COI 11/14

$\dot{x} = -x + u \qquad J = \frac{1}{2}\int_0^1 3x^2 + u^2 \, dt$

Si non précisé $P_{Tf} = Q_{Tf} = 3$

$x = 0 \,|_{t=0} \longrightarrow x = 2 \,|_{t=1}$

$$J = \frac{1}{2}\int_0^1 x^T 3x + u^T 1 u \, dt \Rightarrow \boxed{Q = 3} \quad \boxed{R = 1}$$

$$\hookrightarrow H = -\frac{1}{2}\left(x^T 3x + u^T 1 u\right) + \lambda^T(\cancel{x} + u)$$

avec $\lambda_{f}^T = -x_{Tf} P_{Tf}$

$$H = -\frac{1}{2}\left(3x^2 + u^2\right) + \lambda^T(-x + u)$$

Conditions opt à vérifier :

$$\begin{cases} \dot{x} = \partial_\lambda H = -x_1 + u \\ \dot{\lambda} = -\partial_x H = 3x_1 + \lambda \\ \partial_u H \,|_{u=u^*} = 0 = -u + \lambda^T \\ \partial^2_{u^2} H \,|_{u=u^*} < 0 \quad \text{or } \partial_u^2 = -1 \checkmark \end{cases} \Rightarrow u^* = +\lambda^T$$

Pour trouver λ, on a : $\dot{\lambda}(t) = 3x(t) + \lambda(t)$ et $\lambda_{t=1} = -3x$

on peut partir de $\dot{x} = -x + u^* = -x + \lambda$ or $\lambda = \dot{x} + x$

dérivons $\ddot{x} = -\dot{x} + \dot{\lambda} = -\dot{x} + 3x + \lambda$

$\Leftrightarrow \ddot{x} = -\dot{x} + 3x + \dot{x} + x$

$\Leftrightarrow \ddot{x} = 4x \rightsquigarrow x(t) = K \sinh(\alpha t + \beta)$

D'où $\lambda = \dot{x} + x = \alpha K \cosh(\alpha t + \beta) + K \sinh(\alpha t + \beta) = u^*$