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The emitted spectrum and the standard disk model

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Starting from those scary equations....

1. $\rho = \Sigma / H$
2. $H = c_{\mathrm{s}} R^{3/2} / (GM)^{1/2}$
3. $c_{\mathrm{s}}^2 = P / \rho$
4. $P = \frac{\rho k T_{\mathrm{c}}}{\mu m_{\mathrm{p}}} + \frac{4\sigma}{3c} T_{\mathrm{c}}^4$
5. $\frac{4\sigma T_{\mathrm{c}}^4}{3\tau} = \frac{3GM\dot{M}}{8\pi R^3} \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right]$
6. $\tau = \Sigma \kappa_{\mathrm{R}}(\rho, T_{\mathrm{c}}) = \tau(\Sigma, \rho, T_{\mathrm{c}})$
7. $\nu \Sigma = \frac{\dot{M}}{3\pi} \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right]$
8. $\nu = \nu(\rho, T_{\mathrm{c}}, \Sigma, \alpha, \ \ldots)$

… lets think of the emitted disk radiation.

Providing disc is optically thick in z-direction, each element of the disc face radiates as (approximately) a blackbody of some temperature $T(R)$.

This temperature profile can be obtained from dissipation rate $D(R)$

$\sigma T^4(R) = D(R)$, which gives

$$T(R) = \left\{ \frac{3GM\dot{M}}{8\pi R^3 \sigma} \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right] \right\}^{1/4}$$

For $R >> R_*$

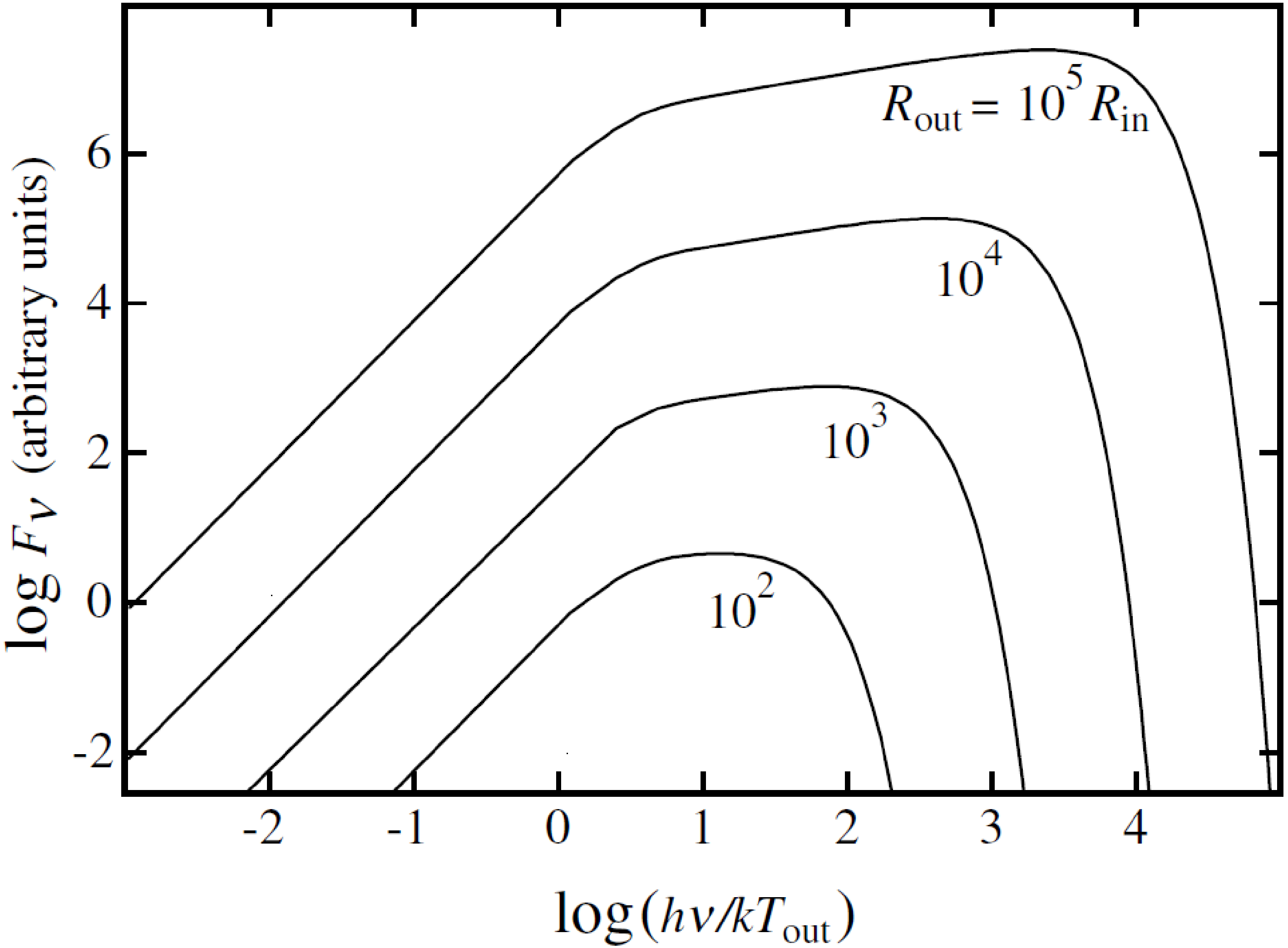
$T = T_*(R/R_*)^{-3/4}$, where

$$T_* = \left(\frac{3GM\dot{M}}{8\pi R_*^3\sigma}^{1/4}\right) = 4.1 \times 10^4 \dot{M}_{16}^{1/4} m_1^{1/4} R_9^{-3/4}\ \mathrm{K}$$

For an observer at distance D

$$F_\nu = \frac{4\pi h \cos i \nu^3}{c^2 D^2} \int_{R_*}^{R_{\rm out}} \frac{R \mathrm{d}R}{\exp[h\nu/kT(R)] - 1}$$

- For low frequencies $\nu << kT(R_{\rm out})/h \implies F_\nu \propto \nu^2$
- For high frequencies $\nu >> kT_*/h \implies F_\nu \propto \nu^3 \exp[-h\nu/kT_*]$
- For intermediate values $F_\nu \propto \nu^{1/3} \int_0^\infty \frac{x^{5/3}}{e^x - 1} \mathrm{d}x, \;\; x = (h\nu/kT_*)(R/R_*)^{3/4}$


$R_{out} = 10^5 R_{in}$
10^4
10^3
10^2
$\log F_\nu$ (arbitrary units)
$\log(h\nu/kT_{out})$
6
4
2
0
-2
-2
-1
0
1
2
3
4

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The Standard Model. Structure of steady $\alpha-$ disk

To solve that scary system we need to rethink equations 6 and 8

- $\tau = \Sigma\kappa_{\mathrm{R}}(\rho, T_{\mathrm{c}}) = \tau(\Sigma, \rho, T_{\mathrm{c}})$
- $\nu = \nu(\rho, T_{\mathrm{c}}, \Sigma, \alpha, \ \ldots)$

Let's introduce $\alpha-$prescription $\nu = \alpha c_{\mathrm{s}} H$

and assume Rosseland mean opacity is well approximated by Kramers' law
$\kappa_{\mathrm{R}} = 5 \times 10^{24} \rho T_{\mathrm{c}}^{-7/2}\ \mathrm{cm}^2\ \mathrm{g}^{-1}$

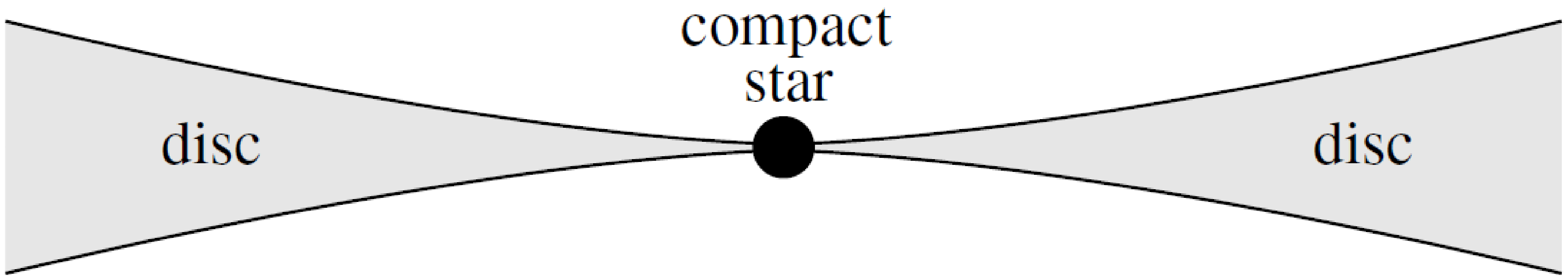
Now equations are algebraic (almost)

1. $\Sigma = 5.2\alpha^{-4/5}\dot{M}_{16}^{7/10}m_1^{1/4}R_{10}^{-3/4}f^{14/5}\ \mathrm{g\ cm^{-2}}$
2. $H = 1.7\times10^{8}\alpha^{-1/10}\dot{M}_{16}^{3/20}m_1^{-3/8}R_{10}^{9/8}f^{3/5}\ \mathrm{cm}$
3. $\rho = 3.1\times10^{-8}\alpha^{-7/10}\dot{M}_{16}^{11/20}m_1^{5/8}R_{10}^{-15/8}f^{11/5}\ \mathrm{g\ cm^{-3}}$
4. $T_\mathrm{c} = 1.4\times10^{4}\alpha^{-1/5}\dot{M}_{16}^{3/10}m_1^{1/4}R_{10}^{-3/4}f^{6/5}\ \mathrm{K}$
5. $\tau = 190\alpha^{-4/5}\dot{M}_{16}^{1/5}f^{4/5},\ \{f=\sqrt[4]{1-(R_*/R)^{1/2}}\}$
6. $\nu = 1.8\times10^{14}\alpha^{4/5}\dot{M}_{16}^{3/10}m_1^{-1/4}R_{10}^{3/4}f^{6/5}\ \mathrm{cm^2\ ^{-1}}$
7. $v_R = 2.7\times10^{4}\alpha^{4/5}\dot{M}_{16}^{3/10}m_1^{-1/4}R_{10}^{-1/4}f^{-14/6}\ \mathrm{cm\ s^{-s}}$

Looking closely

1. $H/R = 1.7 \times 10^{-2} \alpha^{-1/30} \dot{M}_{16}^{3/20} m_1^{-3/8} R_{10}^{1/8} f^{3/5}$
2. $M_{\mathrm{disc}} = 2\pi \int_{R_*}^{R_{\mathrm{out}}} \Sigma R \mathrm{d}R \lesssim (10^{-10} M_\odot) \alpha^{-4/5} \dot{M}_{16}^{7/10}$
3. $\rho << M/R^3$
4. $\kappa_{\mathrm{R}}(Krm) = \tau/\Sigma = 36 \dot{M}_{16}^{-1/2} m_1^{1/4} R_{10}^{3/4} f^{-2}\ \mathrm{cm}^2\ \mathrm{g}$

which dominates for $R \gtrsim 2.5 \times 10^7 \dot{M}_{16}^{2/3} m_1^{1/3} f^{8/3}\ \mathrm{cm}$



$$\frac{P_\mathrm{r}}{P_\mathrm{g}} = 2.8 \times 10^{-3} \alpha^{1/10} \dot{M}_{16}^{7/10} R_{10}^{-3/8} f^{7/5}$$

Radiation pressure exceedes gas pressure at

$$R \lesssim 24 \alpha^{2/21} \dot{M}_{16}^{16/21} m_1^{-3/21} f^{4/21}\ \mathrm{km}$$

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For radiation-pressure-domianted region

$$\tau = \Sigma\kappa_{\mathrm{R}} = \rho H \sigma_{\mathrm{T}}/m_{\mathrm{p}}$$

$$c_{\mathrm{s}}^2 = \frac{3GM\dot{M}\sigma_{\mathrm{T}}H}{8\pi R^3 m_{\mathrm{p}}c}\left[1-\left(\frac{R_*}{R}\right)^{1/2}\right]$$

and $H \cong \frac{2\sigma_{\mathrm{T}}\dot{M}}{8\pi m_{\mathrm{p}}c}\left[1-\left(\frac{R_*}{R}\right)^{1/2}\right]$

$$\dot{M}_{\mathrm{crit}} = \frac{L_{\mathrm{Edd}}R_*}{2\eta M} =$$

$$1.5\times10^{18}\left(\frac{R_*}{3\ \mathrm{km}}\right)\left(\frac{\eta}{0.1}\right)^{-1}\ \mathrm{g\ s^{-1}}$$

$$\dot{M}_{\mathrm{crit}} = 2\pi R_* m_{\mathrm{p}}c/(\eta\sigma_{\mathrm{T}})$$

$$H \cong \frac{3R_*}{4\eta}\frac{\dot{M}}{\dot{M}_{\mathrm{crit}}}\left[1-\left(\frac{R_*}{R}\right)^{1/2}\right]$$

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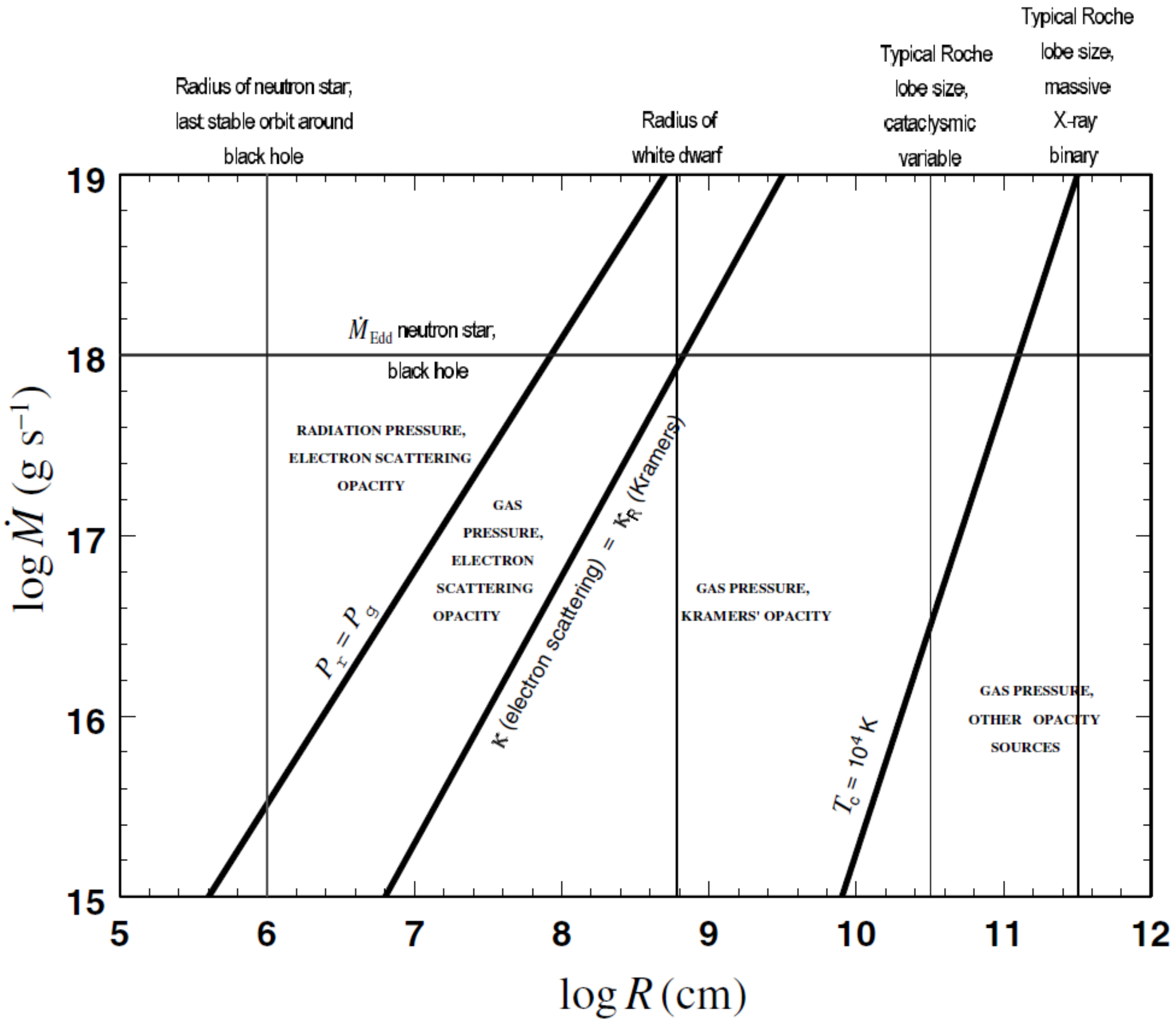
Radius of neutron star, last stable orbit around black hole
Radius of white dwarf
Typical Roche lobe size, cataclysmic variable
Typical Roche lobe size, massive X-ray binary
$\dot{M}_{\rm Edd}$ neutron star, black hole
RADIATION PRESSURE, ELECTRON SCATTERING OPACITY
GAS PRESSURE, ELECTRON SCATTERING OPACITY
GAS PRESSURE, KRAMERS' OPACITY
GAS PRESSURE, OTHER OPACITY SOURCES
$P_r = P_g$
κ (electron scattering) = κ_R (Kramers)
$T_c = 10^4$ K
$\log \dot{M}$ (g s^{-1})
$\log R$ (cm)
19
18
17
16
15
5
6
7
8
9
10
11
12