07-10-2024

1. F

Proof: Take the natural number successively until you arrive at a number ≥ 12.

Take $m = n = 1$

$3m + 5n$

$3 \times 1 + 5 \times 1 = 8$

Take $m = 2,\ n = 1$

$3 \times 2 + 5 \times 1 = 11$

Take $m = 1,\ n = 2$

$3 \times 1 + 5 \times 2 = 13$

Take $m = n = 2$

$3 \times 2 + 5 \times 2 = 16$

Clearly, there is no

$(\nexists m \in N)(\nexists n \in N)(3m + 5n = 12)$

This completes the proof.

2. T

Proof: Let the five consecutive integers be n, $n+1$, $n+2$, $n+3$, and $n+4$.

$n + (n+1) + (n+2) + (n+3) + (n+4)$

$= 5n + 10$

$= 5(n+2)$

Clearly $5(n+2)$ is divisible by 5

This ~~proves the~~ completes the proof.

3. T

Proof:

Assume n is even.

Then, n^2 is even and equals to N

$N + n$ is even (sum of two even numbers).

$(N+n) + 1$ is odd.

Assume n is odd.

n^2 is odd

There $n^2 + n$ is even (sum of two odd numbers is even).

$(n^2 + n) + 1$ is odd.

Therefore, $n^2 + n + 1$ is odd for any integer.

This completes the proof

4.

Proof: By induction.

For $n = 1$

$4n + 1 = 4 \times 1 + 1 = 5$

which is odd.

Assume the theorem holds for n, then for $n+1$

$4(n+1) + 1 = 4n + 4 + 1$

$= 4n + 5$

and $4(n+1) + 3 = 4n + 4 + 3$

$= 4n + 7$

both of which are odd.

This proves the theorem by induction.

Proof:

5. By the Division Theorem,

if n is divisible by 3

then $n = 3k + 0$.

If n is not divisible by 3

then $n = 3k + 1$ or $n = 3k+2$

Therefore, if $n = 3k + 1$

$n+2 = 3k + 1 + 2$

$n+2 = 3k + 3$ which

is divisible by 3.

if $n = 3k + 2$

$n+4 = 3k + 2 + 4$

$n+4 = 3k+6$

$= 3(k+2)$

which is divisible by 3.

This completes the proof.

Proof

6. Suppose p, q is a pair of primes, where $p > 3$. We show that it is impossible to extend p, q to be a prime ~~Aw~~ triple.

Let $A = p \cdot q + 1$.

Then either A is prime or else there is a prime r such that $r \mid A$.

It follows that there is no ~~great~~ prime other than 3,5,7 that can be added to give a prime triple.

7. Proof:

By induction

when $n = 1$

$2^1 = 2^{1+1} - 2$

$2 = 2^2 - 2$

$2 = 4 - 2$

$2 = 2.$

Assume this holds for n, then for $n+1$

$2 + 2^2 + 2^3 + \dots + 2^n + 2^{n+1}$

$= (2^{n+1} - 2) + 2^{n+1}$ (by induction)

$= 2^{n+1} - 2 + 2^{n+1}$

$= 2^{n+1} + 2^{n+1} - 2$

$= 2 . 2^{n+1} - 2$

$= 2^{1+n+1} - 2$

$= 2^{n+2} - 2$

This is the identity at $n+1$.

This completes the proof.

8. Proof:

By the definition of a limit, we can find an N such that

$n \geq N \Rightarrow |a_n - L| < \frac{\varepsilon}{M}$

Then,

$|a_n - L| < \varepsilon/M$

$M|a_n - L| < M \cdot \frac{\varepsilon}{M}$

$|Ma_n - ML| < \varepsilon$

~~The~~ Which by the definition of a limit shows that $\{Ma_n\}_{n=1}^{\infty}$ tends to limit ML.

9. Proof :

Let $A_n = \left(\frac{1}{n+1}, \frac{1}{n}\right)$

For any $x > 0$, we can find an m such that

$1/m < x$

and then

$x \notin \left(\frac{1}{m+1}, \frac{1}{m}\right)$

Hence $\bigcap_{n=1}^{\infty} A_n = \phi$

Proof :

10. Let $A_n = \left(\frac{-1}{n}, +\frac{1}{n}\right)$

For any n, $0 \in A_n$

so $0 \in \bigcap_{n=1}^{\infty} A_n$.

On the other hand, if $x \neq 0$ then there is an m such that $1/m < |x|$, and for that m, $x \notin A_m$,

so $x \notin \bigcap_{n=1}^{\infty} A_n$

Hence $\bigcap_{n=1}^{\infty} A_n = \{0\}$