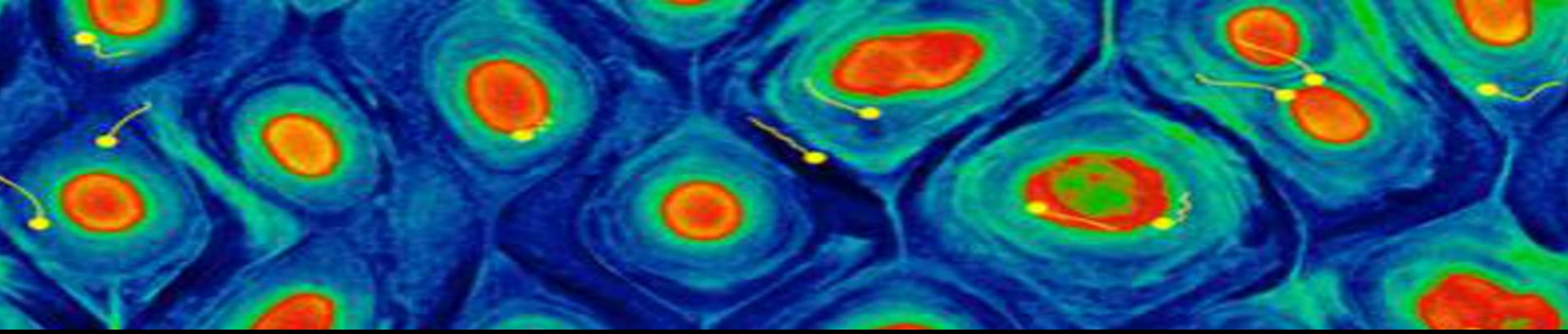
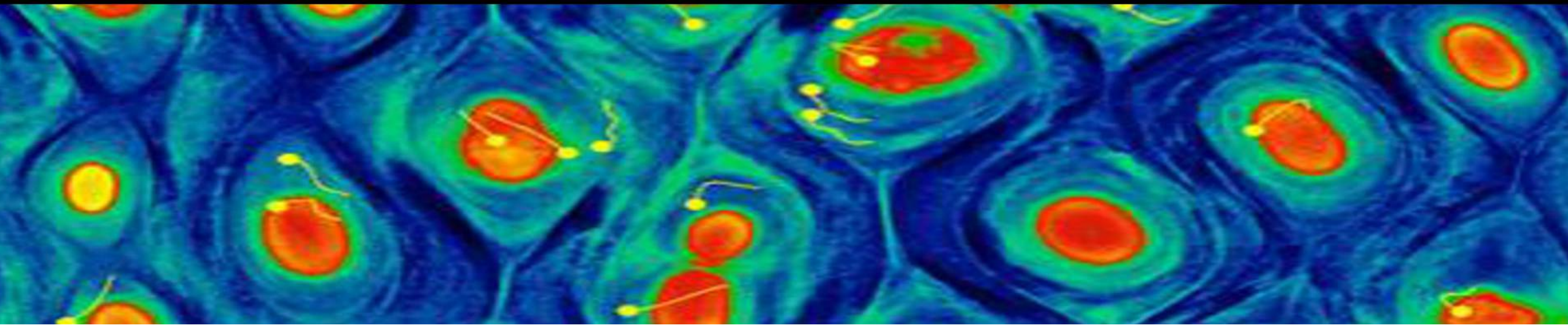


Plasma concepts

Chapter 3. in Accretion Power in Astrophysics



Prelude

- When lengthscales $\propto$ mean free path between collisions: **plasma**
- Mixture of two gases/fluids
  - electron-positron, electron-proton, electron-ion
- Transfer of energy in collisions $\propto m_e/m_i \ll 1$
  - Electrons and ions can have significantly different temperatures

Maxwell's equations (+ few others)

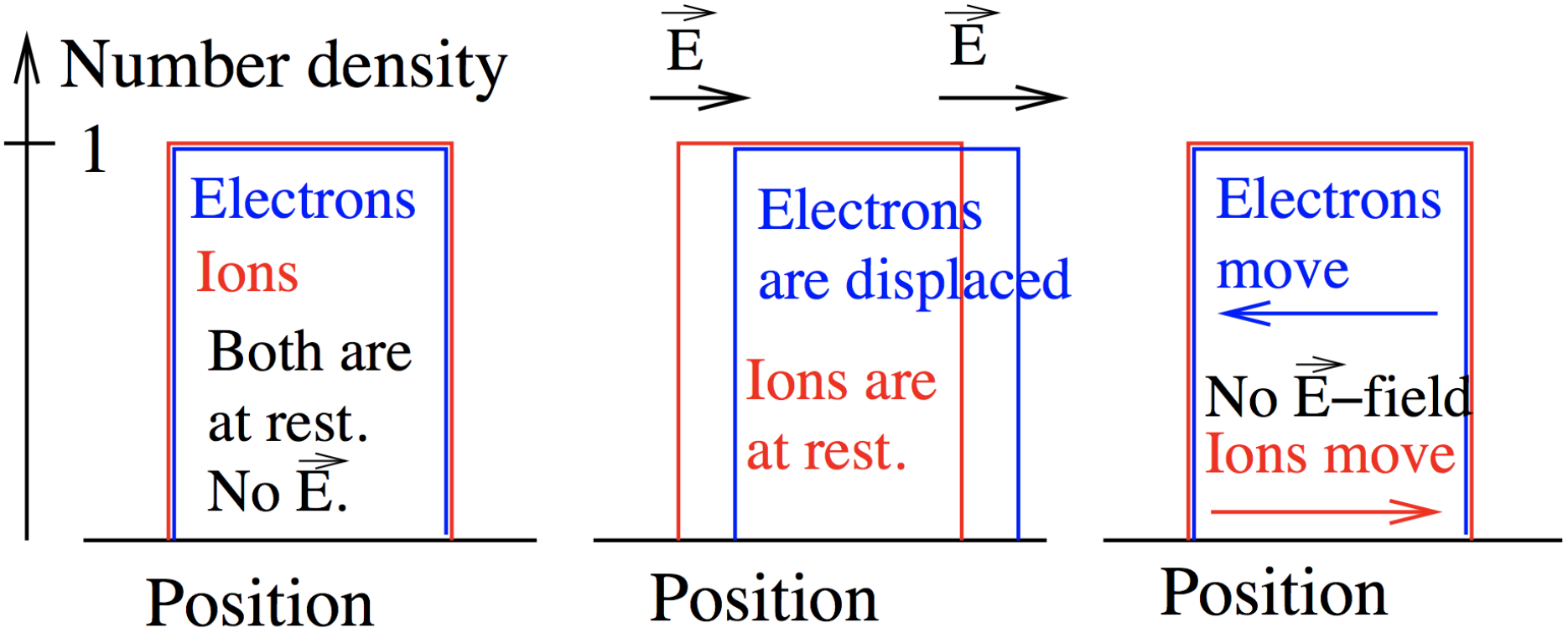
Boring ones:

- $\nabla \cdot B = 0$
- $\nabla \cdot E = \rho / \epsilon_0$

Interesting ones:

- $\nabla \times E = -\partial_t B$
- $\nabla \times B = \mu_0 \epsilon_0 \partial_t E + \mu_0 J$
- Current: $J = qvn$
- Lorentz force:
  $\partial_t v = \frac{q}{m}(E + v \times B)$
- Vlasov: $\partial_t f + v \nabla_x f + a \nabla_u f = 0$

Plasma oscillations



Plasma oscillations

- Characteristic frequency (or timescale) for plasma ($\propto n^{1/2}$ only)
- Lets take ions to have charge $Ze \approx e$ and
  - $n_i = n_0$ (ion family)
  - $n_e = n_0 + n_1(r, t)$ (electron family)
- n_1 gives rise to an E field: $\nabla \cdot E = -\frac{n_1 e}{\epsilon_0}$
- Lorentz force then acts on charged particles: $m_e \partial_t v = -eE$.

- Vlasov equation then gives:
  - $\partial_t n_e + \nabla \cdot (n_e v) = 0$
- Neglecting products of small quantities:
  - $\partial_t n_1 + n_o \nabla \cdot v = 0$
- Take ∂_t of Vlasov: $\partial_t \partial_t n_1 + \partial_t (n_0 \nabla \cdot v) = \partial_t^2 n_1 + n_0 \nabla \cdot \partial_t v = 0$
- Take $\nabla\cdot$ of Force: $\nabla \cdot \partial_t v = -\frac{e}{m} \nabla \cdot E$.
- Combine: $\frac{1}{n_0} \partial_t^2 n_1 - \frac{e}{m} \nabla \cdot E$
- And insert E field expression: $\partial_t^2 n_1 - n_0 \frac{e}{m} \frac{n_1 e}{\epsilon_0} = 0$

Plasma oscillations

In the end, after beautifying we get:

- $\partial_t^2 n_1 - n_1(\frac{e^2 n_0}{m\epsilon_0}) = 0$
- Oscillator with angular frequency: $\omega_p^2 = \frac{e^2 n_0}{m\epsilon_0}$
- $\omega_p \approx 5.7 \times 10^4 n_0^{1/2}$ rad/s. Or more roughly $t_p \sim 10^{-4} n_0^{-1/2}$ s.

	n_0	t_p
ISM	~ 5 cm^{-3}	$\sim 10^{-5}$ s
Earth atmos	$\sim 10^6$ cm^{-3}	$\sim 10^{-7}$ s
Laser plasma	$\sim 10^{15}$ cm^{-3}	$\sim 10^{-12}$ s

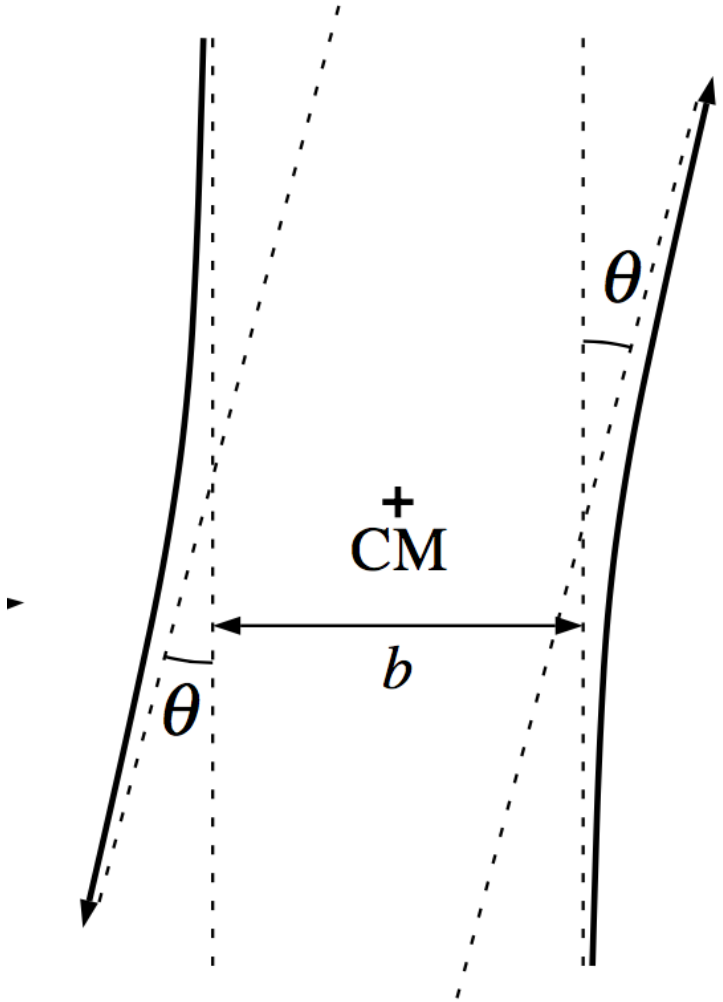
Plasma Shielding: Debye length

- We now know typical plasma response time t_p, what lengthscale does this correspond to?
- Take velocity from thermal plasma: $v_e \sim (kT/m)^{1/2}$
- Then distance covered by electron with velocity v_e in time t_p is
  - $\lambda_{Deb} = \left(\frac{\epsilon_0 kT}{n_0 e^2}\right)^{1/2} \sim 7\,(T[K]\,/\,n_0)^{1/2}$ cm.

	n_0	t_p	T (K)	λ_{Deb}
ISM	~ 5 cm^{-3}	$\sim 10^{-5}$ s	$\sim 5 \times 10^4$	~ 7 m
Earth atmos	$\sim 10^6$ cm^{-3}	$\sim 10^{-7}$ s	~ 300	~ 1.5 mm
Laser plasma	$\sim 10^{15}$ cm^{-3}	$\sim 10^{-12}$ s	$\sim 10^7$	$\sim 7\ \mu$m

Coulomb Collisions

- Deflection angle will be appreciable if Coulomb potential energy $\sim$ kinetic energy:
  - $\frac{e_1 e_2}{\epsilon_0 b} \sim mv^2 \sim kT$
- Change of velocity is $\Delta v \sim v\theta \sim \frac{e_1 e_2}{\epsilon_0 m v b}$ and $\theta \propto 1/v^2$
- Rate of change of energy:
  - $\frac{d}{dt}\Delta v^2 \sim Nv \int_{b_0}^{b_1} (\Delta v)^2 2\pi b db$
    $= \frac{n e_1^2 e_2^2}{8\pi\epsilon_0^2 m^2 v} \ln(\frac{b_1}{b_0})$



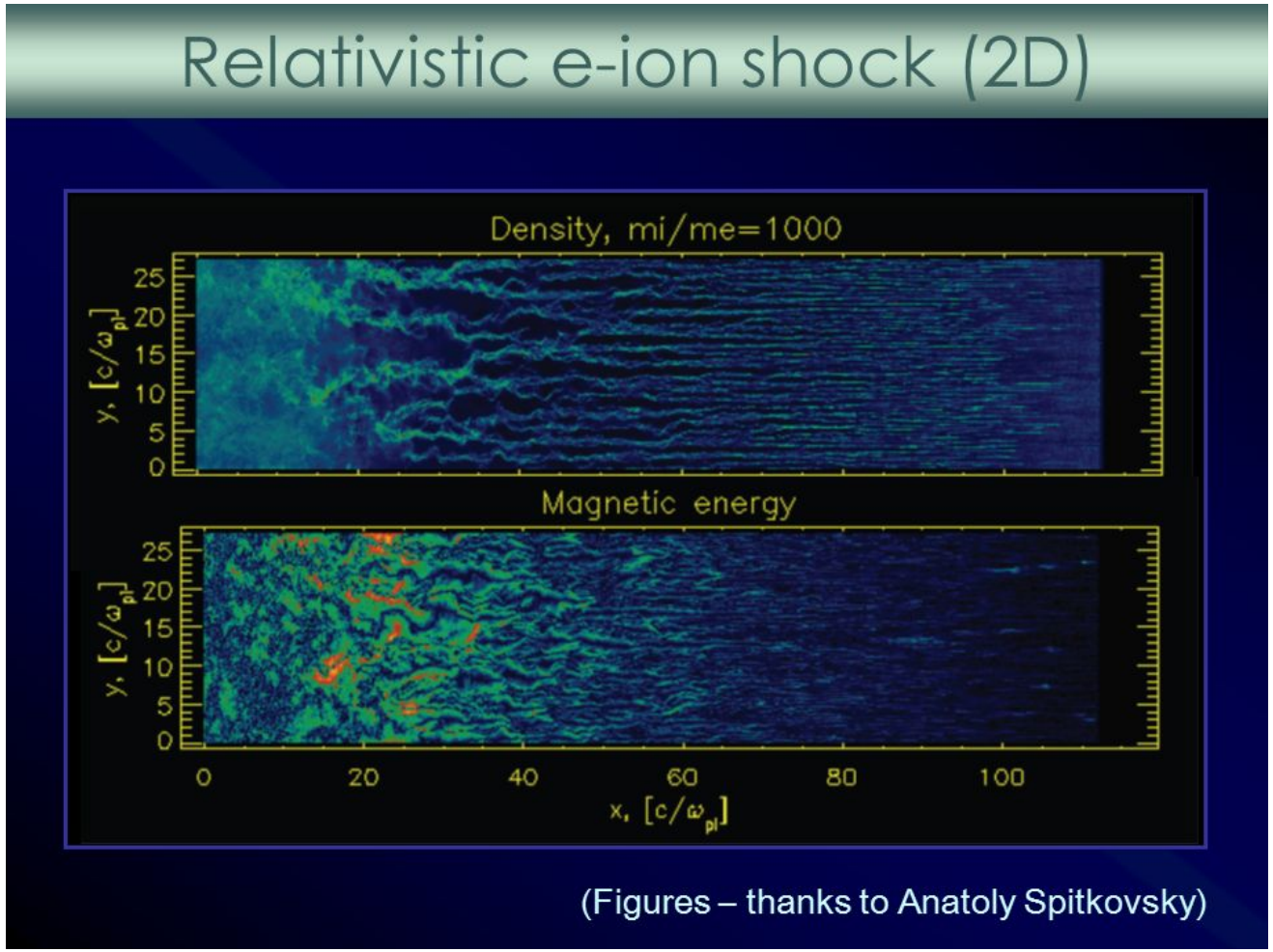
Plasma Coulomb parameter $\ln \Lambda$

- Minimum: b_0 = max$\{\frac{\hbar}{mv}, \frac{2e_1 e_2}{\epsilon_0 m v^2}\}$
  - Other one is quantum mechanical limit from $\Delta x \Delta p \sim \hbar$
  - Second one just from Coulomb pot. E $\sim$ Kinetic energy
- Maximum: $b_1 \sim$ Debye length
- $\ln \Lambda \approx 10 + 3.45 \log T - 1.15 \log n_e$

Collision time

- Typical time $\frac{d\Delta v^2}{dt} \sim \frac{\Delta v^2}{t}$
- So "Deflection time" (=time when on average we have changed direction due to collisions) is $t_D \sim \frac{v^2}{d\Delta v^2/dt} = \frac{8\pi\epsilon_0^2 m^2 v^3}{ne^4 \ln \Lambda}$
- Note that for electron-ion plasma t_d is $\sim m_i/m_e \approx 1836$ greater. Hence it takes time to even out by collisions.

Stopping of fast particles by plasma

- Lets define incoming particle as mass, velocity, and charge as $m_1, U \gg v_t, e_1$.
- Slowing-down timescale: $t_s = -\frac{U}{dU/dt}$
  - $t_s \sim \frac{8\pi\epsilon_0^2 m_1^2 U^3}{n e_1^2 e_2^2 \ln\Lambda} \frac{m_2}{m_1+m_2}$
- Consequence of the mass-term
  - Suprathermal ions are best slowed by electrons in $t_s \sim (m_e/m_p)t_d$
  - Suprathermal electrons are slowed by both species with just $t_s \sim t_d$.

Collisionless shocks