Model-Based Dynamic Event-Triggered Control for Systems With Uncertainty: A Hybrid System Approach

Kun-Zhi Liu, Andrew R. Teel, Xi-Ming Sun, and Xue-Fang Wang

***Abstract*—In this article, the event-triggered control problem for linear systems with uncertainties is addressed. A model-based dynamic event-triggered transmission strategy is proposed for linear systems, and for systems that can be decomposed into interconnected subsystems, a distributed model-based dynamic event-triggered transmission strategy is also proposed with transmission delays and transmission protocols in the networks. The whole systems are modeled into a hybrid system framework by introducing storage variables. Using stability theorems of hybrid systems, explicit designs of the transmission strategies are presented and asymptotic stability is guaranteed. Finally, an example is given to show that the transmissions are significantly reduced by the transmission strategy in this article compared with the transmission strategy by zero-order-hold.**

***Index Terms*—Dynamic event-triggered control, hybrid systems, model-based control.**

I. Introduction

Event-triggered sampled-data control has received a lot of attention because of its potential advantage over time-triggered sampled-data control since it may reduce the data transmission in the network [1]–[17]. Most of the existing literature on event-triggered transmission strategies use zero-order-hold between two updating times on the controller side once the feedback data is received (see, for example, [1], [3]–[6], [18], and references cited therein). The zero-order-hold has the advantage to be implemented easily in sampled-data control systems while it may result in large error between plant state and kept feedback data by the zero-order-hold at the controller side. Differently, a model-based sampled-data controller sufficiently utilizes the model information at the controller side, and between two updating times, the controller predicts the plant state based on the models and received feedback data. With the model-based controller, less data transmission may be expected by the controller designers [7], [19]–[21].

Tabuada [1] proposes an event-triggered scheduling rule based on an inequality relation involving the plant state and kept state by zero-order-hold at the controller side, and proves that Zeno behavior will not happen under the designed transmission condition. Different from Tabuada [1], Girard [9] proposes a dynamic event-triggered transmission strategy which needs to detect whether the state value of a designed dynamic equation achieves zero. Guaranteeing asymptotic stability, Selivanov and Fridman [11] and Abdelrahim *et al.* [17] consider the event-triggered control for systems with output feedback and impose a waiting time between two transmission times to avoid Zeno behavior. Dolk *et al.* [18] consider small delays and transmission protocols in each local network for interconnected nonlinear systems with output feedback, and propose static and dynamic event-triggered transmission strategies by imposing a waiting time in each local network.

The previously mentioned literature uses zero-order-hold at the controller side which maintains the received feedback data between two updating times. Garcia and Antsaklis [7] and Heemels and Donkers [19] consider the model-based event-triggered transmission strategy. Garcia and Antsaklis [7] consider the transmission delays and quantization effects in the network for uncertain linear systems, and adopt the static transmission strategy. Zeno behavior is precluded by proving that there is a lower bound between the interexecution times. Heemels and Donkers [19] propose a periodic event-triggered transmission strategy for discrete-time systems, and for decentralized linear systems, the authors use a reduced model of each subsystem to predict the subsystem state for the packet dropout situation. To the best of the authors' knowledge, a model-based dynamic event-triggered transmission strategy is not considered for uncertain linear systems with transmission delays and protocols in the network. To fill this gap, this article aims at designing centralized and distributed model-based dynamic event-triggered transmission strategies. The literature closest to this article is [7], [18], and [19], while [7] considers static and centralized event-triggered transmission strategies, [19] considers discrete-time systems without delays and protocols, and [18] uses zero-order-hold.

The main contribution of this article can be summarized as follows. A model-based dynamic event-triggered transmission strategy is proposed for uncertain sampled-data linear systems with transmission delays and protocols in the network. By introducing storage variables, the whole systems are modeled into hybrid systems, and then based on stability theorems of hybrid systems, the explicit parameters of dynamic equation involving the transmission strategy are designed such that the hybrid systems are asymptotically stable. Moreover, for uncertain linear systems that can be decomposed into interconnected subsystems, a distributed model-based dynamic event-triggered transmission strategy is also proposed. In each local network, the local controller runs a reduced model that does not use information of other subsystems. Again based on a hybrid system model, the parameter design is explicitly presented and asymptotic stability is guaranteed.

Throughout this article, the following notation is adopted.

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$\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{Z}$ denotes the set of integers. $|\cdot|$ denotes the two-norm of a matrix or vector. x^T denotes the transpose of $x \in \mathbb{R}^n$. For a discontinuous function x that is left-continuous with right limit, we often use $x(s^+)$ to denote the right-hand limit at s. Denote $\mathbb{R}_{\geq 0} := [0, +\infty)$, $\mathbb{R}_{\leq 0} := (-\infty, 0]$, $\mathbb{Z}_{\geq 0} := \{0, 1, 2, \ldots\}$, $\mathbb{Z}_{\leq 0} := \{0, -1, -2, \ldots\}$ and $R^N_{\geq 0} := \{(x_1, \ldots, x_N)^T | x_i \in \mathbb{R}_{\geq 0}\}$, and $\mathbb{Z}^N_{\geq 0} := \{(x_1, \ldots, x_N)^T | x_i \in \mathbb{Z}_{\geq 0}\}$. For any $x \in \mathbb{R}^n$ and a closed subset $\mathcal{W}$ of $\mathbb{R}^n$, $|x|_{\mathcal{W}} := \inf_{y \in \mathcal{W}} |x - y|$. We often write the column vector $(Y_1^T, Y_2^T, \ldots, Y_L^T)^T$ as $(Y_1, Y_2, \ldots, Y_L)$. $\mathbf{0}_n$ denotes the $n \times n$ zero matrix and I_n denotes the n-dimensional identity matrix. $\mathbf{0}$ denotes a zero vector with appropriate dimension and $\mathbf{1}_n$ denotes the n-dimensional vector with all elements being 1. $diag\{M_1, M_2, \ldots, M_N\}$ denotes a block diagonal matrix. A function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is a class-$\mathcal{K}$ function if it is continuous, zero at zero, and strictly increasing. A function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is a class-$\mathcal{KL}$ function if i) it is a class-$\mathcal{K}$ function with respect to its first variable; ii) with respect to its second variable, it is continuous, nonincreasing, and $\lim_{s\to\infty} \beta(\delta, s) = 0, \ \forall \delta \geq 0$.

II. Model Description

Consider the following plant:

$$\dot{x} = Ax + Bu \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ is the state, and $u \in \mathbb{R}^{n_u}$ is the control input. The following model-based controller is used to stabilize system (1)

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u, \quad u = K\hat{x} \tag{2}$$

where $\hat{x} \in \mathbb{R}^{n_x}$ is the state of the model, and $\hat{A}$ and $\hat{B}$ are the available models of system matrices A and B. We will denote $A_0 := \hat{A} + \hat{B}K$, $B_0 := \hat{B}K$, $\Delta A_0 := A + BK - \hat{A} - \hat{B}K$ and $\Delta B_0 := BK - \hat{B}K$ for brevity. We consider the case that the state of system (1) is sampled by the sensors and transmitted via the network at times $t_i \geq 0, i \in \mathbb{Z}_{\geq 0}$ to the controller with $t_i < t_{i+1}$ and $\lim_{i\to\infty} t_i = \infty$. It is assumed that there are ℓ nodes in the network and the state x is separated into $x = (x_1, x_2, \ldots, x_\ell)$ correspondingly with $x_j \in \mathbb{R}^{n_{x_j}}$. At each transmission time t_i, only one node $j(t_i) \in \{1, 2, \ldots, \ell\}$ is authorized by the transmission protocols to transmit the data $x_{j(t_i)}$ to the controller. The transmitted data $x_j(t_i), j \in \{1, 2, \ldots, \ell\}$ will arrive at the controller at $t_i + d_i, d_i \geq 0$ after certain transmission delay $d_i < t_{i+1} - t_i$. Once the controller receives the transmitted data at $t_i + d_i$, $\hat{x}$ will update based on the following equation:

$$\hat{x}((t_i + d_i)^+) = e^{A_0 d_i}((I_{n_x} - \Phi(i))\hat{x}(t_i) + \Phi(i)x(t_i)) \tag{3}$$

where $\Phi(i)$ is a block diagonal matrix defined as

$$\Phi(i) := diag(\mathbf{0}_{n_{x_1}}, \ldots, \mathbf{0}_{n_{x_{j-1}}}, I_{n_{x_j}}, \mathbf{0}_{n_{x_{j+1}}}, \ldots, \mathbf{0}_{n_{x_\ell}})$$

and j in $\Phi(i)$ means that the node j is authorized by the transmission protocol at time t_i.

Update of $\hat{x}$ at $t_i + d_i$ based on (3) can be realized through the time stamp technique in the network. Once the controller receives a packet including the information $x_j(t_i)$ and corresponding time stamp, the controller will compute the transmission delay d_i and $\hat{x}((t_i + d_i)^+)$ by combining the information $\hat{x}(t_i)$ stored in the memory of the controller. $\hat{x}$ updates in the form of (3) because the sensors also run a model

$$\dot{\bar{x}} = A_0\bar{x}, \quad t \in [t_i, t_{i+1}]$$

which updates at t_i according to $\bar{x}(t_i^+) = (I_{n_x} - \Phi(i))\bar{x}(t_i) + \Phi(i)x(t_i)$. Under the same initial condition $\bar{x}(t_0) = \hat{x}(t_0)$, the updating equation (3) will ensure $\hat{x}((t_i + d_i)^+) = \bar{x}((t_i + d_i)^+)$ and thus $\hat{x}(t) = \bar{x}(t), \ t \in (t_i + d_i, t_{i+1}]$ which is used to ensure stability of the closed-loop system if we realize that the transmission protocols, for example, Try-Once-Discard protocols [22] and the event-triggered transmission strategy introduced below can access only the plant state and model state in the sensors. The updating equation (3) can also be found in [7], which considers only one node in the network.

Denote $e := \hat{x} - x \in \mathbb{R}^{n_e}$ with $n_e = n_x$. Then, from (3), we have

$$\hat{x}((t_i + d_i)^+) = e^{A_0 d_i}(h(i, e(t_i)) + x(t_i)) \tag{4}$$

where for brevity, we denote $h(i, e) := (I_{n_x} - \Phi(i))e$ and call $h(i, e)$ the transmission protocol.

From (4), it can be seen that the jumping dynamics at $t_i + d_i$ depend on the past state. Therefore, we introduce a storage variable $\hat{s}$ that flows according to

$$\dot{\hat{s}}(t) = \begin{cases} A_0\hat{s}(t), & t \in [t_i, t_i + d_i) \\ \dot{x}(t), & t \in [t_i + d_i, t_{i+1}] \end{cases} \tag{5}$$

and jumps according to

$$\hat{s}(t_i^+) = h(i, e(t_i)) + x(t_i), \ \hat{s}((t_i + d_i)^+) = x(t_i + d_i). \tag{6}$$

From (4)–(6), we can see that

$$\begin{aligned} \hat{x}((t_i + d_i)^+) &= \hat{s}(t_i + d_i) \\ &= e^{A_0 d_i}(h(i, e(t_i)) + x(t_i)). \end{aligned}$$

Replace variable $\hat{s}$ with $\hat{s} = s + x$. Then, s will flow according to

$$\dot{s}(t) := \begin{cases} A_0 s(t) - \Delta A_0 x(t) - BKe(t), & t \in [t_i, t_i + d_i) \\ 0, & t \in [t_i + d_i, t_{i+1}] \end{cases} \tag{7}$$

and jump according to

$$s(t_i^+) = h(i, e(t_i)), \ s((t_i + d_i)^+) = 0. \tag{8}$$

As a result, the jump dynamics of e can be represented as

$$\begin{aligned} e((t_i + d_i)^+) &= \hat{x}((t_i + d_i)^+) - x(t_i + d_i) \\ &= s(t_i + d_i). \end{aligned} \tag{9}$$

The sequence $\{t_i\}_{i=0}^\infty$ satisfying $T \leq t_{i+1} - t_i, \forall i \in \mathbb{Z}_{\geq 0}$ is assumed to be generated by the following event-triggered transmission strategy:

$$t_{i+1} := \inf\{t | t \geq t_i + T, \upsilon(t) \leq 0\} \tag{10}$$

where $T > 0$ is a waiting time used to avoid the Zeno behavior, and υ is subject to the following dynamic equation to be designed with initial value $\upsilon(t_0) > 0$

$$\dot{\upsilon}(t) = \begin{cases} f_{\upsilon,0}(\upsilon(t), x(t)), & t \in [t_i, t_i + T) \\ f_{\upsilon,1}(\upsilon(t), x(t), e(t)), & t \in [t_i + T, t_{i+1}]. \end{cases} \tag{11}$$

The following assumptions will be used.

Assumption 1: The delays d_i satisfy $0 \leq d_i \leq \bar{d} \leq T$ where $\bar{d}$ is the upper bound of transmission delays. ■

Assumption 2: $\delta_{A_0} > 0$ and $\delta_{B_0} > 0$ are such that the model uncertainties satisfy $|\Delta A_0| \leq \delta_{A_0}$ and $|\Delta B_0| \leq \delta_{B_0}$. ■

Assumption 3: [23] The transmission protocol $h(\kappa, e)$ is uniformly globally exponentially stable (UGES) in the sense that there exist a continuous function $W : \mathbb{Z}_{\geq 0} \times \mathbb{R}^{n_e} \to \mathbb{R}$ which is locally Lipschitz continuous in its second variable, constants $a_i > 0 (i = 1, 2)$ and $\lambda \in$

$(0,1)$ such that for all $\kappa \in \mathbb{Z}_{\geq 0}$ and $e \in \mathbb{R}^{n_e}$, the following conditions hold:

$$a_1|e| \leq W(\kappa, e) \leq a_2|e|,\ W(\kappa+1, h(\kappa, e)) \leq \lambda W(\kappa, e).$$

Moreover, there exist constants $M > 0, \lambda_W \geq 1$ such that for all $\kappa \in \mathbb{Z}_{\geq 0}$ and almost all $e \in \mathbb{R}^{n_e}$, it holds that $|\frac{\partial W(\kappa,e)}{\partial e}| \leq M$ and for all $\kappa \in \mathbb{Z}_{\geq 0}$ and all $e \in \mathbb{R}^{n_e}$, we have that

$$W(\kappa+1, e) \leq \lambda_W W(\kappa, e).$$

■

Remark 1: For the Round-Robin protocol, we have $a_1 = 1$, $a_2 = \sqrt{\ell}$, $\lambda = \sqrt{\frac{\ell-1}{\ell}}$, $\lambda_W = \sqrt{\ell}$, and for the Try-Once-Discard protocol, we have $a_1 = 1$, $a_2 = 1$, $\lambda = \sqrt{\frac{\ell-1}{\ell}}$, $\lambda_W = 1$. More details about the protocols can be found in [22], [23]. △

Combining (1), (2), and (7)–(11), we can model the resulting networked control system into a hybrid system [24]

$$\begin{cases} \dot{z} = f(z), & z \in C \\ z^+ = g(z), & z \in D \end{cases} \tag{12}$$

where $z := (x, e, s, \upsilon, l, \tau, \kappa)$ is the state variable, and $C := C_1 \cup C_2 \cup C_3$, $D := D_1 \cup D_2 \cup D_3$ will be specified in the following description. The flow dynamics are governed by

$$z \in C \quad \begin{cases} \dot{x} = f_1(x, e) \\ \dot{e} = f_2(x, e) \\ \dot{s} = f_3(x, e, s) \\ \dot{\upsilon} = f_\upsilon(x, e, \upsilon) \\ \dot{l} = 0 \\ \dot{\tau} = 1 \\ \dot{\kappa} = 0 \end{cases} \tag{13}$$

where

$$f_1(x, e) := (A_0 + \Delta A_0)x + (B_0 + \Delta B_0)e$$

$$f_2(x, e) := -\Delta A_0 x + (\hat{A} - \Delta B_0)e$$

$$f_3(x, e, s) := \begin{cases} A_0 s - \Delta A_0 x - BKe, & z \in C_1 \\ 0, & z \in C_2 \cup C_3 \end{cases}$$

$$f_\upsilon(x, e, \upsilon) := \begin{cases} f_{\upsilon,0}(\upsilon, x), & z \in C_1 \cup C_2 \\ f_{\upsilon,1}(\upsilon, x, e), & z \in C_3 \end{cases}$$

and

$$C_1 := \mathbb{R}^{n_x} \times \mathbb{R}^{n_e} \times \mathbb{R}^{n_s} \times \mathbb{R}_{\geq 0} \times \{1\} \times [0, \bar{d}] \times \mathbb{Z}_{\geq 0}$$

$$C_2 := \mathbb{R}^{n_x} \times \mathbb{R}^{n_e} \times \{\mathbf{0}\} \times \mathbb{R}_{\geq 0} \times \{0\} \times [0, T] \times \mathbb{Z}_{\geq 0}$$

$$C_3 := \mathbb{R}^{n_x} \times \mathbb{R}^{n_e} \times \{\mathbf{0}\} \times \mathbb{R}_{\geq 0} \times \{2\} \times [T, \infty) \times \mathbb{Z}_{\geq 0}.$$

The state τ is a timer variable, κ is a counter variable that keeps track of the sampling times, and l is a logic variable used to identify whether a transmitted data arrives at the controller.

The jump dynamics are subject to the following equations:

$$\begin{aligned} &G(x, e, s, \upsilon, l, \tau, \kappa) = (x, s, 0, \upsilon, 0, \tau, \kappa), && z \in D_1 \\ &G(x, e, s, \upsilon, l, \tau, \kappa) = (x, e, s, \upsilon, 2, \tau, \kappa), && z \in D_2 \\ &G(x, e, s, \upsilon, l, \tau, \kappa) = (x, e, h(\kappa, e), \upsilon, 1, 0, \kappa+1), && z \in D_3 \end{aligned} \tag{14}$$

where

$$D_1 := \mathbb{R}^{n_x} \times \mathbb{R}^{n_e} \times \mathbb{R}^{n_s} \times \mathbb{R}_{\geq 0} \times \{1\} \times [0, \bar{d}] \times \mathbb{Z}_{\geq 0}$$

$$D_2 := \mathbb{R}^{n_x} \times \mathbb{R}^{n_e} \times \{\mathbf{0}\} \times \mathbb{R}_{\geq 0} \times \{0\} \times \{T\} \times \mathbb{Z}_{\geq 0}$$

$$D_3 := \mathbb{R}^{n_x} \times \mathbb{R}^{n_e} \times \{\mathbf{0}\} \times \{0\} \times \{2\} \times [T, \infty) \times \mathbb{Z}_{\geq 0}.$$

Introducing a storage variable s in the model is inspired by the article [23], which considers the zero-order-hold case while we consider a model-based controller. In [23], the storage variable is subject to $\dot{s} = 0$ in the flow set.

From (10), the transmission is triggered immediately once υ arrives at zero, while from the flow set in system (12), a solution of system (12) may still evolve continuously when υ arrives at zero. The hybrid system (12) can generate a set of solutions which is larger than that generated by (10). Therefore, stability of the closed-loop networked control system under event-triggered transmission strategy (10) can be deduced by stability of hybrid system (12).

III. Design of Triggering Function and Stability Analysis

In Section II, we modeled the event-triggered control system into a hybrid system. In this section, we will give explicit designs of the triggering conditions.

Define the following functions $W_l : \mathbb{Z}_{\geq 0} \times \mathbb{R}^{n_e} \times \mathbb{R}^{n_s} \to \mathbb{R}_{\geq 0}$:

$$W_0(\kappa, e, s) : = W(\kappa, e),\ \ W_2(\kappa, e, s) := W(\kappa, e)$$

$$W_1(\kappa, e, s) : = \max\left\{W(\kappa, s), \frac{\lambda}{\lambda_W} W(\kappa, e)\right\}$$

where W satisfies the conditions in Assumption 3. From the definitions of W_0, W_1, W_2, there exist $b_{l1}, b_{l2} > 0$ such that $b_{l1}|(e,s)|^2 \leq W_l^2(\kappa, e, s) \leq b_{l2}|(e,s)|^2$ for all $z \in C \cup D$ with z specified below (12). Specifically, for $z \in C_1 \cup D_1$, we have $l = 1$ and thus $b_{11}|(e,s)|^2 \leq W_1(\kappa, e, s) \leq b_{12}|(e,s)|^2$ by combining Assumption 3 and the fact that $\frac{1}{\sqrt{2}}|(v_1, v_2)| \leq \max\{|v_1|, |v_2|\} \leq |(v_1, v_2)|$. For $z \in C_2 \cup D_2 \cup C_3 \cup D_3$, we have $s = 0$ and thus $b_{l1}|(e,s)|^2 \leq W_l^2(\kappa, e, s) \leq b_{l2}|(e,s)|^2$.

Using Assumption 3, we can conclude the following lemma.

Lemma 1: Suppose Assumption 3 holds. Then, for all $\kappa \in \mathbb{Z}_{\geq 0}, x \in \mathbb{R}^{n_x}$, and almost all $e \in \mathbb{R}^{n_e}, s \in \mathbb{R}^{n_s}$, it holds that

$$\left|\frac{\partial W_l(\kappa, e, s)}{\partial (e, s)}(f_2(x, e), f_3(x, e, s))\right| \leq L_l W_l + H_l(x)$$

where $L_0 = L_2 := \frac{M(|\hat{A}| + \delta_{B_0})}{a_1}$, $H_l(x) := M\delta_{A_0}|x|$, and

$$L_1 := \max\left\{\frac{M(|\hat{A}| + \delta_{B_0})}{a_1}, \frac{M|A_0|\lambda + M|BK|\lambda_W}{a_1\lambda}\right\}.$$

We are ready to present one of the main conditions in this article before the main result is presented.

Condition 1: There exist constants $\alpha_1, \alpha_2 > 0$, positive definite functions $\phi, \varphi : \mathbb{R}^{n_x} \to \mathbb{R}_{\geq 0}$ and $\chi : \mathbb{R}^{n_e} \to \mathbb{R}_{\geq 0}$, constants $\varepsilon > 0, \mu > 0, \gamma_1 > 0, \gamma_0 = \gamma_2 > 0$, and a continuously differentiable function $V : \mathbb{R}^{n_x} \to \mathbb{R}$ such that the following conditions hold:

(C1) $\alpha_1|x|^2 \leq V(x) \leq \alpha_2|x|^2$ for all $x \in \mathbb{R}^{n_x}$;

(C2) for all $z \in C$

$$\frac{\partial V(x)}{\partial x} f_1(x, e) \leq -\varepsilon|(x, e, s)|^2 - \phi(x) - H_\ell^2(x) + \gamma_\ell^2 W_\ell^2(\kappa, e, s)$$

(C3) for all $\kappa \in \mathbb{Z}_{\geq 0}$, $x \in \mathbb{R}^{n_x}$ and $e \in \mathbb{R}^{n_e}$

$$\begin{aligned} H_0^2(x) + \chi(e) + \phi(x) \geq\ & \gamma_0^2 W^2(\kappa, e) \\ & + 2\mu\gamma_0 W(\kappa, e)(L_0 W(\kappa, e) + H_0(x)) + \varphi(x). \end{aligned}$$

In hybrid system (12), the parameters T and $\bar{d}$ also need to be designed. Consider the following two differential equations:

$$\dot{\pi}_0 = -2L_0\pi_0 - \gamma_0\pi_0^2 - \gamma_0 \tag{15a}$$

$$\dot{\pi}_1 = -2L_1\pi_1 - \gamma_0\pi_1^2 - \frac{\gamma_1^2}{\gamma_0} \tag{15b}$$

where $\gamma_0, \gamma_1 > 0$. With (15a)–(15b), we use the approach in [23] to solve T and $\bar{d}$.

Condition 2: For $\lambda \in (0,1)$, the pair $(\bar{d}, T)$ with $0 \le \bar{d} \le T$ satisfies the following conditions:

$$\begin{aligned} \lambda^2\pi_1(0) &\le \pi_0(T) \\ \pi_0(\theta) &\le \pi_1(\theta), \quad \forall\theta \in [0, \bar{d}] \end{aligned} \tag{16}$$

where π_0, π_1 denote solutions of systems (15a)–(15b) with $\pi_1(0) \ge \pi_0(0) \ge 0$ and $\pi_0(T) = \mu > 0$.

There are many parameters in Conditions 1 and 2. Since the considered system is linear, we can choose a quadratic Lyapunov function and a quadratic function ϕ. We can first solve for γ_0 and γ_1 from (C1) and (C2) of Condition 1 using linear matrix inequalities. Then, we can set $\pi_0(0) = \frac{1}{\lambda}$ and $\pi_1(0) \ge \frac{1}{\lambda}$. By solving intersection of curves, we can find $\bar{d}$ and T satisfying Condition 2. Finally, we find quadratic functions $\chi(e)$ and φ satisfying (C3) of Condition 1 with $0 < \mu = \pi_0(T)$.

We are ready to present one of the main results in this article.

Theorem 1: Consider system (12) and let Assumptions 2 and 3 and Conditions 1 and 2 hold. Suppose that the functions in the transmission strategy (10) are designed as follows:

$$\begin{aligned} f_{\upsilon,0}(\upsilon, x) &:= -\rho(\upsilon) + \phi(x) \\ f_{\upsilon,1}(\upsilon, x, e) &:= -\varrho(\upsilon) + \varphi(x) - \chi(e) \end{aligned}$$

where ρ, ϱ are arbitrarily class-$\mathcal{K}$ functions. Then, the set $\mathcal{W} := \{\mathbf{0}\} \times \{\mathbf{0}\} \times \{\mathbf{0}\} \times \{0\} \times \{0,1,2\} \times \mathbb{R}_{\ge 0} \times \mathbb{Z}_{\ge 0}$ is uniformly globally asymptotically stable for system (12) and all the maximal solutions are complete.

The maximum solution is complete means that each solution z that cannot be extended has unbounded domain $\mathrm{dom} z$.

Proof of Theorem 1: Let $\tilde{\pi}_0 = \pi_0$ and $\tilde{\pi}_1 = \frac{\gamma_0}{\gamma_1}\pi_1$. Then, (15a) and (15b) can be transformed into

$$\begin{aligned} \dot{\tilde{\pi}}_0 &= -2L_0\tilde{\pi}_0 - \gamma_0\tilde{\pi}_0^2 - \gamma_0 \\ \dot{\tilde{\pi}}_1 &= -2L_1\tilde{\pi}_1 - \gamma_1\tilde{\pi}_1^2 - \gamma_1. \end{aligned} \tag{17}$$

Conditions (16) can be transformed into

$$\begin{aligned} \lambda^2\gamma_1\tilde{\pi}_1(0) &\le \gamma_0\tilde{\pi}_0(T) \\ \gamma_1\tilde{\pi}_1(\theta) &\ge \gamma_0\tilde{\pi}_0(\theta), \forall\theta \in [0, \bar{d}]. \end{aligned} \tag{18}$$

Choose the following function:

$$U(z) := \begin{cases} V(x) + \gamma_l\tilde{\pi}_l(\tau)W_l^2(\kappa, e, s) + \upsilon, & z \in \bigcup_{q=1}^{2}(C_q \cup D_q) \\ V(x) + \mu\gamma_0 W_l^2(\kappa, e, s) + \upsilon, & z \in C_3 \cup D_3 \end{cases}$$

where $\tilde{\pi}_l$ are the solutions of (17) satisfying (18). The function U is locally Lipschitz continuous on $C \cup D \cup g(D)$ and there exist $\tilde{\alpha}_1, \tilde{\alpha}_2 \in \mathcal{K}_\infty$ such that $\tilde{\alpha}_1(|z|_{\mathcal{W}}) \le U(z) \le \tilde{\alpha}_2(|z|_{\mathcal{W}})$ for all $z \in C \cup D$. For almost all $z \in C_1$, it holds that

$$\begin{aligned} \langle \nabla U(z), f(z)\rangle \le{}& \frac{\partial V(x)}{\partial x} f_1(x, e) + \gamma_1\dot{\tilde{\pi}}_1 W_1^2(\kappa, e, s) \\ &+ 2L_1\gamma_1\tilde{\pi}_1 W_1(\kappa, e, s) \\ &+ 2\gamma_1\tilde{\pi}_1 W_1(\kappa, e, s)H_1(x) + \dot{\upsilon}. \end{aligned} \tag{19}$$

Using (C2) of Condition 1 and substituting $\dot{\tilde{\pi}}_1$ and $\dot{\upsilon}$ into (19), we can obtain

$$\begin{aligned} \langle \nabla U(z), f(z)\rangle \le{}& -\varepsilon|(x, e, s)|^2 - \phi(x) - H_1^2(x) + \gamma_1^2 W_1^2(\kappa, e, s) \\ &- \gamma_1^2\tilde{\pi}_1^2 W_1^2(\kappa, e, s) - \gamma_1^2 W_1^2(\kappa, e, s) \\ &+ 2\gamma_1\tilde{\pi}_1 W_1(\kappa, e, s)H_1(x) - \rho(\upsilon) + \phi(x) \\ \le{}& -\varepsilon|(x, e, s)|^2 - \rho(\upsilon). \end{aligned}$$

Similarly, for almost all $z \in C_2$, $\langle \nabla U(z), f(z)\rangle \le -\varepsilon|(x, e, s)|^2 - \rho(\upsilon)$. For almost all $z \in C_3$, it holds by combining (C2) and (C3) of Condition 1 that $\langle \nabla U(z), f(z)\rangle \le -\varepsilon|(x, e, s)|^2 - \varrho(\upsilon)$. Since ρ, ϱ are class-$\mathcal{K}$ functions, there exists a positive definite function α such that for almost all $z \in C$, we have $\langle \nabla U(z), f(z)\rangle \le -\alpha(U)$. For $z \in D_1$, it holds that

$$\begin{aligned} U(g(z)) &= V(x) + \gamma_0\tilde{\pi}_0(\tau)W_0^2(\kappa, e, s) + \upsilon \\ &\le V(x) + \gamma_1\tilde{\pi}_1(\tau)W_1^2(\kappa, e, s) + \upsilon \\ &= U(z). \end{aligned}$$

For $z \in D_2$, $U(g(z)) \le U(z)$. For $z \in D_3$, it holds that

$$\begin{aligned} U(g(z)) &= V(x) + \gamma_1\tilde{\pi}_1(0)W_1^2(\kappa + 1, e, h(\kappa, e)) + \upsilon \\ &\le V(x) + \frac{\gamma_0}{\lambda} W_1^2(\kappa + 1, e, h(\kappa, e)) + \upsilon \\ &= U(z) \end{aligned}$$

according to Assumption 3. As a result, $U(g(z)) \le U(z)$ for all $z \in D$. Note that, because of the waiting time $T > 0$, there exist $c_1 > 0$ and $c_2 > 0$ such that for any $(t_1, j_1), (t_2, j_2) \in \mathrm{dom} z$, $j_2 - j_1 \le c_1(t_2 - t_1) + c_2$. Therefore, system (12) has persistent flow dynamics, and by similar arguments to that of [25], $\mathcal{W}$ is uniformly globally asymptotically stable.

Note that the data of system (12) satisfies the basic assumptions of hybrid system (Assumption 6.5 in [24]). We next show $f(z) \cap T_C(z) \ne \emptyset$ for any $z \in C \setminus D$, where $T_C(z)$ denotes the tangent cone to the set C at the point z. For $l = 0, \tau = 0, \upsilon > 0$ and $z \in C \setminus D$, $T_C(z) = \mathbb{R}^{n_x} \times \mathbb{R}^{n_e} \times \{0\} \times \mathbb{R} \times \{0\} \times \mathbb{R}_{\ge 0} \times \{0\}$ and thus $f(z) \cap T_C(z) \ne \emptyset$. For $l = 0, \tau = 0, \upsilon = 0$ and $z \in C \setminus D$, $T_C(z) = \mathbb{R}^{n_x} \times \mathbb{R}^{n_e} \times \{0\} \times \mathbb{R}_{\ge 0} \times \{0\} \times \mathbb{R}_{\ge 0} \times \{0\}$ and $f(z) \cap T_C(z) \ne \emptyset$ also holds. Similarly, for other cases, we can also have $f(z) \cap T_C(z) \ne \emptyset$. In addition, $g(z) \in C$ for all $z \in D$. As a result, Proposition 6.10 of [24] implies that all the maximal solutions of system (12) are complete. □

Remark 2: Once υ arrives at the value 0, it may increase because of the nonnegative term $\phi(x)$. This mechanism may avoid the possibility of periodic sampling. △

IV. Distributed Model-Based Dynamic Event-Triggered Control

In this section, we consider the distributed case. Consider the following plant consisting of N subsystems:

$$\dot{x}_i = \sum_{j=1} A_{ij}x_j + B_i u_i, \quad i \in Q \tag{20}$$

where $x_i \in \mathbb{R}^{n_{x_i}}$ is the state of ith subsystem, and $Q := \{1, 2, \ldots, N\}$ is an index set of the subsystems. A distributed model-based controller is given to stabilize systems (20)

$$\dot{\hat{x}}_i = \hat{A}_{ii}\hat{x}_i + \hat{B}_i u_i, \quad u_i = K_i\hat{x}_i \tag{21}$$

where $\hat{A}_{ii}$ and $\hat{B}_i$ are the available models of system matrices A_{ii} and B_i, respectively. Each controller (21) uses a model that omits the information of other subsystems. We assume that each subsystem sends its state x_i to the controller via a network $\mathcal{N}_i$ and there exist $\ell_i \in \mathbb{Z}_{\geq 0}$ nodes in the network $\mathcal{N}_i$. Let $\{t_k^i\}_{k=0}^{\infty}$ be the sampling sequence of network $\mathcal{N}_i$ with $t_k^i < t_{k+1}^i$ and $\lim_{k\to\infty} t_k^i = \infty$. x_i can be divided into $x_i = (x_{i1}, x_{i2}, \ldots, x_{i\ell_i})$ correspondingly with $x_{ij} \in \mathbb{R}^{n_{x_{ij}}}$. In network $\mathcal{N}_i$, the transmission order of nodes is determined by the transmission protocols. The transmitted data x_{ij} at time t_k^i will arrive at the controller at $t_k^i + d_k^i$, where d_k^i is the transmission delay with $0 \leq d_k^i < t_{k+1}^i - t_k^i$. The sampling times of network $\mathcal{N}_i$ satisfying $T_i \leq t_{k+1}^i - t_k^i, \forall k \in \mathbb{Z}_{\geq 0}$ are generated by the following transmission strategy:

$$t_{k+1}^i := \inf\{t \geq t_k^i + T_i | \upsilon_i \leq 0\} \tag{22}$$

where $T_i > 0$ is the waiting time in $\mathcal{N}_i$. The variable υ_i is subject to the following equation with $\upsilon_i(t_0^i) > 0$:

$$\dot{\upsilon}_i(t) = \begin{cases} f_{\upsilon_i,0}(\upsilon_i(t), x_i(t)), & t \in [t_k^i, t_k^i + T_i) \\ f_{\upsilon_i,1}(\upsilon_i(t), x_i(t), e_i(t)), & t \in [t_k^i + T_i, t_{k+1}^i) \end{cases}$$

where $e_i := \hat{x}_i - x_i \in \mathbb{R}^{n_{e_i}}$, and $\hat{x}_i$ updates at $t_k^i + d_k^i$ as

$$\begin{aligned} &\hat{x}_i((t_k^i + d_k^i)^+) \\ &= e^{(\hat{A}_{ii} + \hat{B}_i K_i) d_k^i}((I_{n_{x_i}} - \Phi_i(k))\hat{x}_i(t_k^i) + \Phi_i(k) x_i(t_k^i)) \end{aligned} \tag{23}$$

with a block diagonal matrix $\Phi_i(k)$. $h_i(k, e_i) := (I_{n_{x_i}} - \Phi_i(k)) e_i$ is called the transmission protocol of network $\mathcal{N}_i$.

Some assumptions are also needed as presented below.

Assumption 4: The delays satisfy $0 \leq d_i \leq \bar{d}_i \leq T_i$. ∎

Assumption 5: For each network $\mathcal{N}_i (i \in Q)$, the transmission protocols are UGES which means that there exist functions $W_i : \mathbb{Z}_{\geq 0} \times \mathbb{R}^{n_{e_i}} \to \mathbb{R}_{\geq 0}$ which are locally Lipschitz continuous in their second variables, constants $a_{i1}, a_{i2} > 0$, and $\lambda_i \in (0, 1)$ such that the following conditions hold for all $e_i \in \mathbb{R}^{n_{e_i}}$ and $k \in \mathbb{Z}_{\geq 0}$:

$$\begin{aligned} &a_{i1}|e_i| \leq W_i(k, e_i) \leq a_{i2}|e_i| \\ &W_i(k+1, h_i(k, e_i)) \leq \lambda_i W_i(k, e_i). \end{aligned}$$

Moreover, there exist constants $M_i > 0, \lambda_{W_i} \geq 1$ such that for all $k \in \mathbb{Z}_{\geq 0}$ and almost all $e_i \in \mathbb{R}^{n_{e_i}}$, it holds that $|\frac{\partial W_i(k,e_i)}{\partial e_i}| \leq M_i$ and for all $k \in \mathbb{Z}_{\geq 0}$ and all $e_i \in \mathbb{R}^{n_{e_i}}$, we have $W_i(k+1, e_i) \leq \lambda_{W_i} W_i(k, e_i)$. ∎

The entire closed-loop system can be modeled into the following hybrid system by using a similar modeling approach to the centralized case:

$$\begin{cases} \dot{z} \in F(z), & z \in C \\ z^+ \in G(z), & z \in D \end{cases} \tag{24}$$

where $z := (x, e, s, \upsilon, l, \tau, \kappa) \in \mathbb{R}^{n_z}$, $x := (x_1, x_2, \ldots, x_N) \in \mathbb{R}^{n_x}$, $e := (e_1, e_2, \ldots, e_N) \in \mathbb{R}^{n_e}$, $s := (s_1, s_2, \ldots, s_N) \in \mathbb{R}^{n_s}$, $\upsilon := (\upsilon_1, \upsilon_2, \ldots, \upsilon_N) \in \mathbb{R}^N$, $l := (l_1, l_2, \ldots, l_N)$, $\tau := (\tau_1, \tau_2, \ldots, \tau_N)$, and $\kappa := (\kappa_1, \kappa_2, \ldots, \kappa_N)$. The flow dynamics are subject to

$$z \in C \quad \begin{cases} \dot{x} = f_1(x, e) \\ \dot{e} = f_2(x, e) \\ \dot{s} = f_3(x, e, s) \\ \dot{\upsilon} = f_4(x, e, \upsilon) \\ \dot{l} = \mathbf{0} \\ \dot{\tau} = \mathbf{1}_N \\ \dot{\kappa} = \mathbf{0} \end{cases} \tag{25}$$

where the flow set is defined as

$$\begin{aligned} C &:= \bigcap_{i \in Q} \bigcup_{q \in \{1,2,3\}} C_{iq} \\ C_{i1} &:= \{z | \upsilon_i \in \mathbb{R}_{\geq 0}, l_i = 1, \tau_i \in [0, \bar{d}_i], \kappa_i \in \mathbb{Z}_{\geq 0}\} \\ C_{i2} &:= \{z | \upsilon_i \in \mathbb{R}_{\geq 0}, l_i = 0, \tau_i \in [0, T_i], \kappa_i \in \mathbb{Z}_{\geq 0}\} \\ C_{i3} &:= \{z | \upsilon_i \in \mathbb{R}_{\geq 0}, l_i = 2, \tau_i \in [T_i, \infty), \kappa_i \in \mathbb{Z}_{\geq 0}\} \end{aligned}$$

and

$$\begin{aligned} f_1(x, e) &:= A^* x + B^* \sum_{i=1}^N e_i \\ f_2(x, e) &:= (f_{21}(x, e), \ldots, f_{2N}(x, e)) \\ f_3(x, e, s) &:= (f_{31}(x, e, s), \ldots, f_{3N}(x, e, s)) \\ f_4(x, e, \upsilon) &:= (f_{41}(x, e, \upsilon), \ldots, f_{4N}(x, e, \upsilon)) \\ f_{2i}(x, e) &:= (\hat{A}_{ii} - \Delta B_{i0}) e_i - \Delta A_{i0} x_i - E_i x \\ f_{3i}(x, e, s) &:= \begin{cases} A_{i0} s_i - \Delta A_{i0} x_i - B_i K_i e_i - E_i x, \ z \in C_{i1} \\ 0, \ z \in C_{i2} \cup C_{i3} \end{cases} \\ f_{4i}(x, e, \upsilon) &:= \begin{cases} f_{\upsilon_i,0}(\upsilon_i, x_i), & z \in C_{i1} \cup C_{i2} \\ f_{\upsilon_i,1}(\upsilon_i, x_i, e_i), & z \in C_{i3} \end{cases} \\ A^* &:= \begin{pmatrix} A_1 & A_{12} & \cdots & A_{1N} \\ A_{21} & A_2 & \cdots & A_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ A_{N1} & \cdots & A_{NN-1} & A_N \end{pmatrix} \\ B^* &:= \mathrm{diag}\{B_1 K_1, \ldots, B_N K_N\} \\ E_i &:= \begin{pmatrix} A_{i1} & \cdots & A_{ii-1} & 0_{n_{x_i}} & A_{ii+1} & \cdots & A_{1N} \end{pmatrix} \\ A_i &:= A_{ii} + B_i K_i \\ A_{i0} &:= \hat{A}_{ii} + \hat{B}_i K_i \\ \Delta A_{i0} &:= A_{ii} + B_i K_i - \hat{A}_{ii} - \hat{B}_i K_i \\ \Delta B_{i0} &:= B_i K_i - \hat{B}_i K_i. \end{aligned}$$

Denote

$$\begin{aligned} &D_{i1} := C_{i1}, \ D_{i2} := \{z | \upsilon_i = 0, l_i = 0, \tau_i = T_i, \kappa_i \in \mathbb{Z}_{\geq 0}\} \\ &D_{i3} := \{z | \upsilon_i = 0, l_i = 2, \tau_i \in [T_i, \infty), \kappa_i \in \mathbb{Z}_{\geq 0}\}. \end{aligned}$$

Then, the jump set is specified as $D := \bigcup_{i \in Q} \bigcup_{q \in \{1,2,3\}} D_{iq}$. Let Γ_i denote an N-dimensional diagonal matrix with the ith diagonal element being zero and the other diagonal elements being 1. Let Γ_i^* denote an n_e-dimensional block diagonal matrix consisting of N diagonal blocks as follows:

$$\mathrm{diag}\{I_{n_{e_1}}, \ldots, I_{n_{e_{i-1}}}, 0_{n_{e_i}}, I_{n_{e_{i+1}}}, \ldots, I_{n_{e_N}}\}.$$

The jump map is specified as $G(z) := \bigcup_{i \in Q} G_i(z)$ where

$$G_i(z) := \begin{cases} \widetilde{G}_i(z), & z \in \bigcup_{q \in \{1,2,3\}} D_{iq} \\ \emptyset, & \text{otherwise} \end{cases}$$

and

$$\begin{aligned} &\widetilde{G}_i(z) := (x, \Gamma_i^* e + (I_{n_e} - \Gamma_i^*) s, \Gamma_i^* s, \upsilon, \Gamma_i l, \tau, \kappa), && z \in D_{i1} \\ &\widetilde{G}_i(z) := (x, e, s, \upsilon, l + (I_N - \Gamma_i) 1_N, \tau, \kappa), && z \in D_{i2} \\ &\widetilde{G}_i(z) := (x, e, \Gamma_i^* s + (I_{n_e} - \Gamma_i^*) h(\kappa, e), \upsilon \\ &\Gamma_i l + (I_N - \Gamma_i) 1_N, \Gamma_i \tau, \Gamma_i \kappa + (I_N - \Gamma_i)(\kappa + 1_N)), && z \in D_{i3} \end{aligned}$$

with $h(\kappa, e) := (h_1(\kappa_1, e_1), \ldots, h_N(\kappa_N, e_N))$.

The following assumptions will be needed.

Assumption 6: The constants $\delta_{A_{i0}} > 0, \delta_{B_{i0}} > 0, i \in Q$ are such that $|\Delta A_{i0}| \leq \delta_{A_{i0}}, |\Delta B_{i0}| \leq \delta_{B_{i0}}$. ■

To analyze stability of the subsystems (e_i, s_i), consider the following functions:

$$W_{i1}(\kappa_i, e_i, s_i) : = \max\{W_i(\kappa_i, s_i), \frac{\lambda_i}{\lambda_{W_i}} W_i(\kappa_i, e_i)\}$$

$$W_{i0}(\kappa_i, e_i, s_i) : = W_i(\kappa_i, e_i),\ W_{i2}(\kappa_i, e_i, s_i) := W_i(\kappa_i, e_i).$$

Under Assumption 5, we can conclude the following lemma.

Lemma 2: Suppose Assumption 5 holds. Then, for all $\kappa_i \in \mathbb{Z}_{\geq 0}, x \in \mathbb{R}^{n_x}$ and almost all $e_i \in \mathbb{R}^{n_{e_i}}$, $s_i \in \mathbb{R}^{n_{s_i}}$, it holds that $|\frac{\partial W_{il_i}(\kappa_i, e_i, s_i)}{\partial(e_i, s_i)}(f_{2i}(x, e), f_{3i}(x, e, s))| \leq L_{il_i} W_{il_i} + H_{il_i}(x)$ where $L_{i0} = L_{i2} := \frac{M_i(|\hat{A}_{ii}| + \delta_{B_{i0}})}{a_{i1}}$, $\quad H_{il_i}(x) := M_i \delta_{A_{i0}} |x_i| + M_i |E_i x|$, and

$$L_{i1} := \max\left\{\frac{M_i(|\hat{A}_{ii}| + \delta_{B_{i0}})}{a_{i1}}, \frac{M_i(\lambda_i |A_{i0}| + \lambda_{W_i} |B_i K_i|)}{\lambda_i a_{i1}}\right\}.$$

We will use the following condition.

Condition 3: Suppose that there exist $\alpha_j > 0 (j = 1, 2)$, positive definite functions $\phi_i, \varphi_i : \mathbb{R}^{n_{x_i}} \to \mathbb{R}_{\geq 0}$ and $\chi_i : \mathbb{R}^{n_{e_i}} \to \mathbb{R}_{\geq 0} (i \in Q)$, constants $\varepsilon > 0, \mu_i > 0, \gamma_{i1} > 0, \gamma_{i0} = \gamma_{i2} > 0 (i \in Q)$, and a continuously differentiable function $V : \mathbb{R}^{n_x} \to \mathbb{R}_{\geq 0}$ such that the following conditions hold:

(C1) $\alpha_1 |x|^2 \leq V(x) \leq \alpha_2 |x|^2$ for all $x \in \mathbb{R}^{n_x}$;

(C2) for all $z \in C$, it holds that

$$\frac{\partial V}{\partial x} f_1(x, e) \leq -\varepsilon |(x, e, s)|^2 - \sum_{i=1}^{N} \phi_i(x_i) - \sum_{i=1}^{N} (H_{il_i}^2(x) - \gamma_{il_i}^2 W_{il_i}^2(\kappa_i, e_i, s_i))$$

(C3) for all $x \in \mathbb{R}^{n_e}, e \in \mathbb{R}^{n_e}$ and $i \in Q$

$$H_{i0}^2(x) + \chi_i(e_i) + \phi_i(x_i) \geq \gamma_{i0}^2 W_i^2(\kappa_i, e_i) + 2\mu_i \gamma_{i0} W_i(\kappa_i, e_i)(L_{i0} W_i(\kappa_i, e_i) + H_{i0}(x)) + \varphi_i(x_i).$$

Remark 3: V and ϕ_i are chosen as quadratic functions. We can first solve γ_{i0} and then obtain $\gamma_{i1} = \frac{\lambda_{W_i}}{\lambda_i} \gamma_{i0}$ based on the inequality relation between W_{i1} and W_{i0}. Selection of the remaining parameters is similar to the centralized case. △

Consider the following equations for each $i \in Q$:

$$\dot{\pi}_{i0} = -2L_{i0}\pi_{i0} - \gamma_{i0}\pi_{i0}^2 - \gamma_{i0}$$
$$\dot{\pi}_{i1} = -2L_{i1}\pi_{i1} - \gamma_{i0}\pi_{i1}^2 - \frac{\gamma_{i1}^2}{\gamma_{i0}} \tag{26}$$

where $\gamma_{i0}, \gamma_{i1} > 0$.

The parameters T_i and $\bar{d}_i$ satisfy the following conditions.

Condition 4: The pair $(T_i, \bar{d}_i)$ with $0 \leq \bar{d}_i < T_i$ satisfies the following conditions:

$$\lambda_i^2 \pi_{i1}(0) \leq \pi_{i0}(T_i)$$
$$\pi_{i0}(\theta) \leq \pi_{i1}(\theta), \quad \forall \theta \in [0, \bar{d}_i]$$

where π_{i0}, π_{i1} denote the solutions of (26) satisfying $\pi_{i1}(0) \geq \pi_{i0}(0)$ and $\pi_{i0}(T_i) = \mu_i > 0$.

The same as the explanation in the paragraph below Condition 2, we can also find out a pair of $(T_i, \bar{d}_i)$ satisfying Condition 4. We are ready to present another main theorem of this article.

Theorem 2: Consider System (24) under Assumptions 4–6. Suppose Conditions 3 and 4 hold. Then, the set $\mathcal{W} := \{\mathbf{0}\} \times \{\mathbf{0}\} \times \{\mathbf{0}\} \times \{\mathbf{0}\} \times \{0, 1, 2\}^N \times \mathbb{R}_{\geq 0}^N \times \mathbb{Z}_{\geq 0}^N$ is uniformly globally asymptotically stable for system (24) and all the maximal solutions are complete if the functions in the strategy (22) are designed as

$$f_{\upsilon_i,0}(\upsilon_i, x_i) : = -\rho_i(\upsilon_i) + \phi_i(x_i)$$
$$f_{\upsilon_i,1}(\upsilon_i, x_i, e_i) : = -\varrho_i(\upsilon_i) + \varphi_i(x_i) - \chi_i(e_i).$$

Proof: Consider the following function $U : C \cup D \to \mathbb{R}$:

$$U(z) := V(x) + \sum_{i=1}^{N} U_i(e_i, s_i, \upsilon_i, l_i, \tau_i, \kappa_i)$$

where

$$U_i(e_i, s_i, \upsilon_i, l_i, \tau_i, \kappa_i) := \begin{cases} \gamma_{il_i}\pi_{il_i}(\tau_i) W_{il_i}(\kappa_i, e_i, s_i) + \upsilon_i, & z \in C_{i1} \cup C_{i2} \cup D_{i1} \cup D_{i2} \\ \mu_i \gamma_{i0} W_{il_i}(\kappa_i, e_i, s_i) + \upsilon_i, & z \in C_{i3} \cup D_{i3}. \end{cases}$$

Using Lemma 2, we can derive from Condition 3 that, for almost all $z \in C$ and $f \in F(z)$, $\langle \nabla U(z), f \rangle \leq -\varepsilon |(x, e, s)|^2 - \sum_{i=1}^{N} \min\{\rho_i(\upsilon_i), \varrho_i(\upsilon_i)\}$, and using Assumption 5, we can conclude that for all $z \in D$ and $g \in G(z)$, $U(g) \leq U(z)$. The rest of the proof is similar to that of Theorem 1. ■

Remark 4: The article [18] considers a decentralized dynamic event-triggered transmission strategy for nonlinear systems with output feedback. The difference is that the controller in [18] uses the zero-order-hold while this article considers model-based dynamic event-triggered control. △

V. Example and Simulation

Consider the linearized model of the interconnected pendulum used in [26], [27]. The matrix parameters are as follows:

$$A_{11} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 2.9156 & 0 & -0.0005 & 0 \\ 0 & 0 & 0 & 1 \\ -1.6663 & 0 & 0.0002 & 0 \end{bmatrix}$$

$$A_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.0011 & 0 & 0.0005 & 0 \\ 0 & 0 & 0 & 0 \\ -0.0003 & 0 & -0.0002 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ -0.0042 \\ 0 \\ 0.0167 \end{bmatrix}, \quad B_2 = B_1, A_{22} = A_{11}, A_{21} = A_{12}.$$

Such system is open-loop unstable. To stabilize this system, choose the gains as $K_1 = [11396\ 7196.2\ 573.96\ 1199.0]$, $K_2 = [29241\ 18135\ 2875.3\ 3693.9]$. Suppose that each network has two nodes and the transmission protocols are Try-Once-Discard protocols, then $a_{11} = a_{12} = a_{21} = a_{22} = 1$, $M_1 = M_2 = 1$, and $\lambda_1 = \lambda_2 = \sqrt{\frac{1}{2}}, \lambda_{W_1} = \lambda_{W_2} = 1$. We can compute $L_{10} = 3.3582$, $L_{11} = 561.0889$, $L_{20} = 3.3582$, $L_{21} = 1441.7$, and $H_{il_i}(x) = 0.0013|x|$ in the absence of disturbances. Choose $V(x) := x^T P x$, $\varepsilon = 0.001$, $\phi_i(x_i) = 0.02|x_i|^2$, and transform (C2) of Condition 3 into linear matrix inequality. Then, we can obtain $\gamma_{10} = \gamma_{20} = 8.3$ by using the LMI toolbox in MATLAB (these two gains can be optimized by function mincx in LMI toolbox). We can also obtain the gains $\gamma_{11} = \gamma_{21} = 12$. Set $\pi_{10}(0) = \pi_{20}(0) = \sqrt{2}$ and $\pi_{11}(0) =$

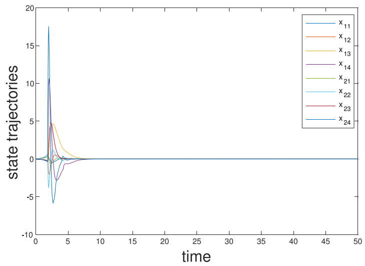

Fig. 1. State trajectories based on model-based controller.

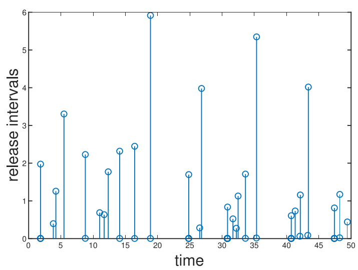

Fig. 2. Release intervals in network $\mathcal{N}_1$ under model-based controller.

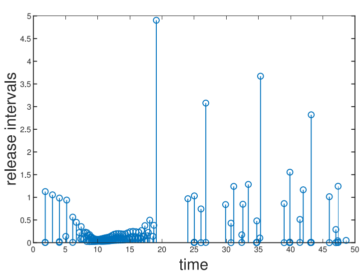

Fig. 3. Release intervals in network $\mathcal{N}_2$ under model-based controller.

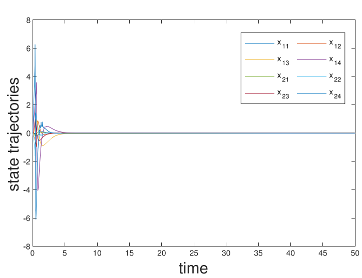

Fig. 4. State trajectories with controller based on zero-order-hold.

$\pi_{21}(0) = 2.5$. Then, $T_i = 4$ ms and $\bar{d}_i = 0.2$ ms satisfy Condition 4. We have $\mu_i = \pi_{i0}(T_i) = 1.25$. Therefore, we can choose $\rho_i(\upsilon_i) = \varrho_i(\upsilon_i) = -2\upsilon_i$, $\varphi_i(x_i) = 0.02|x_i|^2$ and $\chi_i(e_i) = 240|e_i|^2$ such that (C3) of Condition 3 holds. We run simulations with $x_1(0) = x_2(0) = (0.02, 0.02, -0.02, -0.02)$ for two cases: one uses model-based controllers for both subsystems and the other uses controller based on zero-order-hold for both subsystems. Both cases use Try-Once-Discard protocols in networks $\mathcal{N}_1$ and $\mathcal{N}_2$. Figs. 1–3 illustrate the state trajectories and release intervals in both networks under model-based controller, and Figs. 4–6 illustrate the state trajectories and release intervals in both networks with controller based on zero-order-hold.

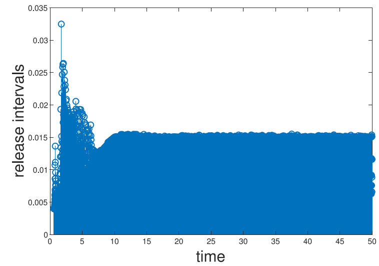

Fig. 5. Release interval of $\mathcal{N}_1$ with controller based on zero-order-hold.

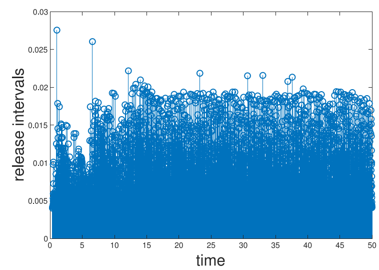

Fig. 6. Release interval of $\mathcal{N}_2$ with controller based on zero-order-hold.

By the model-based controller, there are 72 and 203 transmissions in network $\mathcal{N}_1$ and network $\mathcal{N}_2$, respectively. By the controller based on zero-order-hold, there are 4977 and 7027 transmissions in networks $\mathcal{N}_1$ and network $\mathcal{N}_2$, respectively. The proposed transmission strategy greatly reduces data transmission in the network compared with the controller based on zero-order-hold. This illustrates the effectiveness of the proposed controllers. We are also curious about the static event-triggered transmission strategy by using model-based controllers for networks $\mathcal{N}_1$ and $\mathcal{N}_2$. All parameters are the same with the dynamic transmission strategy, except that (22) is replaced by $t^i_{k+1} := \inf\{t \geq t^i_k + T_i | 240|e_i|^2 \geq 0.02|x_i|^2\}$. There are 81 and 47 transmissions in the networks $\mathcal{N}_1$ and $\mathcal{N}_2$. This shows that with the distributed model-based transmission strategy, the static strategy can also reduce the network transmission greatly.

VI. Conclusion

The results established in the previous sections consider only linear systems and state feedback; however, it is not hard to extend the proposed results to nonlinear systems with special forms, for example,

$$\dot{x} = Ax + \tilde{f}(x) + Bu. \tag{27}$$

For system (27), a model-based controller can be given as follows:

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u, \; u = K\hat{x}. \tag{28}$$

Following the same approaches as previous sections, we can also design centralized and decentralized dynamic event-triggered transmission strategies for (27) and (28). This article has focused on the small delay cases, and in the future, an interesting research topic will be the large delay case where the delays can be larger than the transmission intervals [28].

References

[1] P. Tabuada, “Event-triggered real-time scheduling of stabilizing control tasks,” *IEEE Trans. Autom. Control*, vol. 52, no. 9, pp. 1680–1685, Sep. 2007.

[2] L. Wu, Y. Gao, J. Liu, and H. Li, "Event-triggered sliding mode control of stochastic systems via output feedback," *Automatica*, vol. 82, pp. 79–92, 2017.

[3] X. Wang and M. D. Lemmon, "Event-triggering in distributed networked control systems," *IEEE Trans. Autom. Control*, vol. 56, no. 3, pp. 586–601, Mar. 2011.

[4] D. V. Dimarogonas, E. Frazzoli, and K. H. Johansson, "Distributed event-triggered control for multi-agent systems," *IEEE Trans. Autom. Control*, vol. 57, no. 5, pp. 1291–1297, May 2012.

[5] D. Yue, E. Tian, and Q. L. Han, "A delay system method for designing event-triggered controllers of networked control systems," *IEEE Trans. Auto. Control.*, vol. 58, no. 2, pp. 475–481, Feb. 2013.

[6] M. C. F. Donkers and W. P. M. H. Heemels, "Output-based event-triggered control with guaranteed $\mathcal{L}_\infty$-gain and improved and decentralized event-triggering," *IEEE Trans. Autom. Control.*, vol. 57, no. 6, pp. 1362–1376, Jun. 2012.

[7] E. Garcia and P. J. Antsaklis, "Model-based event-triggered control for systems with quantization and time-varying network delays," *IEEE Trans. Autom. Control.*, vol. 58, no. 2, pp. 422–434, Feb. 2013.

[8] R. Postoyan, P. Tabuada, D. Nešić, and A. Anta, "A framework for the event-triggered stabilization of nonlinear systems," *IEEE Trans. Autom. Control.*, vol. 60, no. 4, pp. 982–996, Apr. 2015.

[9] A. Girard, "Dynamic triggering mechanisms for event-triggered control," *IEEE Trans. Autom. Control.*, vol. 60, no. 7, pp. 1992–1997, Jul. 2015.

[10] H. Li and Y. Shi, "Event-triggered robust model predictive control of continuous-time nonlinear systems," *Automatica*, vol. 50, no. 5, pp. 1507–1513, 2014.

[11] A. Selivanov and E. Fridman, "Event-triggered h_∞ control: A switching approach," *IEEE Trans. Autom. Control.*, vol. 61, no. 10, pp. 3221–3226, Oct. 2016.

[12] X. Yi, K. Liu, D. V. Dimarogonas, and K. H. Johansson, "Distributed dynamic event-triggered control for multi-agent systems," in *Proc. IEEE 56th Annu. Conf. Decis. Control*, 2017, pp. 6683–6698.

[13] H. Yu, F. Hao, and T. Chen, "A uniform analysis on input-to-state stability of decentralized event-triggered control systems," *IEEE Trans. Autom. Control.*, vol. 64, no. 8, pp. 3423–3430, Aug. 2019.

[14] M. Mazo and P. Tabuada, "Decentralized event-triggered control over wireless sensor/actuator networks," *IEEE Trans. Autom. Control.*, vol. 56, no. 10, pp. 2456–2461, Oct. 2011.

[15] E. Garcia, Y. Cao, and D. W. Casbeer, "Decentralized event-triggered consensus with general linear dynamics," *Automatica*, vol. 50, no. 10, pp. 2633–2640, 2014.

[16] D. Yang, W. Ren, X. Liu, and W. Chen, "Decentralized event-triggered consensus for linear multi-agent systems under general directed graphs," *Automatica*, vol. 69, pp. 242–249, 2016.

[17] M. Abdelrahim, R. Postoyan, J. Daafouz, and D. Nešić, "Stabilization of nonlinear systems using event-triggered output feedback controllers," *IEEE Trans. Autom. Control*, vol. 61, no. 9, pp. 2682–2687, Sep. 2016.

[18] V. S. Dolk, D. P. Borgers, and W. P. M. H. Heemels, "Output-based and decentralized dynamic event-triggered control with guaranteed $\mathcal{L}_p$-gain performance and zeno-freeness," *IEEE Trans. Autom. Control.*, vol. 62, no. 1, pp. 34–49, Jan. 2017.

[19] W. P. M. H. Heemels and M. C. F. Donkers, "Model-based periodic event-triggered control for linear systems," *Automatica*, vol. 49, no. 3, pp. 698–711, 2013.

[20] Y. Yin, D. Yue, S. Hu, C. Peng, and Y. Xue, "Model-based event-triggered predictive control for networked systems with data dropout," *SIAM J. Control Optim.*, vol. 54, no. 2, pp. 567–586, 2016.

[21] L. A. Montestruque and P. J. Antsaklis, "On the model-based control of networked systems," *Automatica*, vol. 39, no. 10, pp. 1837–1843, 2003.

[22] D. Nešić and A. R. Teel, "Input–output stability properties of networked control systems," *IEEE Trans. Autom. Control.*, vol. 49, no. 10, pp. 1650–1667, Oct. 2004.

[23] W. P. M. H. Heemels, A. R. Teel, N. van de Wouw, and D. Nešić, "Networked control systems with communication constraints: Tradeoffs between transmission intervals, delays and performance," *IEEE Trans. Autom. Control.*, vol. 55, no. 8, pp. 1781–1796, Aug. 2010.

[24] R. Goebel, R. G. Sanfelice, and A. R. Teel, *Hybrid Dynamical Systems: Modeling, Stability, and Robustness*. Princeton, NJ, USA: Princeton University Press, 2012.

[25] D. Carnevale, A. R. Teel, and D. Nešić, "A Lyapunov proof of an improved maximum allowable transfer interval for networked control systems," *IEEE Trans. Autom. Control.*, vol. 52, no. 5, pp. 892–897, May 2007.

[26] K. Z. Liu, X. M. Sun, and M. Krstic, "Distributed predictor-based stabilization of continuous interconnected systems with input delays," *Automatica*, vol. 91, pp. 69–78, 2018.

[27] W. P. M. H. Heemels, D. P. Borgers, N. van de Wouw, D. Nešić, and A. R. Teel, "Stability analysis of nonlinear networked control systems with asynchronous communication: A small-gain approach," in *Proc. IEEE 52nd Annu. Conf. Decis. Control*, 2013, pp. 4631–4637.

[28] D. Freirich and E. Fridman, "Decentralized networked control of systems with local networks: A time-delay approach," *Automatica*, vol. 69, pp. 201–209, 2016.