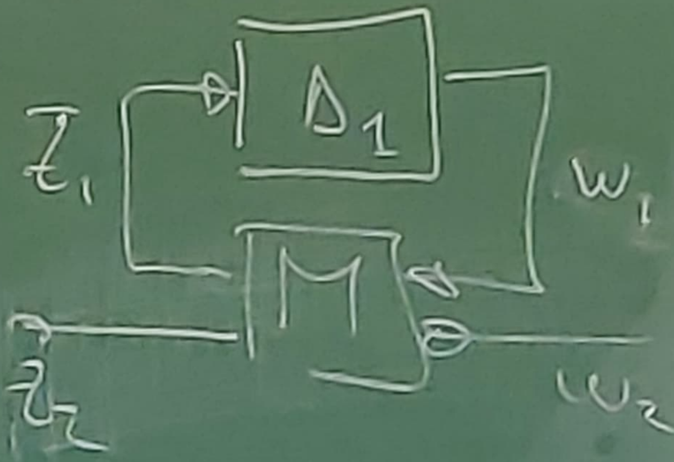
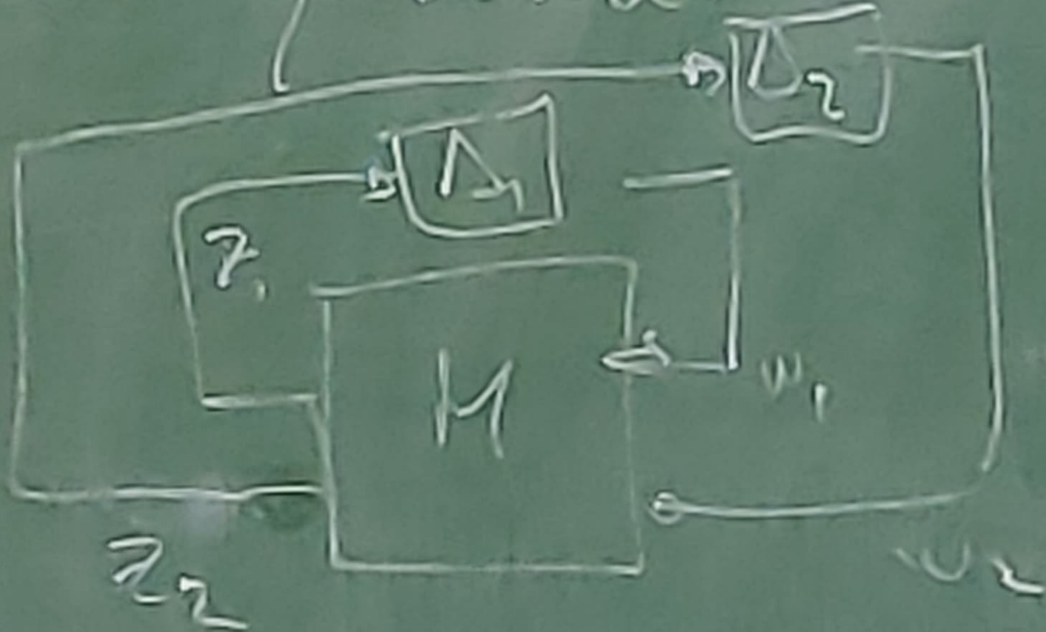
* Exemple d'utilisation



Je veux prouver que

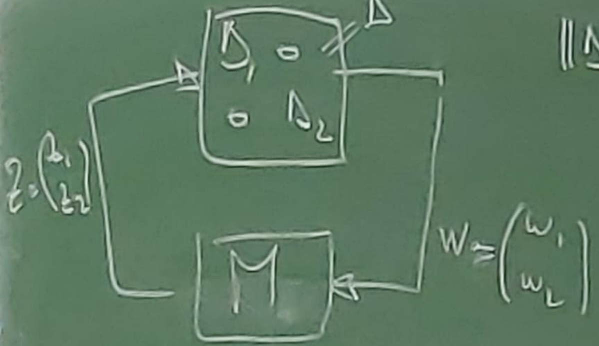
$$\|M_{w_2 \to z_2}\|_2^2 = \frac{\|z_2\|^2}{\|w_2\|^2} < \gamma^2 \quad \text{pour tout } \|\Delta_1\|_2^2 \leq k^2$$

Problème equivalent :



robustement stable $\|\Delta_2\|^2 \leq 1/\gamma^2$

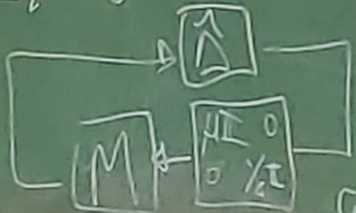
$\|\Delta_1\|^2 \leq \mu^2$



$$\|\Delta\| \leq \max\left(\|\Delta_1\|, \|\Delta_2\|\right)$$

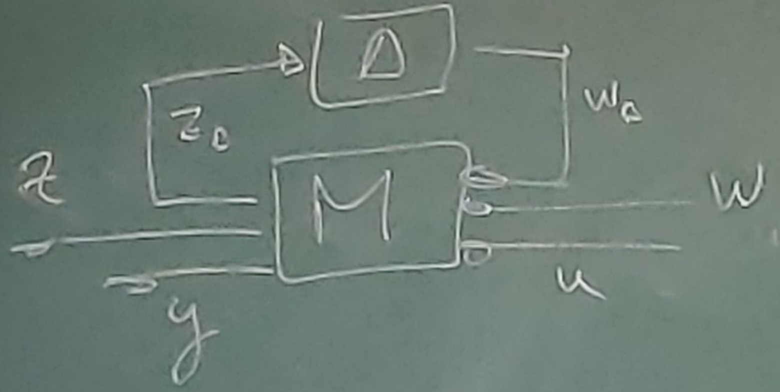
$$\hat{\Delta}_1 = \frac{1}{\mu}\Delta_1 \qquad \|\hat{\Delta}_1\|^2 \leq 1$$

$$\hat{\Delta}_2 = \gamma\, \Delta_2 \qquad \|\hat{\Delta}_2\|^2 \leq 1$$



$$\hat{\Delta} = \begin{pmatrix} \hat{\Delta}_1 & 0 \\ 0 & \hat{\Delta}_2 \end{pmatrix}$$

r. stable $\forall \|\hat{\Delta}\| \leq 1$ $\Leftarrow$ vrai si $\left\| M \begin{pmatrix} \mu I & 0 \\ 0 & \varepsilon I \end{pmatrix} \right\|_\infty^2 < 1$



$$
M: \quad
\begin{aligned}
\dot{x} &= Ax + B_\Delta w_\Delta + B_w w + B_u u \\
z_\Delta &= C_\Delta x + D_{\Delta\Delta} w_\Delta + D_{\Delta w} w + D_{\Delta u} u \\
z &= C_z x + D_{z\Delta} w_\Delta + D_{zw} w + D_{zu} u \\
y &= C_y x + D_{y\Delta} w_\Delta + D_{yw} w + D_{yu} u
\end{aligned}
$$

$$\begin{pmatrix} \dot{x} \\ z \\ y \end{pmatrix} = \begin{pmatrix} A & B_w & B_u \\ C_z & D_{zw} & D_{zu} \\ C_y & D_{yw} & D_{yu} \end{pmatrix} \begin{pmatrix} x \\ w \\ u \end{pmatrix} + \begin{pmatrix} B_\Delta \\ D_{z\Delta} \\ D_{y\Delta} \end{pmatrix} \overset{\Delta z_\Delta}{\overset{\parallel}{w_\Delta}}$$

$$z_\Delta = \begin{pmatrix} C_\Delta & D_{\Delta w} & D_{\Delta u} \end{pmatrix} \begin{pmatrix} x \\ w \\ u \end{pmatrix} + D_{\Delta\Delta} w_\Delta$$

$$z_\Delta = \begin{bmatrix} C_\Delta & D_{\Delta w} & D_{\Delta u} \end{bmatrix} \begin{pmatrix} x \\ w \\ u \end{pmatrix} + D_{\Delta\Delta} \Delta z_\Delta$$

$$(I - D_{\Delta\Delta} \Delta) z_\Delta = \begin{bmatrix} C_\Delta & D_{\Delta w} & D_{\Delta u} \end{bmatrix} \begin{pmatrix} x \\ w \\ u \end{pmatrix} \qquad z_\Delta = (I - D_{\Delta\Delta} \Delta)^{-1} \begin{bmatrix} C_\Delta & D_{\Delta w} & D_{\Delta u} \end{bmatrix} \begin{pmatrix} x \\ w \\ u \end{pmatrix}$$

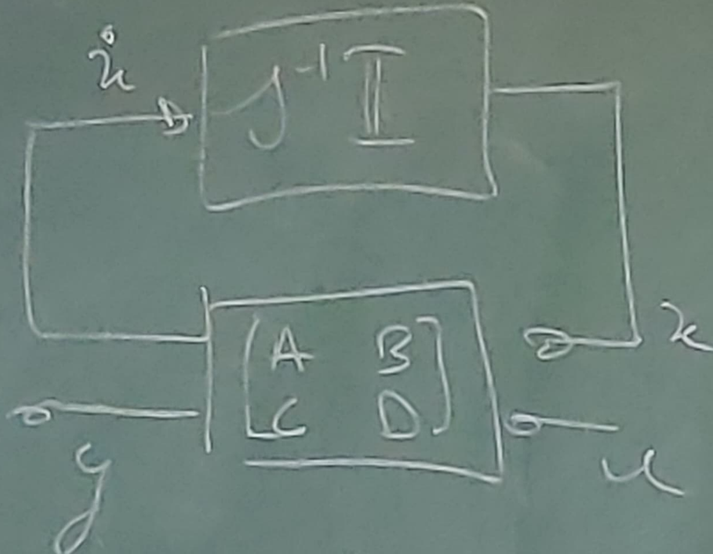
$$
\begin{pmatrix} \dot{x} \\ z \\ y \end{pmatrix} = \left[\begin{bmatrix} A & B_w & B_u \\ C_z & D_{zw} & D_{zu} \\ C_y & D_{yw} & D_{yu} \end{bmatrix} + \begin{bmatrix} B_\Delta \\ D_{z\Delta} \\ D_{y\Delta} \end{bmatrix} \Delta \left(I - D_{\Delta\Delta} \Delta \right)^{-1} \begin{bmatrix} C_\Delta & D_{\Delta w} & D_{\Delta u} \end{bmatrix} \right] \begin{pmatrix} x \\ w \\ u \end{pmatrix}
$$

$$
\begin{aligned}
\dot{x} &= A(\Delta) x + B_w(\Delta) w + B_u(\Delta) u \\
z &= C_z(\Delta) x + D_{zw}(\Delta) w + D_{zu}(\Delta) u \\
y &= C_y(\Delta) x + D_{yw}(\Delta) w + D_{yu}(\Delta) u
\end{aligned}
$$

$$
A(\Delta) = A + B_\Delta \Delta (I - D_{\Delta\Delta} \Delta)^{-1} C_\Delta
$$

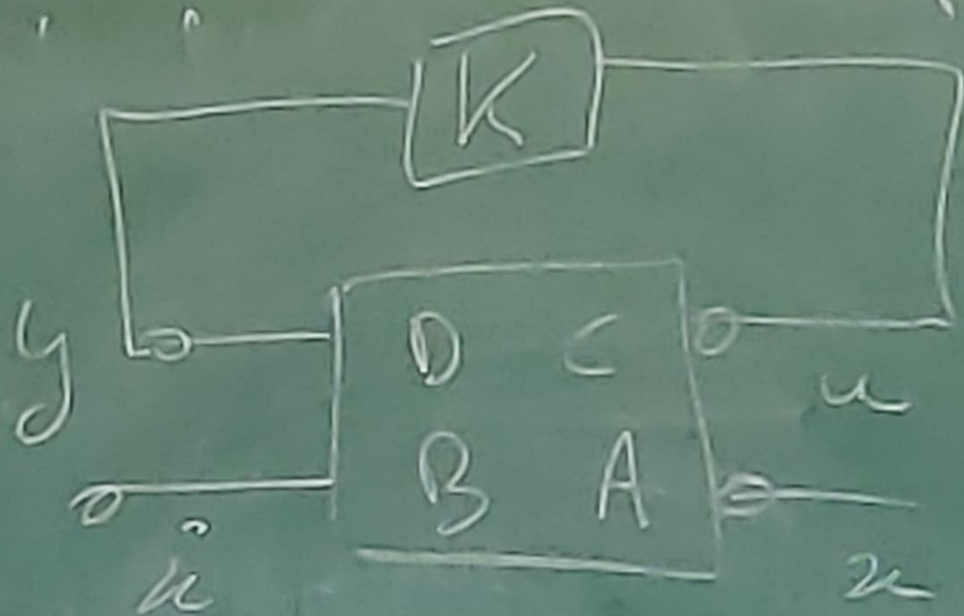
$$
D_{zu}(\Delta) = D_{zu} + D_{z\Delta} \Delta (I - D_{\Delta\Delta} \Delta)^{-1} D_{\Delta u}
$$

$$
\delta (1 - d\delta)^{-1} = \frac{\delta}{1 - d\delta}
$$



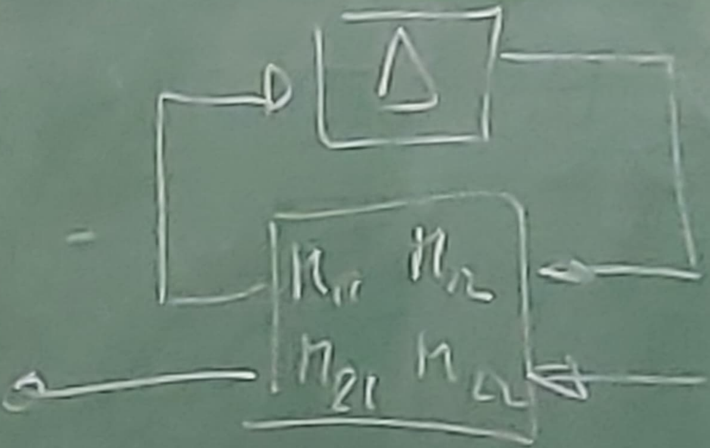
$$y = \left(D + C \left(s^{-1}\mathbb{I} \right) \left(\mathbb{I} - A \left(s^{-1}\mathbb{I} \right) \right)^{-1} B \right) u$$

$$= \left(D + C \left(s\mathbb{I} - A \right)^{-1} B \right) u$$



$$\dot{x} = \left(A + BK(I - DK)^{-1}C\right)x$$

LFT =



linear fractional Transform

$$\Delta \star \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12}$$

$$\Delta = \begin{bmatrix} \delta & 0 \\ 0 & \delta \end{bmatrix} = \delta \mathbb{I}_2$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} w_\Delta + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$

$$z_\Delta = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} w_\Delta + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 1 \end{bmatrix} w_\Delta$$

u

$$\Delta_a = \Delta\left(I - D_{\Delta\Delta}\Delta\right)^{-1}$$

$$= \delta\left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\delta\right)^{-1}$$

$$= \delta\begin{bmatrix}1-\delta & 0\\ 0 & 1\end{bmatrix}^{-1} = \begin{bmatrix}\delta/_{1-\delta} & 0\\ 0 & \delta\end{bmatrix}$$

$$A(\Delta) = A + B_\Delta \Delta_a C_\Delta = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix} + \begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}\delta/_{1-\delta} & 0\\ 0 & \delta\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix} + \begin{bmatrix}0 & \delta\\ \delta/_{1-\delta} & \delta\end{bmatrix} = \begin{bmatrix}0 & 1+\delta\\ \delta/_{1-\delta} & \delta\end{bmatrix}$$

$$B_u(\Delta) = B_u + B_\Delta \Delta_a D_{\Delta u} = \begin{bmatrix}0\\ 1\end{bmatrix} + \begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}\begin{pmatrix}\delta/_{1-\delta} & 0\\ 0 & \delta\end{pmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}$$

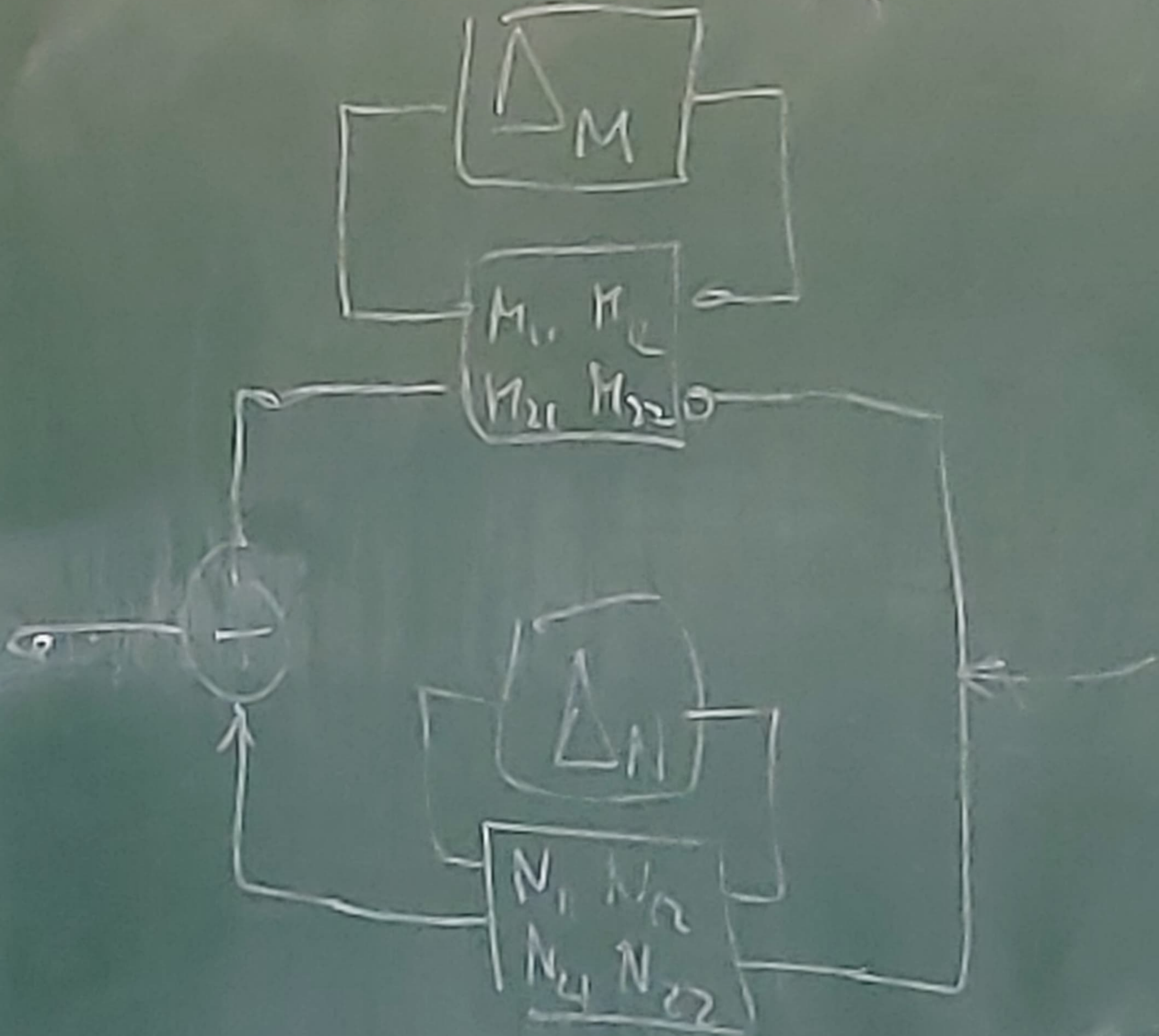
$$= \begin{bmatrix}0\\ 1\end{bmatrix} + \begin{bmatrix}0 & \delta\\ \delta/_{1-\delta} & \delta\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = \begin{bmatrix}0\\ 1+\delta/_{1-\delta}\end{bmatrix} = \begin{bmatrix}0\\ \frac{1}{1-\delta}\end{bmatrix}$$

$$C_y(\Delta) = C_y + D_{y\Delta}\Delta_a C_\Delta$$

$$= \begin{bmatrix}1 & 0\end{bmatrix} + \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}\delta/1.5 & 0\\ 0 & \delta\end{bmatrix}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$= \begin{bmatrix}1 & 0\end{bmatrix} + \begin{bmatrix}0 & \delta\end{bmatrix} = \begin{bmatrix}1 & \delta\end{bmatrix}$$

$$D_{yu}(\Delta) = D_{yu} + D_{y\Delta}\Delta_a \Delta_{\Delta u} = 0 + \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}\delta/1.5 & 0\\ 0 & \delta\end{bmatrix}\begin{pmatrix}1\\ 0\end{pmatrix} = 0$$



Δ_M
M11 M12
M21 M22
Δ_N
N11 N12
N21 N22

$$\Delta_M \star \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} + \Delta_N \star \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix}$$

$$= \begin{bmatrix} \Delta_M & 0 \\ 0 & \Delta_N \end{bmatrix} \star \left[\begin{array}{cc|c} M_{11} & 0 & M_{12} \\ 0 & N_{11} & N_{12} \\ \hline M_{21} & N_{21} & M_{22}+N_{22} \end{array}\right]$$

$$[\delta_1 \ \ \delta_2] = 1 \cdot \delta_1 \cdot [1 \ \ 0] + 1 \cdot \delta_2 \cdot [0 \ \ 1]$$

$$= \delta_1 \star \left[\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 1 & 0 & 0 \end{array}\right] + \delta_2 \star \left[\begin{array}{c|cc} 0 & 0 & 1 \\ \hline 1 & 0 & 0 \end{array}\right]$$

$$\star \left[\begin{array}{c|c} 0 & M_{12} \\ \hline M_{21} & M_{22} \end{array}\right] = \begin{bmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{bmatrix} \star \left[\begin{array}{cc|cc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hline 1 & 1 & 0 & 0 \end{array}\right]$$

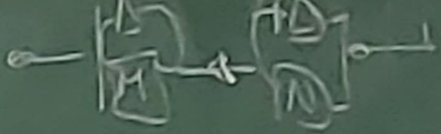
$$M_{22} + M_{21}\Delta M_{12} = \Delta \star \left[\begin{array}{c|c} 0 & M_{12} \\ \hline M_{21} & M_{22} \end{array}\right]$$

$$M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12}$$

$$- \Delta^{\star} \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

$$= \Delta^{\star} \begin{bmatrix} M_{11} & M_{12} \\ -M_{21} & -M_{22} \end{bmatrix}$$

$$\left(\Delta_M \star \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22}\end{bmatrix}\right)\cdot\left(\Delta_N \star \begin{bmatrix} N_{11} & N_{12}\\ N_{21} & N_{22}\end{bmatrix}\right)$$



$$\begin{bmatrix}\Delta_M & 0\\ 0 & \Delta_N\end{bmatrix} \star \left[\begin{array}{cc|c} M_{11} & M_{12}N_{21} & M_{12}N_{22}\\ 0 & N_{11} & N_{12}\\ \hline M_{21} & M_{22}N_{21} & M_{22}N_{22}\end{array}\right]$$

$$\delta^2 = \left(\delta \star \left[\begin{array}{c|c} 0 & 1\\ \hline 1 & 0\end{array}\right]\right)\left(\delta\star\left[\begin{array}{c|c}0&1\\ \hline 1&0\end{array}\right]\right) = \begin{bmatrix}\delta&0\\0&\delta\end{bmatrix}\star\left[\begin{array}{cc|c}0&1&0\\0&0&1\\ \hline 1&0&0\end{array}\right]$$

$$\left(\Delta \star \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}\right)^{-1} = \Delta \star \left[\begin{array}{c|c} M_{11} - M_{12} M_{22}^{-1} M_{21} & -M_{12} M_{22}^{-1} \\ \hline M_{22}^{-} M_{21} & M_{22}^{-1} \end{array}\right]$$

$$\frac{\delta}{1-\delta} = \left(\delta \star \left[\begin{array}{c|c} 0 & 1 \\ \hline 1 & 0 \end{array}\right]\right)\left(\delta \star \left[\begin{array}{c|c} 0 & 1 \\ \hline -1 & 1 \end{array}\right]\right)^{-1}$$

$$= \left(\delta \star \left[\begin{array}{c|c} 0 & 1 \\ \hline 1 & 0 \end{array}\right]\right)\left(\delta \star \left[\begin{array}{c|c} 0 - 1 \cdot 1^{-1} \cdot (-1) & -1 \cdot 1^{-1} \\ \hline 1^{-1}(-1) & 1^{-1} \end{array}\right]\right)$$

$$= \left(\delta \star \left[\begin{array}{c|c} 0 & 1 \\ \hline 1 & 0 \end{array}\right]\right)\left(\delta \star \left[\begin{array}{c|c} 1 & -1 \\ \hline -1 & 1 \end{array}\right]\right)$$

$$= \begin{pmatrix} \delta & 0 \\ 0 & \delta \end{pmatrix} \star \left[\begin{array}{cc|c} 0 & -1 & 1 \\ 0 & 1 & -1 \\ \hline 1 & 0 & 0 \end{array}\right]$$

$$\dot{x} = \frac{\delta_2}{1+\delta_1} x + u$$

$$y = x$$

y

u

$$(1+\delta_1)\mathring{x} = \delta_2 x + (1+\delta_1) u$$

$$\mathring{x} = 0\, x + 1 \cdot u + \underbrace{\delta_1 (u - \dot{x})}_{W_{\Delta_1}} + \underbrace{\delta_2 x}_{W_{\Delta_2}}$$

$$\tilde{z}_{\Delta_2} = x$$

$$\tilde{z}_{\Delta_1} = v - \dot{x} = v - \begin{bmatrix} 1 & 1 \end{bmatrix} w_\Delta - u$$

$$= \begin{bmatrix} 1 & 1 \end{bmatrix} w_\Delta$$

$$z_\Delta = \begin{pmatrix} z_{\Delta 1} \\ z_{\Delta 2} \end{pmatrix}$$

$$\begin{bmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{bmatrix}$$

w_Δ

$$\dot{x} = 0 \cdot x + \begin{bmatrix} 1 & 1 \end{bmatrix} w_\Delta + 1 \cdot u$$

$$z_\Delta = \begin{bmatrix} 0 \\ 1 \end{bmatrix} x + \begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix} w_\Delta$$

$$y = 1 \cdot x$$

u

$$\dot{x} = \begin{bmatrix} \delta - 1 & 0 \\ \delta / 1+\delta & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w$$

$$z = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1+\delta \end{bmatrix} \dot{x} = \begin{bmatrix} \delta-1 & 0 \\ \delta & -1-\delta \end{bmatrix} x + \begin{bmatrix} 0 \\ 1+\delta \end{bmatrix} w$$

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w + \begin{bmatrix} 0 & 0 \\ 0 & -\delta \end{bmatrix} \dot{x} + \begin{bmatrix} \delta & 0 \\ \delta & -\delta \end{bmatrix} x + \begin{bmatrix} 0 \\ \delta \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix} x + \begin{bmatrix}0\\ 1\end{bmatrix} w + \underbrace{\begin{bmatrix}0 & 0\\ 0 & -\delta\end{bmatrix}\dot{x} + \begin{bmatrix}\delta & 0\\ \delta & -\delta\end{bmatrix} x + \begin{bmatrix}0\\ \delta\end{bmatrix} w}$$

$$\begin{bmatrix}1\\ 0\end{bmatrix} \delta \begin{bmatrix}1 & 0\end{bmatrix} x \qquad w_{\delta,1} = \delta z_{\delta,1}$$

$$+ \begin{bmatrix}0\\ 1\end{bmatrix} \delta \left(\begin{bmatrix}0 & -1\end{bmatrix}\dot{x} + \begin{bmatrix}1 & -1\end{bmatrix} x + w \right)$$

$w_{\delta 2}$

$$z_{\Delta 2} = \begin{bmatrix} 0 & -1 \end{bmatrix} \dot{x} + \begin{bmatrix} 1 & -1 \end{bmatrix} x + w$$

$$= \begin{bmatrix} 0 & -1 \end{bmatrix} \left(-x + w_\Delta + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w \right) + \begin{bmatrix} 1 & -1 \end{bmatrix} x + w$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & -1 \end{bmatrix} w_\Delta + (-1+1) w$$

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} w_\Delta + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w$$

$$z_\Delta = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} w_\Delta + \begin{bmatrix} 0 \\ 0 \end{bmatrix} w$$

$$m\ddot{p} + c\dot{p} + kp = w$$

$$z = p$$

je veux $\frac{\|z\|_2^2}{\|w\|_2^2} < \gamma^2$ pour tous

$1 \leq m \leq 3$

$0{,}1 \leq c \leq 0{,}3$

$0{,}9 \leq k \leq 1{,}1$

$$\frac{\|z\|^2}{\|\hat{w}\|^2} \leq 1$$

$$\hat{w} = \gamma w$$

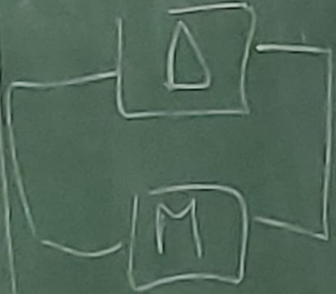
$$1 \leq m \leq 3 \qquad m = 2 + \delta_m \qquad |\delta_m| \leq 1$$

$$0{,}1 \leq c \leq 0{,}3 \qquad c = 0{,}2 + 0{,}1\,\delta_c \qquad |\delta_c| \leq 1$$

$$0{,}9 \leq k \leq 1{,}1 \qquad k = 1 + 0{,}1\,\delta_k \qquad |\delta_k| \leq 1$$

$$2\ddot{\rho} = -\underbrace{\delta_M \ddot{\rho}}_{w_{\Delta_1} = \delta_M z_0} - 0{,}2\dot{\rho} - 0{,}1\underbrace{\delta_c \dot{\rho}}_{w_{\Delta_2}} - \rho - 0{,}1\underbrace{\delta_k \rho}_{w_{\Delta_3}} + \frac{1}{\gamma}\underbrace{\hat{w}}_{w_{\Delta_4}}$$

$$\Delta = \begin{bmatrix} \delta_M & & & 0 \\ & \delta_c & & \\ & & \delta_c & \\ 0 & & & \hat{\Delta} \end{bmatrix}$$



$\|\Delta\| \le 1$

robstable $\xLeftarrow{\text{th}}$ $\|M\|_\infty \le 1$

$$2\ddot{\rho} = -\underbrace{\delta_m \ddot{\rho}}_{w_{\Delta_1} = \delta_m z_{\Delta_1}} - 0{,}2\,\dot{\rho} - 0{,}1\,\underbrace{\delta_c \dot{\rho}}_{w_{\Delta_2}} - \rho - 0{,}1\,\underbrace{\delta_k \rho}_{w_{\Delta_3}} + \frac{1}{\gamma}\underbrace{\hat{w}}_{w_{\Delta_4}}$$

$$X = \begin{pmatrix} \rho \\ \dot{\rho} \end{pmatrix}$$

$$M:\quad \dot{x} = \begin{bmatrix} 0 & 1 \\ -1/2 & -0{,}1 \end{bmatrix} x + \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1/2 & 0{,}05 & -0{,}05 & \frac{1}{2\gamma} \end{bmatrix} w_\Delta$$

$$z_\Delta = \begin{bmatrix} -1/2 & -0{,}1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} -1/2 & -0{,}05 & -0{,}05 & \frac{1}{2\gamma} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} w_\Delta$$

$\Delta = \{ \|\Delta\| \leq 1$ et Δ : diagonale $\}$

$\cancel{\|M\|_\infty < 1}$

$\mu_{\text{diag}}(M) < 1$