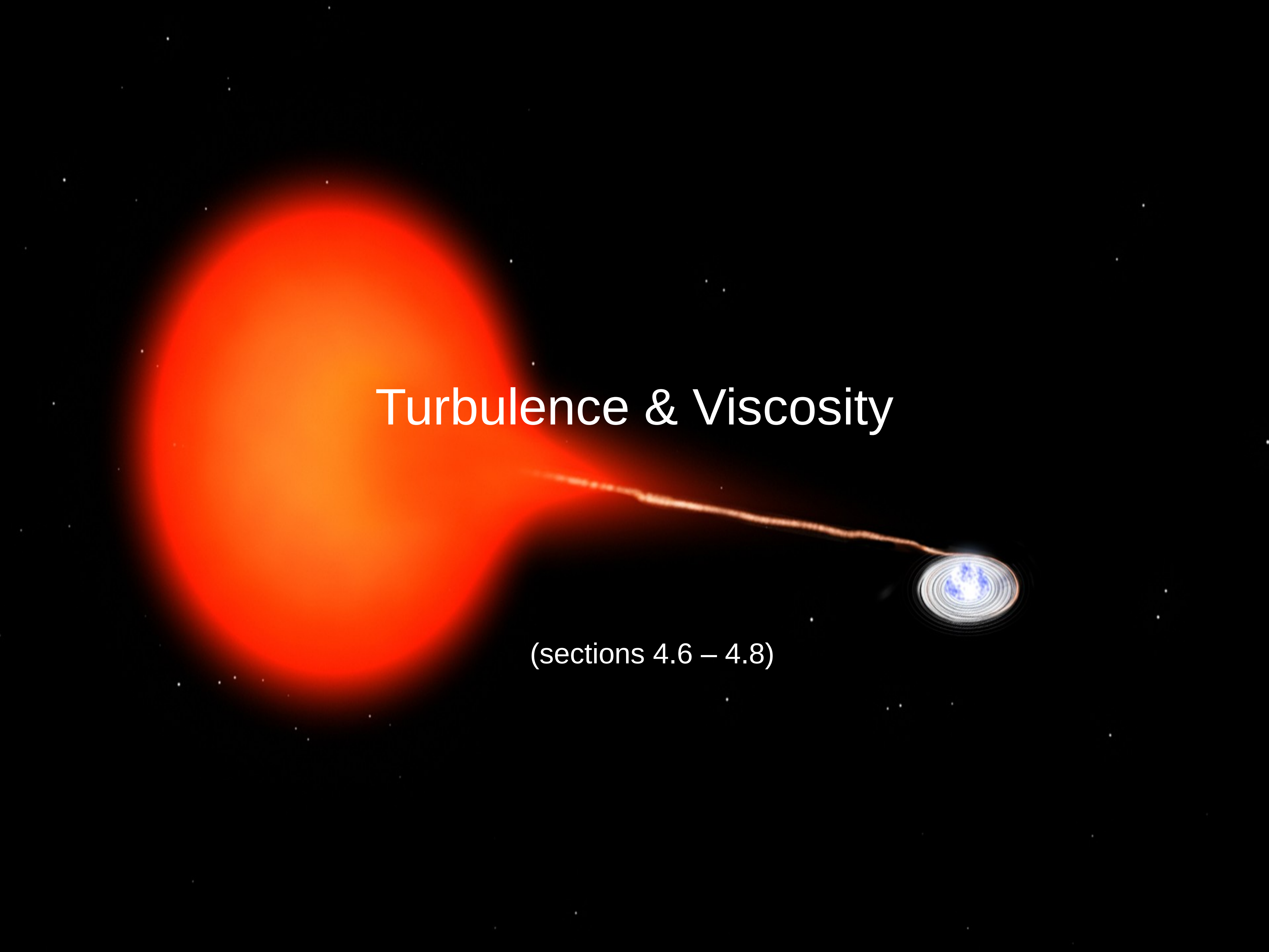
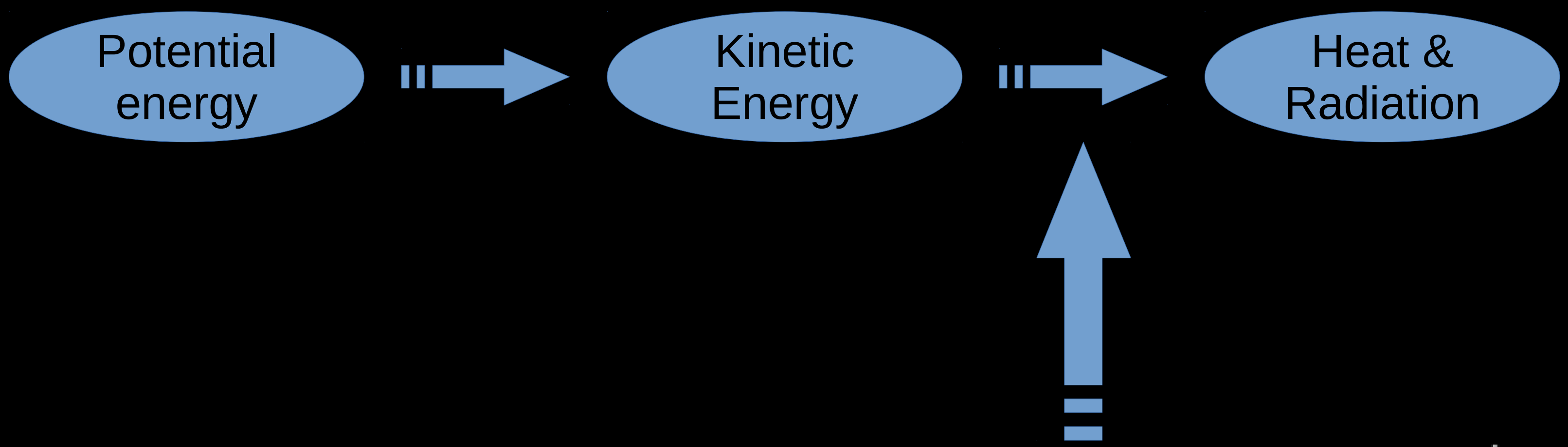


Turbulence & Viscosity

(sections 4.6 – 4.8)



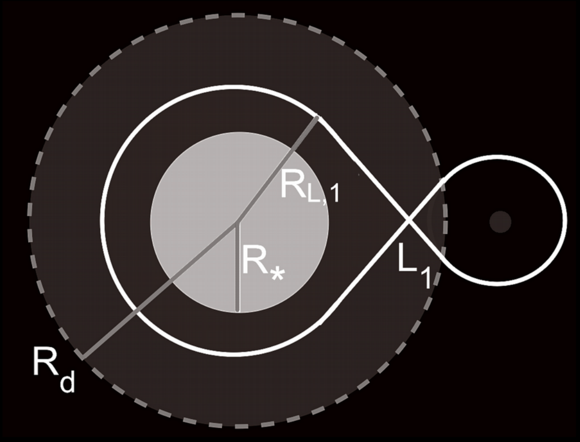
$$L_{\rm disc} = \frac{1}{2}\frac{GM\dot{M}}{R}.$$

$$E_{\rm GR}(r) = -\frac{GMm}{r}.$$

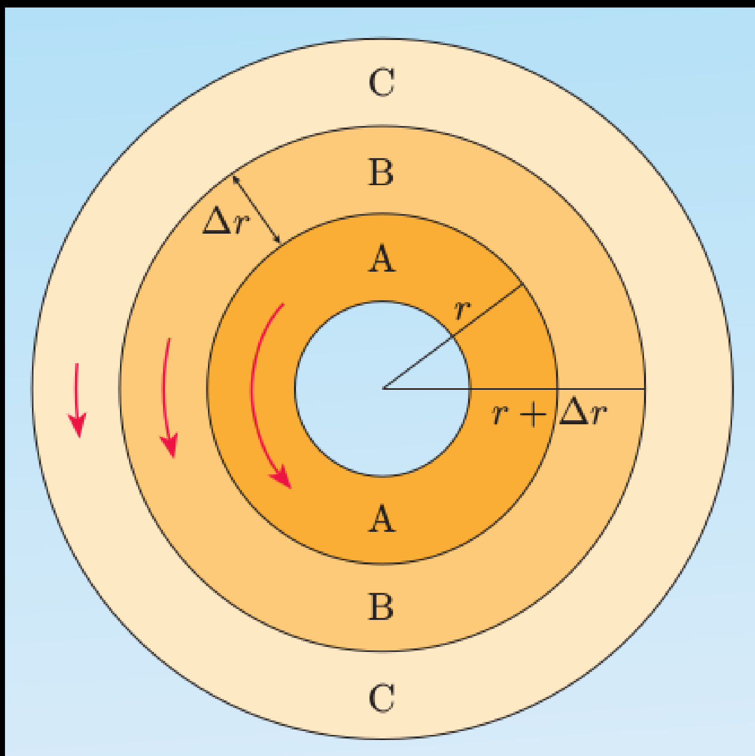
$$E_{\rm tot} = E_{\rm K} + E_{\rm GR} = \frac{1}{2}E_{\rm GR}.$$

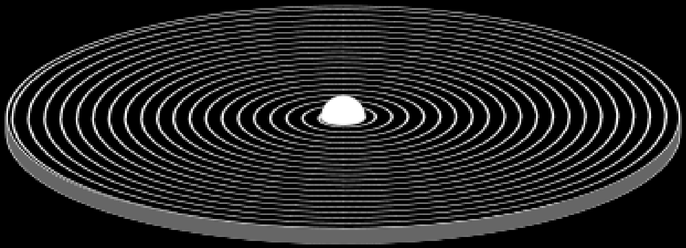
Recap accretion disks

- Stellar disk formation: Roche overflow
- Conservation of angular momentum
- Other accretion disks:
  - AGN
  - Proto-stars



Accretion Disk II





Geometrically thin, with scale height H

H << r

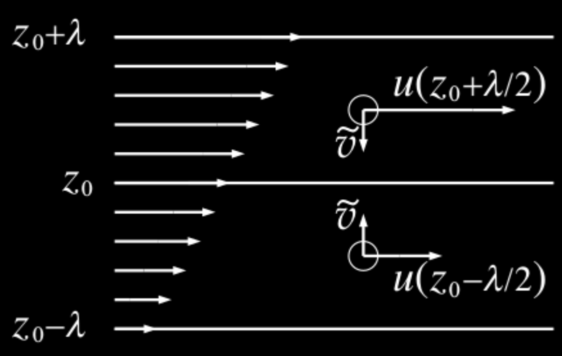
Surface density:

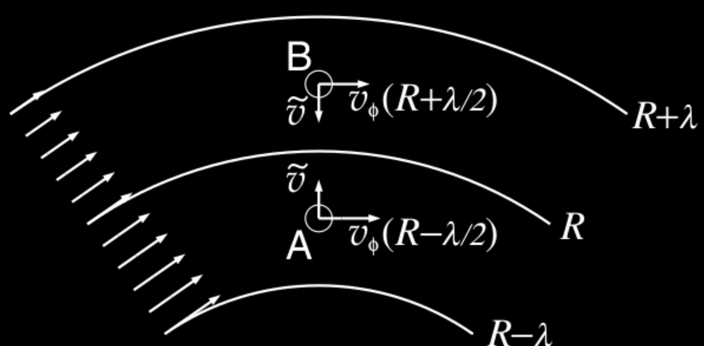
$$\Sigma(r) = \int_{-\infty}^{+\infty} \rho(r, z)\, \mathrm{d}z.$$

Viscous stress

- Transportation process
  - Angular momentum
- Radial velocity gradient due to differential rotation (Keplerian)

$$\omega_{\mathrm{K}} = \left(\frac{GM}{r^3}\right)^{1/2} .$$





Stress, strain and viscosity

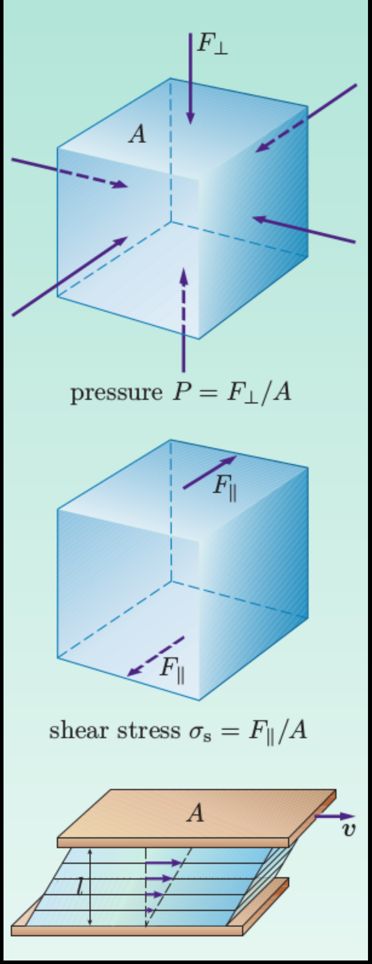
- Shear viscosity: orthogonal to bulk motion

'Stress causes strain'

Stress measures the *force* applied over the surface of a body, while **strain** is a measure of the *deformation* that the stress is causing. In the case of a fluid flow — such as accreting plasma — the 'deformation' manifests itself as a change in the velocity field in the flow.

In the simplest case, the applied stress is proportional to the resulting strain, and the viscosity is just the constant of proportionality:

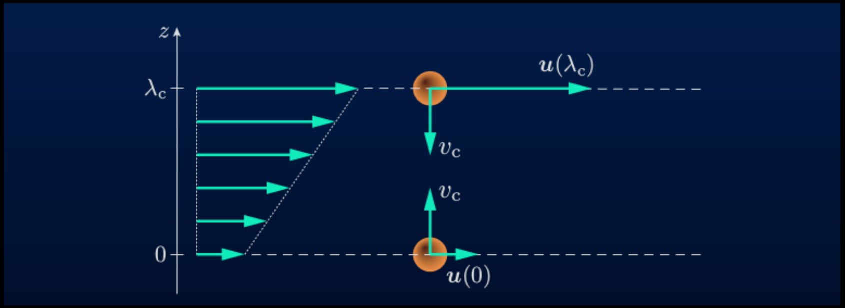
$$\text{stress} = \text{viscosity} \times \text{strain}.$$



From shear stress to viscous torque

$$\sigma_{\rm s} = \frac{F_{\parallel}}{A} = -\nu_{\rm vis}\,\rho\,\frac{\partial v}{\partial z},$$

$$\nu_{\rm vis} \simeq \lambda_{\rm c} \times v_{\rm c}.$$



$$G_{\rm vis} = -Fr = -A\sigma_{\rm s} \times r = 2\pi r H \times \nu_{\rm vis}\,\rho r\,\frac{\partial \omega}{\partial r} \times r$$

$$G_{\rm vis} = 2\pi r\,\nu_{\rm vis}\,\Sigma r^2\,\frac{\partial \omega}{\partial r}.$$

From torque to viscous dissipation

$$D(r) = \tfrac{1}{2}\nu_{\mathrm{vis}}\,\Sigma\left(r\,\frac{\partial\omega}{\partial r}\right)^2.$$

or

$$D(R) = \frac{G\Omega'}{4\pi R} = \frac{1}{2}\nu\Sigma(R\Omega')^2$$

In case of Keplerian rotation:

$$\Omega = \Omega_{\mathrm{K}} = \left(\frac{GM}{R^3}\right)^{1/2}$$

This becomes:

$$D(R) = \frac{9}{8}\nu\Sigma\frac{GM}{R^3}.$$

Constraining viscosity

$$Re = \frac{\text{inertia}}{\text{viscous}} \sim \frac{v_\phi^2/R}{\lambda\tilde{v}v_\phi/R^2} = \frac{Rv_\phi}{\lambda\tilde{v}}$$

If Re << 1, viscous forces are dominant
If Re >> 1, viscosity is irrelevant

$$\lambda_{\rm d} \cong \frac{7\times 10^5}{\ln\Lambda}\frac{T^2}{N}\,\text{cm}.$$

Check:

$$c_{\rm s} \cong 10(T/10^4K)^{1/2}\ \text{km s}^{-1}$$

$$\Omega_{\rm K}(R) = (GM_1/R^3)^{1/2}.$$

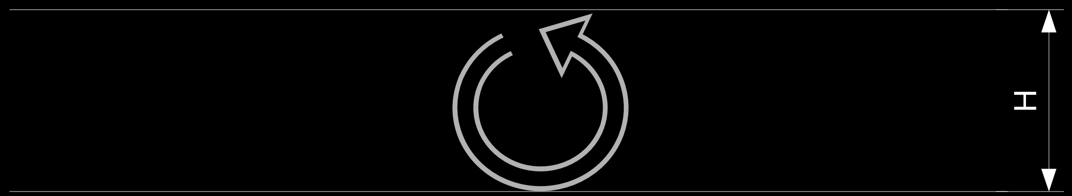
$$Re_{\rm mol} \sim 0.2Nm_1^{1/2}R_{10}^{1/2}T_4^{-5/2}.$$

$$Re_{\rm mol} > 10^{14}.$$

Conclusion: “classical” molecular viscosity cannot be responsible for the transportation of angular momentum and dissipation

Introducing alpha-viscosity

- We must assume that there is turbulence in the accretion disks
- Molecular viscosity seems not the be able to do the trick
- So Shakura & Sunyaev came up with the alfa viscosity trick:
  - $\nu = \alpha c_s H$



- Maximum size of eddies: H (scale height of the disk)
- Maximum velocity of the eddies: c_s (speed of sound)
- Alpha is just a scale factor, with no clear theoretical foundation

Beyond alfa-viscosity

- Blandford & Paine (1982) hydromagnetic model
- (this I need to work out, is not covered in the book, and I will not go to deep)