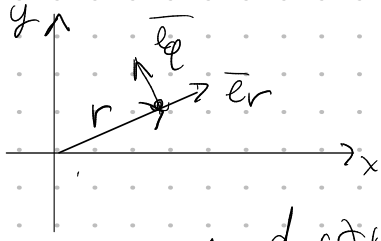
## N 1.18

**Дано:**

$r^2\dot{\varphi} = const$

**Доказ:**

$\bar{w} \parallel \bar{r}$




1) $|dr_\varphi| = r\,d\varphi \Rightarrow H_\varphi = r$

2) $|dr_r| = dr \Rightarrow H_r = 1$

3) $w_\varphi = \frac{1}{H_\varphi}\frac{d}{dt}\left(\frac{\partial(v^2/2)}{\partial\dot{\varphi}} - \frac{\partial(v^2/2)}{\partial\varphi}\right)$

4) $v^2 = \dot{r}^2 + r^2\dot{\varphi}^2 \Rightarrow W_\varphi = \frac{1}{r}\cdot\frac{d(r^2\dot{\varphi})}{dt} = 0 \Rightarrow \bar{w} \parallel \bar{r}$ – ч.т.д.

## N 1.25

**Дано:**

$W = \alpha(v \times r)$

$\alpha = const > 0$

**Найти:**

$\rho(\bar{r}, \bar{v})$ – ?

1) $W = \alpha(\bar{v} \times \bar{r}) \Rightarrow \bar{W} \perp \bar{V};\ \bar{W} \perp \bar{r}$

2) $\bar{W} = \dot{V}\cdot\bar{\tau} + \frac{V^2}{\rho}\cdot\bar{n} \Rightarrow \bar{W}\cdot\bar{V} = \dot{V}\cdot\bar{\tau}\cdot\bar{V} + \frac{V^2}{\rho}\cdot\bar{n}\cdot\bar{V} = 0$

т.к. $W \perp V$

3) $\dot{V}\cdot\bar{\tau}\cdot\bar{V} = 0 \Rightarrow \dot{V} = 0 \Rightarrow W = \frac{V^2}{\rho}\bar{n} \Rightarrow \rho = \frac{V^2}{\alpha(\bar{v}\times\bar{r})}$

ОТВЕТ

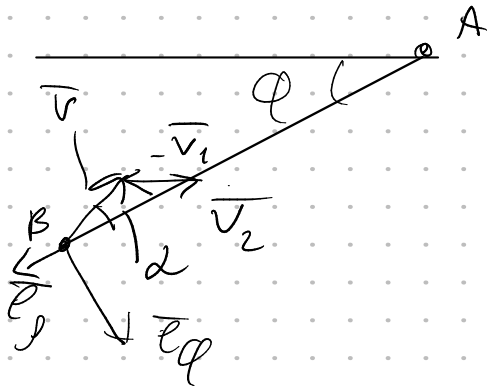
## N 1.31

**Дано:**

$V_1, V_2, \varphi_0 \neq 0$

**Найти:**

$r(\varphi)$ – ?




$\bar{r} = \begin{pmatrix} -\rho\cos\varphi \\ -\rho\sin\varphi \end{pmatrix}$ радиус вектор B в СО A

1) $V_\rho = \frac{d\rho}{dt} = -V_2 + V_1\cos\varphi$

$V_\varphi = \rho\cdot\frac{d\varphi}{dt} = V_1\sin\varphi$

$\Rightarrow \frac{d\rho}{\rho} = \frac{V_1\cos\varphi - V_2}{V_1\sin\varphi}d\varphi$

2) $\int_{\rho_0}^{\rho}\frac{d\rho}{\rho} = \underbrace{\int_{\varphi_0}^{\varphi} ctg\,\varphi\, d\varphi}_{①} - \frac{V_2}{V_1}\underbrace{\int_{\varphi_0}^{\varphi}\frac{d\varphi}{\sin\varphi}}_{②}$

① : $\int_{\varphi_0}^{\varphi} ctg\,\varphi\, d\varphi = \int_{\sin\varphi_0}^{\sin\varphi}\frac{d\sin\varphi}{\sin\varphi} = \ln\left(\frac{\sin\varphi}{\sin\varphi_0}\right)$

② : $\int_{\varphi_0}^{\varphi} \frac{d\varphi}{\sin\varphi} \underset{t=\operatorname{tg}\frac{\varphi}{2}}{=} \int_{\operatorname{tg}\frac{\varphi_0}{2}}^{\operatorname{tg}\frac{\varphi}{2}} \frac{2dt}{1+t^2} \cdot \frac{1+t^2}{2t} = \int_{\operatorname{tg}\frac{\varphi_0}{2}}^{\operatorname{tg}\frac{\varphi}{2}} \frac{dt}{t} = \ln\left(\frac{\operatorname{tg}\frac{\varphi}{2}}{\operatorname{tg}\frac{\varphi_0}{2}}\right)$

3) $\ln\left(\frac{\rho}{\rho_0}\right) = \ln\left(\frac{\sin\varphi}{\sin\varphi_0}\right) + \ln\left(\left(\frac{\operatorname{tg}\frac{\varphi_0}{2}}{\operatorname{tg}\frac{\varphi}{2}}\right)^{\frac{v_2}{v_1}}\right)$

$\frac{\rho}{\rho_0} = \frac{\sin\varphi}{\sin\varphi_0} \cdot \left(\frac{\operatorname{tg}\varphi_0/2}{\operatorname{tg}\varphi/2}\right)^{\frac{v_2}{v_1}} \Rightarrow$

$\Rightarrow r(\varphi) = \frac{\sin\varphi}{\sin\varphi_0} \cdot \left(\frac{\operatorname{tg}\varphi_0/2}{\operatorname{tg}\varphi/2}\right)^{\frac{v_2}{v_1}} \cdot \rho_0$ / ОТВЕТ

## Т7

Дано:

$\bar{r} = \begin{pmatrix} r\cos\varphi\cos\lambda \\ r\cos\varphi\sin\lambda \\ r\sin\varphi \end{pmatrix}$

$V = const$

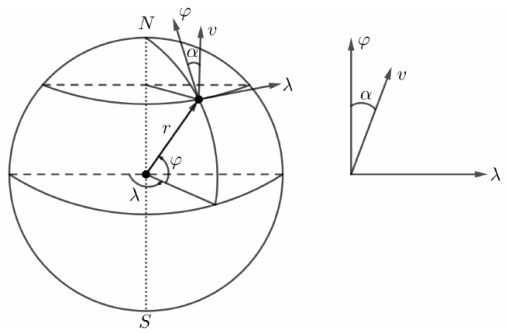
$\angle(\bar{e}_\varphi, \bar{V}) = \alpha$

Найти:

ур-е кривой - ?

$W_r$ - ? $W_\lambda$ - ? $W_\varphi$ - ?

$|W|$ - ? $\rho$ - ?




1) $V_\varphi = V\cos\alpha = \frac{d\varphi}{dt} r$

$V_\lambda = V\sin\alpha = \frac{d\lambda}{dt} \cdot r\cos\varphi$

$\Rightarrow \begin{cases} \frac{d\varphi}{dt} = \frac{V\cos\alpha}{r} \\ \frac{d\lambda}{dt} = \frac{V\sin\alpha}{r\cos\varphi} \end{cases} \Rightarrow \frac{d\varphi}{d\lambda} = \frac{\cos\alpha}{\sin\alpha} \cdot \cos\varphi$

2) $\frac{d\varphi}{\cos\varphi} = \frac{\cos\alpha}{\sin\alpha} d\lambda \Rightarrow \int_{\varphi_0}^{\varphi} \frac{d\varphi}{\cos\varphi} = \int_{-\varphi_0}^{\varphi} \frac{d\varphi}{\sin(\varphi+\frac{\pi}{2})} \underset{u=\varphi+\frac{\pi}{2}}{=} \int_{\varphi_0+\frac{\pi}{2}}^{\varphi+\frac{\pi}{2}} \frac{du}{\sin u} =$

$= \int_{\operatorname{tg}(\frac{\varphi_0}{2}+\frac{\pi}{4})}^{\operatorname{tg}(\frac{\varphi}{2}+\frac{\pi}{4})} \frac{dt}{t} = \ln\left(\frac{\operatorname{tg}(\varphi/2+\pi/4)}{\operatorname{tg}(\varphi_0/2+\pi/4)}\right)$

из пред. задачи

3) $\ln\left(\frac{\operatorname{tg}(\varphi/2+\pi/4)}{\operatorname{tg}(\varphi_0/2+\pi/2)}\right) = c\operatorname{tg}\alpha(\lambda-\lambda_0)$

4) $\operatorname{tg}(\varphi/2+\pi/4) = \operatorname{tg}(\varphi_0/2+\pi/4) \cdot \exp(c\operatorname{tg}\alpha \cdot (\lambda-\lambda_0))$ - ур-е кривой

5) $H_r = 1$ $\quad V_r = \dot r$

$H_\varphi = r \quad \Rightarrow \quad V_\varphi = r\dot\varphi$ $\quad \Rightarrow V^2 = \dot r^2 + r^2\dot\varphi^2 + r^2\cos^2\varphi\,\dot\lambda^2$

$H_\lambda = r\cos\varphi \quad V_\lambda = \dot\lambda r\cos\varphi$

6) $W_r = \ddot r - r\dot\varphi^2 - r\cos^2\varphi\cdot\dot\lambda^2 = -\dfrac{V^2}{r}$

7) $W_\varphi = \dfrac{1}{r}\left(r^2\ddot\varphi + 2r\dot r\dot\varphi + r^2\dot\lambda^2\cdot\cos\varphi\sin\varphi\right) = r\ddot\varphi + 2\dot r\dot\varphi + r\dot\lambda^2\cos\varphi\sin\varphi =$

$= \dfrac{\sin^2\varphi}{\cos^2\varphi}\cdot\dfrac{V^2}{r^2}\,r\cos\varphi\sin\varphi = \dfrac{V^2\sin^2\alpha\cdot tg\varphi}{r}$

8) $W_\lambda = \dfrac{r^2}{r\cos\varphi}\left(\ddot\lambda\cos^2\varphi - 2\cos\varphi\sin\varphi\,\dot\varphi\dot\lambda\right) = r\left(\ddot\lambda\cos\varphi - 2\sin\varphi\,\dot\varphi\dot\lambda\right) =$

$= r\left(\cos\varphi\,\dfrac{V\sin\alpha}{r}\cdot\dfrac{V\cos\alpha}{r\cos\alpha}\sin\varphi - 2\sin\varphi\cdot\dfrac{V\cos\alpha}{r}\cdot\dfrac{V\sin\alpha}{r\cos\varphi}\right) =$

$= \dfrac{1}{2r}V^2 tg\alpha\cdot\sin 2\alpha - \dfrac{1}{r}V^2 tg\varphi\sin 2\alpha = -\dfrac{V^2 tg\varphi\sin 2\alpha}{2r}$

9) $W = \sqrt{\dfrac{V^4}{r^2} + \dfrac{V^4\sin^4\alpha\, tg^2\varphi}{r^2} + \dfrac{V^4 tg^2\varphi\sin^2 2\alpha}{4r^2}} =$

$= \dfrac{V^2}{r}\sqrt{1 + tg^2\varphi\sin^2\alpha(\sin^2\alpha + \cos^2\alpha)} = \dfrac{V^2}{r}\sqrt{1 + \sin^2\alpha\, tg^2\varphi}$

10) $V = const \Rightarrow a = a_n = \dfrac{V^2}{\rho} \Rightarrow \rho = V^2\cdot\dfrac{r}{V^2}\cdot\dfrac{1}{\sqrt{1+\sin^2\alpha\, tg^2\varphi}} \Rightarrow$

$\Rightarrow \rho = \dfrac{r}{\sqrt{1 + \sin^2\alpha\cdot tg^2\varphi}}$

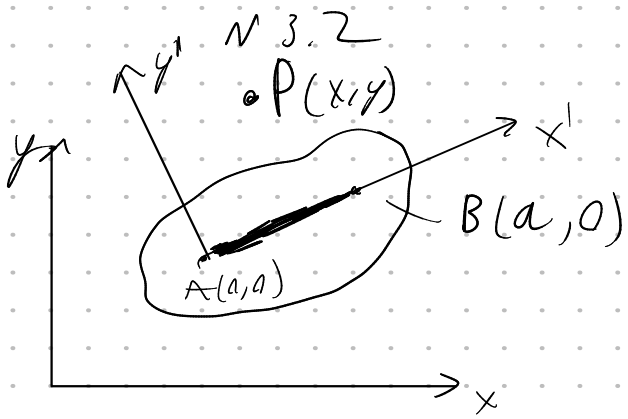
N 3.2

Дано:

$\dfrac{V_A}{V_B} = \lambda \neq 1$

$AB = a$

$P(x,y)$ — ?



1) В $A_{x'y'}$:

Т.к. P – мг. с., то $\frac{V_A}{AP} = \frac{V_B}{BP}$, $V_A = \lambda V_B$ $\Rightarrow \frac{\lambda}{AP} = \frac{1}{BP} \Rightarrow AP = \lambda BP$

2) $AP^2 = x^2 + y^2$

$BP^2 = (a-x)^2 + y^2$

$AP = \lambda BP$

$$\Rightarrow x^2 + y^2 = \lambda^2 (a-x)^2 + \lambda^2 y^2 \Rightarrow$$

$$x^2 + y^2 = \lambda^2 a^2 - 2\lambda^2 ax + \lambda^2 x^2 + \lambda^2 y^2$$

$$(1-\lambda^2)x^2 + 2\lambda^2 ax - \lambda^2 a^2 + (1-\lambda^2) y^2 = 0$$

$$x^2 + 2\frac{\lambda^2 a}{1-\lambda^2} x + y^2 = \frac{\lambda^2 a^2}{1-\lambda^2}$$

$$\left(x + \frac{\lambda^2 a}{1-\lambda^2}\right)^2 + y^2 = \frac{\lambda^2 a^2}{1-\lambda^2} + \frac{\lambda^4 a^2}{(1-\lambda^2)^2} =$$

$$= \frac{\lambda^2 a^2 - \lambda^4 a^2 + \lambda^4 a^2}{(1-\lambda^2)^2} = \left(\frac{\lambda a}{1-\lambda^2}\right)^2$$

Ответ: $\left(x + \frac{\lambda^2 a}{1-\lambda^2}\right)^2 + y^2 = \left(\frac{\lambda a}{1-\lambda^2}\right)^2$ в $A_{x'y'}$

**N3.20**

Дано:

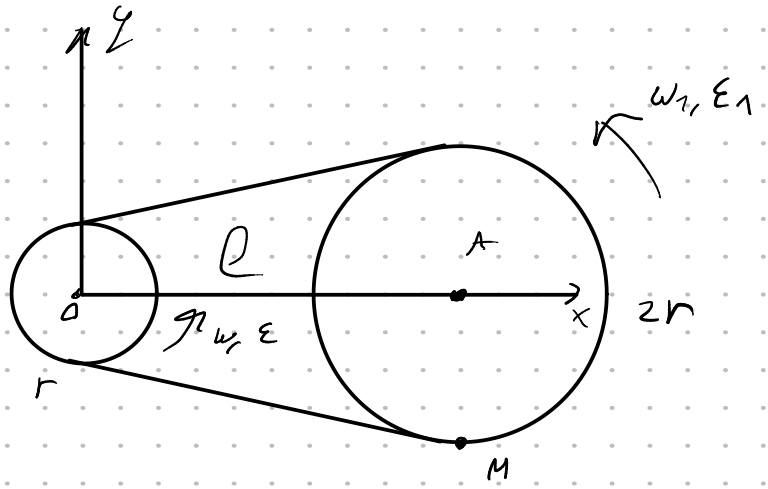
$l, r, R = 2r, \omega, \varepsilon$

$AM \perp OA$

Найти:

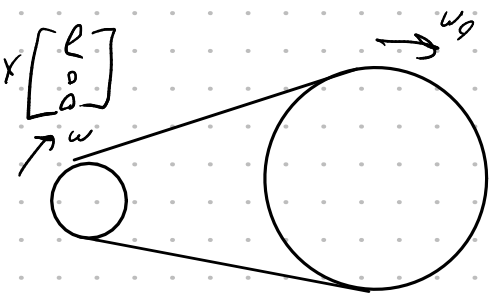
$V_M$ – ?

$W_M$ – ?




1) $\overline{V_M} = \overline{V_A} + \overline{\omega_1} \times \overline{AM}$

$\overline{V_A} = \overline{V_O} + \overline{\omega} \times \overline{OA} = \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} \times \begin{bmatrix} l \\ 0 \\ 0 \end{bmatrix}$




2) В (O) кривошипа: $\omega r = \omega_0 \cdot 2r \Rightarrow \omega_0 = \frac{1}{2}\omega$

В МСО: $\overline{\omega_1} = \begin{bmatrix} 0 \\ 0 \\ \omega_1 \end{bmatrix} \times \begin{bmatrix} l \\ -2r \\ 0 \end{bmatrix} = \begin{bmatrix} r\omega \\ 0 \\ 0 \end{bmatrix}$

$$\bar V_M = \begin{bmatrix} r\omega \\ \ell\omega \\ 0 \end{bmatrix} \Rightarrow \boxed{V_M = \omega\sqrt{\ell^2+r^2}}$$

ОТВЕТ

3) $\bar W_M = \bar W_A + \bar\varepsilon_1 \times \bar\rho - \omega_1^2 \bar\rho$

$$\bar W_A = \bar\varepsilon \times \bar\ell - \omega^2 \bar\ell = \begin{bmatrix} -\omega^2\ell \\ \ell\varepsilon \\ 0 \end{bmatrix}$$

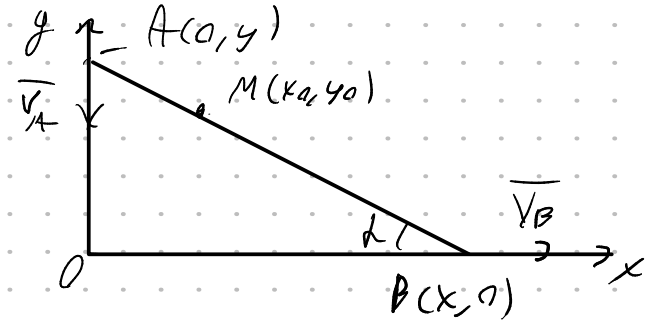
$$\Rightarrow \bar W_M = \begin{bmatrix} -\omega^2\ell \\ \varepsilon\ell \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \varepsilon/2 \end{bmatrix} \times \begin{bmatrix} 0 \\ -2r \\ 0 \end{bmatrix} - \frac{\omega^2}{\ell}\begin{bmatrix} 0 \\ -2r \\ 0 \end{bmatrix} = \boxed{\begin{bmatrix} -\omega^2\ell + \varepsilon r \\ \ell\varepsilon + \omega^2 r/2 \\ 0 \end{bmatrix}}$$

$$\boxed{W_M = \sqrt{(\omega^2\ell - \varepsilon r)^2 + \left(\ell\varepsilon + \frac{\omega^2 r}{2}\right)^2}}$$

**N 3.21**

$V_A = const$

Док-ть:
$W_M \perp O_y$
$W_M \sim x^{-3}$




1) $A$ – МЦУ, т.к. $\bar V_A = const$

$\bar{AM} \parallel \bar{AB} \Rightarrow \bar W_M \parallel \bar W_B \Rightarrow \boxed{\bar W_M \perp O_y}$

2) $AM$, $AB$ – „балки"

$$\bar W_M \cdot \bar i = \bar W_A \cdot \bar i + \bar\varepsilon \times \bar{AM} \cdot \bar i - \omega^2 \bar{AM} \cdot \bar i$$

$$W_M = 0 + \begin{bmatrix} 0 \\ 0 \\ \varepsilon \end{bmatrix} \times \begin{bmatrix} AM\cos\alpha \\ -AM\sin\alpha \\ 0 \end{bmatrix} \cdot \bar i - \omega^2 \cdot AM \cdot \cos\alpha = \varepsilon \cdot AM \cdot \sin\alpha - \omega^2 AM\cos\alpha$$

Т.к. $A$ – МЦУ, то $\operatorname{tg}\alpha = \frac{\varepsilon}{\omega^2} \Rightarrow W_A = \omega^2 AM \cdot \operatorname{tg}\alpha \cdot \sin\alpha - \omega^2 AM\cos\alpha =$

$$= \omega^2 AM\left(\frac{\sin^2\alpha}{\cos\alpha} - \cos\alpha\right) = \omega^2 AM \cdot \frac{1-2\cos^2\alpha}{\cos\alpha} = \omega^2 AM\left(\frac{1}{\cos x} - 2\cos\alpha\right)$$

$$\bar V_B \cdot \bar i = \bar V_A \cdot \bar i + \bar\omega \times \bar{AB} \cdot \bar i \Rightarrow V_B = \omega y$$

$$V_A \sin\alpha = V_B \cos\alpha \Rightarrow \operatorname{tg}\alpha = \frac{y}{x} \Rightarrow V_B = \frac{y}{x} V_A$$

$$\Rightarrow \omega = \frac{V_B}{y} = \frac{V_A}{x} = \frac{AM}{AB} \cdot \frac{V_A}{x_0}$$

$$\cos\alpha = \frac{x_0}{AM} = \frac{x}{AB} \Rightarrow x = \frac{AB}{AM} x_0 \Rightarrow W_M = \left(\frac{AM}{AB} \cdot \frac{V_A}{x_0}\right)^2 \cdot AM\left(\frac{AM}{x_0} - 2\frac{AM}{x_0}\right)$$

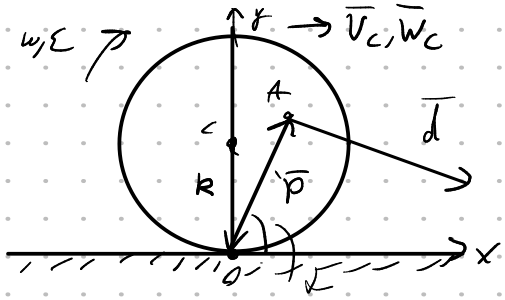
$$\boxed{W_M = \frac{AM^4}{AB^2} \cdot V_A^2\left(\frac{1}{x_0^3} - \frac{2}{x_0}\right) \sim \frac{1}{x_0^3}}$$

/ОТВЕТ

**N 3.25**

$V_c, W_c, R$

$W_n, W_\tau - ?$




Т.к. без проскальз.: $\omega = \frac{V_c}{R}$, $\varepsilon = \frac{W_c}{R}$

$$W_A = W_C + \bar\varepsilon \times \overline{CA} - \omega^2 \cdot \overline{CA} =$$

$$= \begin{bmatrix} \varepsilon R \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\varepsilon \end{bmatrix} \times \begin{bmatrix} x \\ y-R \\ 0 \end{bmatrix} - \omega^2 \cdot \begin{bmatrix} x \\ y-R \\ 0 \end{bmatrix} =$$

$$= \begin{bmatrix} y\varepsilon - \omega^2 x \\ \omega^2 R - x\varepsilon - \omega^2 y \\ 0 \end{bmatrix}$$

$$\bar\rho = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} : W_\rho = \frac{1}{\rho} \bar W_c \cdot \bar\rho = \frac{1}{\sqrt{x^2+y^2}} \left[ (y\varepsilon - \omega^2 x)\, x - (y\varepsilon + \omega^2 y - \omega^2 R)\, y \right] =$$

$$= -\frac{\omega^2}{\sqrt{x^2+y^2}} (x^2+y^2-yR) \Rightarrow$$

$$\Rightarrow \boxed{W_n = \frac{V_c^2 (x^2+y^2-yR)}{R^2 \sqrt{x^2+y^2}}} \qquad \bar d = \begin{bmatrix} y \\ -x \\ 0 \end{bmatrix} \perp \bar\rho$$

$$W_d = \frac{1}{d} \cdot \bar W_c \cdot \bar d = \frac{1}{d} \left[ (y\varepsilon - \omega^2 x)\, y + (x\varepsilon + \omega^2 y - \omega^2 R)\, x \right] = \frac{\varepsilon (x^2+y^2) - \omega^2 R x}{d} \Rightarrow$$

$$\Rightarrow \boxed{W_\tau = \frac{W_c (x^2+y^2) - V_c^2 x}{R \sqrt{x^2+y^2}}}$$

ОТВЕТ

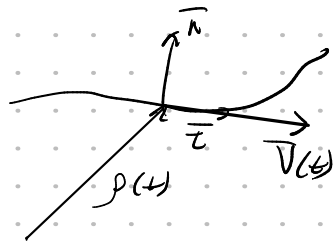
**N 3.36**

$\bar V(t), \rho(t)$

$\bar\varepsilon, \bar w - ?$

$$\boxed{\begin{aligned} &\omega = \frac{V}{\rho} \Rightarrow \bar w = \bar b \cdot \frac{|\bar V|}{\rho} = \bar b \cdot \frac{\bar V \cdot \bar\tau}{\rho} \\ &\bar\varepsilon = \dot{\bar w} = \bar b \left( \frac{\dot{\bar V} \cdot \bar\tau}{\rho} - \frac{\bar V \cdot \bar\tau\, \dot\rho}{\rho^2} \right) \end{aligned}}$$

ОТВЕТ




**N 4.4**

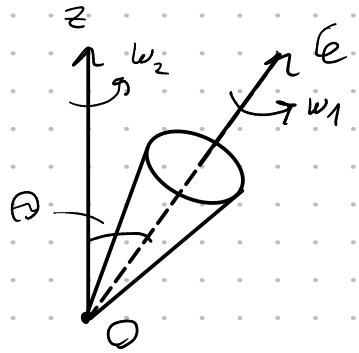
$\omega_1, \omega_2, \theta$

$\bar\omega, \bar\varepsilon - ?$

$$\bar\omega = \bar\omega_1 + \bar\omega_2 \Rightarrow \boxed{\omega = \sqrt{\omega_1^2 + \omega_2^2 + 2\omega_1\omega_2 \cos\theta}}$$

Введем $\bar e_{\omega_1} = \frac{\bar\omega_1}{\omega_1}$

ОТВЕТ

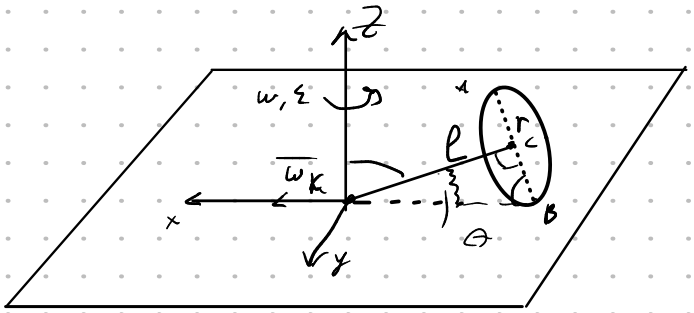



$$\bar\varepsilon = (\omega_1 \cdot \bar e_{\omega_1})^{\cdot} = \dot\omega_1 \bar e_{\omega_1} + \omega_1 \dot{\bar e}_{\omega_1} = \omega_1 \cdot \bar\omega_2 \times \bar e_{\omega_1} = \bar\omega_2 \times \bar\omega_1 \Rightarrow \boxed{\varepsilon = \omega_1 \cdot \omega_2 \cdot \sin\theta}$$

**N4.10**

$r,\ l = r\sqrt{3},\ \omega,\ \varepsilon$

$\omega_K,\ \varepsilon_K,\ V_A,\ V_B - ?$




1) $V_C = \overline{V_O} + \overline{\omega} \times \overline{OC} = \overline{V_B} + \overline{\omega_K} \times \overline{BC} \Rightarrow \omega \cdot OC \cdot \cos\theta = \omega_K \cdot BC \cdot \cos\theta$

$\omega_K = \omega \cdot \frac{OC}{BC} = \frac{l}{r}\omega \Rightarrow \boxed{\omega_K = \sqrt{3}\,\omega}$

2) $\overline{e_{\omega_K}} = \frac{\overline{\omega_K}}{\omega_K} \Rightarrow \overline{\varepsilon_K} = \dot{\omega}\,\overline{e_{\omega_K}} + \omega_K\,\dot{\overline{e}}_{\omega_K} = \sqrt{3}\,\varepsilon \cdot \overline{e_{\omega_K}} + \overline{\omega} \times \overline{\omega_K}$

$\varepsilon_K = \sqrt{(\sqrt{3}\cdot\varepsilon)^2 + (\omega\cdot\sqrt{3}\,\omega)^2} \Rightarrow \boxed{\varepsilon_K = \sqrt{3(\varepsilon^2 + \omega^4)}}$

3) $\overline{W_B} = \overline{W_O} + \overline{\varepsilon} \times \overline{OB} + \overline{\omega_K} \times (\overline{\omega_K} \times \overline{OB}) = \overline{\varepsilon_K} \times \overline{OB} = \begin{bmatrix} \sqrt{3}\,\varepsilon \\ \omega_K \cdot \omega \\ 0 \end{bmatrix} \times \begin{bmatrix} -r/\sin\theta \\ 0 \\ 0 \end{bmatrix} =$

$= \begin{bmatrix} 0 \\ 0 \\ \frac{\omega_K \cdot \omega \cdot r}{\sin\theta} \end{bmatrix} \xrightarrow[tg\,\theta = \frac{\sqrt{3}}{3}]{} \boxed{W_B = 2\sqrt{3}\,\omega^2 r}$

4) $\overline{W_A} = \underset{①}{\overline{W_B}} + \underset{②}{\overline{\varepsilon_K} \times \overline{BA}} + \underset{③}{\overline{\omega_K} \times (\overline{\omega_K} \times \overline{BA})} = \begin{bmatrix} 0 \\ 0 \\ 2\sqrt{3}\,\omega^2 r \end{bmatrix} + \begin{bmatrix} 3\omega^2 r \\ -3\varepsilon r \\ -\sqrt{3}\,\omega^2 r \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -3\sqrt{3}\,\omega^2 r \end{bmatrix} \ominus$

① $\begin{bmatrix} \sqrt{3}\,\varepsilon \\ \sqrt{3}\,\omega^2 \\ 0 \end{bmatrix} \times \begin{bmatrix} 2r\sin\theta \\ 0 \\ 2r\cos\theta \end{bmatrix} = \begin{bmatrix} 3\omega^2 r \\ -3\varepsilon r \\ -\sqrt{3}\,\omega^2 r \end{bmatrix}$

② $\begin{bmatrix} \sqrt{3}\,\omega \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 2r\sin\theta \\ 0 \\ 2r\cos\theta \end{bmatrix} = \begin{bmatrix} 0 \\ -3\omega r \\ 0 \end{bmatrix}$

③ $\begin{bmatrix} \sqrt{3}\,\omega \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ -3\omega r \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -3\sqrt{3}\,\omega^2 r \end{bmatrix}$

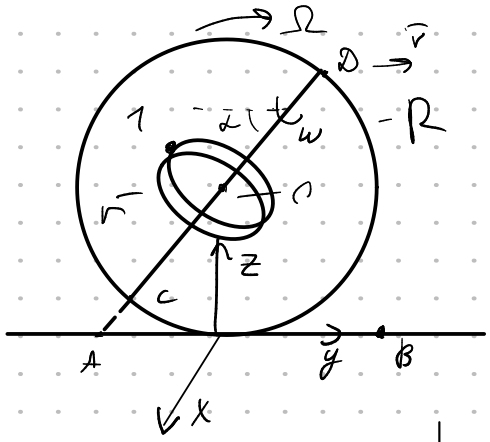
$\ominus \begin{bmatrix} 3\omega^2 r \\ -3\varepsilon r \\ -2\sqrt{3}\,\omega^2 r \end{bmatrix} \Rightarrow \boxed{W_A = r\sqrt{9\varepsilon^2 + 21\omega^4}}$

$\boxed{\phantom{xx}}$ – ОТВЕТ

## N 4.12

$\bar{V}, R, r, \alpha$

$V_1, W_1 - ?$




1) $\bar{\omega}_A = \bar{\omega} + \bar{\Omega}$

$\Omega = \frac{V}{R}$ т.к. чист качение $\Rightarrow \bar{\Omega} = \begin{bmatrix} -\frac{V}{R} \\ 0 \\ 0 \end{bmatrix} \Rightarrow$

$= \bar{\omega}_A = \begin{bmatrix} -V/R \\ \omega\cos\alpha \\ \omega\sin\alpha \end{bmatrix}$

$$\bar{V}_1 = \bar{V} + \bar{\omega}_a \times \bar{O1} = \begin{bmatrix} 0 \\ V \\ 0 \end{bmatrix} + \begin{bmatrix} -V/R \\ \omega\cos\alpha \\ \omega\sin\alpha \end{bmatrix} \times \begin{bmatrix} 0 \\ -r\sin\alpha \\ r\cos\alpha \end{bmatrix} = \begin{bmatrix} \omega r \\ V + V/R \cdot r\cos\alpha \\ V/R \cdot r\sin\alpha \end{bmatrix}$$

$$V_1^2 = \omega^2 r^2 + \left(V + \frac{V}{R} r\cos\alpha\right)^2 + V^2\left(\frac{r}{R}\right)^2 \sin^2\alpha = \omega^2 r^2 + V^2 + 2V^2 \frac{r}{R}\cos\alpha + V^2\frac{r^2}{R^2} \Rightarrow$$

$$\Rightarrow \boxed{V_1 = \sqrt{\omega^2 r^2 + V^2 + 2V^2 \frac{r}{R}\cos\alpha + \left(V\frac{r}{R}\right)^2}}$$ — ОТВЕТ

2) $\bar{\varepsilon}_a = \bar{\Omega} \times \bar{\omega}$

$$\boxed{\bar{W}_1} = \bar{W} + \bar{\varepsilon}_a \times \bar{O1} + \bar{\omega}_a \times (\bar{\omega}_a \times \bar{O1}) = \bar{\omega}_a \times (\bar{\omega}_a \times \bar{O1}) = \boxed{\begin{bmatrix} 0 \\ r\sin\alpha\left(\omega^2 + \frac{V^2}{R^2}\right) \\ -r\cos\alpha\left(\omega^2 + \frac{V^2}{R^2}\right) \end{bmatrix}}$$

## N 4.23

$\bar{W}_1, \bar{W}_2, X_1, X_2$

$\bar{W}(x) - ?$

$$\bar{W}_2 = \bar{W}_1 + \bar{\varepsilon}_1 \times \overline{X_1X_2} + \omega_1 \times (\omega_1 \times \overline{X_1X_2}) \Rightarrow$$

$$\Rightarrow \bar{W}_2 - \bar{W}_1 = \bar{\varepsilon}_1 \times \overline{X_1X_2} + \bar{\omega}_1 \times (\bar{\omega}_1 \times \overline{X_1X_2})$$

$$\overline{X_1X_2} = k \cdot \overline{X_1X} \Rightarrow k = \frac{X_2 - X_1}{X - X_1}$$

$$\bar{W} = \bar{W}_1 + \bar{\varepsilon}_1 \times \overline{X_1X_2} + \bar{\omega}_1(\bar{\omega}_1 \times \overline{X_1X}) = \bar{W}_1 + \frac{1}{k}\left[\bar{\varepsilon} \times \overline{X_1X_2} + \omega_1 \times (\omega_1 \times \overline{X_1X_2})\right] =$$

$$= \bar{W}_1 + \frac{1}{k}(\bar{W}_2 - \bar{W}_1) \Rightarrow$$

$$\Rightarrow \boxed{\bar{W} = \frac{X_2 - X_1}{X_2 - X_1}\bar{W}_1 + \frac{X - X_1}{X_2 - X_1}\bar{W}_2}$$ — ОТВЕТ

## № 4.30

$\bar{\omega}, \bar{\varepsilon}$

Доказ:

$\left.\begin{array}{l}\bar{W}_{вр} = \bar{W}_t \\ \bar{W}_{ос} = \bar{W}_n\end{array}\right| \Leftrightarrow \bar{\varepsilon}, \bar{\omega}, \bar{r}$ – компланар.

$\bar{W}_{вр} = \bar{\varepsilon} \times \bar{r}$, $\bar{W}_t = \dot{\bar{v}} \cdot \bar{\tau} = ([\bar{\varepsilon} \times \bar{r}] \cdot \bar{\tau}) \cdot \bar{\tau}$

$\bar{W}_{вр} = \bar{W}_t \Leftrightarrow \bar{\varepsilon} \times \bar{r} = ([\bar{\varepsilon} \times \bar{r}] \cdot [\bar{\omega} \times \bar{r}]) \cdot \dfrac{\bar{\omega} \times \bar{r}}{|\bar{\omega} \times \bar{r}|^2}$ $\Big|$ $\cdot\, \bar{\varepsilon}$ слева

$\bar{\varepsilon} \cdot (\bar{\varepsilon} \times \bar{r}) = \bar{\varepsilon} \cdot [\bar{\omega} \times \bar{r}] \cdot ([\bar{\varepsilon} \times \bar{r}] \cdot [\bar{\omega} \times \bar{r}]) \cdot \dfrac{1}{|\bar{\omega} \times \bar{r}|^2}$

$\bar{\varepsilon} \cdot (\bar{\varepsilon} \times \bar{r}) = 0 \Rightarrow \bar{W}_{вр} = \bar{W}_t \Leftrightarrow \bar{\varepsilon} \cdot [\bar{\omega} \times \bar{r}] = 0 \Leftrightarrow \bar{\varepsilon}, \bar{\omega}, \bar{r}$ – компл.

$\bar{W} = \bar{W}_{вр} + \bar{W}_{ос} = \bar{W}_t + \bar{W}_n = \bar{W}_{вр} + \bar{W}_n \Rightarrow$ $\boxed{\bar{W}_n = \bar{W}_{ос} \text{ – ч.т.д.}}$

## № 4.56

Найдем собств вектор $\bar{u}$

$A\bar{u} = \bar{u}$

$\det(A\bar{u} - \bar{u}) = 0 \Rightarrow \lambda^3 - \lambda^2 \operatorname{tr} A + \lambda \operatorname{tr} A - 1 = 0$

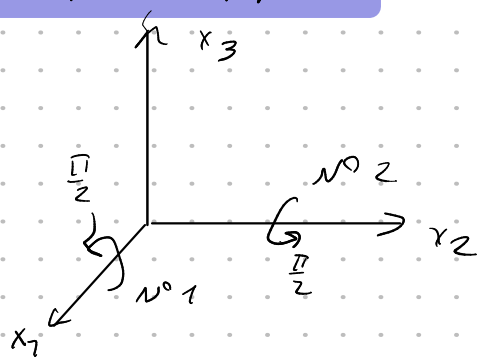
$\lambda_1 = 1 \Rightarrow \bar{u}$ – остается неизм. при повороте тела $\Rightarrow$

$\Rightarrow$ поворот вокруг $\bar{u}$

## Т2

$\frac{\pi}{2}$ отн $x_1$, $\frac{\pi}{2}$ отн $x_2$, $x$ и $\xi$ – совп. в $t = 0$

$A$ – ? ; углы Эйлера




1) $A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

$A_2 = \begin{bmatrix} \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -\sin\varphi & 0 & \cos\varphi \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$\Rightarrow A = A_2 \times A_1 = \boxed{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}}$

2) $\psi = 0$, $\theta = \frac{\pi}{2}$, $\varphi = \frac{\pi}{2}$

$\Rightarrow \Lambda = \Lambda_\theta \circ \Lambda_\varphi = (\cos\frac{\pi}{4} + \bar{i}\sin\frac{\pi}{4}) \circ (\cos\frac{\pi}{4} - \bar{k}\sin\frac{\pi}{4}) =$

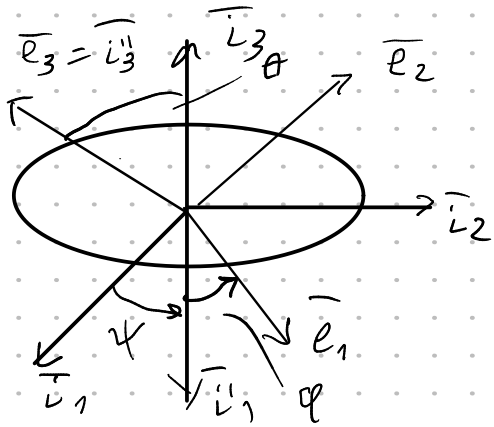
$$= \frac{1}{2}(1+\bar{i})\circ(1-\bar{k}) = \frac{1}{2}(1-\bar{k}+\bar{i}+\bar{j}) \Rightarrow \bar{e} = \begin{bmatrix}1\\1\\-1\end{bmatrix} \Rightarrow$$

$$\Rightarrow \alpha = 2\arccos\lambda_0 = 60^\circ \ldots$$

**№ 4.66**

$$\bar{i}_3 \xrightarrow[\psi]{} \bar{i}_1' \xrightarrow[\theta]{} \bar{i}_3'' \xrightarrow[\varphi]{}$$

Напр. вос. оси коорд. пов.




$$\Lambda_\psi = \overset{c_1}{\cos\frac{\psi}{2}} + \bar{i}_3 \overset{s_1}{\sin\frac{\psi}{2}}$$

$$\Lambda_\theta = \overset{c_2}{\cos\frac{\theta}{2}} + \bar{i}_1' \overset{s_2}{\sin\frac{\theta}{2}}$$

$$\Lambda_\varphi = \overset{c_3}{\cos\frac{\varphi}{2}} + \bar{i}_3'' \overset{s_3}{\sin\frac{\varphi}{2}}$$

Углы Эйлера $\Rightarrow$ пассивная т.з.р.

$$\Lambda = \Lambda_\psi \circ \Lambda_\theta \circ \Lambda_\varphi$$

Штрихи можно убрать т.к. кват. поворота имеет один. коорд в $i$ $i'$ и $i''$

$$\Lambda_\psi \circ \Lambda_\theta = c_1c_2 + c_1s_2\bar{i}_1 + c_2s_1\bar{i}_3 + s_1s_2\bar{i}_2$$

$$\Lambda = c_1c_2c_3 - c_2s_1s_3 + c_1c_2s_3\bar{i}_3 + c_3c_1s_2\bar{i}_1 + c_3c_2s_1\bar{i}_3 + c_3s_1s_2\bar{i}_2 +$$

$$+ s_2s_1s_3\bar{i}_1 - c_1s_2s_3\bar{i}_2 = c_2(c_1c_3 - s_1s_3) + \bar{i}_3\cdot c_2(c_1s_3 + s_1c_3) +$$

$$+ \bar{i}_2\cdot s_2(s_1c_3 - c_1s_3) + \bar{i}_1\cdot s_2(c_1c_3 + s_1s_3)$$

Компоненты кватерниона

$$\lambda_0 = \cos\frac{\theta}{2}\cdot\cos\frac{\psi+\varphi}{2} = c_+$$

$$\lambda_1 = \sin\frac{\theta}{2}\cdot\cos\frac{\psi-\varphi}{2} = c_-$$

Нормировка:

$$s_2^2\cdot c_-^2 + s_2^2\cdot s_-^2 + c_2^2\cdot s_+^2 =$$

$$= \ldots = 1 - c_2^2c_+^2 = 1 - \lambda_0$$

$\lambda_2 = \sin\frac{\theta}{2} \cdot \sin\frac{\psi - \varphi}{2} = s_-$

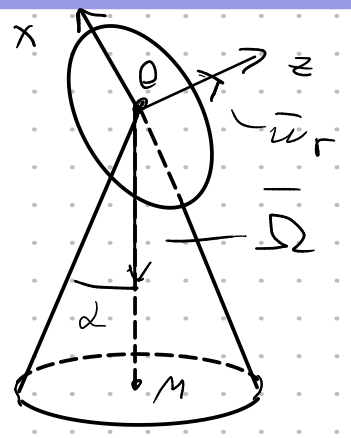
$\lambda_3 = \cos\frac{\theta}{2} \cdot \sin\frac{\psi + \varphi}{2} = s_+$

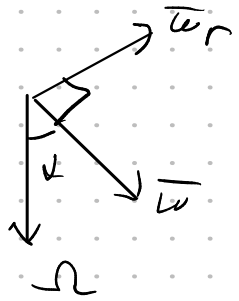
**Ответ:** $\alpha = \arccos\lambda_0$ ; $\gamma_i = \frac{\lambda_i}{\sqrt{1-\lambda_0^2}}$ $i = 1,2,3$

**№4.70**

$\Omega$, $t = \frac{\pi}{2}$, $\alpha = \frac{\pi}{4}$

$\alpha, \gamma_i$ – ?




$k = \Omega\sin\alpha = \frac{\Omega}{\sqrt{2}}$

$\omega_r = \omega = \frac{\Omega}{\sqrt{2}}$

пассивная т.зр

Поворот вокруг верт.:

$\varphi_1 = \Omega t = \pi$, $\Lambda_1 = \cos\frac{\pi}{2} - \frac{1}{\sqrt{2}}(\bar{i} + \bar{k})\sin\frac{\pi}{2} = -\frac{1}{\sqrt{2}}\bar{i} - \frac{1}{\sqrt{2}}\bar{k}$

Поворот вокруг $O_z$:

$\varphi_2 = \omega_r t = \frac{\pi}{\sqrt{2}}$, $\Lambda_2 = \underbrace{\cos\frac{\pi}{2\sqrt{2}}}_{c} + \bar{k}\underbrace{\sin\frac{\pi}{2\sqrt{2}}}_{s}$

$\Lambda = \Lambda_1 \circ \Lambda_2 = \frac{1}{\sqrt{2}}[+s - c\bar{i} - c\bar{k} + s\bar{j}]$

$\varphi = 2\arccos\lambda_0 \Rightarrow \cos\varphi = 2\cdot\frac{s^2}{2} - 1 = s^2 - 1 = -c^2 = -\cos^2\frac{\pi}{2\sqrt{2}}$

$\gamma_1 = \gamma_3 = \frac{\cos\frac{\pi}{2\sqrt{2}}}{\sqrt{1+\cos^2\frac{\pi}{2\sqrt{2}}}}$ $\gamma_2 = \frac{\sin\frac{\pi}{2\sqrt{2}}}{\sqrt{1+\cos^2\frac{\pi}{2\sqrt{2}}}}$

**№4.87**

$\Lambda = \underbrace{\cos\frac{\varphi(t)}{2}}_{c} + \bar{e}(t)\cdot\underbrace{\sin\frac{\varphi(t)}{2}}_{s}$

$\bar{\omega}$ – ?

$\bar{R} = \Lambda \circ \bar{r} \circ \tilde{\Lambda}$

$\bar{V} = \dot{\bar{r}} = \Lambda \circ \bar{r} \circ \dot{\tilde{\Lambda}} + \dot{\Lambda} \circ \bar{r} \circ \tilde{\Lambda} =$

$$= \dot{\Lambda} \circ \dot{\tilde{\Lambda}} \circ \bar{R} - \bar{R} \circ \dot{\Lambda} \circ \tilde{\Lambda}$$

$$|\Lambda| = 1 \Rightarrow \dot{\Lambda} \circ \tilde{\Lambda} + \Lambda \circ \dot{\tilde{\Lambda}} = 0, \text{ где } \dot{\Lambda} = \dot{\lambda}_0 + \sum_1^3 \dot{\lambda}_k \bar{i}_k$$

$$\overbrace{(\dot{\Lambda} \circ \tilde{\Lambda})} = (\Lambda \circ \dot{\tilde{\Lambda}}) = -(\dot{\Lambda} \circ \tilde{\Lambda}) \Rightarrow \text{sqal}(\dot{\Lambda} \circ \Lambda) = 0 \Rightarrow$$

$$\Rightarrow V = 2(\dot{\Lambda} \circ \tilde{\Lambda}) \times \bar{R} = \bar{\omega} \times \bar{R}$$

$$\boxed{\bar{\omega}} = (-s\dot{\varphi} + c \cdot \dot{\varphi}\bar{e} + 2s\dot{\bar{e}}) \circ (c - s\bar{e}) \underset{\text{sqal}=0}{=\!=\!=} s^2\dot{\varphi}\bar{e} + c^2\dot{\varphi}\bar{e} + 2sc\dot{\bar{e}} - 2s^2\dot{\bar{e}} \times \bar{e} =$$

$$= \boxed{\bar{e}\dot{\varphi} + \dot{\bar{e}}\sin\varphi + \bar{e} \times \dot{\bar{e}}(1 - \cos\varphi)}$$ /ОТВЕТ

T3

T4

## N2.9

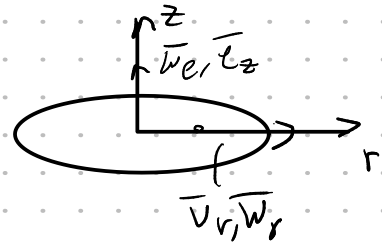
$\bar{\omega}_e = \bar{e}_z \omega_e$

$\bar{\varepsilon}_e = \bar{e}_z \varepsilon_e$

$\bar{W}_a = \bar{0}$

по оси Oz, прямл. Oxz

$\bar{V}_r, \bar{W}_r - ?$




$$\bar{W}_a = \bar{W}_r + \bar{W}_e + \bar{W}_k = \bar{0}$$

$$\bar{W}_e = \cancel{\bar{W}_0}^{\,0} + \bar{\varepsilon}_e \times \bar{\rho} + \bar{\omega}_e \times (\bar{\omega}_e \times \bar{\rho}) =$$

$$= \begin{bmatrix} 0 \\ 0 \\ \varepsilon_e \end{bmatrix} \times \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \omega_e \end{bmatrix} \times \left( \begin{bmatrix} 0 \\ 0 \\ \omega_e \end{bmatrix} \times \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} -r\omega_e^2 \\ r\varepsilon_e \\ 0 \end{bmatrix}$$

$$\bar{W}_k = 2\bar{\omega}_e \times \bar{V}_r = 2 \cdot \begin{bmatrix} 0 \\ 0 \\ \omega_e \end{bmatrix} \times \begin{bmatrix} V_r \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2V_r\omega_e \\ 0 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \omega_r \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -r\omega_e^2 \\ r\varepsilon_e \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2V_r\omega_e \\ 0 \end{bmatrix} = \bar{0} \Rightarrow$$

$$\boxed{\begin{aligned} \bar{W}_r &= \bar{e}_r \omega^2 r \\ \bar{V}_r &= -\bar{e}_r \frac{\varepsilon_e r}{2\omega_e} \end{aligned}}$$

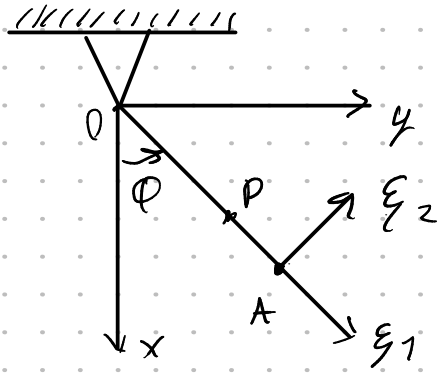
/ОТВЕТ

## N2.16

$\varphi = \varphi_0 \sin(\omega t)$

$OP = \frac{1}{2} a t^2$

$V_a, W_a - ?$




Введем $O\xi$, связ со стержнем

$$\bar{V}_a = \bar{V}_r + \bar{V}_e$$

$$\bar{V}_r = \begin{bmatrix} a\xi \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{V}_e = \cancel{\bar{V}_0}^{\,0} + \bar{\omega} \times \overline{OP} = \begin{bmatrix} 0 \\ 0 \\ \dot{\varphi} \end{bmatrix} \times \begin{bmatrix} OP \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ OP \cdot \dot{\varphi} \\ 0 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \bar{V}_a = \begin{bmatrix} at \\ \frac{1}{2} a t^2 \dot{\varphi} \\ 0 \end{bmatrix} \Rightarrow \boxed{V_a = \frac{at}{2} \sqrt{4 + \varphi_0^2 \omega^2 \cos^2(\omega t)}}$$

ОТВЕТ 1

$$\bar{W}_a = \bar{W}_r + \bar{W}_e + \bar{W}_k$$

$$\bar{W}_r = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}$$

$$W_e = \overline{W}_0 + \overline{\varepsilon} \times \overline{OP} + \overline{\omega}_e \times (\overline{\omega}_e \times \overline{OP}) = \begin{bmatrix} 0 \\ 0 \\ \ddot{\varphi} \end{bmatrix} \times \begin{bmatrix} OP \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\varphi} \end{bmatrix} \times \begin{bmatrix} 0 \\ OP \cdot \dot{\varphi} \\ 0 \end{bmatrix} =$$

$$= \begin{bmatrix} -\dot{\varphi}^2 \, OP \\ \ddot{\varphi} \, OP \\ 0 \end{bmatrix}$$

$$\overline{W}_k = 2\overline{\omega} \times \overline{V}_r = 2\begin{bmatrix} 0 \\ 0 \\ \dot{\varphi} \end{bmatrix} \times \begin{bmatrix} at \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2\dot{\varphi} at \\ 0 \end{bmatrix}$$

---

$$\overline{W}_a = \begin{bmatrix} a - \dot{\varphi}^2 OP \\ \ddot{\varphi} OP + 2\dot{\varphi} at \\ 0 \end{bmatrix} \Rightarrow W_a^2 = \left(a - \frac{1}{2}\dot{\varphi}^2 at^2\right)^2 + \left(\frac{1}{2}\ddot{\varphi} + 2\dot{\varphi} at\right)^2 =$$

$$= \frac{a}{4}\left[\left(2 - [\varphi_0 \omega t \cdot \cos(\omega t)]^2\right)^2 + \left(4\varphi_0 \omega t \cos(\omega t) - \varphi_0 \omega^2 t^2 \sin(\omega t)\right)^2\right] \Rightarrow$$

$$\Rightarrow \boxed{W_a = \frac{a}{2}\sqrt{[2 - (\varphi_0 \omega t \cos(\omega t))]^2 + \varphi_0^2 \omega^2 t^2 [4\cos(\omega t) - \omega t \sin(\omega t)]^2}}$$

ОТВЕТ 2

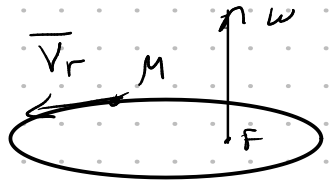
**№2.38**

$r = \dfrac{p}{1 + e\cos\varphi}$

$r^2\dot{\varphi} = C, \ \overline{\omega} = const$

$V_a(r) - ?$

$W_a(r) - ?$




Введем $a_{r\varphi z}$ сказ с эллипсом

1) $\overline{V}_a = \overline{V}_r + \overline{V}_e$

$$V_e = \cancel{\overline{V}_0}^{\,0} + \overline{\omega} \times \overline{r} = \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} \times \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ r\omega \\ 0 \end{bmatrix}$$

$$\dot{r} = \frac{\dot{\varphi} p e \sin\varphi}{(1 + e\cos\varphi)^2} = V_r$$

$$\dot{\varphi} = \frac{C}{r^2} \Rightarrow V_e = r\dot{\varphi}$$

$$r + re\cos\varphi = p \Rightarrow \cos\varphi = \frac{p - r}{re}$$

$$\dot{r}^2 = \left(\frac{C}{r^2} \cdot r^2 \frac{e\sin\varphi}{p}\right)^2 = \left(\frac{Ce}{p}\right)^2\left[1 - \left(\frac{p - r}{re}\right)^2\right] = C^2\left[\left(\frac{e}{p}\right)^2 - \left(\frac{p - r}{pr}\right)^2\right] =$$

$$= C^2\left[\left(\frac{e}{p}\right)^2 - \left(\frac{1}{r} - \frac{1}{p}\right)^2\right]$$

$$\overline{V}_a = \begin{bmatrix} \dot{r} \\ r\dot{\varphi} + r\omega \\ 0 \end{bmatrix} \Rightarrow \boxed{V_a = \sqrt{\left(\omega r + \frac{C}{r}\right)^2 + C^2\left[\left(\frac{e}{p}\right)^2 - \left(\frac{1}{r} - \frac{1}{p}\right)^2\right]}}$$ – ОТВЕТ 1

2) $\overline{W_a} = \overline{W_r} + \overline{W_e} + \overline{W_k}$

$\overline{W_e} = \cancel{\overline{W_0}}^{\,0} + \cancel{\overline{\varepsilon_0} \times \overline{r}}^{\,0} + \overline{\omega} \times (\overline{\omega} \times \overline{r}) = \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} \times \begin{bmatrix} 0 \\ r\omega \\ 0 \end{bmatrix} = \begin{bmatrix} -\omega^2 r \\ 0 \\ 0 \end{bmatrix}$

$V^2 = \dot{r}^2 + r^2 \dot{\varphi}^2$

$W_r = \ddot{r} - r\dot{\varphi}^2$

По з-ну площадей: $W_\varphi = 0$

$\overline{W_k} = 2\overline{\omega} \times \overline{V_r} = 2\begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} \times \begin{bmatrix} \dot{r} \\ r\dot{\varphi} \\ 0 \end{bmatrix} = 2\omega \begin{bmatrix} -r\dot{\varphi} \\ \dot{r} \\ 0 \end{bmatrix}$

$2\dot{r}\ddot{r} = \left[-c^2\left(\frac{1}{r} - \frac{1}{\rho}\right)^2\right]' = 2c^2 \cdot \left(\frac{1}{r} - \frac{1}{\rho}\right) \cdot \frac{\dot{r}}{r^2}$

$\ddot{r} = c^2\left(\frac{1}{r} - \frac{1}{\rho}\right) \cdot \frac{1}{r^2}$

$W_r = c^2\left(\frac{1}{r} - \frac{1}{\rho}\right) \cdot \frac{1}{r^2} - r\frac{c^2}{r^4} = -\frac{c^2}{\rho r^2}$

$\overline{W_a} = \begin{bmatrix} W_r \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -\omega^2 r \\ 0 \\ 0 \end{bmatrix} + 2\omega \begin{bmatrix} -r\dot{\varphi} \\ \dot{r} \\ 0 \end{bmatrix}$

$W_a^2 = \left(2\frac{c^2}{r^3} + \omega^2 r + 2\omega r\dot{\varphi}\right)^2 + (2\omega\dot{r})^2$

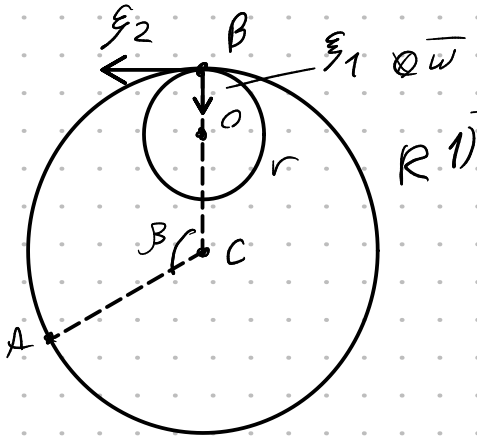
$$\boxed{W_a = \sqrt{\left(\omega^2 r + 2\omega\frac{c}{r} + \frac{c^2}{\rho r^2}\right)^2 + 4\omega^2 c^2\left[\left(\frac{c}{\rho}\right)^2 - \left(\frac{1}{r} - \frac{1}{\rho}\right)^2\right]}}$$

ОТВЕТ 2

**N 3.30**

$r, R, \beta, V = const$

$V_A, W_A$ -?




R 1) $\overline{V_A} = \overline{V_{Ae}} + \overline{V_{Ar}} = \overline{0} \Rightarrow \overline{V_{Ar}} = -\overline{V_{Ae}}$

$V_{Ae} = \cancel{\overline{V_B}}^{\,0} + \overline{\omega} \times \overline{BA}$

$\omega = \frac{V}{r}$

$AB = 2R\sin\frac{\beta}{2}$

$\Rightarrow$

$\Rightarrow \boxed{V_{Ar} = 2V\frac{R}{r}\sin\frac{\beta}{2}}$ – ОТВЕТ 1

2) $\overline{W_A} = \overline{W_e} + \overline{W_r} + \overline{W_k} = \overline{0}$

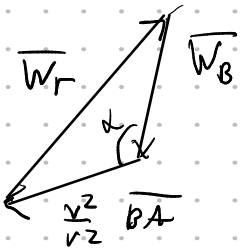
$\overline{W_k} = 2\overline{\omega} \times \overline{V_{Ar}} = -2\overline{\omega} \times (\overline{\omega} \times \overline{BA}) = 2\omega^2\overline{BA}$

$$\overline{W}_r = -\overline{W}_e - \overline{W}_k = -\left(\overline{W}_B + \frac{V^2}{r^2}\overline{BA}\right)$$

$$\overline{W}_B = \overline{W}_O - \omega^2 \overline{OB}$$

$$W_O = -\frac{V^2}{R-r}\frac{\overline{OB}}{OB}$$

$$\Rightarrow \overline{W}_B = \overline{BO}\cdot V^2\left(\frac{1}{(R-r)r} + \frac{1}{r^2}\right) = \frac{RV^2}{(R-r)r^2}\cdot\overline{BO}$$




$$\varphi = 90 - \frac{\beta}{2}$$

$$\alpha = 180 - \varphi = 90 + \frac{\beta}{2}$$

$$\cos\alpha = -\sin\frac{\beta}{2} = -s$$

$$W_r^2 = \left(\frac{RV^2}{(R-r)r^2}\cdot BO\right)^2 + \left(\frac{V^2}{r^2}\cdot BA\right)^2 + 2\sin\frac{\beta}{2}\frac{V^2}{r^2}BA\cdot\frac{Rr}{(R-r)r^2}V^2 =$$

$$= \frac{R^2V^4}{r^4(R-r)^2}\left[r^2 + 4(R-r)^2s^2 + 4s^2(R-r)\cdot r\right] \Rightarrow$$

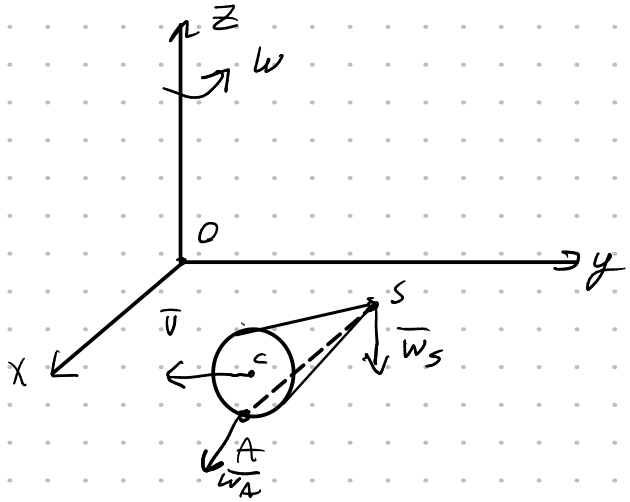
$$\Rightarrow \boxed{W_r = \frac{RV^2}{r^2(R-r)}\sqrt{r^2 + 4R(R-r)\sin^2\frac{\beta}{2}}}$$ / ОТВЕТ 2

**№4.25**

$\overline{\omega}(t)$, $V = const$

$h$, $2\beta$

$w_a$, $\varepsilon_a$ – ?




1) $\overline{w}_a = \overline{w} + \overline{w}_r$

$V_c = \overline{w}_A \times \overline{AC}$ $\Rightarrow$

$$\Rightarrow V = w_a \cdot SC \cdot \sin\beta \Rightarrow \boxed{w_A = \frac{V}{h\sin\beta} = \sqrt{\omega^2 + \left(\frac{V}{h\sin\beta}\right)^2}}$$ / ОТВЕТ 1

2) $\overline{\varepsilon}_a = \overline{\varepsilon}_e + \overline{\varepsilon}_r + \overline{\omega}_e \times \overline{w}_r$

$\overline{\varepsilon}_r = \cancel{\dot{\omega}_A}^{\,0}\overline{e} + \omega_A \cdot \dot{\overline{e}} = \overline{\omega}_S \times \overline{w}_A$

$\overline{V}_c = \overline{\omega}_S \times \overline{SC} \Rightarrow V = \omega_S \cdot SC \cdot \sin(90+\beta) \Rightarrow \omega_S = \frac{V}{h\cos\beta}$ $\Rightarrow$

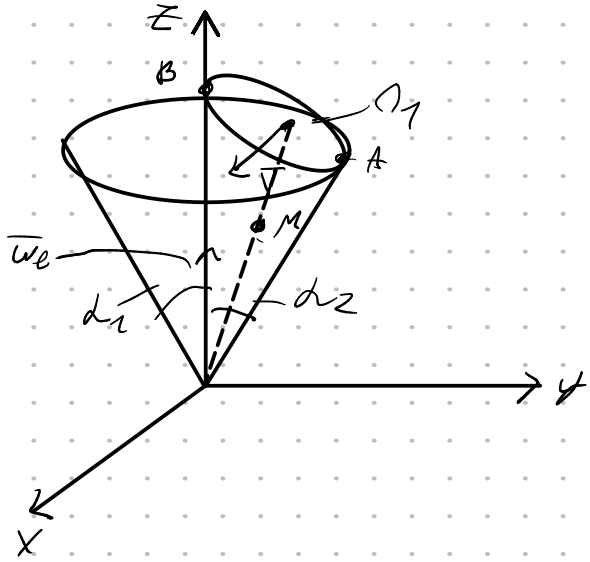
$$\Rightarrow \overline{\varepsilon}_A = \dot{\overline{\omega}} + \overline{\omega}_S \times \overline{w}_A + \overline{\omega} \times \overline{w}_A \Rightarrow$$

$$\Rightarrow \boxed{\varepsilon_A = \sqrt{\dot{\omega}^2 + \left[\frac{V}{h\sin\beta}\left(\omega - \frac{V}{h\cos\beta}\right)\right]^2}}$$ / ОТВЕТ 2

# T5

$\alpha_2 = 45^0$
$\alpha_1 = 90^0$
$OO_1 = l$
$V_{OM} = f(t)$
$OM = MO_1$

$V_A, W_A - ?$




1) $\bar{V}_A = \bar{V}_e + \bar{V}_r$

$\bar{V}_e = \frac{1}{2}\bar{V}$

$\bar{V}_r = \begin{bmatrix} 0 \\ \dot{f}_y \\ \dot{f}_z \end{bmatrix}$

$\Rightarrow \bar{V}_A = \begin{bmatrix} \frac{1}{2}V \\ \dot{f}_y \\ \dot{f}_z \end{bmatrix} \Rightarrow \boxed{V_A = \sqrt{\frac{1}{4}v^2 + (\dot{f}(t))^2}}$ ОТВЕТ 1

2) $\bar{V}_{O_1} = \begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix} = \bar{\omega}_e \times \overline{OO_1} = \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} \times \begin{bmatrix} 0 \\ OO_{1y} \\ OO_{1x} \end{bmatrix} = \begin{bmatrix} -\omega OO_{1y} \\ 0 \\ 0 \end{bmatrix}$

$s = \sin\frac{\alpha}{2}$
$c = \cos\frac{\alpha}{2}$

$\Rightarrow \omega = \frac{V}{ls}$

$\bar{W}_A = \bar{W}_e + \bar{W}_r + \bar{W}_k$

$\bar{W}_k = 2\bar{\omega}_e \times \bar{V}_r = \begin{bmatrix} -2\omega\dot{f}_y \\ 0 \\ 0 \end{bmatrix}$

$\bar{W}_r = \begin{bmatrix} 0 \\ \ddot{f}_y \\ \ddot{f}_z \end{bmatrix}$

$\bar{W}_e = \cancel{\bar{\varepsilon}_e \times OM}^{\,0} + \bar{\omega}_e \times (\bar{\omega}_e \times \overline{OM}) = \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} \times \begin{bmatrix} -\frac{1}{2}\omega OO_{1y} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{1}{2}\omega^2 OO_{1y} \\ 0 \end{bmatrix}$

$\Rightarrow$

$\Rightarrow \bar{W}_A = \begin{bmatrix} 0 + 0 - 2\omega\dot{f}_y \\ -\frac{1}{2}\omega^2 OO_{1y} + \ddot{f}_y + 0 \\ 0 + \ddot{f}_z + 0 \end{bmatrix}$

$W_A^2 = (\ddot{f}\cdot c)^2 + (\ddot{f}\cdot s - \frac{1}{2}\omega^2 ls)^2 + (2\omega\dot{f}\cdot s)^2 = \ddot{f}^2 - \ddot{f}\omega^2 ls^2 + \frac{1}{4}\omega^4 l^2 s^2 + 4\omega^2\dot{f}^2\cdot s^2$

$$W_A = \sqrt{\dot{f}^2 - \dot{f}\frac{v^2}{l} + \frac{1}{f^2}\frac{v^4}{2-\sqrt{2}} + \frac{4v^2 f^2}{l^2}}$$ /ОТВЕТ 2

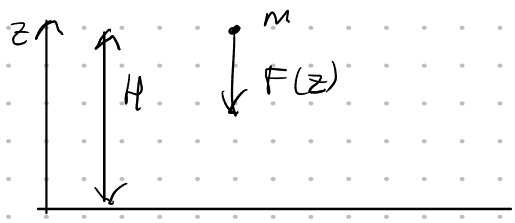
**N5.7**

$m, H, T, h$

$F(z) = \frac{\gamma m M}{(R+z)^2}$

$T - ?$

$\lim_{R\to\infty} T - ?$




1) $m\ddot{z} = -\frac{\gamma m M}{(R+z)^2}$

$x(z) = \dot{z}(t)$

$\ddot{z}(t) = x \cdot x'(z)$

$$x \cdot x' = -\frac{\gamma M}{(R+z)^2} \Rightarrow \int_0^v x\,dx = -\int_H^z \frac{\gamma M}{(R+z)^2}\,dz$$

$$\frac{v^2}{2} = \gamma M\left(\frac{1}{R+z} - \frac{1}{R+H}\right) = 2\gamma M \frac{1}{R+H}\cdot\left(\overbrace{\frac{R+H}{R+z}}^{a} - 1\right) = \frac{2\gamma M}{a}\cdot\left(\frac{a}{R+z} - 1\right)$$

$u = R+z$

$$dt = \sqrt{\frac{a}{2\gamma M}}\cdot\frac{du}{\sqrt{\frac{a}{u}-1}} = -\sqrt{\frac{a}{2\gamma M}}\cdot\frac{2a}{(v^2+1)^2}$$

$v = \sqrt{\frac{a}{u}-1}$

$u = \frac{a}{v^2+1}$

$du = -\frac{2av}{(v^2+1)^2}dv$

$v = tg\,t$

$tg^2 t + 1 = \frac{1}{\cos^2 t}$

$dv = \frac{dt}{\cos^2 t}$

$$\Rightarrow \sqrt{\frac{2\gamma M}{a^3}}\int_a^T dt = -2\int_{arctg\,v}^{0}\frac{\cos 2t + 1}{2}\,dt = -\left(\frac{\sin 2t}{2} + t\right)\Bigg|_{arctg\,v}^{0} =$$

$$= \frac{1}{2}\sin(2\,arctg\,v) + arctg\,v$$

$$\sin(2\,arctg\,v) = 2\sin(arctg\,v)\cdot\cos(arctg\,v) = 2\cdot\frac{v}{\sqrt{1+v^2}}\cdot\frac{1}{\sqrt{1+v^2}} =$$

$$= \frac{2v}{1+v^2} = \frac{2v}{1+v^2} = \frac{2uv}{a}$$

$q = arctg\sqrt{x-1}$

$$tg\,\varphi = \sqrt{x-1} \quad \Big| \Rightarrow \cos\varphi = \frac{1}{\sqrt{1+tg^2\varphi}} = \frac{1}{\sqrt{x}} \Rightarrow \varphi = \arccos\frac{1}{\sqrt{x}}$$

$$T = \sqrt{\frac{a^3}{2\gamma M}}\left(\frac{uV}{a} + arctg\,V\right) = \sqrt{\frac{a^3}{2\gamma M}}\cdot\left(\frac{u}{a}\sqrt{\frac{a}{u}-1} + arctg\,V\right) =$$

$$= \sqrt{\frac{a^3}{2\gamma M}}\left(\sqrt{\frac{u^2}{a^2}\left(\frac{a}{u}-1\right)} + \arccos\sqrt{\frac{u}{a}}\right)$$

$$\gamma M = gR^2 \Rightarrow \boxed{T = \sqrt{\frac{(R+M)^3}{2gR^2}}\left(\sqrt{\frac{(R+h)(M-h)}{(R+M)^2}} + \arccos\sqrt{\frac{R+h}{R+M}}\right)}$$

/ ОТВЕТ 1

2) $R \to \infty$: $\frac{R+h}{R+M} = 1 - \frac{M-h}{R+M} = 1 - b,\ b \to 0$

$$\sqrt{1-b} = 1 - \frac{b}{2} + o(b),\ b \to 0$$

$$\arccos(\sqrt{1-b}) = \arccos\left(1 - \frac{b}{2} + o(b)\right) = \arcsin\left(\sqrt{1-\left(1-\frac{b}{2}\right)^2 + o(b^2)}\right) =$$

$$= \arcsin\left(\sqrt{b + o(b)}\right) = \sqrt{b + o(b)},\ b \to 0$$

$$\boxed{\lim_{R\to\infty} T} = \lim\left[\sqrt{\frac{(R+M)^3}{2gR^2}\cdot\frac{(R+h)(M-h)}{(R+M)^2}} + \sqrt{\frac{(R+M)^3}{2gR^2}\cdot\frac{M-h}{R+M}}\right] =$$

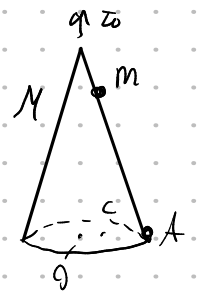
$$= \sqrt{\frac{M-h}{2g}} + \sqrt{\frac{M-h}{2g}} = \boxed{\sqrt{\frac{2(M-h)}{g}}}$$

/ ОТВЕТ 2

**№6.13**

$\omega_0,\ M = 4m$

$\omega - ?$




Рассм сист. колеса + м.т

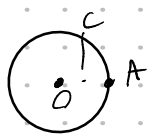
$R_z = (M+m)g - N = 0 \Rightarrow$

$\Rightarrow \dot{\bar{p}} = \bar{R}^e = \bar{0} \Rightarrow V_C = const$

$V_C(0) = 0 \Rightarrow \bar{V}_C = const$

$F^e \parallel$ оси вр. $\Rightarrow \dot{\bar{K}}_C = 0 \Rightarrow K_C = const \Rightarrow K_C^{нач} = K_C^{кон}$

$$\bar{K}_C^{нач} = J_C\omega_0 = \frac{3}{2}MR^2\bar{\omega}_0$$




$$\overline{OC} = \frac{mR}{m+M}$$

$$CA = OA - OC = \frac{MR}{m+M}$$

$$\bar{K}_C^{кон} = \bar{K}_C^{1} + \bar{K}_C^{2}$$

кон / м.т.

$\overline{K_C^1} = \overline{K_O} + \overline{Q} \times \overline{OC} = \frac{3}{10} M R^2 \omega + M \overline{V_O} \times \overline{OC}$

$\overline{V_O} = \cancel{\overline{V_C}}^{\,0} + \overline{\omega} \times \overline{CO} \Rightarrow \overline{V_O} \times \overline{OC} = -(\overline{\omega} \times \overline{OC}) \times \overline{OC} = \overline{OC} \times (\overline{\omega} \times \overline{OC}) = OC^2 \overline{\omega} - \overline{OC}(\cancelto{0}{\overline{OC} \cdot \overline{\omega}}) =$

$= OC^2 \cdot \overline{\omega}$

$\overline{K_C^1} = \left(\frac{3}{10} M R^2 + M OC^2\right) \overline{\omega} = M R^2 \left(\frac{3}{10} + \left(\frac{m}{m+M}\right)^2\right) \overline{\omega}$

$\overline{K_C^2} = m \overline{V_A} \times \overline{AC} = m \cdot (\overline{\omega} \times \overline{CA}) \times \overline{AC} = m \cdot \overline{CA} \times \overline{\omega} \times \overline{CA} = m\, CA^2 \overline{\omega} = m \left(\frac{MR}{m+M}\right)^2 \overline{\omega}$ $\Rightarrow$

$\Rightarrow K_C^{кон} = \overline{\omega} \cdot M R^2 \left(\frac{3}{10} + \frac{m^2}{(m+M)^2} + \frac{mM}{(m+M)^2}\right) = \overline{\omega} M R^2 \left(\frac{3}{10} + \frac{m}{m+M}\right) \Rightarrow$

$\Rightarrow \omega M R^2 \left(\frac{3}{10} + \frac{m}{m+M}\right) = \frac{3}{10} M R^2 \overline{\omega_0} \Rightarrow$

$\Rightarrow \omega_0 = \frac{3/10 + m/(m+M)}{3/10} \overline{\omega} = \frac{13m + 3M}{3(m+M)} \overline{\omega} = \frac{13 + 3k}{3(1+k)} \overline{\omega} \Rightarrow \boxed{\omega = \frac{3(1+k)}{13+3k} \omega_0}$

ОТВЕТ

**№637**

$T = \frac{1}{2} P, \alpha$

$f = \frac{1}{3}$

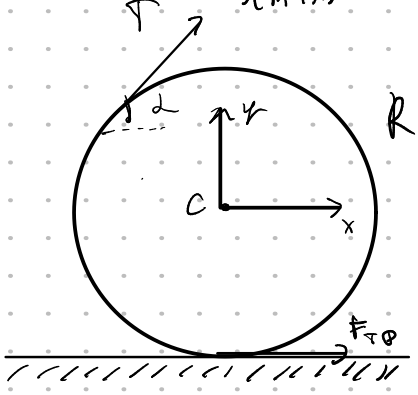
$\alpha^*$ – начало скольж

$\alpha^* - ?$

$\varepsilon, W_C$ при:

$\alpha < \alpha^*$

$\alpha > \alpha^*$




1) $m W_C = T \cos\alpha + F_{тр}$

$T \sin\alpha + N = P \Rightarrow N = \frac{P}{2}(2 - \sin\alpha)$

$m W_C = T \cos\alpha + F_{тр}$

$K_C = J_C \varepsilon = R(T - F_{тр})$

Без проск.: $W_C = \varepsilon R$, $F_{тр} \le f N \Rightarrow \frac{1}{2} m R^2 \varepsilon = R(T - F_{тр})$

$m \varepsilon R = T \cos\alpha + F_{тр} = 2T - 2F_{тр}$

$F_{тр} = \frac{1}{3} T (2 - \cos\alpha) \le f N = \frac{1}{3} \cdot \frac{P}{2}(2 - \sin\alpha) \Rightarrow \cos\alpha \le \sin\alpha$

при $\alpha = \alpha^*$: $\cos\alpha^* = \sin\alpha^* \Rightarrow \boxed{\alpha^* = 45^\circ}$ ОТВЕТ 1

2) $\alpha < \alpha^*$: $F_{тр} = \frac{1}{3} T (2 - \cos\alpha)$

$\boxed{W_C} = \frac{1}{m}(T \cos\alpha + F_{тр}) = \frac{P}{2m}\left(\cos\alpha + \frac{1}{3}(2 - \cos\alpha)\right) = \boxed{\frac{g}{3}(\cos\alpha + 1)}$

$\boxed{\varepsilon = \frac{W_C}{R} = \frac{g}{3R}(1 + \cos\alpha)}$ ОТВЕТ 2

3) $\alpha > \alpha^*$: $F_{тр} = f N = \frac{P}{6}(2 - \sin\alpha)$

$W_C = \frac{1}{m}(T \cos\alpha + F_{тр}) = \frac{P}{6m}(3\cos\alpha + 2 - \sin\alpha) \Rightarrow \boxed{W_C = \frac{g}{6}(3\cos\alpha + 2 - \sin\alpha)}$

$\varepsilon = \frac{2R}{mR^2}(T - F_{тр}) = \frac{2P}{2mR}\left(1 - f(2 - \sin\alpha)\right) = \boxed{\frac{g}{R}\left(1 - \frac{1}{3}(2 - \sin\alpha)\right) = \varepsilon}$

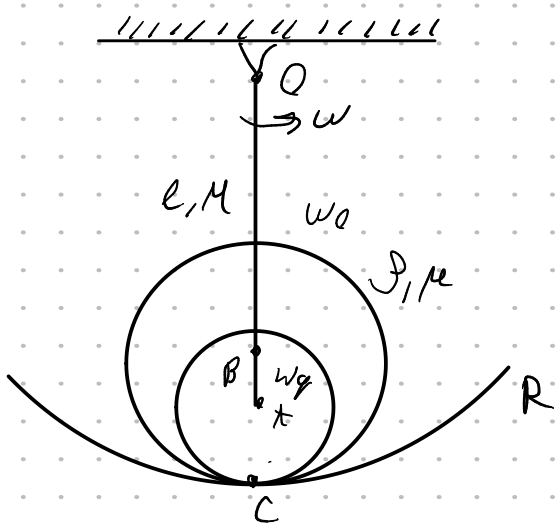
ОТВЕТ 3

## № 7.4

$l, M, \omega, r, m$

$R = l + r, \rho, \mu$

$T - ?$




B – центр обруча

$V_A = \omega_0 r = \omega l \Rightarrow$

$\Rightarrow \omega_1 = \frac{l}{r}\omega$

$V_B = \omega_0 \rho = \omega(l + r - \rho) \Rightarrow$

$\Rightarrow \omega_2 = \frac{l + r - \rho}{\rho}\omega$

$$T = T_{ст} + T_g + T_o = \frac{1}{2} J_{ст}\omega^2 + \frac{1}{2} V_A^2 m + \frac{1}{2} J_g \omega_g^2 + \frac{1}{2} V_B^2 \mu + \frac{1}{2} J_o \omega_o^2$$

$\bar{V}_A = \cancel{\bar{V}_O}^{\,0} + \bar{\omega} \times \bar{OA} \Rightarrow V_A = \omega l$

$\bar{V}_B = \cancel{\bar{V}_C}^{\,0} + \bar{\omega}_o \times \bar{CA} \Rightarrow V_B = \omega_o \rho = \omega(l + r - \rho)$

$\Rightarrow$

$$\Rightarrow T = \frac{1}{2}\cdot\frac{1}{3} M l^2 \omega^2 + \frac{1}{2}\omega^2 l^2 m + \frac{1}{2}\cdot\frac{1}{2} m r^2 \cdot \left(\frac{l}{r}\right)^2 \omega^2 + \frac{1}{2}\mu(l + r - \rho)^2\omega^2 +$$

$$+ \frac{1}{2}\mu\rho^2 \left(\frac{l + r - \rho}{\rho}\right)^2 \omega^2 = \frac{1}{2}\omega^2\left[\frac{1}{3}M\rho^2 + l^2 m + \frac{1}{2} m l^2 + \mu(l + r - \rho) + \mu(l + r - \rho)\right.$$

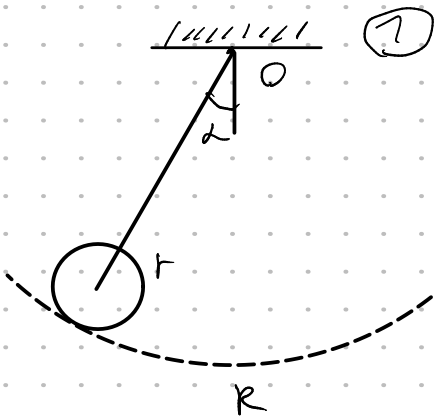
$$T = \frac{1}{2}\omega^2\left[\frac{1}{3}Ml^2 + \frac{3}{2}ml^2 + 2\mu(l + r - \rho)^2\right]$$ ответ

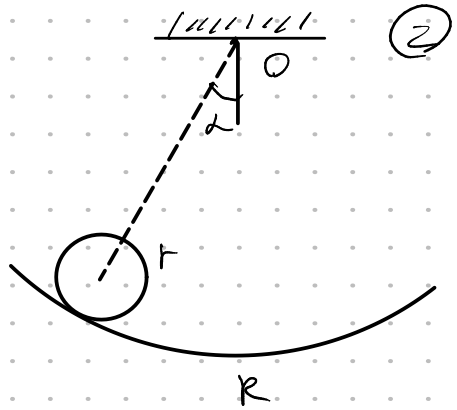
## № 7.28

$r, R$

1) $\frac{V_2}{V_1} - ?$

2) $V_1 = V_2$, $\frac{R}{r} - ?$




ЗСЭ: $\Pi_м = T_м$

$\Pi_м$ в обоих случаях один $\Rightarrow T_{м_1} = T_{м_2}$

1) $\Omega$ – угл. ск. в нижней точке $\Rightarrow \Omega = \frac{V_1}{R - r}$

$$T_{м_1} = \frac{1}{2}\left[J_м + m(R - r)^2\right]\Omega^2 = \frac{1}{2} m\left[\frac{2}{5} r^2 \cdot \frac{V_1^2}{(R - r)^2} + V_1^2\right] \Rightarrow T_{м_1} = \frac{1}{10} m V_1^2\left[2\cdot\left(\frac{r}{R + r}\right)^2 + 5\right]$$

② $\omega$ – угл. ск. в механ. точке $\Rightarrow \omega = \frac{V_2}{r}$

$$T_{M_2} = \frac{1}{2} m V_2^2 + \frac{1}{2} J_w \omega^2 = \frac{1}{2} m V_2^2 \left[1 + \frac{2}{5}\right] = \frac{7}{10} m V_2^2$$

1) $T_{M_1} = T_{M_2} \Rightarrow \left(\frac{V_2}{V_1}\right)^2 = \frac{1}{7} \cdot \frac{2r^2 + 5R^2 - 10Rr + 5r^2}{(R-r)^2} \Rightarrow \boxed{\frac{V_2}{V_1} = \frac{\sqrt{5R^2 - 10Rr + 7r^2}}{(R-r)\sqrt{7}}}$

/ОТВЕТ 1

2) $\frac{V_2}{V_1} = 1 \Rightarrow 5R^2 - 10Rr + 7r^2 = 7R^2 - MRr + 7r^2 \Rightarrow$

$\Rightarrow 2R = 4r \Rightarrow \boxed{R = 2r}$ /ОТВЕТ

## N 7.32

Закреп. C

$\Delta \bar{P}, \Delta T$ – ?

$\bar{P} = m\bar{V}_C \Rightarrow \boxed{\Delta \bar{P} = 0 - m\bar{V}_C}$ — ОТВЕТ

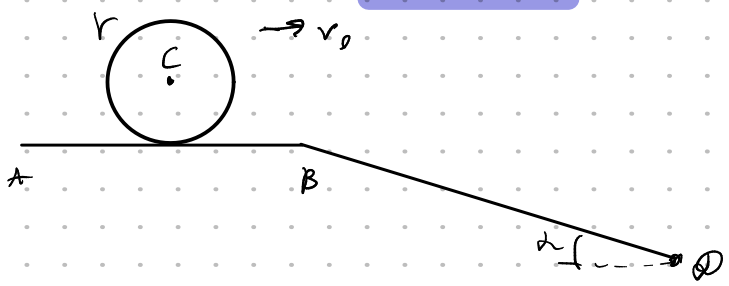
$T = T_C + T_{отм}$

$T_{отм} = const \Rightarrow \Delta T = 0 - \frac{mV_C}{2} \Rightarrow \boxed{2\Delta T = mV_C}$

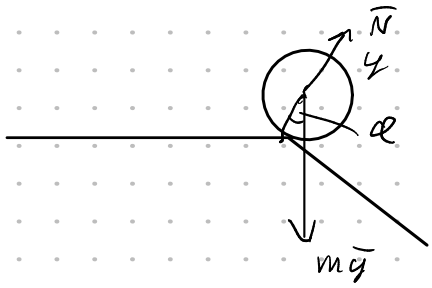
## N 7.92

$r, V_0$, без отр.

$\alpha$ – ?




в момент отрыва через B




$W_{C_0} = \frac{V_0^2}{r}$

$\bar{N} + m\bar{g} = m\bar{W}$

$N - mg\cos\varphi = -m\frac{V^2}{r}$

Усл. ск. $N = 0$: $\cos\varphi_0 = \frac{V^2}{rg} \Rightarrow \frac{V^2}{gr} \leq 1$

$mgr(1 - \cos\varphi_0) = \Delta T$ (1)

1) $T_1$ – по горизонт. плоск, $\omega_1 = \frac{V_0}{r}$

$$T_1 = \frac{1}{2} J_w \omega_1^2 + \frac{1}{2} m V_0^2 = \frac{1}{2} m\left(\frac{2}{5} r^2 \cdot \frac{V_0^2}{r^2} + V_0^2\right) = \frac{7}{10} m V_0^2$$

2) $T_2$ – время отн B, аналог $T_1$: $T_2 = \frac{7}{10} m V^2$

$V^2 = gr\cos\varphi_0 \Rightarrow T_2 = \frac{7}{10} mgr\cos\varphi_0$

Подставив в (1):

$mgr(1 - \cos\varphi_0) = \frac{7}{10} m(gr\cos\varphi_0 - V_0^2)$

$$gr - gr\cos\varphi_0 = \frac{7}{10}gr\cos\varphi_0 - \frac{7}{10}v_0^2 \Rightarrow \cos\varphi_0 = \frac{10}{17} + \frac{7}{17}\frac{v_0^2}{gr} \Rightarrow$$

$$\Rightarrow \boxed{0 \le \alpha \le \arccos\left(\frac{10}{17} + \frac{7}{17}\frac{v_0^2}{gr}\right), \quad \frac{v_0^2}{gr} \le 1} \quad /\text{ОТВЕТ}$$

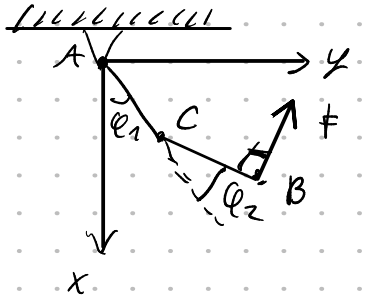
**N 7.59**

$AC = CB = l$

$\varphi_1, \varphi_2$

$F = const$

F-потенц. -?




$\cos\varphi_1 = c_1 \quad \sin\varphi_1 = s_1$

$\cos\varphi_2 = c_2 \quad \sin\varphi_2 = s_2$

$$\delta A = (\bar F \cdot \bar V_C)dt + ([CB \times \bar F]\cdot\dot{\bar\varphi}_2)dt$$

$$V_C = \cancel{V_A}^{\,0} + \bar\varphi_1 \times \overline{AC} = \begin{bmatrix} 0 \\ 0 \\ \dot\varphi \end{bmatrix} \times \begin{bmatrix} l c_1 \\ l s_1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\dot\varphi l s_1 \\ \dot\varphi l c_1 \\ 0 \end{bmatrix} \Rightarrow$$

$$F \cdot \bar V_C = F s_2 \dot\varphi l s$$

$$\Rightarrow \delta A = F l \dot\varphi c_{12} dt + F l \dot\varphi_2 dt = F l c_{12} d\varphi_1 + F l d\varphi_2$$

Если F-пот., то $\delta A = -dU$

$$\frac{dU}{d\varphi_1} = F l c_{12} \quad \frac{dU}{d\varphi_2} = F l, \quad \frac{\partial^2 U}{\partial\varphi_1 \partial\varphi_2} \ne \frac{\partial^2 U}{\partial\varphi_2\partial\varphi_1} = F l s_{12} \Rightarrow \nexists U \Rightarrow$$

$\Rightarrow$ **F-непот.** /ОТВЕТ

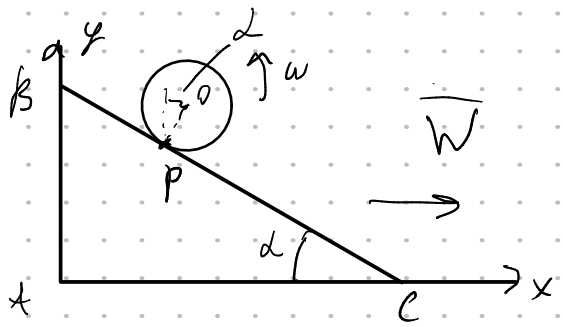
**N 9.8**

$\bar W = const$

$\alpha$, без скольж.

движ. вверх

W-?




В СО диска $m\bar W^r = \underbrace{\bar F - m\bar W^e}_{\bar J^e} \underbrace{- m\bar W^c}_{\bar J^K}$

$$\bar J^e = -m(\bar W + \bar\varepsilon \times \bar\rho - \omega^2\bar\rho) = -m\bar W$$

$$\bar J^K = -2m\bar\omega \times \bar V^r = \bar 0$$

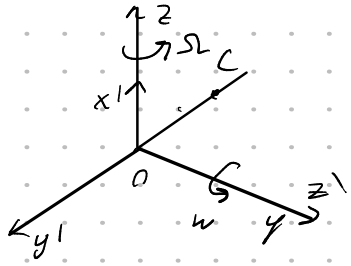
Т.к. $\bar J^e$ – сферопод., то $J^e$ проходит через т.O

$$\dot K_P = M_P^{J^e} + M_P^{mg} = mW \cdot r\sin(90-\alpha) - mgr\sin\alpha = 0 \Rightarrow \boxed{W = g\,\mathrm{ctg}\,\alpha} \quad /\text{ОТВЕТ}$$

## № 9.25

$\omega = const$
$\Omega = const$
$OC = s$, $f$
$\theta$ $t=0$ – верт

Найти диф. ур-ие движ кольца




$$m\overline{W^r} = m\bar g + \bar N + \bar F_{тр} + \bar J^e + \bar J^c$$

$$OC = s,\ \overline{V^r} = \dot s,\ \overline{W^r} = \ddot s$$

$$\overline{\omega^r} = \bar\omega,\ \overline{\omega^e} = \Omega,\ \overline{\omega^a} = \bar\omega + \bar\Omega = \begin{bmatrix} \Omega\cos\omega t \\ \Omega\sin\omega t \\ 0 \end{bmatrix} \text{ в } O_{x'}$$

$$\overline{\varepsilon^a} = \bar\Omega \times \bar\omega = \begin{bmatrix} \Omega\cos\omega t \\ \Omega\sin\omega t \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} = \begin{bmatrix} \omega\Omega\sin\omega t \\ -\omega\Omega\cos\omega t \\ 0 \end{bmatrix}$$

$$\overline{W^e} = \overline{W_0} + \overline{\varepsilon^a} \times \overline{OC} + \omega^a \times (\omega^a \times \overline{OC}) = \begin{pmatrix} \omega\Omega\sin\omega t \\ \omega\Omega\cos\omega t \\ 0 \end{pmatrix} \times \begin{pmatrix} s \\ 0 \\ 0 \end{pmatrix} +$$

$$+ \begin{pmatrix} \Omega\cos\omega t \\ \Omega\sin\omega t \\ \omega \end{pmatrix} \times \left( \begin{pmatrix} \Omega\cos\omega t \\ \Omega\sin\omega t \\ \omega \end{pmatrix} \times \begin{pmatrix} s \\ 0 \\ 0 \end{pmatrix} \right) = s \cdot \begin{bmatrix} -(\omega^2 + \Omega^2\sin^2\omega t) \\ \Omega^2\sin\omega t\cos\omega t \\ 2\omega\Omega\cos\omega t \end{bmatrix}$$

$$\overline{W^k} = 2\cdot\overline{\omega^e} \times \overline{V^r} = 2\begin{bmatrix} \Omega\cos\omega t \\ \Omega\sin\omega t \\ \omega \end{bmatrix} \times \begin{bmatrix} \dot s \\ 0 \\ 0 \end{bmatrix} = 2\dot s \begin{bmatrix} 0 \\ \omega \\ -\Omega\sin\omega t \end{bmatrix} \qquad \bar g = -g\begin{bmatrix} \cos\omega t \\ \sin\omega t \\ 0 \end{bmatrix}$$

$$\overline{F_{тр}} = -fN\begin{bmatrix} \operatorname{sign}(\dot s) \\ 0 \\ 0 \end{bmatrix}$$

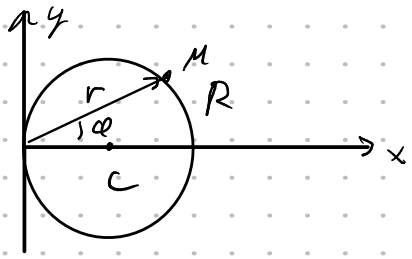
N уравновешивает проекции сил на y' и z'

$$N^2 = \left[m(-s\Omega^2\sin\omega t + 2\dot s\omega + g\sin\omega t)\right]^2 + \left[2m\Omega(s\omega\cos\omega t + \dot s\sin\omega t)\right]^2$$

2 ЗН на x': $\ddot s = -g\cos\omega t + s(\omega^2 + \Omega^2\sin^2\omega t) - \frac{1}{m}fN\operatorname{sign}(\dot s)$ /ОТВЕТ

## № 8.11

$F = -\frac{\alpha m}{r^5}$
$\alpha = const$
$r = 2R\cos\varphi$




$K_0$ – ?

(1) ф-ла Бине: $u'' + u = -\frac{F(u)}{mC^2u^2}$, где $u = \frac{1}{r} = \frac{1}{2R\cos\varphi}$, $C = \frac{K_0}{m}$

$$u'_\varphi = u'_r \cdot r'_\varphi = -\frac{1}{r^2}(-2R\sin\varphi) = 2Ru^2\sin\varphi$$

$$u''_{\varphi\varphi} = 2R(2u\cdot 2Ru^2\sin^2\varphi + u^2\cos\varphi) = 8R^2u^3 - 8R^2u^3\cdot\frac{1}{4R^2u^2} +$$

$$\cos\varphi = \frac{1}{2Ru} \qquad \sin^2\varphi = 1 - \frac{1}{4R^2u^2}$$

$$+2Ru^2 \cdot \frac{1}{2Ru} = 8R^2u^3 - u$$

Подставим в (1): $8R^2u^3 - u + u = -\frac{F(u)}{mc^2u^2} \Rightarrow F(u) = -8R^2c^2u^5m$

$$F(r) = -\frac{8R^2c^2m}{r^5} \Rightarrow$$ **только для сил вида $F = \frac{-\alpha m}{r^5}$**

/ОТВЕТ

$$-\frac{8R^2c^2m}{r^5} = -\frac{\alpha m}{r^5} \Rightarrow \alpha = 8R^2c^2 = 8R^2\frac{K_0^2}{m^2} \Rightarrow \boxed{K_0 = \frac{m}{R}\sqrt{\frac{\alpha}{8}}}$$

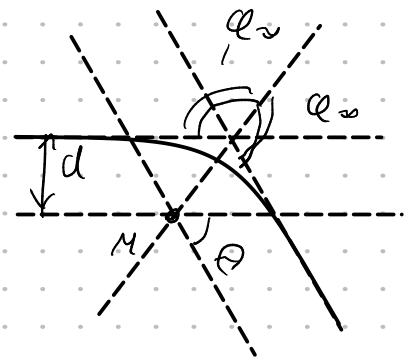
№8.22

$M \gg m_1$

$V_\infty, d$

гр-ие траект

$\theta$ - ?




$M \gg m \Rightarrow$ ракета не влияет на звезду

$r = \frac{p}{1+e\cos(\varphi+\varphi_0)}$, при $\varphi = 0$ - перицентр $\Rightarrow$

$\Rightarrow \varphi_0 = 0$

$$\boxed{r = \frac{p}{1+e\cos\varphi}}$$ - ОТВЕТ 1, $e > 1$, т.к. $V_\infty > 0$

$r \to \infty$ при $1 + e\cos\varphi \to 0 \Rightarrow \cos\varphi \to -\frac{1}{e} \Rightarrow \varphi_\infty = \arccos\left(-\frac{1}{e}\right)$

$2\varphi_\infty - \theta = \pi \Rightarrow \theta = 2\varphi_\infty - \pi = 2\arccos\left(-\frac{1}{e}\right) - \pi$

$\cos\theta = -\cos 2\varphi_\infty = 1 - 2\cos^2\varphi_\infty$

$K_\infty = dmV_\infty$, $E_\infty = \frac{mV_\infty^2}{2} = E_0$

$$e^2 = \frac{d^2m^3V_0^4}{m\alpha^2} = \frac{d^2V_\infty^4 + \gamma^2\mu^2}{\gamma^2\mu^2}$$

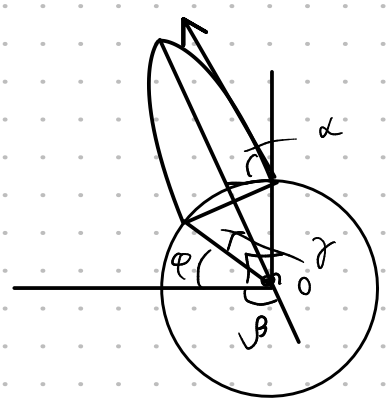
$$\cos\theta = 1 - 2\cdot\frac{\gamma^2\mu^2}{d^2V_\infty^4 + \gamma^2\mu^2} = \frac{d^2V_\infty^4 - \gamma^2\mu^2}{d^2V_\infty^4 + \gamma^2\mu^2}$$

$$\boxed{\theta = \arccos\left(\frac{d^2V_\infty^4 - \gamma^2\mu^2}{d^2V_\infty^4 + \gamma^2\mu^2}\right)}$$ /ОТВЕТ 2

**№8.24**

$\alpha, \varphi_0, R, e, p$

$\alpha(\varphi_0)$ - ?




$$r = \frac{p}{1+e\cos(\varphi+\beta)}$$

Выбираем $\beta$ так, чтобы при $\varphi = 0 \Rightarrow r = r_{min}$

$$\gamma = \left(\frac{\pi}{2} - \varphi\right)\cdot\frac{1}{2} = \frac{\pi}{4} - \frac{\varphi}{2} \Rightarrow \beta =$$

$$= \pi - \gamma + \varphi = \frac{3\pi}{4} - \frac{\varphi}{2}$$

(1) $r = \frac{p}{1+e\cos(\varphi+\beta)}$ => (2) $\dot r = \dot\varphi \cdot \frac{e p \sin(\varphi+\beta)}{(1+e\cos(\varphi+\beta))^2}$

При $\varphi = \frac{\pi}{2}$: (1) $R = \frac{p}{1-e\sin\beta}$ => $p = R(1-e\sin\beta)$

(2) $V_a \sin\alpha = -\frac{V_a}{R}\cos\alpha \cdot \frac{e p \cos\beta}{(1-e\sin\beta)^2}$ => $\sin\alpha = -\cos\alpha \cdot \frac{e\cos\beta}{1-e\sin\beta}$

$\sin\alpha - e\sin\beta\cdot\sin\alpha = -e\cos\beta\cos\alpha$ => $e\cos(\alpha+\beta) = -\sin\alpha$

$\cos(\alpha+\beta) = \cos\left(\alpha + \frac{3\pi}{4} - \frac{\varphi}{2}\right) = \cos\left(\frac{\pi}{4}+\alpha-\frac{\varphi}{2}+\frac{\pi}{2}\right) = -\sin\left(\frac{\pi}{4}+\alpha-\frac{\varphi}{2}\right)$

$$\boxed{e = \frac{\sin\alpha}{\sin\left(\frac{\pi}{4}+\alpha-\frac{\varphi}{2}\right)}}$$

$$\boxed{p} = R\left(1+\frac{\sin\alpha\sin\beta}{\cos(\alpha+\beta)}\right) = R\left(\frac{\cos\alpha\cdot\cos\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}\right) =$$

$$= R\left(\frac{1}{1-\operatorname{tg}\alpha\operatorname{tg}\beta}\right) = \boxed{\frac{R}{1+\operatorname{tg}\alpha\operatorname{tg}\left(\frac{\varphi}{2}+\frac{\pi}{4}\right)}}$$ / ОТВЕТ 1+2

Чтобы снаряд попал на широту $\varphi_0$, $0<e<1$

$e=1$: $\sin\left(\frac{\pi}{4}+\alpha-\frac{\varphi_0}{2}\right) = \sin\alpha$

$\sin\alpha - \sin\left(\alpha+\frac{\pi}{4}-\frac{\varphi_0}{2}\right) = 0$

$2\cos\left(\alpha+\frac{\pi}{8}-\frac{\varphi_0}{4}\right)\sin\left(\frac{\varphi_0}{4}-\frac{\pi}{8}\right) = 0$ => $\alpha+\frac{\pi}{8}-\frac{\varphi_0}{4} = \frac{\pi}{2}$ => $\alpha = \frac{3\pi}{8}+\frac{\varphi_0}{4}$ =>

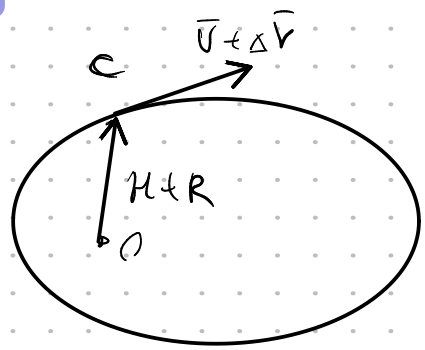
=> $\boxed{0<\alpha<\frac{3\pi}{8}+\frac{\varphi_0}{4}}$ / ОТВЕТ 3

№ 8.52

$m, e_a, p_0, \bar V$

$\Delta\bar\varphi = \lambda m\bar V, M, R$

$p, e$ - ?




$r = M+R$

$p_0 = \frac{c_0^2}{K} = \frac{r^2 V_\varphi^2}{K}$

$\Delta\varphi = \lambda m\bar V = m\Delta\bar V \Rightarrow \Delta\bar V = \lambda\bar V$

$C = r(1+\lambda)V_\varphi$ $\quad p = \frac{C^2}{K} = \frac{r^2(1+\lambda)^2 V_\varphi^2}{K}$ => $\boxed{p = p_0(1+\lambda)^2}$

/ ОТВЕТ 1

$E_0 = -\frac{mK}{R+M} + \frac{mV_0^2}{2}$

$E = -\frac{mK}{R+M} + (1+\lambda)\frac{mV_0^2}{2} = (1+\lambda)^2\left[-\frac{mK}{R+M}+\frac{mV_0^2}{2}\right] - (\lambda^2+2\lambda)\left(-\frac{mK}{R+M}\right) =$

$= (1+\lambda)^2 E_0 + (\lambda^2+2\lambda)\cdot\frac{mK}{R+M}$

$$e_0^2 - 1 = \frac{2E_0 c_0^2}{mk^2}, \quad c = (1+\lambda)\, c_0$$

$$e^2 = 1 + \frac{2Ec^2}{mk^2} = 1 + (1+\lambda)^2 \left[ (1+\lambda)^2 \cdot \frac{2E_0 c_0^2}{mk^2} + 2(\lambda^2 + 2\lambda) \cdot \frac{mk}{R+M} \cdot \frac{c_0^2}{mk^2} \right] \Rightarrow$$

$$\Rightarrow \boxed{e^2 = 1 + (1+\lambda)^2 \left[ (1+\lambda)^2 (e_0^2 - 1) + (\lambda^2 + 2\lambda) \cdot \frac{2\rho_0}{R+M} \right]} \quad / \text{ОТВЕТ 2}$$