$$\begin{cases} x_1^2 - x_2^2 = 0 \\ x_1 x_2 = 0 \end{cases}$$

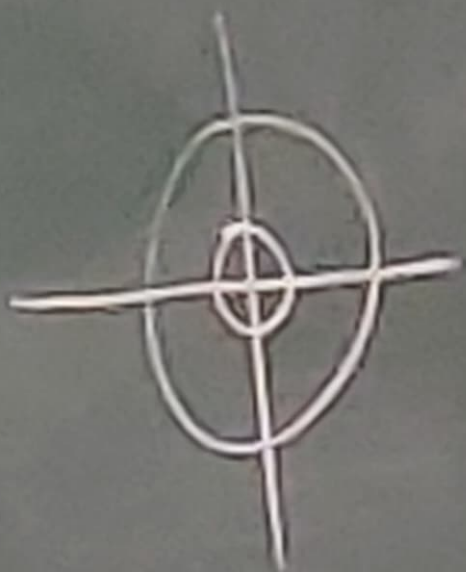
$$\Longrightarrow \quad \begin{cases} x_1^2 = x_2^2 \\ \text{et} \\ x_1 x_2 = 0 \end{cases}$$

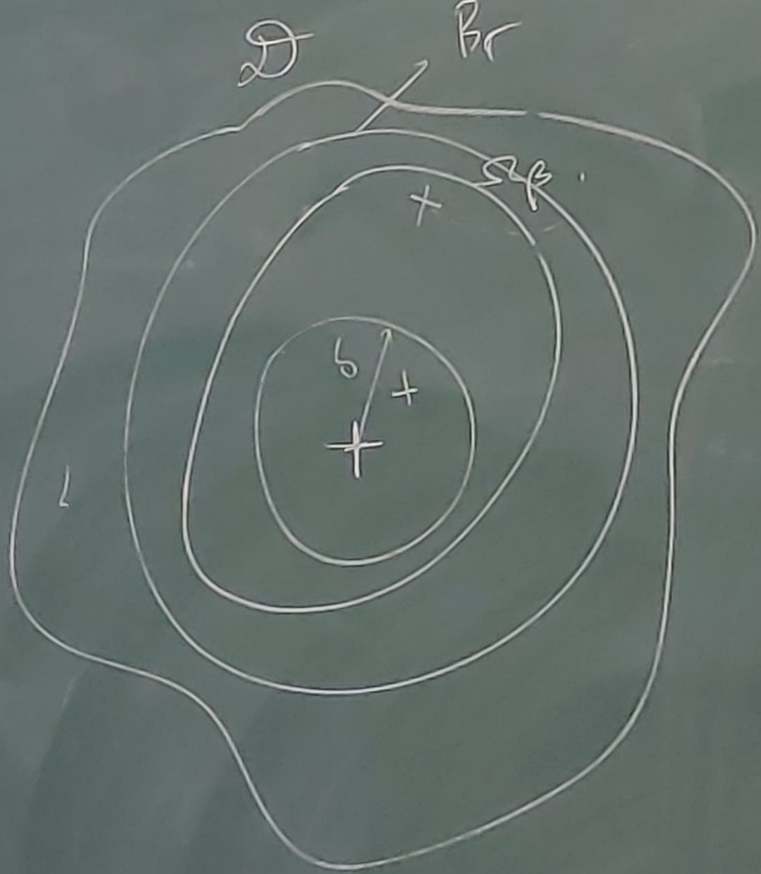
$$\Longrightarrow \quad \begin{cases} x_1 = x_2 \text{ ou } x_1 = -x_2 \\ \text{et} \\ x_1 = 0 \text{ ou } x_2 = 0 \end{cases}$$

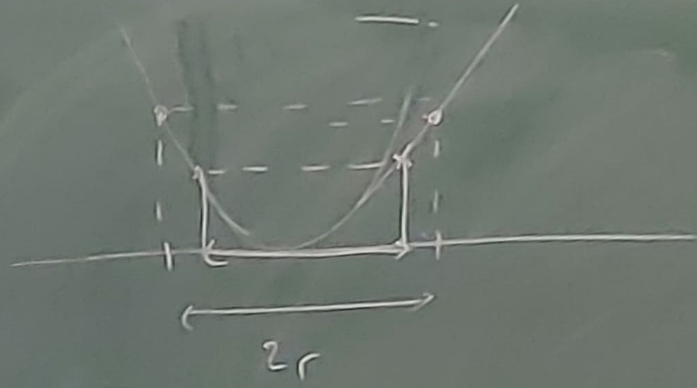
$$\Longrightarrow \quad \begin{cases} x_1 = 0 \\ x_2 = 0 \end{cases}$$

$$\begin{cases} \dot{x}_1 = -x_2 \\ \dot{x}_2 = x_1 \end{cases} \qquad A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$A\cos(t) + B\sin(t) \qquad \mathrm{vp}(A) = \{i, -i\}$$







$\forall \varepsilon > 0, \exists \delta(\varepsilon) > 0,$

$\|x(0)\| < \delta, \; x(t) \in \Omega_\beta.$

or $\Omega_\beta \subset B_r$ et

donc $\|x(t)\| < r < \varepsilon$

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$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \underbrace{-a\, x_1^3(t)}_{\text{potentiel?}} - \underbrace{b\, x_2(t)}_{\text{frottement}} \end{cases} \qquad a, b > 0$$

Pt équilibre :

$$\begin{cases} \dot{x}_1 = 0 \\ \dot{x}_2 = 0 \end{cases} \Rightarrow \begin{cases} x_2 = 0 \\ \text{et} \\ -a x_1^3 = 0 \end{cases} \Rightarrow \begin{cases} x_2 = 0 \\ x_1 = 0 \end{cases}$$

Trouver une Lyapunov :

$$V_1 = \gamma x_1^4 + \frac{1}{2} x_2^2 \rightarrow \boxed{V_1 = \frac{a}{4} x_1^4 + \frac{1}{2} x_2^2}$$ DP sur $\mathbb{R}^2$ et RNB (*)

$$\dot{V}_1 = x_2 \dot{x}_2 = -a\, x_2 x_1^3 - b x_2^2 + 4\gamma x_2 x_1^3$$

$$\gamma = \frac{a}{4}$$

$$\dot{V} = -b x_2^2 \leq 0$$

(*) Montrer que $\|x\| \to +\infty \Rightarrow V(x) \to +\infty$

$W(x) = \{ x \mid x_2 = 0 \}$ on sait qu'il existe des invariants pour ce système

Supposons que $x_2 \equiv 0$, cherchons les trajectoires possibles ?

$$\begin{cases} \dot{x}_1(t) = 0 \\ \dot{x}_2(t) = -a\, x_1^3(t) = 0 \end{cases} \Rightarrow$$ l'unique trajectoire est $x = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $x(0) =$

Donc 0 est attracteur pour le principe de Lasalle