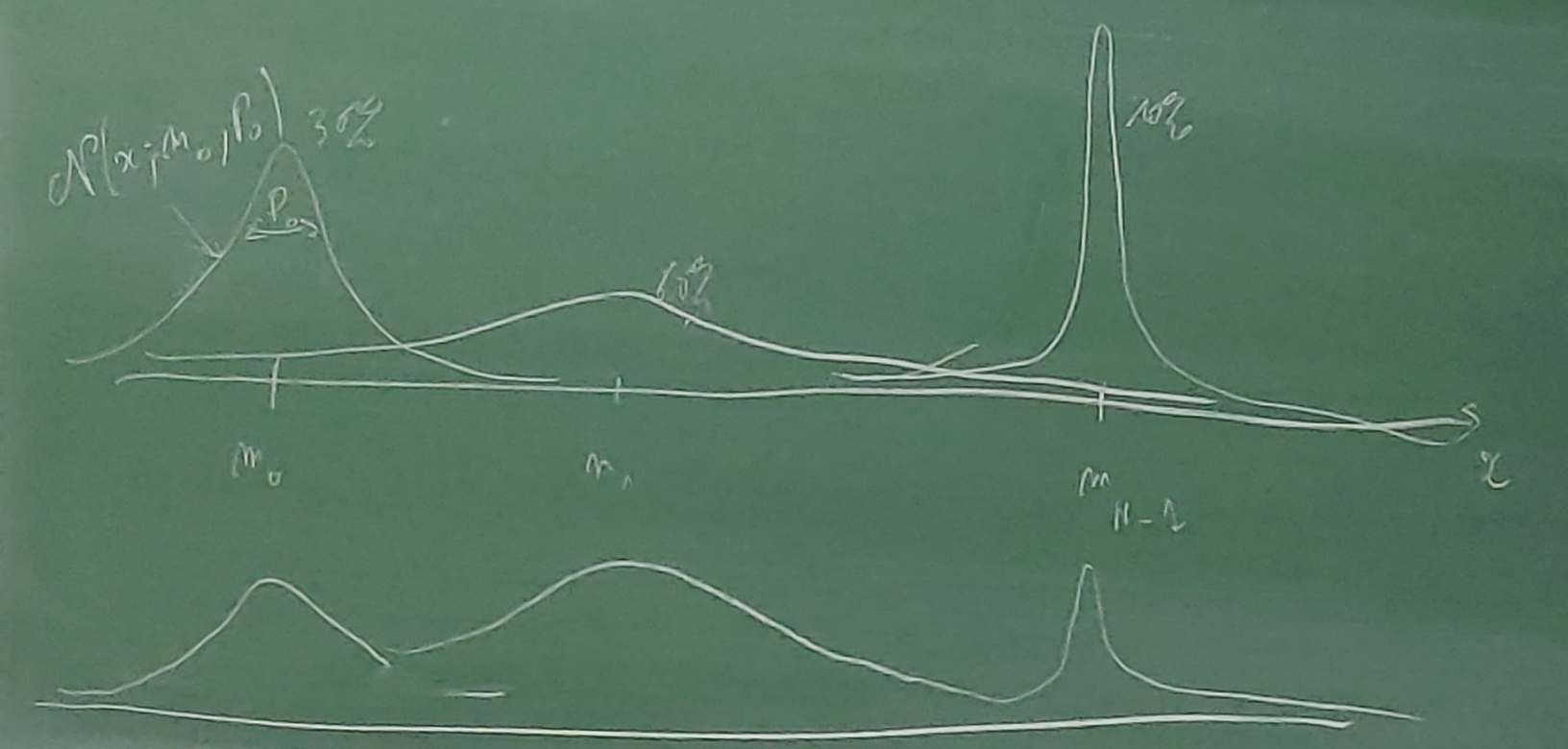
où $\lambda_0, \lambda_1, \lambda_2 \geq 0$

$\lambda_0 + \lambda_1 + \lambda_2 = 1$

$H \in \{H_0, H_1, H_2\}$

Théo des probas totales

$$q(x_k|x_{k-1}, z_k) = \sum_{h \in \{0,1,2\}} \mathbb{P}(H = H_i)\, q(x_k|x_{k-1}, z_k, H_i)$$


$\mathcal{N}(x; m_0, \rho_0)$
3%
ρ_0
15%
10%
m_0
m_1
m_{N-1}
x

* $x_k = F_{k-1} x_{k-1} + G_{k-1} u_{k-1} + v_{k-1}$

$z_k = H_k x_k + v_k$

$x_0, w_{0\,k-1}, v_{1\,k-1}$ mut° ind, $\mathcal{G}$ dans leur ensemble

$x_0 \sim \mathcal{N}(m_0, P_0)$; $w_{k-1} \sim \mathcal{N}(0, Q_{k-1})$, $v_k \sim \mathcal{N}(0, R_k)$

* Implémentons l'algo CONDENSATION à N particules

→ Échantillonnage (idéal) du nuage initial et Pondération ($k=0$)

- $x_0^{(1)}$ —— $x_0^{(N)}$ $\overset{\text{iid}}{\sim}$ $\mathcal{N}(m_0, P_0)$ ⊛

- $w_0^{(1)} = \dots = w_0^{(N)} = \frac{1}{N}$

pour réaliser ⊗, cf $y_0^{(1)} \dots y_0^{(N)}$ iid $\sim \mathcal{N}\left(0_{m_x \times 1}, \mathbb{1}_{m_x \times m_x}\right)$

puis

$$x_0^{(i)} = m_0 + L_0 y_0^{(i)} \quad \text{où} \quad L_0 \text{ tq } L_0 L_0^T = P_0$$

- Alors $\overline{x_{0|0}} = \sum_{i=1}^{N} w_0^{(i)} x_0^{(i)} = \frac{1}{N} \sum_i x_0^{(i)}$ est un estimé de $\hat{x}_{0|0} := m_0$

$$\overline{P_{0|0}} = \sum_i w_0^{(i)} \left(x_0^{(i)} - \overline{x_{0|0}}\right)\left(\cdot\right)^T \quad \longrightarrow \quad P_{0|0} := P_0$$

$\longrightarrow$ Condensation $\Leftrightarrow$ $q(x_k | x_{k-1}, z_k) := p(x_k | x_{k-1}) = \mathcal{N}(x_k ; F_{k-1} x_{k-1} + G_{k-1} u_{k-1}, Q_{k-1})$

- $x_k^{(i)} \overset{ind}{\sim} p(x_k | x_{k-1}^{(i)}) \Leftrightarrow x_k^{(i)} \overset{ind}{\sim} \mathcal{N}(F_{k-1} x_{k-1}^{(i)} + G_{k-1} u_{k-1}, Q_{k-1})$

ssi $y_k^{(i)} \sim \mathcal{N}(0_{m_x \times 1}, \mathbb{1}_{m_x \times m_x})$

puis

$$x_k^{(i)} = F_{k-1} x_{k-1}^{(i)} + G_{k-1} u_{k-1} + M_{k-1} y_k^{(i)} \quad \text{où} \quad M_{k-1} M_{k-1}^T = Q_{k-1}$$

- $w_k^{(i)} \propto w_{k-1}^{(i)} q(z_k | x_k^{(i)})$ (A)

(4) or $p(z_k|x_k) = \mathcal{N}(z_k, H_k x_k, R_k)$

donc $w_k^{(i)} \propto w_{k-1}^{(i)} \; \mathcal{N}(z_k, H_k x_k^{(i)}, R_k)$

=

$$w_{k-1}^{(i)} \; \frac{1}{\sqrt{\det(2\pi R_k)}} \exp\left[-\frac{1}{2} \left(z_k - H_k x_k^{(i)}\right)^T R_k^{-1} \left(\Diamond\right) \right]$$

$$\hookrightarrow \boxed{w_k^{(i)} \propto w_{k-1}^{(i)} \exp\left[-\frac{1}{2} \left(z_k - H_k x_k^{(i)}\right)^T R_k^{-1} \left(z_k - H_k x_k^{(i)}\right) \right]}$$

NOTA. $\overline{x}_{k|k} = \sum_{i=1}^{N} w_k^{(i)} x_k^{(i)}$ est un estimé de $\hat{x}_{k|k}$ exact donné par le filtre de Kalman ; $\overline{P}_{k|k} = \ldots$ est un estimé de $P_{k|k}$, etc.

* Quid de l'implémentation du filtre particulaire optimal ?

$$\cdot\ q^*(x_k \mid x_{k-1}, y_k) \propto p(y_k \mid x_k)\, p(x_k \mid x_{k-1}) = \mathcal{N}(y_k ; H_k x_k, R_k)\, \mathcal{N}(x_k ; F_{k-1} x_{k-1} + G_{k-1} u_{k-1}, Q_{k-1})$$

$$= \mathcal{N}(x_k ; \rho_k, \Sigma_k)$$

où $\rho_k = \left(F_{k-1} x_{k-1} + G_{k-1} u_{k-1}\right) + N_k \left(y_k - H_k \left(F_{k-1} x_{k-1} + G_{k-1} u_{k-1}\right)\right)$; $\Sigma_k = Q_{k-1} - N_k H_k Q_{k-1}$ où

$$N_k = Q_{k-1} H_k^T \left(R_k + H_k Q_{k-1} H_k^T \right)^{-1}$$

$$w_k^{(i)} \propto w_{k-1}^{(i)} \; p(z_k \mid x_{k-1}^{(i)})$$

où $p(z_k \mid x_{k-1}) = \mathcal{N}(z_k ; H_k(F_{k-1} x_{k-1} + G_{k-1} u_{k-1}), H_k Q_{k-1} H_k^T + R_k)$

en vertu du fait que

$$z_k = H_k(F_{k-1} x_{k-1} + G_{k-1} u_{k-1} + w_{k-1}) + v_k$$

$$= H_k(F_{k-1} x_{k-1} + G_{k-1} u_{k-1}) + H_k w_{k-1} + v_k$$

IV - Filtre particulaire Rao-Blackwellisé

* Position du pb : $x_k = \left(r_k^T , m_k^T \right)^T$

  • conditionnellement à r_k, la loi de filtrage de m_k peut être calculée/approx analytiquement (via F Kalman, EKF, UKF ...)

  ⇓

  on sait calculer/approx $p(m_{0:k} | r_{0:k}, y_{1:k})$

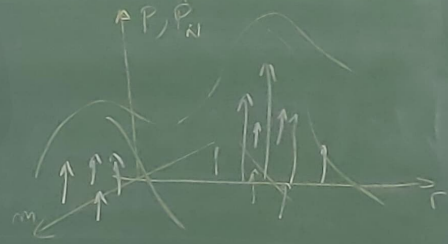
  • On suppose que $p(r_k | r_{k-1}, m_{k-1}) = p(r_k | r_{k-1})$

c à d. que m_{k-1} n'intervient pas dans la dyn a priori de r_k

$$p(x_k|x_{k-1}) = p(r_k, m_k | r_{k-1} m_{k-1}) = p(m_k | r_k r_{k-1} m_{k-1})\ p(r_k | r_{k-1} \cancel{m_{k-1}})$$

s'écrit

$$p(x_k|x_{k-1}) = p(m_k | m_{k-1} r_k r_{k-1})\quad p(r_k | r_{k-1})$$



* "FP classique"

$$\hookrightarrow p(x_{0:k} \mid z_{1:k}) = p(r_{0:k}\, m_{0:k} \mid z_{1:k}) \approx \sum_{i=1}^{N} w_{0:k}^{(i)}\, \delta\left(x_{0:k} - x_{0:k}^{(i)}\right) =: \hat{p}_N\left(\quad x_{0:k} \mid z_{1:k}\right)$$

$$= \sum_{i=1}^{N} w_{0:k}^{(i)}\, \delta\left(r_{0:k} - r_{0:k}^{(i)}\right) \delta\left(m_{0:k} - m_{0:k}^{(i)}\right.$$

* L'idée est d'approximer $p(r_{0:k} | z_{1:k})$ par

$$p(r_{0:k} | z_{1:k}) \approx \hat{p}_N(r_{0:k} | z_{1:k}) := \sum_{i=1}^{N} \bar{w}_{0:k}^{(i)} \delta\left(r_{0:k} - r_{0:k}^{(i)}\right)$$

où $r_{0:k}^{(i)} \sim q(r_{0:k} | z_{1:k})$

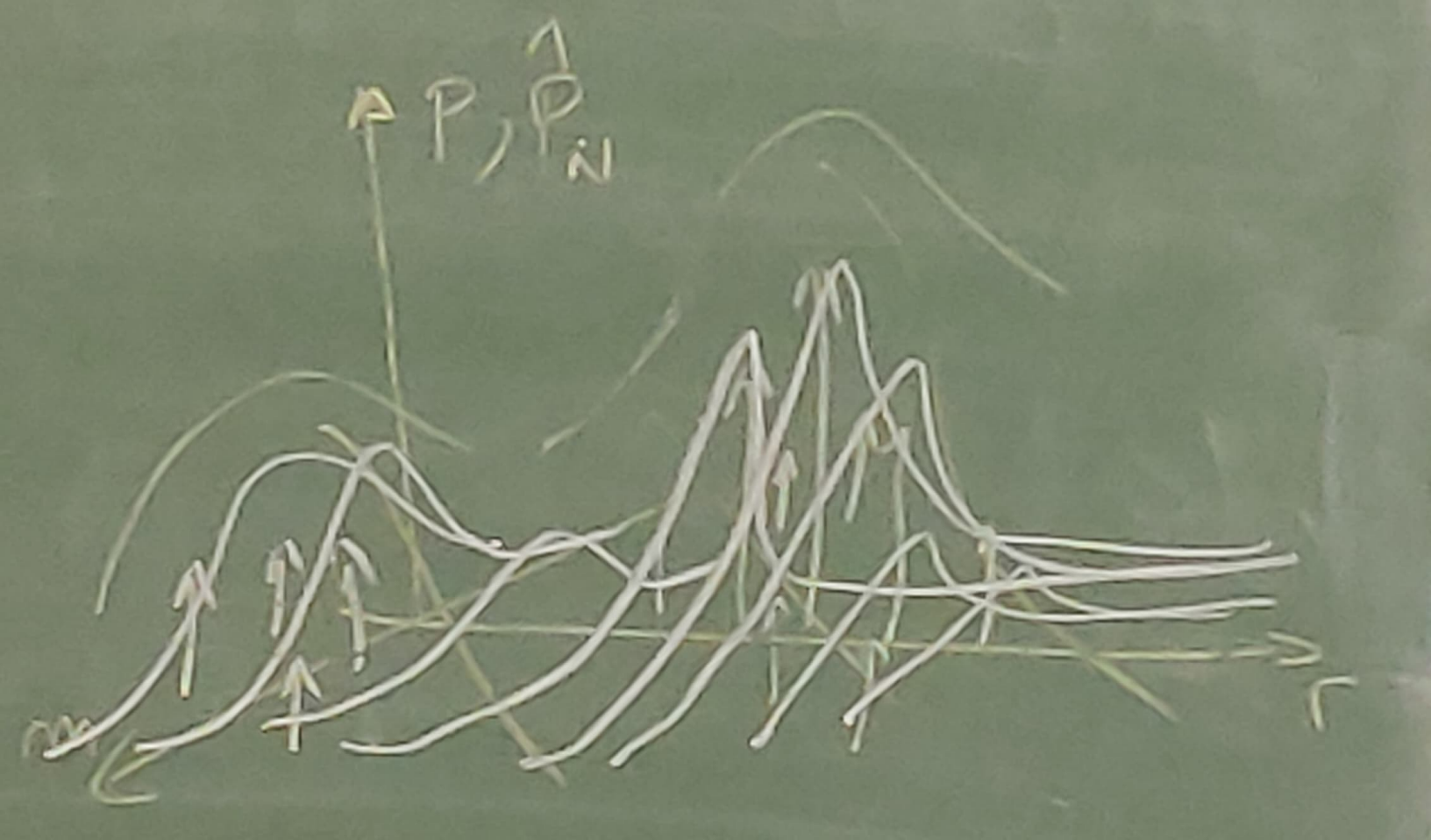
et de déduire l'approximation de

$$p(r_{0:k}, m_{0:k} | z_{1:k}) = p(m_{0:k} | r_{0:k}, z_{1:k})\, p(r_{0:k} | z_{1:k})$$

comme $p(r_{0:k}, m_{0:k} | z_{1:k}) \approx p(m_{0:k} | r_{0:k}, z_{1:k})\, \hat{p}_N(r_{0:k} | z_{1:k})$

$$= \sum_{i=1}^{N} \bar{w}_{0:k}^{(i)}\, p\left(m_{0:k} \middle| r_{0:k}^{(i)}, z_{1:k}\right) \delta\left(r_{0:k} - r_{0:k}^{(i)}\right)$$

où $\varphi(m_{0k} | r_{0k}^{(i)} \; \mathcal{Z}_{1k})$ est obtenu analytiquement par KF, EKF, UKF, etc.


P, P_N
r

$$\bar{w}_{0:k}^{(i)} \propto \bar{w}_{0:k-1}^{(i)} \frac{p(z_k \mid z_{1:k-1}, r_{0:k}^{(i)})\, p(r_k^{(i)} \mid r_{k-1}^{(i)})}{q(r_k^{(i)} \mid r_{k-1}^{(i)}, z_{1:k})}$$