$$\begin{cases} \dot{x}_1 = f_1(x_1, \phi(x_2)) \\ \dot{x}_2 = f_2(x_2, \phi(x_2)) \\ \boxed{CI} \begin{cases} x_1 \\ x_2 \end{cases} \end{cases}$$

$\begin{cases} \text{si "tout se passe bien",} \\ \\ x_2 \xrightarrow[t\to\infty]{} \psi(x_1, x_2) \end{cases}$

$$\begin{cases} x_1 \in \mathcal{M} \\ x_2 = x_1 \end{cases}$$

① $u = -x_2$ et donc 0 est GAS pour $\dot{x}_1 = -x_2$

$$V = \frac{1}{2} x_1^2$$

$$\dot{x}_2 = x_1$$

$$\begin{cases} \dot{X}_1(t) = -X_1 \\ X_1(0) = x_1 \end{cases}$$

$$\dot{X}_2 = e^{-t} x_1$$

$$X_1(t) = e^{-t} x_1$$

$$X_2(t) = \int_0^t e^{-s} x_1 \, ds + x_2.$$

$$X_2(t) = -\left[e^{-s} \right]_0^t x_1 + x_2 = -(e^{-t} - 1) x_1 + x_2$$

donc

$$x_2(t) = (1 - e^{-t})\, x_2 + x_2$$

$$\xrightarrow[t \to +\infty]{} \quad x_1 + x_2$$

on choisit comme fonction de Lyapunov
candidate :

RNB
DP

$$W(x_1, x_2) = \frac{1}{2} x_1^2 + \frac{1}{2}(x_1 + x_2)^2$$

$$\overset{\circ}{W}_{(x_1, x_2)} = x_1 \overset{\circ}{x}_1 + (x_1 + x_2)(\overset{\circ}{x}_1 + \overset{\circ}{x}_2)$$

$\overset{\circ}{\alpha}(x_2, x_2) = x_1(-x_1 + v)$

$\qquad + (x_1 + x_2)(-x_1 + v + x_1)$

$\qquad = -x_1^2 + v(x_1 + x_2 + x_1)$

$\qquad = -x_1^2 + v(2x_1 + x_2)$

on choisit $v(x_2, x_2) = -(2x_1 + x_2)$

$\overset{\circ}{W} = -x_1^2 - (2x_1 + x_2)^2 \quad DN$

$u = -x_2 + v$

ℓ_2 ℓ_1 se commandes étis

$u = -x_1 - 2x_1 - x_2$

$\begin{cases} \overset{\circ}{x_1} = -2x_1 - x_2 \\ \overset{\circ}{x_1} = x_1 \end{cases}$

$A = \begin{pmatrix} -3 & -1 \\ 1 & 0 \end{pmatrix}$