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TD : Linéarisation Entrée-Etats

Exercice : Résolution type à la fin du poly

1. $\begin{cases} \dot{x_1}(t) = x_2^2 - x_2^3 + x_2 \\ \dot{x_2}(t) = u \end{cases}$

$$f(x) = \begin{bmatrix} x_2^2 - x_2^3 + x_2 \\ 0 \end{bmatrix}$$

$$g(x)u = \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$[f, g] = \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} \cdot g = 0 - \begin{bmatrix} 2x_2 - 3x_2^2 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= - \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

2. On cherche $y = h(x)$

$\dot{y} = L_f h(x)$

$\vdots$

$y^{(n-2)} = L_g L_f^{(n-2)} h(x) = 0$ or $n = 2$ donc $n-2 = 0$

Ainsi $y^{(0)} = L_g L_f^0 h(x) = 0$ g est involutif avec lui-même

On a $n=2$ $[g, [f,g]] = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ et $[g] = 0$

De même, $[f, f] = \frac{\partial f}{\partial x} \cdot f - \frac{\partial f}{\partial x} f = 0$

$\frac{\partial T_1}{\partial x} \cdot g = 0$ et $\begin{bmatrix} \frac{\partial T_1}{\partial x_1} & \frac{\partial T_1}{\partial x_2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0 \iff \frac{\partial T_1}{\partial x_2} = 0$

Par exemple $T_1(x) = x_1$

$T(x) = \begin{bmatrix} T_1(x) \\ L_f T_1(x) \end{bmatrix}$ on pose le changement de variable $\begin{cases} z_1 = x_1 \\ z_2 = \dot{x_1} = x_2^2 - x_2^3 + x_2 \end{cases}$

bijectif car équivalent à $\begin{cases} x_1 = z_1 \\ x_2 = z_2 - z_2^2 + z_2^3 \end{cases}$

$$\frac{\partial T}{\partial x} = \begin{bmatrix} 1 & 0 \\ 2x_1 - 3x_1^2 & 1 \end{bmatrix}$$

déterminant non nul sur $\mathbb{R}^2$
$\Rightarrow$ difféomorphisme
$\mathbb{R}^2 = T(\mathbb{R}^2)$ donc difféomorphisme global.

<u>Rappel:</u> $y_1 = T_1(x)$

$$\dot{y}_1 = \frac{d}{dt}[T_1(x)] = \frac{\partial T_1}{\partial x} \cdot \dot{x}$$

$$= \frac{\partial T_1}{\partial x} f(x) + g(x) u$$

$$= \frac{\partial T_1}{\partial x} f(x) = L_f T_1 + \cancel{\frac{\partial T_1}{\partial x} g(x) u}^{\,0}$$

$$\dot{y}_2 = \frac{d}{dt} L_f T_1(x)$$

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = v \end{cases} \qquad v = -k_1 y_1 - k_2 y_2$$

<u>Exercice :</u>

$$\begin{cases} \dot{x}_1(t) = -x_1 + x_1^2 x_2 \\ \dot{x}_2(t) = u \end{cases} \qquad f(x) = \begin{bmatrix} -x_1 + x_1^2 x_2 \\ 0 \end{bmatrix}$$

$$[g \quad ad_f g] = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$[f, g] = \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g$$

$$= 0 - \begin{bmatrix} \quad \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$=$$