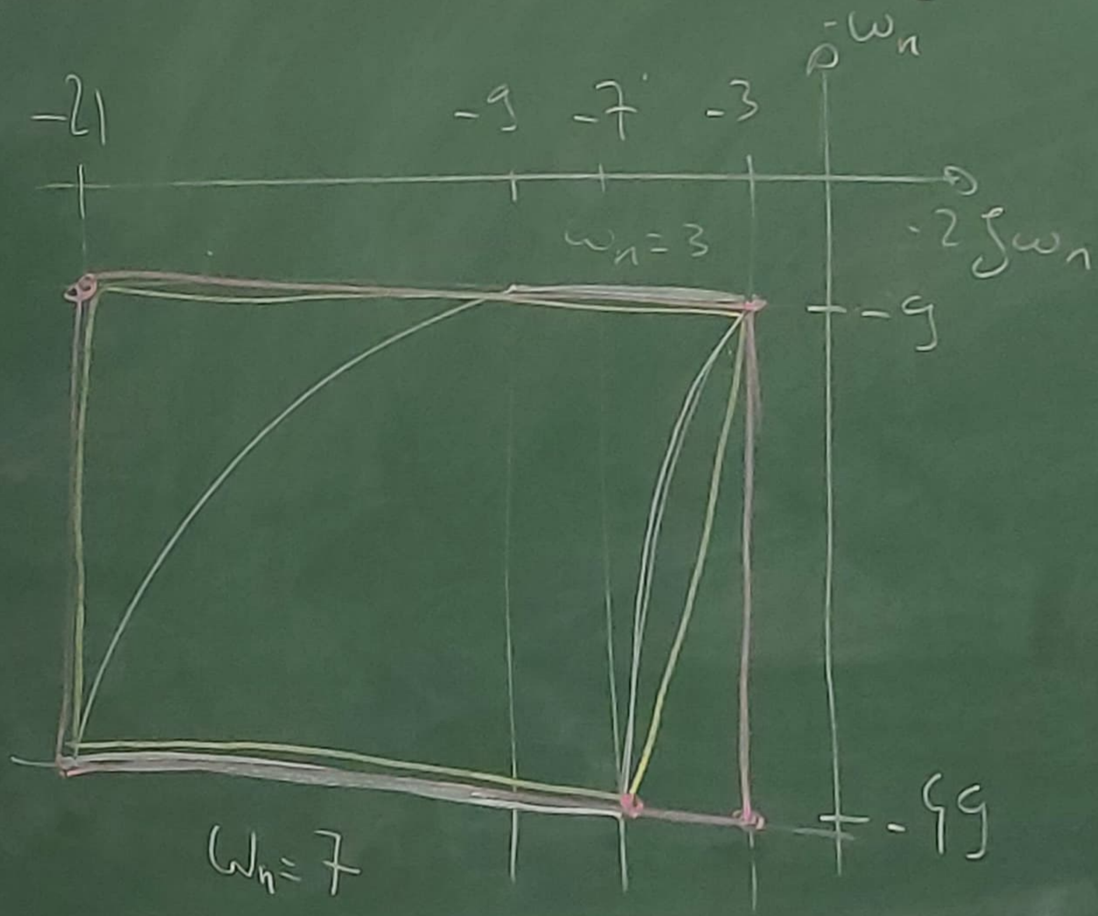
$$A = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix}$$

$$\frac{1}{2} \leq \zeta \leq \frac{3}{2}$$

$$3 \leq \omega_n \leq 7$$



Inégalités matricielles linéaires LMI

$\exists P \succ \varepsilon_1 \mathbb{I} : \forall v = 1 \dots \overline{V}$

si $A^{[v]T} P + P A^{[v]} \prec -\varepsilon_2 \mathbb{I}$

alors le système $\dot{x} = A(t)\, x(t)$

r.S $\forall\, A(t) \in Co\{A^{[v]}\}$

$$\iff \begin{cases} \hat{P} \succeq \mathbb{I} \\ A^{(v)T} \hat{P} + \hat{P} A^{(v)} \preceq -\mathbb{I} \end{cases}$$

$$v = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \xrightarrow{M} y = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\|M\|_2^2 \geq 0$$

$$v = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \xrightarrow{M} y = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\|v\|^2 = 2 \qquad \|y\|^2 = 8$$

$$\|M\|_2^2 \geq 4$$

$$\frac{\|y\|^2}{\|u\|^2} = \frac{u^* M^* M u}{u^* u}$$

$$= \frac{u^* T^* D T u}{u^* u}$$

$$M^*M = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\det M^*M = 0 = \lambda_1 \lambda_2$$

$$v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\mathrm{Tr}\,(M^*M) = 4 = \lambda_1 + \lambda_2$$

$$\lambda_1 = 0 \qquad \lambda_2 = 4$$

* Signaux $t \longmapsto x(t) \in \mathbb{C}^n$

$s \longmapsto X(s) \in \mathbb{C}^n$

$$X(s) = \int_0^\infty x(t) e^{-st} \, dt$$

$$\|x\|_2^2 = \int_0^\infty \|x(t)\|_2^2 \, dt = \int_{-\infty}^{+\infty} \|X(j\omega)\|_2^2 \, d\omega$$

$$= \|X\|_2^2$$

* Systèmes $H \begin{cases} \dot{x} = f(x, u) \\ y = g(x, u) \end{cases}$

Norme induite $\mathcal{L}_2$

$$\|H\|_2^2 = \sup \left\{ \frac{\|y\|_2^2}{\|u\|_2^2} \quad \begin{matrix} x(0) = 0 \\ \|u\|_2 \neq 0 \end{matrix} \right\}$$

Système LTI $\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$ $\rightarrow$ $Y(s) = \underbrace{\left[D + C(sI - A)^{-1}B \right]}_{H(s)} U(s)$

$x(0) = 0$ $\quad \mathcal{L} \Big($

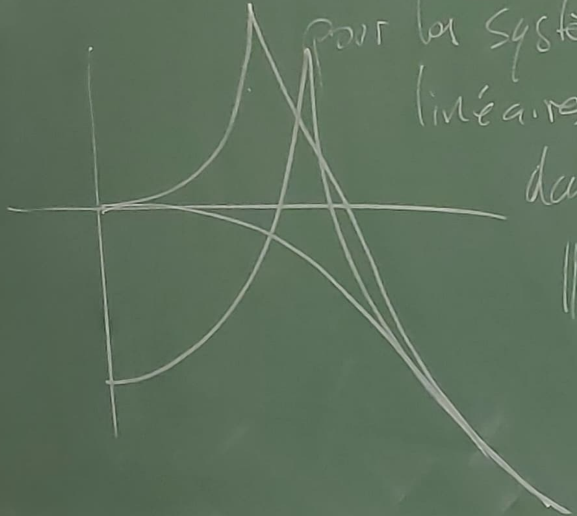
$$sX = AX + BU$$

$$(sI - A)X = BU$$

$$X = (sI - A)^{-1}BU$$

$$\|H\|_2^2 = \sup_{\omega \in \mathbb{R}} \|H(j\omega)\|_2^2 = \sup_{\omega \in \mathbb{R}} \overline{\sigma}^2(H(j\omega)) = \|H\|_\infty^2$$

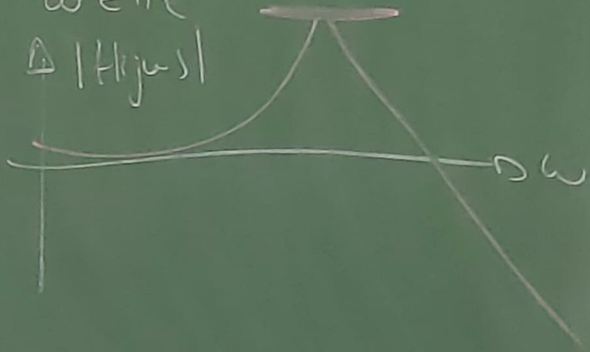
$$= \sup_{\omega \in \mathbb{R}} \lambda_{max}(H^*(j\omega)H(j\omega))$$

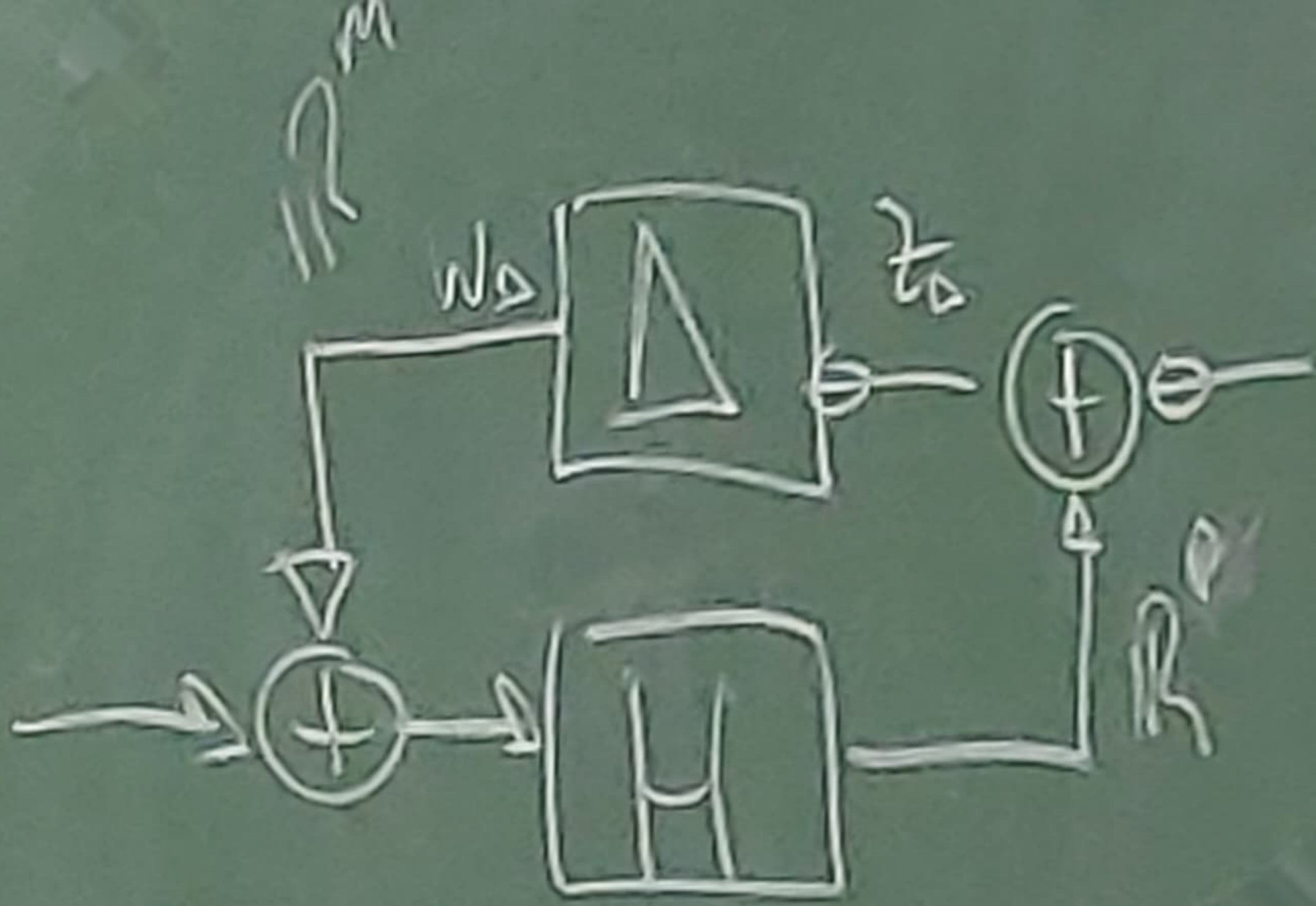


pour les systèmes linéaires invariant dans le temps

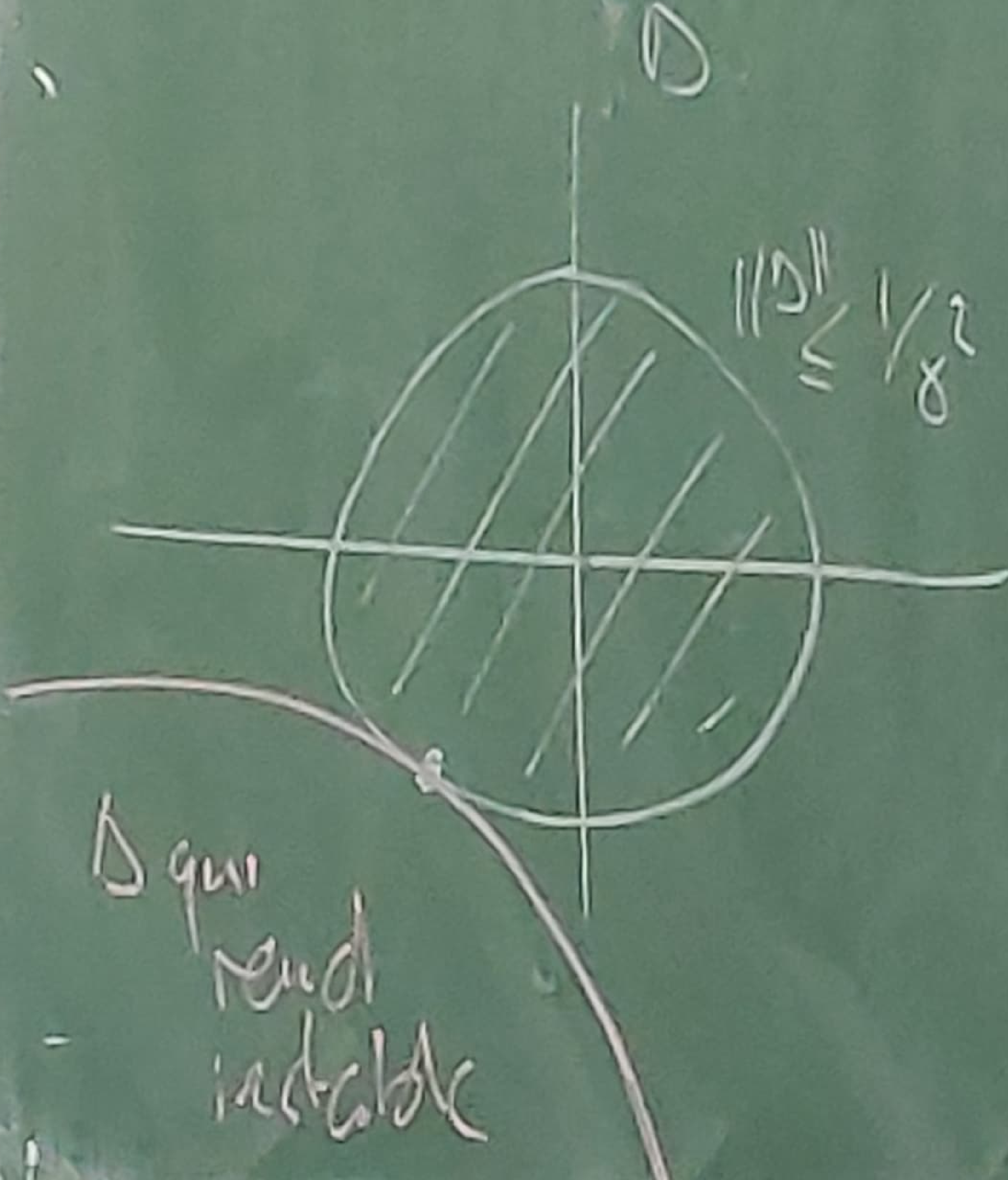
$$\|H\|_2 = \|H\|_\infty$$

norme H_∞

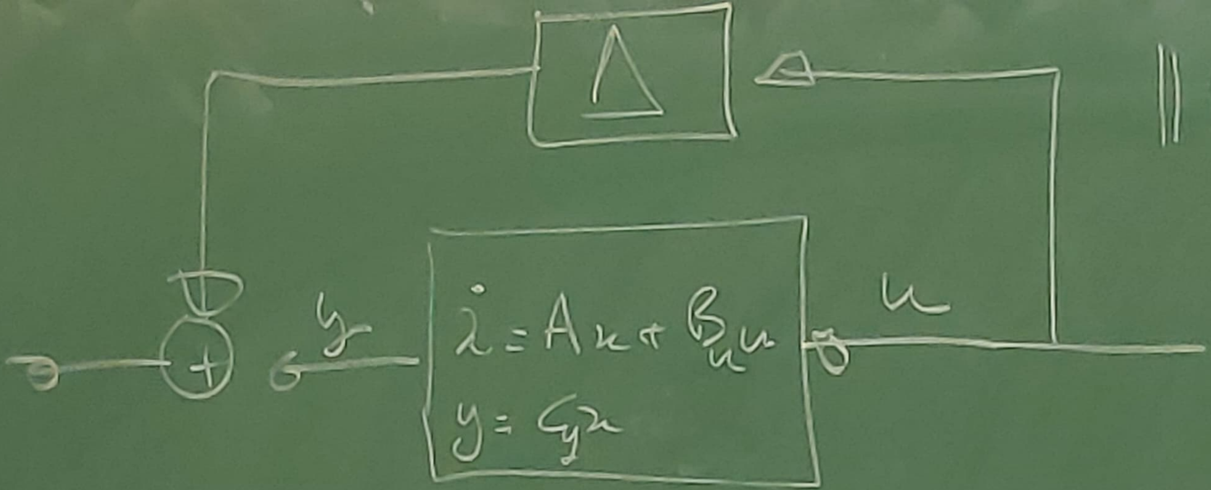




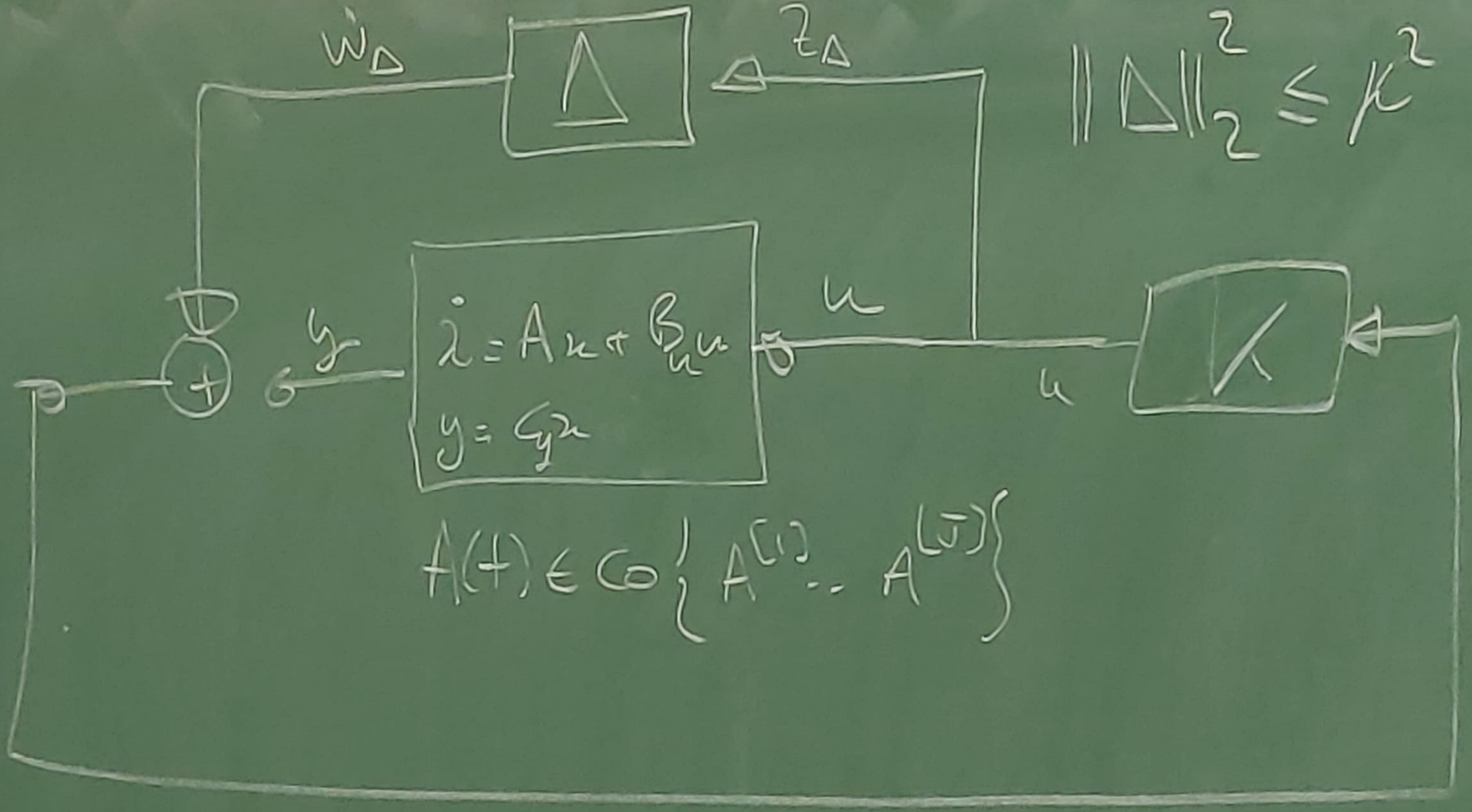
$\mathbb{R}^m$
w_Δ
Δ
z_Δ
H


Δ
‖Δ‖ ≤ 1/γ²
Δ qui rend instable

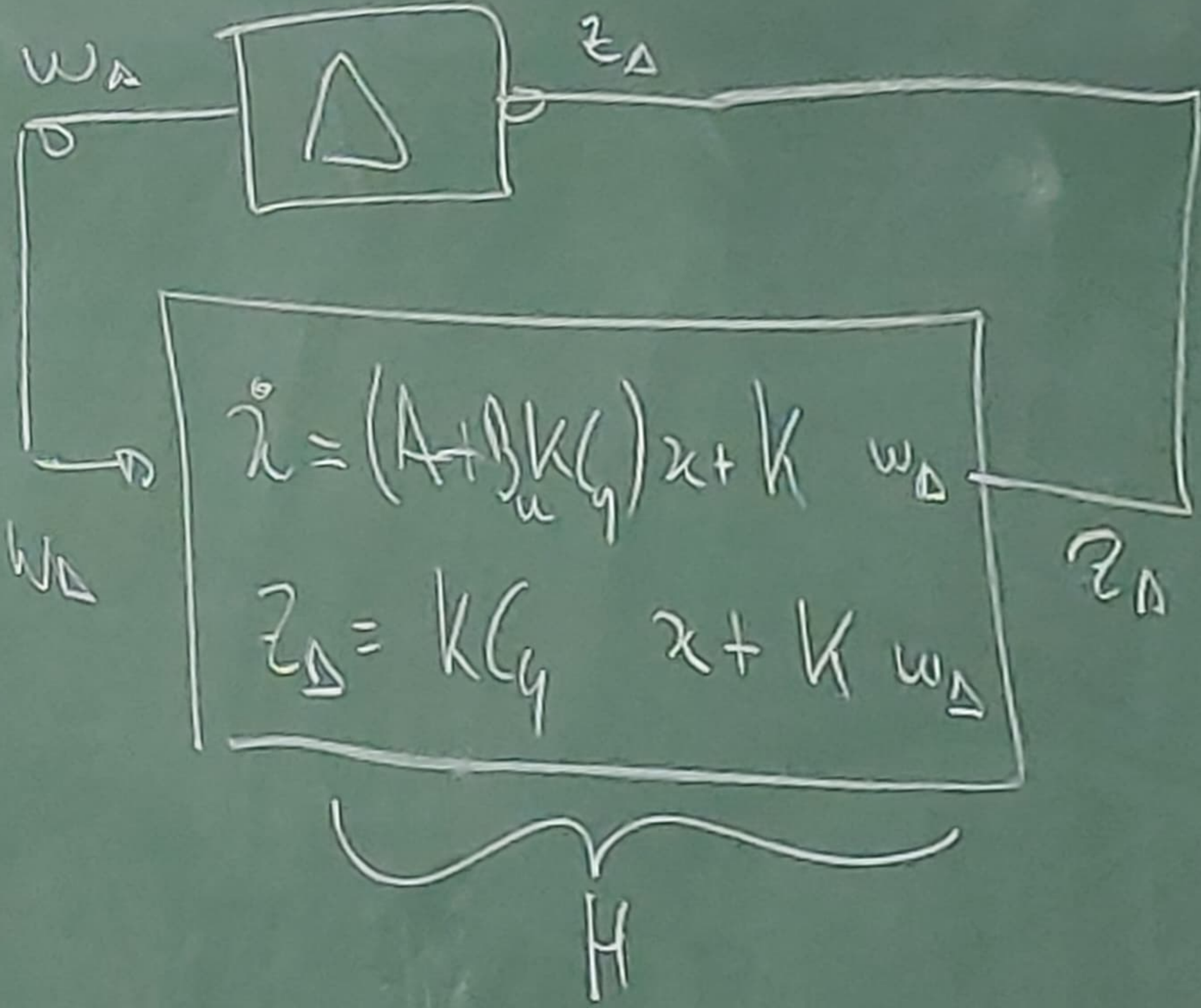
$$\|\Delta\|_2^2 \le \mu^2$$



$$A(t) \in \mathrm{Co}\{A^{[1]} \ldots A^{[J]}\}$$



w_Δ
Δ
z_Δ
$\|\Delta\|_2^2 \leq \kappa^2$
u
$\dot{x} = Ax + B_u u$
$y = C_y x$
y
$+$
u
K
$A(t) \in Co\{A^{[1]} \ldots A^{[J]}\}$


w_Δ
Δ
z_Δ
$\dot{x} = (A + B_u K C_y) x + K \; w_\Delta$
$z_\Delta = K C_y \; x + K \; w_\Delta$
w_Δ
z_Δ
H

THM. Si $\exists P \succ 0$ tq $\forall v = 1 \dots \bar{v}$

$$(1) \quad \begin{bmatrix} A^{[v]T} P + P A^{[v]} & P B_\Delta \\ B_\Delta^T P & -\gamma^2 \mathbb{I} \end{bmatrix} + \begin{bmatrix} C_\Delta^T \\ D_{\Delta\Delta}^T \end{bmatrix} \begin{bmatrix} C_\Delta & D_{\Delta\Delta} \end{bmatrix} \prec 0$$

alors $\dfrac{\|z_\Delta\|^2}{\|w_\Delta\|^2} < \gamma^2$ pour le système

$$\dot{x} = A(t) x + \overset{B_w}{B_\Delta} w_\Delta$$
$$z_\Delta = \underset{C_z}{C_\Delta} x + \underset{D_{zw}}{D_{\Delta\Delta}} w_\Delta$$

avec $A(t) \in \mathrm{Co}\{A^{[1]} \dots A^{[\bar{v}]}\}$

$$\sum_{v=1}^{\bar{v}} \xi_v(\bot) < 0$$

$$\begin{pmatrix} x^T & w_\Delta^T \end{pmatrix}\left(\begin{bmatrix} A(t)^T P + P A(t) & P B_\Delta \\ B_\Delta^T P & -\gamma^2 \mathbb{I} \end{bmatrix} + \begin{bmatrix} C_\Delta^T \\ D_{\Delta\Delta}^T \end{bmatrix}\begin{bmatrix} C_\Delta & D_{\Delta\Delta} \end{bmatrix}\right)\begin{pmatrix} x(t) \\ w_\Delta(t) \end{pmatrix} < 0 \quad \forall A(t) \in Co\{A^{(v)}\}$$

$$x^T(t) P \left(A x(t) + B_\Delta w_\Delta(t)\right) - \gamma^2 w_\Delta^T(t) w_\Delta(t) + z_\Delta^T(t) z_\Delta(t) < 0$$
$$+ (Ax + B_\Delta w_\Delta)^T P x(t)$$

$V = x^T P x$

$$\frac{dV}{dt}(t) - \gamma^2 \| w_\Delta(t) \|^2 + \| z_\Delta(t) \|^2 < 0$$

$$\int_0^{t} \left(\frac{dV}{dt}(t) + \|z_\Delta(t)\|^2 < \gamma^2 \|w_\Delta(t)\|^2 \right) dt$$

$\forall \bar{t} > 0$

$$\underbrace{V(\bar{t})}_{\geq 0} - \underbrace{V(0)}_{= 0} + \int_0^{\bar{t}} \|z_\Delta(t)\|^2 dt < \gamma^2 \int_0^{\bar{t}} \|w_\Delta(t)\|^2 dt$$

$$\|z_\Delta\|^2 < \gamma^2 \|w_\Delta\|^2$$

$\square$