Stability and Optically thin ADAFs – similarity solutions

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Stability

- Consider small perturbations: If perturbations continues to grow rather than being damped, the supposed steady state solution is said to unstable.
- In case of instability, small perturbation could lead to a disc that would break up into disconnected rings.

Stability

- The condition for viscous instability from previous talks:
- All branches with positive slopes in $\partial \dot{M}/\partial\Sigma$ (or in $\partial T(R)/\partial\Sigma$) are viscously stable.
- Qualitive argument:
  - In case of positive slope: Increasing the local surface density in a narrow annulus, will increase the local mass transfer through the annulus depleting the increased mass.
- To check the viscous stability of accretion discs: check the relation between T^4 and Σ at a given disc radius R.

Stability

- Viscous stability is connected to thermal stability as seen also previously (S-curve)
- Thermal response shown with upward pointing arrows where heating dominates, and downwoard pointing arrows, where cooling (radiation+advection) dominates.

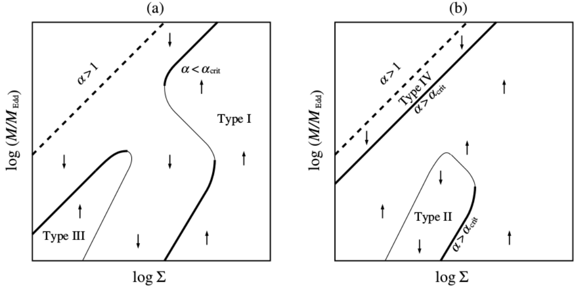

Fig. 11.2. Possible local disc solutions at some arbitrary radius are shown schematically for (a) $\alpha < \alpha_{\rm crit}$ and (b) $\alpha > \alpha_{\rm crit}$. Thick lines identify branches which are thermally and viscously stable. The arrows show the thermal-timescale evolution of a disc annulus which is not on the local equilibrium curve.

Stability

- Solutions of Shapiro, Lightman and Eardly [SLE] obey $\dot{M} \propto \alpha^{-1}\Sigma^6$ are viscosly stable and thermally unstable.

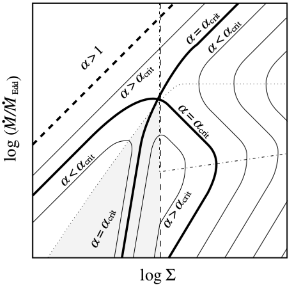

Fig. 11.4. Accretion disc solutions shown schematically on a local $(\dot{M}/\dot{M}_{\rm Edd}, \Sigma)$-plane. The shaded region corresponds to the two-temperature, optically thin, Compton cooled, gas pressure dominated SLE solutions.

Optically thin ADAFs -similarity solutions

- Standard thin disc cools radiatively, ADAF by advection (heat captured by moving matter)
- Advection can dominate either at very low (concentrated here) or high optical depths (low or high $\dot{M}$).
- See Narayan and Yi (1994 and 1995)
- In 1994 they study ADAFs using height(/vertically)-integrated equations, and in 1995 they show that their original conclusions agreed more exact spherically avereged approach.
- The height-integrated approach shown in the next slides:

Continuity, momentum and energy eqs:

$$\frac{d}{dR}(\rho R H v) = 0 \, , \tag{1}$$

$$v\frac{dv}{dR} - \Omega^2 R = -\Omega_K^2 R - \frac{1}{\rho}\frac{d}{dR}(\rho c_s^2) \, , \tag{2}$$

$$v\frac{d(\Omega R^2)}{dR} = \frac{1}{\rho R H}\frac{d}{dR}\left(\frac{\alpha \rho c_s^2 R^3 H}{\Omega_K}\frac{d\Omega}{dR}\right) , \tag{3}$$

$$\Sigma v T \frac{ds}{dR} = \frac{3 + 3\epsilon}{2} 2\rho H v \frac{dc_s^2}{dR} - 2c_s^2 H v \frac{d\rho}{dR} = Q^+ - Q^- \, . \tag{4}$$

Self-similar solution (1):

$$Q^+ - Q^- = \frac{2\alpha\rho c_s^2 R^2 H}{\Omega_K}\left(\frac{d\Omega}{dR}\right)^2 - Q^- \equiv f\,\frac{2\alpha\rho c_s^2 R^2 H}{\Omega_K}\left(\frac{d\Omega}{dR}\right)^2 . \tag{5}$$

The parameter f measures the degree to which the flow is advection-dominated. In the extreme limit of no radiative cooling, we have $f = 1$, while in the opposite limit of very efficient cooling, $f = 0$. Finally, we define $\epsilon' \equiv \epsilon/f$. The parameter ε' plays a critical role in determining the nature of the flow.

Let us for simplicity assume that ϵ' is independent of R. Equations (1)–(4) then permit a self-similar solution of the form (cf. Spruit et al. 1987)

$$\rho \propto R^{-3/2}\,, \quad v \propto R^{-1/2}\,, \quad \Omega \propto R^{-3/2}\,, \quad c_s^2 \propto R^{-1}\,, \tag{6}$$

Self-similar solution (2):

where

$$v = -(5 + 2\epsilon')\,\frac{g(\alpha, \epsilon')}{3\alpha}\,v_K \approx -\frac{3\alpha}{(5 + 2\epsilon')}\,v_K\,, \qquad (7)$$

$$\Omega = \left[\frac{2\epsilon'(5 + 2\epsilon')g(\alpha, \epsilon')}{9\alpha^2}\right]^{1/2}\Omega_K \approx \left(\frac{2\epsilon'}{5 + 2\epsilon'}\right)^{1/2}\Omega_K\,, \qquad (8)$$

$$c_s^2 = \frac{2(5 + 2\epsilon')}{9}\,\frac{g(\alpha, \epsilon')}{\alpha^2}\,v_K^2 \approx \frac{2}{5 + 2\epsilon'}\,v_K^2\,, \qquad (9)$$

$$g(\alpha, \epsilon') \equiv \left[1 + \frac{18\alpha^2}{(5 + 2\epsilon')^2}\right]^{1/2} - 1\,. \qquad (10)$$

Positive Bernoulli allows outward flows

Gas can escape to infinity if b>0, and for ADAFs it is when $f > \frac{1}{3}$ for any $\gamma < \frac{5}{3}$.

From this we can compute the normalized parameter $b \equiv \mathrm{Be}/v_{\mathrm{K}}^2$, where Be is the Bernoulli constant:

$$b = \frac{1}{v_{\mathrm{K}}^2}\left(\frac{1}{2}v^2 + \frac{1}{2}\Omega^2 R^2 - \Omega_{\mathrm{K}}^2 R^2 + \frac{\gamma}{\gamma-1}c_s^2\right)$$
$$= -\frac{\Omega^2 R^2}{2v_{\mathrm{K}}^2} + \left(\frac{\gamma}{\gamma-1} - \frac{5}{2}\right)\frac{c_s^2}{v_{\mathrm{K}}^2} = \frac{3\epsilon - \epsilon'}{5 + 2\epsilon'} . \qquad (12)$$

Convection causes a correction to the eqs above:

- Dynamical instability due to convection happens when $f > \frac{2}{3} + \frac{2}{3}\epsilon$.
- ADAFs thus convectively unstable, however strong advection ensures that convection can only be a moderate perturbation. It also only enhances the properties that are caused by advection.

$$\epsilon' = \frac{\epsilon}{f}\left(1 + \frac{\alpha_c c_s^2}{v v_K}\right) = \frac{\epsilon}{f}\left(1 - \frac{2}{3}\frac{\alpha_c}{\alpha}\right). \qquad (18)$$

ADAFs -conclusions

- 1. Accretion flow is not disklike in morphology, more like a Bondi spherical accretion instead (altgough more complicated).
- 2. Angular velocity is significantly sub-Keplerian, implying that the central star may not spin up to the "break-up limit" as it does with standard disk.
- 3. Typically high radial accretion velocity

ADAFs -conclusions

- 4. Positive Bernoulli parameter over most of the flow, and the likely violent convection close to rotational axes can cause a substantial bipolar outflow (jets).
- 5. The convective motions will transport both energy and angular momentum and will be a source of viscosity.
- 6. ADAFs are underluminous relative to the mass accretion rate.
- 7. The spectrum of optically thin ADAF is harder than for standard disc.

The End