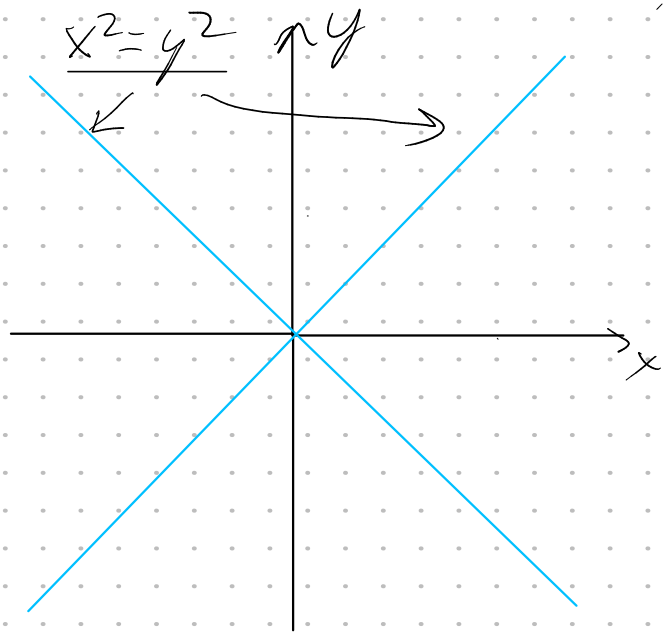
T1

$x^2 = y^2$



a) $y = x$, $y = -x$

$y = |x|$, $y = -|x|$

$\forall x \in U \to f(x) = x$

$\forall x \in R \setminus U \to f(x) = -x$

Итог: ∞

б) $y = x$; $y = -x$ | Итог: 4

$y = |x|$; $y = -|x|$

в) $y = x$; $y = |x|$ | Итог: 2

г) $y = x$

при $y \in [1;2]$ и $y(1) = 1$ | Итог: 1

N3.61(1)

$u(x,y)$;

$(x,y) = (0,1)$: $0 \cdot \cos 1 + 1 \cdot \cos u + u \cos 0 = 1$

$u + \cos u = 1 \xrightarrow[u=0]{} F(0,1,0) = 0$

u'_x: $\cos y - y u'_x \sin u + u'_x \cos x - u \sin x = 0 \Rightarrow \cos 1 + 0 + u'_x - 0 = 0 \Rightarrow$

$\begin{pmatrix} x \\ y \\ u \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Rightarrow u'_x = -\cos 1$

u'_y: $-x \sin y + \cos u - y u'_y \sin u + u'_y \cos x = 0 \Rightarrow$

$\Rightarrow 0 \cdot \sin 1 + \cos 0 - u'_y \cdot \sin 0 + u'_y \cos 0 = 0$

$\begin{pmatrix} x \\ y \\ u \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $u'_y = -1 \neq 0$

Ответ: $u'_x = -\cos 1$

$u'_y = -1$

$x=1;\ y=1$

№ 3.64 (2)

$1+2+u^3-3u+2-3=0 \Rightarrow u^3-3u-2=0$

а) $u=1$: $1-3-2=-4\neq 0 \Rightarrow$ опр не сущ.

б) $u=-2$: $1+2-8+6+2-3=0 \Rightarrow$

$\Rightarrow u'_x: 3x^2+0+3u'_x u^2-3yu-3xyu'_x+0-0=0$

$\begin{pmatrix} x \\ y \\ u \end{pmatrix}=\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$: $3+12u'_x+6-3u'_x=0 \Rightarrow u'_x=-1$

$u'_y: 0+6y^2+3u'_y u^2-3xu-3xyu'_y+2-0=0$

$\begin{pmatrix} x \\ y \\ u \end{pmatrix}=\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$: $6+12u'_y+6-3u'_y+2=0 \Rightarrow u'_y=-\frac{14}{9}$

Ответ: б) $du=-dx-\frac{14}{9}dx$

а) $\nexists\, du$

№ 3.69

$f(x-y;\ y-z;\ z-x)=0=f(u,z,w)$

$f'_u(dx-dy)+f'_v(dy-dz)+f'_w(dz-dx)=0$

$f'_u\,dx-f'_u dy+f'_v dy-f'_v dz+f'_w dz-f'_w dx=0$

$$dz=\frac{(f'_u-f'_w)dx+(f'_v-f'_u)dy}{f'_v-f'_w}$$ / Ответ

№ 3.75

75. Найти в точке $(1;2)$ частные производные дифференцируемых функций $u(x;y)$ и $v(x;y)$, заданных неявно уравнениями

$$\underbrace{xe^{u+v}+2uv=1}_{f_1},\quad \underbrace{ye^{u-v}-\frac{u}{1+v}=2x}_{f_2},\quad u(1;2)=v(1;2)=0.$$

$\frac{df_1}{dx}: e^{u+v}+x(u'_x+v'_x)e^{u+v}+2u'_x v+2uv'_x=0$

$\frac{df_1}{dy}: xe^{u+v}(u'_y+v'_y)+2u'_y v+2uv'_y=0$

$\frac{df_2}{dx}: ye^{u+v}(u'_x+v'_x)-(u'_x(1+v)+uv'_x)(1+v)^{-2}=2$

$\frac{df_2}{dy}: e^{u+v}+e^{u+v}(u'_y+v'_y)-(u'_y(1+v)+uv'_y)(1+v)^{-2}=0$

$x=1,\ y=2,\ u=v=0:\ \begin{cases} 1+u'_x+v'_x=0 \\ u'_y+v'_y=0 \\ 2(u'_x-v'_x)-u'_x=2 \\ 1+2(u'_y-v'_y)-u'_y=0 \end{cases}$ $\begin{cases} -u'_x=1+v'_x \\ u'_y=-v'_y \\ u'_x=2+2v'_x \\ u'_y=2v'_y-1 \end{cases} \Rightarrow \begin{cases} u'_x=0 \\ u'_y=-\frac{1}{3} \\ v'_x=-1 \\ v'_y=\frac{1}{3} \end{cases}$ ОТВЕТ

№ 4.43 (5)

$d:\ 3u^2du+2dyu+2ydu+dxy+xdy=0$

$\begin{pmatrix} x \\ y \\ u \end{pmatrix}=\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}:\ 3du-2dy-2du-dx+dy=0 \Rightarrow du=dx+dy$

$6u\,du^2+3u^2d^2u+2du\,dy+2yd^2u+2dy\,du+dx\,dy+dx\,dy=0$

$-6dx^2-6dy^2-12dx\,dy+3d^2u+4dy^2+4dx\,dy-2d^2u+2dx\,dy=0$

$d^2u=6dx^2+2dy^2+6dx\,dy$ / ОТВЕТ

№ 4.46 (1)

$f=(x+u;y+u)=0$

$f_1'=f'_{x+u},\ f_2'=f'_{y+u},\ f''_{11}=f''_{(x+u)},\ f''_{12}=f''_{(x+u),(y+u)},\ f''_{22}=f''_{(y+u)}$

$f_1'dx+f_1'du+f_2'dy+f_2'du=0 \Rightarrow$

$\Rightarrow du=\dfrac{-(f_1'dx+f_2'dy)}{(f_1'+f_2')}$

$d^2u=f''_{11}\underbrace{(dx+du)}_{(1)}{}^2+2f''_{12}\underbrace{(dy+du)}_{(2)}(dx+du)+f''_{22}(dy+du)^2 \ominus$

(1) $dx+du=\dfrac{f_1'dx+f_2'dx-f_1'dx-f_2'dy}{f_1'+f_2'}=\dfrac{f_2'(dx-dy)}{f_1'+f_2'}$

(2) $dy+du=\dfrac{f_1'dy+f_2'dy-f_1'dx-f_2'dy}{f_1'+f_2'}=\dfrac{f_1'(dx-dy)}{f_1'+f_2'}$

$\ominus\ \dfrac{(dx-dy)^2}{(f_1'+f_2')^2}\cdot\left(f''_{11}\cdot(f_2')^2+2f''_{12}f_1'f_2'+f''_{22}\cdot(f_1')^2\right)$ / ОТВЕТ

№ 3.105

$x=r\cos^p\varphi\cos^q\psi \quad y=r\sin^p\varphi\cos^q\varphi \quad z=r\sin^p\psi \quad p,q\in\mathbb{N}$

$$\frac{d(x,y,z)}{d(r,\varphi,\psi)}=\begin{vmatrix} \cos^p\varphi\cos^q\psi & -rp\cos^{p-1}\varphi\cdot\sin\varphi\cdot\cos^q\psi & -rq\cos^p\varphi\cos^{q-1}\psi\sin\psi \\ \sin^p\varphi\cos^q\psi & rp\sin^{p-1}\varphi\cdot\cos\varphi\cdot\cos^q\psi & -rq\sin^p\varphi\cos^{q-1}\psi\sin\psi \\ \sin^q\psi & 0 & q\sin^{q-1}\psi \end{vmatrix}=$$

$=r^2\sin\psi\cos^{2q-1}(\psi)\cdot pq\cdot\cos^{p-1}(\varphi)\cdot\sin^{p-1}(\varphi)\begin{vmatrix} -\sin\varphi & -\cos\varphi \\ \cos\varphi & -\sin\varphi \end{vmatrix}$ $=pqr^2(\sin\varphi\cos\varphi)^{p-1}\cdot\cos^{2q-1}\varphi\cdot\sin^{q-1}\psi$ / ОТВЕТ

N T3

$u = e^x \cos y \qquad v = e^x \sin y$

1) $J = \begin{vmatrix} e^x \cos y & -e^x \sin y \\ e^x \sin y & e^x \cos y \end{vmatrix} = e^{2x} > 0$, но т.к. $u(x,y) = u(x, y+2\pi)$, $v(x,y) = v(x, y+2\pi)$ отобр. не биект т.к. период.

2) $|\vec{r}| = \sqrt{e^{2x}\cos^2 y + e^{2x}\sin^2 y} = e^x > 0 \Rightarrow$

$\Rightarrow f(\vec{R}) = R^2 \setminus \{0,0\}$

N T.4

$x = r\cos\varphi \quad y = r\sin\varphi, \quad r > 0$

$$\begin{cases} 1 = r'_x \cos\varphi + r(-\sin\varphi)\varphi'_x \\ 0 = r'_y \cos\varphi + r(-\sin\varphi)\varphi'_y \\ 0 = r'_x \sin\varphi + r\cos\varphi \cdot \varphi'_x \\ 1 = r'_y \sin\varphi + r\cos\varphi \cdot \varphi'_y \end{cases}$$

$$\begin{cases} r'_x \cos\varphi + r(-\sin\varphi)\varphi'_x = 1 \\ r'_x \sin\varphi + r\cos\varphi \cdot \varphi'_x = 1 \end{cases}$$

$$\begin{cases} r'_y \cos\varphi - \varphi'_y r\sin\varphi = 0 \\ r'_y \sin\varphi + \varphi'_y r\cos\varphi = 1 \end{cases}$$

$\Delta_1 = \begin{vmatrix} \cos\varphi & -r\sin\varphi \\ \sin\varphi & r\cos\varphi \end{vmatrix} = r$

$\Delta\varphi_1 = \begin{vmatrix} \cos\varphi & 1 \\ \sin\varphi & 0 \end{vmatrix} = -\sin\varphi$

$\Delta r_1 = \begin{vmatrix} 1 & r\sin\varphi \\ 0 & r\cos\varphi \end{vmatrix} = r\cos\varphi$

$\Delta_2 = \begin{vmatrix} \cos\varphi & r\sin\varphi \\ \sin\varphi & r\cos\varphi \end{vmatrix} = r$

$\Delta\varphi_2 = \begin{vmatrix} \cos\varphi & 0 \\ \sin\varphi & 1 \end{vmatrix} = \cos\varphi$

$\Delta r_2 = \begin{vmatrix} 0 & r\sin\varphi \\ 1 & r\cos\varphi \end{vmatrix}$

$\Downarrow \qquad \Leftarrow$

$r'_x = \frac{\Delta r_1}{\Delta_1} = \cos\varphi$

$\varphi'_x = \frac{\Delta\varphi_1}{\Delta_1} = -\frac{\sin\varphi}{r}$

$r'_y = \frac{\Delta r_2}{\Delta_2} = \sin\varphi$

$\varphi'_y = \frac{\Delta\varphi_2}{\Delta_2} = \frac{\cos\varphi}{r}$

ОТВЕТ

N 3.86

$x\frac{du}{dy} - y\frac{du}{dx}$

$x u'_y - y u'_x = 0$

$x = r\cos\varphi$

$y = r\sin\varphi$

$\Rightarrow$

$u'_x = u'_r \cdot r'_x + u'_\varphi \cdot \varphi'_x = u'_r \cos\varphi - u'_\varphi \frac{\sin\varphi}{r}$

$u'_y = u'_r \cdot r'_y + u'_\varphi \cdot \varphi'_x = u'_r \sin\varphi + u'_\varphi \frac{\cos\varphi}{r}$

u_z T4

$r\cos\varphi\left(u'_r \sin\varphi + u'_\varphi \frac{\cos\varphi}{r}\right) - r\sin\varphi\left(u'_r \cos\varphi - u'_\varphi \frac{\sin\varphi}{r}\right) = 0 \Rightarrow u'_\varphi = 0 \Rightarrow$

$\Rightarrow u(r,\varphi) = u(r) = u(x^2+y^2)$ ОТВЕТ

N3. 88(1)

$$\frac{dz}{dx} - \frac{dz}{dy} = 0 \; ; \; u = x+y, \; v = x-y$$

$$u'_x = 1 \quad u'_y = 1 \qquad z'_x = z'_u u'_x + z'_v v'_x = z'_u + z'_v$$
$$v'_x = 1 \quad v'_y = -1 \qquad z'_y = z'_u u'_y + z'_v v'_y = z'_u - z'_v \Rightarrow$$

$$\Rightarrow z'_u + z'_v - z'_u + z'_v = 0 \Rightarrow z'_v = 0 \Rightarrow \boxed{z = z(x+y)}$$ — ОТВЕТ

N 3.91

$$(y-z)\frac{dz}{dx} + (y+z)\frac{dz}{dy} = 0 \; ; \; x = x(u,v) \; ; \; u = y-z \quad v = y+z$$

$$dx = x'_u du + x'_v dv = x'_u dy - x'_u dz + x'_v dy + x'_v dz = (x'_u + x'_v)dy + (x'_v - x'_u)dz$$

$$dz = (x'_v - x'_u)^{-1} dx + (x'_u + x'_v)(x'_u - x'_v)^{-1} dy$$

$$u(x'_v - x'_u)^{-1} + v(x'_u + x'_v)(x'_u - x'_v)^{-1} = 0 \Rightarrow u = v(x'_u + x'_v) \Rightarrow \boxed{\frac{u}{v} = x'_u + x'_v}$$

ОТВЕТ

N4.51(2)

$$x^2 \frac{d^2u}{dx^2} + 2xy\frac{d^2u}{dx\,dy} + y^2\frac{d^2u}{dy^2} = 0$$

$x = r\cos\varphi$ | $u'_x = u'_r \cos\varphi - u'_\varphi \frac{\sin\varphi}{r}$ | $r'_x = \cos\varphi$

$y = r\sin\varphi$ | $u'_y = u'_r \sin\varphi + u'_\varphi \frac{\cos\varphi}{r}$ | $\varphi'_x = -\frac{\sin\varphi}{r}$

$r'_y = \sin\varphi$

$\varphi'_y = \frac{\cos\varphi}{r}$

u_z ТУ

u

3.86

$$u''_{xx} = (u'_x)'_r \cdot r'_x + (u'_x)'_\varphi \cdot \varphi'_x = \left(u''_{rr}\cos\varphi - \frac{u''_{\varphi r}\sin\varphi}{r} + \frac{u'_\varphi \sin\varphi}{r^2}\right)\cdot\cos\varphi +$$
$$+\left(u''_{r\varphi}\cos\varphi - u'_r\sin\varphi - \frac{u''_{\varphi\varphi}\sin\varphi}{r} + \frac{u'_\varphi\cos\varphi}{r}\right)\cdot\left(-\frac{\sin\varphi}{r}\right) =$$

$$= u''_{rr}\cos^2\varphi - u''_{\varphi r}\frac{\sin\varphi\cos\varphi}{r} + u'_r\frac{\sin^2\varphi}{r} + u''_{\varphi\varphi}\frac{\sin^2\varphi}{r^2} - u''_{r\varphi}\frac{\sin\varphi\cos\varphi}{r}$$

$$u''_{yy} = (u'_y)'_r \cdot r'_y + (u'_y)'_\varphi \cdot \varphi'_y = \left(u''_{rr}\sin\varphi + \frac{u''_{\varphi r}\cos\varphi}{r} - \frac{u'_\varphi\cos\varphi}{r^2}\right)\cdot\sin\varphi +$$
$$+\left(u''_{r\varphi}\sin\varphi + u'_r\cos\varphi + u''_{\varphi\varphi}\frac{\cos\varphi}{r} - u'_\varphi\frac{\sin\varphi}{r}\right)\cdot\frac{\cos\varphi}{r} =$$

$$= u''_{rr}\sin^2\varphi + 2\frac{\cos\varphi\cdot\sin\varphi}{r}u''_{\varphi r} + u''_{\varphi\varphi}\frac{\cos^2\varphi}{r^2} + u'_r\cdot\frac{\cos^2\varphi}{r} - 2\frac{\sin\varphi\cdot\cos\varphi}{r}u'_\varphi$$

$$u''_{yx} = (u'_y)_r \cdot r'_x + (u'_y)'_\varphi \cdot \varphi'_x = \cos\varphi\cdot\left(u''_{rr}\cdot\sin\varphi + u''_{\varphi r}\frac{\cos\varphi}{r} - u'_\varphi\cdot\frac{\cos\varphi}{r^2}\right) -$$
$$-\frac{\sin\varphi}{r}\left(u''_{r\varphi}\cdot\sin\varphi + u'_r\cos\varphi + u''_{\varphi\varphi}\cdot\frac{\cos\varphi}{r} - u'_\varphi\frac{\sin\varphi}{r}\right) =$$

$$= u''_{rr}\cdot\sin\varphi\cdot\cos\varphi + u''_{\varphi r}\frac{(\cos^2\varphi-\sin^2\varphi)}{r} - u''_{\varphi\varphi}\cdot\frac{\sin\varphi\cos\varphi}{r} + u'_{\varphi}\frac{\sin^2\varphi-\cos^2\varphi}{r^2} - u'_r\frac{\sin\varphi\cos\varphi}{r} =$$

Итоги:

$$w''_{xx}r^2\cos^2\varphi + u''_{yx}r^2\cdot 2\sin\varphi\cos\varphi + u''_{yy}\cdot r^2\sin^2\varphi = 0$$

$$\left(u''_{rr}\cos^2\varphi - 2\frac{\sin\varphi\cos\varphi}{r}u''_{\varphi r} + \frac{\sin^2\varphi}{r^2}u''_{\varphi\varphi} + \frac{\sin^2\varphi}{r}u'_r + 2\frac{\cos\varphi\cdot\sin\varphi}{r^2}u'_{\varphi}\right)r^2\cos^2\varphi +$$

$$+\left(u''_{rr}\cdot\sin^2\varphi + 2\frac{\cos\varphi\cdot\sin\varphi}{r}u''_{\varphi r} + u''_{\varphi\varphi}\frac{\cos^2\varphi}{r^2} + u'_{\varphi}\left(\frac{\sin^2\varphi-\cos^2\varphi}{r^2} - u'_r\frac{\sin\varphi\cdot\cos\varphi}{r}\right)\right)\cdot$$

$$\cdot r^2\cdot 2\cdot\sin\varphi\cdot\cos\varphi = 0$$

$$u''_{rr}\cdot r^2(\cos^4\varphi + \sin^4\varphi + 2\sin^2\varphi\cdot\cos^2\varphi) = 0$$

$$u''_{rr}\cdot r^2 = 0 = r^2\cdot\frac{d^2u}{dr^2}$$

№3.52 (1)

$$\frac{d^2z}{dt^2} = a^2\frac{d^2z}{dx^2}$$
$$u = x - at$$
$$v = x + at$$

$$z''_{tt} = a^2 z''_{xx}$$

$$z'_x = z'_u u'_x + z'_v v'_x = z'_u + z'_v$$

$$z'_t = z'_u u'_t + z'_v v'_t = a(-z'_u + z'_v)$$

$$z''_{xx} = z''_{uu} + z''_{vu} + z''_{uv} + z''_{vv} = z''_{uu} + 2z''_{uv} + z''_{vv}$$

$$z''_{tt} = a\left[(-z''_{uu} + z''_{vu})\cdot(-a) + (-z''_{uv} + z''_{vv})\cdot a\right] = a^2(z''_{uu} - 2z''_{uv} + z''_{vv})$$

$$a^2(z''_{uu} - 2z''_{uv} + z''_{vv}) = a^2(z''_{uu} + 2z''_{uv} + z''_{vv}) \Rightarrow z''_{uv} = 0$$

№5.2 (2)

$$u = 3x^2y + y^3 - 12x - 15y + 3$$

$$u'_x = 6xy - 12 = 0$$
$$u'_y = 3x^2 + 3y^2 - 15 = 0$$

$$\Rightarrow \begin{cases} xy = 2 \\ x^2 + y^2 = 5 \end{cases} \Rightarrow (2,1);\ (-2,-1);\ (1,2);\ (-1,-2)$$

$$u''_{xx} = 6y$$
$$u''_{yy} = 6y$$
$$u''_{xy} = 6x$$

$$\Rightarrow d^2u = 6(y(dx^2 + dy^2) + 2x\,dx\,dy)$$

$(2,1)$: $d^2u = 6(dx^2 + dy^2 + 4dx\,dy)\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ $\Delta_1 > 0$, $\Delta_2 < 0$ $\Rightarrow$ нет экстр.

$(-2,-1)$: $d^2u = -6(dx^2 + dy^2 + 4dx\,dy)\begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix}$ $\Delta_1 < 0$, $\Delta_2 < 0$ $\Rightarrow$ нет экстр.

$(1,2)$: $d^2u = 12(dx^2 + dy^2 + dx\,dy)\begin{bmatrix} 1 & 0{,}5 \\ 0{,}5 & 1 \end{bmatrix}$ $\Delta_1 > 0$, $\Delta_2 > 0$ $\Rightarrow$ лок. мин. $= -25$

$(-1,-2)$: $d^2u = -12(dx^2 + dy^2 + dx\,dy)\begin{bmatrix} -1 & -0{,}5 \\ -0{,}5 & -1 \end{bmatrix}$ $\Delta_1 < 0$, $\Delta_2 > 0$ $\Rightarrow$ лок. макс $= 37$

№3.9

$u = x^4 + y^4 - 2x^2$

$u'_x = 4x^3 - 4x = 0 \Rightarrow x(x^2-1) = 0$
$u'_y = 4y^3 = 0 \Rightarrow y = 0$
$\Rightarrow (0,0)\ (1,0)\ (-1,0)$

$u''_{xx} = 12x^2 - 4$
$u''_{yy} = 12y^2$
$u''_{xy} = 0$
$\Rightarrow d^2u = (12x^4 - 4)dx^2 + 12y^2 dy^2$

$(0,0): d^2u = -4dx^2$ – стр. макс.

$\Delta u(0,0) = u(\Delta x, \Delta y) - u(0,0) = (\Delta x)^4 + (\Delta y)^4 - 2(\Delta x)^2 = (\Delta x)^2((\Delta x)^2 - 2) + (\Delta y)^4$

Если $0 < |\Delta x| < \sqrt{2}$, $\Delta y = 0$, то $\ominus$
Если $\Delta x = 0$, $\Delta y \neq 0$, то $\oplus$
нет экстр

$(\pm 1, 0): d^2u = 8dx^2$ – полож макс

$\Delta u(\pm 1, 0) = u(\pm 1 + \Delta x, \Delta y) - u(\pm 1, 0) = (\pm 1 + \Delta x)^4 + (\Delta y)^4 - 2(\pm 1 + \Delta x)^2 + 1 =$

$= 1 + 4\Delta x + o(\Delta x)^2 \pm 4(\Delta x^3) + (\Delta x)^4 + (\Delta y)^4 - 2 \mp 4\Delta x - 2(\Delta x)^2 + 1 =$

$= 4(\Delta x)^2 \pm 4(\Delta x^3) + (\Delta x)^4 + (\Delta y)^4 = (\Delta x)^2(2 \pm \Delta x)^2 + (\Delta y)^4 \geq 0$

Если $\Delta u = 0$, то $\Delta y = 0$ и $\Delta x = 0 \Rightarrow$ 2 стр. лок. мин $u(\pm 1, 0) = -1$

№5.13(1)

$u = x^2 + y^2 + (z+1)^2 - xy + x$

$u'_x = 2x - y + 1 = 0$
$u'_y = 2y - x = 0$
$u'_z = 2z + 2 = 0$
$\Rightarrow \left(-\frac{2}{3}, -\frac{1}{3}, -1\right)$ – экстр.

$u''_{xx} = 2$ $\quad u''_{xz} = 0$
$u''_{yy} = 2$ $\quad u''_{xy} = -1$
$u''_{zz} = 2$ $\quad u''_{yz} = 0$
$\Rightarrow d^2u = 2(dx^2 + dy^2 + dz^2 - dx\,dy)$ – квадр. от xyz

$$\begin{bmatrix} 1 & 1/2 & 0 \\ 1/2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} \Delta_1 > 0 \\ \Delta_2 > 0 \\ \Delta_3 > 0 \end{matrix}$$

$\Rightarrow$ мин $u\left(-\frac{2}{3}, -\frac{1}{3}, -1\right) = -\frac{1}{3}$

№5.18(1)

$x^2 + y^2 + u^2 + 2x - 2y + 4u - 3 = 0$

$2x\,dx + 2y\,dy + 2u\,du + 2dx - 2dy + 4du = 0$

$(x+1)dx + (y-1)dy = -(u+2)du \quad (1)$

$du = ((x+1)dx + (y-1)dy)(-u-2)^{-1}$

$1+1+u^2-2-2+4u-3=0$

$u^2+4u-5=0 \Rightarrow \left[\begin{array}{l} u=1 \\ u=5 \end{array}\right.$

Диф (1), dx и dy - const : $dx^2+dy^2=-(u+2)\,d^2u-\cancel{du^2}^{\,0}$ т.к. ст. т.

$d^2u=-(dx^2+dy^2)(u+2)^{-1}$

$(-1,1,1)$: $d^2u=-\frac{1}{3}(dx^2+dy^2)$ отр. опр. $\Rightarrow$ лок max

$(-1,1,5)$: $d^2u=\frac{1}{3}(dx^2+dy^2)$ полож опр. $\Rightarrow$ лок. min

Т5

а) да (пример в 5.9)

б) нет

в) да (пример в 5.9)

№5.19(1)

$u=xy,\ x+y-1=0$

$y=1-x$

$u=x(1-x)=x-x^2$ $\Rightarrow$ max в т. $\left(\frac{1}{2},\frac{1}{2}\right) \Rightarrow u\left(\frac{1}{2},\frac{1}{2}\right)=\frac{1}{4}$

№5.21(2)

$u=1-4x-8y,\ x^2-8y^2=8$

$L=1-4x-8y+\lambda(x^2-8y^2-8)$

$L'_x=-4+2\lambda x=0$

$L'_y=-8-16\lambda y=0$

$x^2-8y^2=8$

$\Rightarrow \lambda=0$ - не подх $\Rightarrow x=\frac{2}{\lambda}\quad y=-\frac{1}{2\lambda}$

$\frac{4}{\lambda^2}-\frac{2}{\lambda^2} \Rightarrow \lambda=\pm\frac{1}{2} \Rightarrow x=\pm 4\quad y=\mp 1$

$L''_{xx}=2\lambda$

$L''_{yy}=-16\lambda$

$L''_{xy}=0$

$\Rightarrow d^2L=2\lambda(dx^2-8dy^2)$

диф. усл. св. : $2x\,dx-16y\,dy=0 \Rightarrow dx=8\frac{y}{x}dy$

$\lambda=\frac{1}{2},\ x=4,\ y=-1$: $d^2L=4dy^2-8dy^2=-4dy^2$ - отр. опр. $\Rightarrow u(4,-1)=-7$ - max

$\lambda=-\frac{1}{2},\ x=-4,\ y=1$: $d^2L=-4dy^2+8dy^2=4dy^2$ - полож. опр. $\Rightarrow u(-4,1)=9$ - min

№3.25(6)

$u = x - y + 2z, \; x^2 + y^2 + 2z^2 = 16$

$L = x - y + 2z + \lambda(x^2 + y^2 + 2z^2 - 16)$

$L'_x = 1 + 2\lambda x = 0$

$L'_y = -1 + 2\lambda y = 0$

$L'_z = 2 + 4\lambda z = 0$

$x^2 + y^2 + 2z^2 = 16$

$\Rightarrow$ $x = -\frac{1}{2\lambda}$ ($\lambda = 0$ – не подх.)

$y = \frac{1}{2\lambda}$

$z = -\frac{1}{2\lambda}$

$4 \cdot \frac{1}{4\lambda^2} = 16 \Rightarrow \lambda = \pm\frac{1}{4}$

$\Rightarrow x = -y = z = \mp 2$

$L''_{xx} = 2\lambda \quad L''_{xy} = 0$

$L''_{yy} = 2\lambda \quad L''_{xz} = 0$

$L''_{zz} = 4\lambda \quad L''_{yz} = 0$

$\Rightarrow d^2L = 2\lambda(dx^2 + dy^2 + 2dz^2)$

$\lambda = \frac{1}{4}$ – полож. опр. $\Rightarrow u(-2, 2, -2) = -8$ min

$\lambda = -\frac{1}{4}$ – отр. опр. $\Rightarrow u(2, -2, 2) = 8$ max

№5.31 (3)

$u = x + y + z, \; x^2 + y^2 \le z \le 1$ G

Стац. в G: $u'_x = 1 \quad u'_y = 1 \quad u'_z = 1 \Rightarrow$ нет стац. т. в G

1) $u = x + y + z, \; z = x^2 + y^2 < 1$

$u = x + y + x^2 + y^2$

$u'_x = 1 + 2x = 0$

$u'_y = 1 + 2y = 0$

$\Rightarrow u\left(-\frac{1}{2}, -\frac{1}{2}\right) = -\frac{1}{2}$

2) $u = (x + y + z) \; z = 1, \; x^2 + y^2 < 1$

$u = x + y + 1$ – нет стац. т.

3) $u = x + y + z, \; x^2 + y^2 = z = 1 \Rightarrow u = x + y + 1, \; x^2 + y^2 = 1$

$L = x + y + 1 + \lambda(x^2 + y^2 - 1)$

$L'_x = 1 + 2\lambda x = 0$

$L'_y = 1 + 2\lambda y = 0$

$x^2 + y^2 = 1$

$\Rightarrow$ $2\lambda = -\frac{1}{x} = -\frac{1}{y}$

$x = y = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow$ $u\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = 1 + \sqrt{2}$

$u\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = 1 - \sqrt{2}$

$\Rightarrow$ $M = 1 + \sqrt{2}$, $m = -\frac{1}{2}$

№8.23

$$f(x,y) = \begin{cases} \frac{1}{y^2}, & 0 < x < y < 1 \\ -\frac{1}{x^2}, & 0 < y < x < 1 \\ 0, & \text{вне ш.} \end{cases} \Rightarrow G = [0,1] \times [0,1]$$

$$\int_0^1 dy \int_0^1 f\,dx = \int_0^1 dy\left(\int_0^y f\,dx + \int_y^1 f\,dx\right) = \int_0^1 dy\left(\int_0^y \frac{dx}{y^2} - \int_y^1 \frac{dx}{x^2}\right) = \int_0^1 dy\left(\frac{x}{y^2}\Big|_0^y + \frac{1}{x}\Big|_y^1\right) = \int_0^1 dy = 1$$

$$\int_0^1 dx \int_0^1 f\,dy = \int_0^1 dx\left(\int_0^x f\,dy + \int_x^1 f\,dy\right) = \int_0^1 dx\left(-\int_0^x \frac{dy}{x^2} + \int_x^1 \frac{dx}{y^2}\right) = \int_0^1 dx\left(-\frac{y}{x^2}\Big|_0^x - \frac{1}{y}\Big|_x^1\right) =$$

$$= \int_0^1 dx\left(-\frac{1}{x} - 1 + \frac{1}{x}\right) = -\int_0^1 dx = -1$$

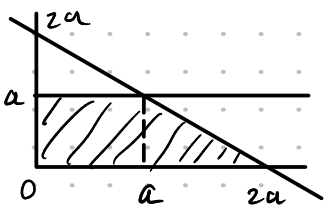
Т.к. $\int_0^1 dx \int_0^1 f\,dy \neq \int_0^1 dy \int_0^1 f\,dx \Rightarrow$ не инт. на $[0,1]\times[0,1]$

Т6

а) нет (8,23)

б) нет: $f(0,y) = \begin{cases} 1, & y \in \mathbb{Q} \\ 0, & y \in \mathbb{R}\setminus\mathbb{Q} \end{cases}$ — не инт. при $x=0$

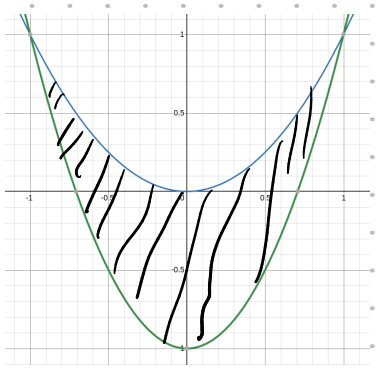
№ 8.79(1) $\boxed{x=0,\ y=0,\ y=a,\ x+y=2a}$



$$\int_0^a dx \int_0^a f(x,y)\,dy + \int_a^{2a} dx \int f(x,y)\,dy = \int_0^a dy \int_0^{2a-y} f(x,y)\,dx$$

№ 8.83(15)

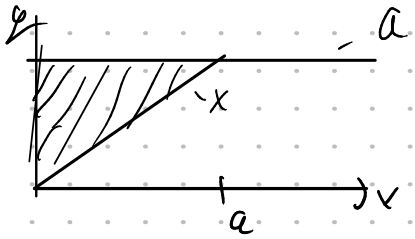
$$\int_{-1}^1 dx \int_{x^2}^{2x^2-1} f(x,y)\,dy$$



$$\int_{-1}^1 dx \int_{x^2}^{2x^2-1} f(x,y)\,dy = \int_0^1 dy \int_{-\sqrt{\frac{y+1}{2}}}^{-\sqrt{y}} f(x,y)\,dx + \int_0^1 dy \int_{\sqrt{\frac{y+1}{2}}}^{\sqrt{y}} f(x,y)\,dx +$$

$$+ \int_{-1}^0 dy \int_{-\sqrt{\frac{y+1}{2}}}^{\sqrt{\frac{y+1}{2}}} f(x,y)\,dx$$

№ 8.85(3) $\int_0^a dx \int_x^a (a^2-y^2)^\alpha\,dy,\ a>0,\ \alpha>0$



$$\int_0^a dx \int_x^a (a^2-y^2)^\alpha\,dy = \int_0^a dy \int_0^y (a^2-y^2)^\alpha\,dx =$$

$$= \int_0^a y(a^2-y^2)^\alpha\,dy = -\frac{1}{2}\int_0^a (a^2-y^2)^\alpha\,d(a^2-y^2) =$$

$$= -\frac{1}{2}\,\frac{(a^2-y^2)^{\alpha+1}}{\alpha+1}\Big|_0^a = \frac{a^{2(\alpha+1)}}{2(\alpha+1)}$$

N 8.90 (1)

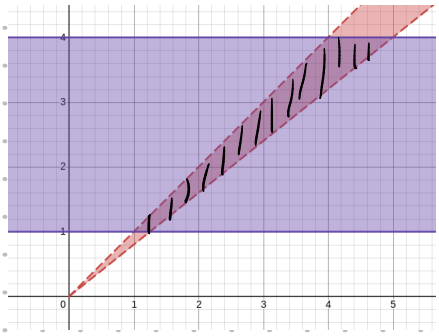
$$\iint_G (x\sin y + y\cos x)\,dx\,dy, \quad G=\left[0,\frac{\pi}{2}\right]\times\left[0,\frac{\pi}{2}\right]$$

$$I=\iint_G (x\sin y+y\cos x)\,dx\,dy=\int_0^{\pi/2}dx\int_0^{\pi/2}(x\sin y+y\cos x)\,dy=$$

$$=\int_0^{\pi/2}dx\left(-x\cos y+\frac{1}{2}y^2\cos x\right)\Big|_0^{\frac{\pi}{2}}=\int_0^{\pi/2}\left(\frac{\pi^2}{8}\cos x+x\right)dx=\left(\frac{\pi^2}{8}\sin x+\frac{1}{2}x^2\right)\Big|_0^{\pi/2}=\boxed{\frac{\pi^2}{4}}$$

N 8.90 (8)

$$\iint_G \sqrt{x-y}\,dx\,dy, \quad G=\left\{\frac{4}{5}x\le y\le x,\ 1\le y\le 4\right\}$$



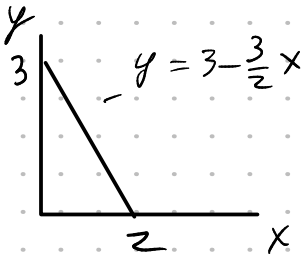
$$I=\iint_G\sqrt{x-y}\,dx\,dy=\int_1^4 dy\underbrace{\int_y^{\frac{5}{4}y}\sqrt{x-y}\,dx}_{I_0}$$

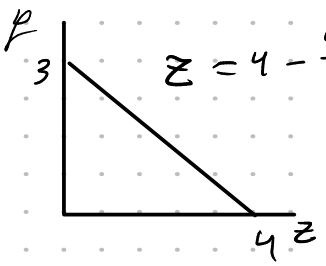
$$I_0=\frac{2}{3}(x-y)^{\frac{3}{2}}\Big|_y^{\frac{5}{4}y}=\frac{2}{3}\left(\frac{1}{4}y\right)^{\frac{3}{2}}$$

$$I=\frac{2}{3}\cdot\frac{1}{8}\cdot\int_1^4 y^{\frac{3}{2}}\,dy=\frac{1}{30}(32-1)=\boxed{\frac{31}{30}}$$

N 8.133

$$\int_0^4 dz\int_0^{3-\frac{3z}{4}}dy\int_0^{2-\frac{2y}{3}-\frac{z}{2}}f(x,y,z)\,dx=\boxed{\int_0^2 dx\int_0^{3-\frac{3}{2}x}dy\int_0^{4-2x-\frac{4}{3}y}f(x,y,z)\,dz}$$





N 8.134 (2)

$$f(x,y,z)=(1+x+y+z)^{-3}$$

G: exp: $x+y+z=1$, $x=0$, $y=0$, $z=0$

(Отрезок: (1,0,0), (0,1,0), (0,0,1), (0,0,0))

$$I=\iiint_G f\,dx\,dy\,dz=\iint_S dx\,dy\int_0^{1-x-y}f\,dz=\int_0^1 dx\underbrace{\int_0^{1-x}dy}_{I_1}\underbrace{\int_0^{1-x-y}(1+x+y+z)^{-3}\,dz}_{I_0}$$

$$I_0 = -\frac{1}{2}(1+x+y+z)^{-2}\Big|_0^{1-x-y} = -\frac{1}{2}\left(2^{-2} - (1+x+y)^{-2}\right)$$

$$I_1 = -(2+x+y)^{-1}\Big|_0^{1-x} - \frac{1}{4}y\Big|_0^{1-x} = -2^{-1} + (1+x)^{-1} - \frac{1}{4}(1-x)$$

$$I = \frac{1}{2}\int_0^1 \left[-2^{-1} + (1+x)^{-1} - \frac{1}{4}(1-x)\right]dx = \frac{1}{2}\left(-\frac{1}{2} + \ln 2 - \frac{1}{8}\right) = \frac{1}{2}\ln 2 - \frac{5}{8} = \boxed{\frac{8\ln 2 - 5}{16}}$$

№8.175 (2)

$Q_n = [0,a]^n \in R^n,\ n \geq 2 \qquad \int\limits_{Q_n} \sum\limits_{k=1}^{n} x_k\, dx$

$$\int\limits_{Q_n} \sum_{k=1}^{n} x_k\, dx = \int_0^a dx_1 \int_0^a \dots \int_0^a \sum_{k=1}^{n} x_k\, dx_k = \int_0^a x_1\, dx_1 \dots \int_0^a dx_n + \dots \int_0^a dx_1 \dots \int_0^a x_n\, dx_n =$$

$$= n \cdot a^{n-1} \cdot \frac{a^2}{2} = \boxed{\frac{n a^{n+1}}{2}}$$

№8.176 (2)

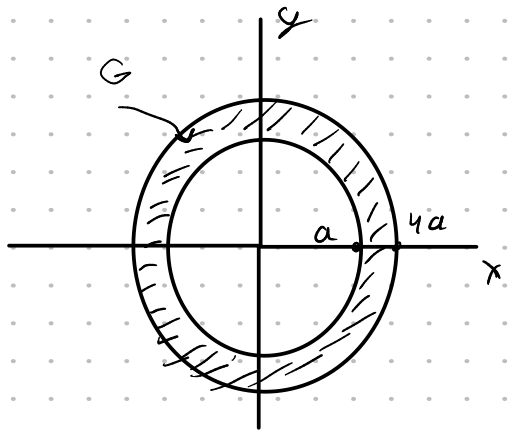
$\Pi_n = \{0 \leq x_n \leq x_{n-1} \leq \dots \leq x_1 \leq a\} \qquad \int x_1 x_2 \dots x_n\, dx$

$$\int\limits_{\Pi_n} x_1 x_2 \dots x_n\, dx = \int_0^a x_1\, dx_1 \int_0^{x_1} x_2\, dx_2 \dots \int_0^{x_{n-1}} x_n\, dx_n = \int_0^a x_1\, dx_1 \int_0^{x_1} x_2\, dx_2 \dots \int_0^{x_{n-2}} \frac{x_{n-1}^3}{2}\, dx_n =$$

$$= \boxed{\frac{a^{2n}}{(2n)!!}}$$ т.к. $n=1: \int_0^a x_1\, dx_1 = \frac{a^2}{2}$ $\quad n=2: \int_0^a \frac{x_1^3}{2} = \frac{a^4}{2\cdot 4}$ $\quad n=3: \int_0^a \frac{x^5}{2\cdot 4} = \frac{a^6}{2\cdot 4\cdot 6}$ и т.д.

№8.106 (3)

$\iint\limits_G |xy|\, dx\, dy, \quad G = \{a^2 \leq x^2 + y^2 \leq 4a^2$

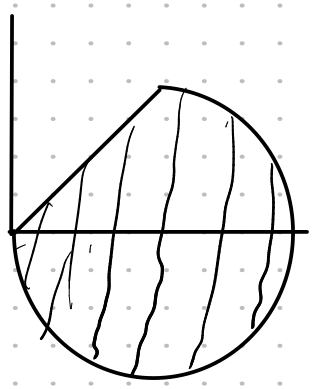


$$\int_0^{2\pi} d\varphi \int_a^{2a} r^3 |\cos\varphi \sin\varphi|\, dr = \frac{15a^4}{4} \cdot 4 \int_0^{\frac{\pi}{2}} \cos\varphi \sin\varphi\, d\varphi =$$

$$= 15a^2 \int_0^{\frac{\pi}{2}} \sin\varphi\, d\sin\varphi = \boxed{\frac{15}{2}a^4}$$

№8.107 (2)

$\iint\limits_G y\, dx\, dy, \quad G = \{x^2 + y^2 \leq 2x,\ x > y\}$



$x^2 + y^2 \leq 2x \Rightarrow (x-1)^2 + y^2 \leq 1, \quad r^2 \leq 2r\cos\varphi$

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} d\varphi \int_0^2 r^2 \sin\varphi\, dr + \int_{-\frac{\pi}{2}}^{-\pi/4} d\varphi \int_0^{2\cos\varphi} r^2 \sin\varphi\, dr =$$

$$= \frac{8}{3}\left[\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cancel{\sin\varphi}^{\,0}\, d\varphi + \int_{-\frac{\pi}{2}}^{-\pi/4} \cos^3\varphi \sin\varphi \, d\varphi\right] = -\frac{8}{3}\cdot\frac{1}{4}\cos^4\varphi \Big|_{\frac{\pi}{2}}^{-\frac{\pi}{4}} = \boxed{-\frac{1}{6}}$$

№8.110(3)

$$\int_0^{+\infty} e^{-x^2}\,dx$$

№8.124(1)

$f(x,y) = x+y$, G огр. $xy=a$, $xy=b$, $y=x$, $y=x-c$ где $0<a<b$, $0<c$

$$\begin{matrix} xy = u \\ y-x = v \end{matrix} \Bigg| \Rightarrow \frac{D(u,v)}{D(x,y)} = \begin{vmatrix} y & x \\ -1 & 1 \end{vmatrix} = y+x = f(x,y)$$

$$\iint_G du\,dv = \int_a^b du \int_{-c}^{0} dv = \boxed{(b-a)\cdot c}$$

№8.124(5)

$f(x,y) = x^4 - y^4$, $G = \{x>0,\ 1\le xy\le 2,\ 1\le x^2-y^2\le 2\}$

$$\begin{matrix} xy = u \\ x^2-y^2 = v \end{matrix} \Bigg| \Rightarrow \frac{D(u,v)}{D(x,y)} = \begin{vmatrix} y & x \\ 2x & -2y \end{vmatrix} = -2(x^2+y^2) \Rightarrow \frac{x^4-y^4}{2(y^2+x^2)} = \frac{x^2-y^2}{2} = \frac{v}{2}$$

$$\frac{1}{2}\iint_G v\,du\,dv = \frac{1}{2}\int_1^2 du \int_1^2 v\,dv = \frac{1}{2}\cdot\frac{3}{2}\int_1^2 du = \boxed{\frac{3}{4}}$$

№8.144(6)

$$\iiint f(x,y,z)\,dx\,dy\,dz$$

$r^2 < r\sin\psi \Rightarrow r < \sin\psi$

$f(x,y,z) = \sqrt{x^2+y^2+z^2}$, $G = \{x^2+y^2+z^2 \le z\}$

$$\begin{aligned} x &= r\cos\psi\cos\varphi \\ y &= r\sin\psi \\ z &= r\cos\psi\sin\varphi \end{aligned}$$

$-\frac{\pi}{2} \le \psi \le \frac{\pi}{2}$

$-\pi \le \varphi \le \pi$

$$\frac{D(x,y,z)}{D(r,\varphi,\psi)} = r^2\cos\psi$$

$$x^2+y^2+\left(z-\frac{1}{2}\right)^2 < \frac{1}{4}$$

$$\int_{-\pi}^{\pi} d\varphi \int_0^{\pi/2} d\psi \int_0^{\sin\varphi} r^3\cos\psi\,dr = 2\pi\int_0^{\frac{\pi}{2}} \cos\psi\,d\psi \int_0^{\sin\psi} r^3\,dr = 2\pi\int_0^{\frac{\pi}{2}} \cos\psi\cdot\frac{\sin^4\psi}{4}\,d\psi =$$

$$= \frac{\pi}{2}\int_0^1 t^4\,dt = \boxed{\frac{\pi}{10}}$$

N 8.145 (1)

$$\iiint_G f\left(\sqrt{x^2+y^2+z^2}\right)dx\,dy\,dz$$

$$G=\left\{\sqrt{x^2+y^2}\le z\le\sqrt{2-x^2-y^2}\right\}$$

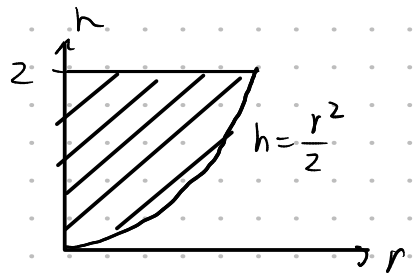
$$\left.\begin{aligned} x&=r\cos\psi\cos\varphi\\ y&=r\cos\psi\sin\varphi\\ z&=r\sin\psi\end{aligned}\right| \Rightarrow \begin{aligned} &r\cos\psi\le r\sin\psi\le\sqrt{2-r^2\cos^2\varphi}\\ &\cos\psi\le\sin\psi\\ &\operatorname{tg}\psi\ge 1,\ \psi\ge\frac{\pi}{4}\\ &r^2\cos^2\psi\le 2\Rightarrow r\in[0,\sqrt{2}]\end{aligned}$$

$$\iiint_G f(r)r^2\cos\psi\,dr\,d\varphi\,d\psi=\int_0^{2\pi}d\varphi\int_{\pi/4}^{\pi/2}\cos\psi\,d\psi\int_0^{\sqrt{2}}f(r)r^2\,dr=2\pi\left(1-\frac{1}{\sqrt{2}}\right)\int_0^{\sqrt{2}}r^2f(r)\,dr=$$

$$=\boxed{\pi(2-\sqrt{2})\int_0^{\sqrt{2}}r^2f(r)\,dr}$$

N 8.146 (3)

$$\left.\begin{aligned} x&=r\cos\varphi\\ y&=r\sin\varphi\\ z&=h\end{aligned}\right| \quad G=\left\{\frac{r}{2}\le h\le 2\right\}$$



$$\iiint_G(x^2+y^2)\,dx\,dy\,dz=\int_0^{2\pi}d\varphi\int_0^2 dh\int_0^{\sqrt{2h}}r^3\,dr=2\pi\int_0^2 dh\cdot\frac{r^4}{4}\Big|_0^{\sqrt{2h}}=\frac{\pi}{2}\int_0^2 4h\,dh=$$

$$=2\pi\frac{h^3}{3}\Big|_0^2=\boxed{\frac{16\pi}{3}}$$

N 8.147

$$\begin{aligned} x&=ar\cos\varphi\cos\psi\\ y&=br\sin\varphi\cos\psi\\ z&=cr\sin\psi\end{aligned} \quad J=abc\,r^2\cos\psi$$

$$\frac{D(x,y,z)}{D(r,\varphi,\psi)}=\begin{vmatrix} a\cos\varphi\cos\psi & -ar\sin\varphi\cos\psi & -ar\cos\varphi\sin\psi\\ b\sin\varphi\cos\psi & br\cos\varphi\cos\psi & -br\sin\varphi\sin\psi\\ c\sin\psi & 0 & cr\cos\psi\end{vmatrix}=$$

$$=c\sin\psi\cdot r^2ab\cdot\cos\psi\sin\psi+cr^2\cos\psi\,ab(\cos^2\varphi\cos^2\psi+\cos^2\psi\sin^2\varphi)=\boxed{abcr^2\cos\psi}$$

N 8.148 (2)

$$G=\left\{x^2/a^2+y^2/b^2+z^2/c^2\le 1\right\}$$

$$\iiint_G(x^2+y^2)\,dx\,dy\,dz$$

$x = ar\cos\psi\cos\varphi \qquad 0 \le r \le 1 \qquad x^2+y^2 = r^2\cos^2\psi(a^2\cos^2\varphi + b^2\sin^2\varphi)$

$y = br\cos\psi\sin\varphi \qquad 0 \le \varphi \le 2\pi$

$z = cr\sin\varphi \qquad -\frac{\pi}{2} \le \psi \le \frac{\pi}{2}$

$$\frac{D(x,y,z)}{D(r,\varphi,\psi)} = abcr^2\cos\psi$$

$$\int_0^{2\pi} d\varphi \int_{-\frac{\pi}{2}}^{\pi/2} d\psi \int_0^1 abcr^4\cos^3\psi\left(b^2+\cos^2\varphi(a^2-b^2)\right)dr =$$

$$= \int_0^{2\pi}(a^2\cos^2\varphi + b^2\sin^2\varphi)\,d\varphi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^3\psi\,d\psi \int_0^1 abcr^4\,dr = \pi(a^2+b^2)\cdot\frac{4}{3}\frac{abc}{5} =$$

$$\int_{-\frac{\pi}{2}}^{\pi/2}\cos^3\psi\,d\psi = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\psi(1-\sin^2\psi)\,d\psi = \int_{-1}^{1}(1-t^2)\,dt$$

$$= \boxed{\frac{4\pi abc(a^2+b^2)}{15}}$$

№ 9.6 (2)

$(x^2+y^2)^2 = 2a^2(x^2-y^2)$, $x^2+y^2=a^2$

$r^4 = 2a^2r^2(\cos^2\varphi - \sin^2\varphi) \Rightarrow r^2 = 2a^2\cos 2\varphi \Rightarrow \cos 2\varphi = \frac{1}{2} \Rightarrow \varphi = \pm\frac{\pi}{6} + \pi n$

т.к. $r^2 = a^2$

$$S = 4\int_0^{\frac{\pi}{6}} d\varphi \int_a^{a\sqrt{2\cos 2\varphi}} r\,dr = 2a^2\int_0^{\pi/6}(2\cos 2\varphi - 1)\,d\varphi = 2a^2\left(\sin 2\varphi\Big|_0^{\frac{\pi}{6}} - \frac{\pi}{6}\right) =$$

$$= 2a^2\left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right) = \boxed{\frac{3\sqrt{3}-\pi}{3}a^2}$$

№ 9.8 (6)

$\sqrt[4]{x/a} + \sqrt[4]{y/b} = 1$, $x=0$, $y=0$ $\quad a>0\ b>0$

$x = ar\cos^8\varphi$

$y = br\sin^8\varphi$

$\Rightarrow \sqrt[4]{r}(\cos^2\varphi + \sin^2\varphi) = 1 \Rightarrow G = \left\{\begin{matrix} 0<r<1 \\ 0<\varphi<\frac{\pi}{2}\end{matrix}\right\}$

$$\frac{D(x,y)}{D(r,\varphi)} = \begin{vmatrix} a\cos^8\varphi & -8ar\cos^7\varphi\sin\varphi \\ b\sin^8\varphi & 8br\sin^7\varphi\cos\varphi \end{vmatrix} = 8abr\cos^7\varphi\sin^7\varphi$$

$$S=\iint_G dx\,dy=\int_0^{\pi/2} d\varphi\int_0^1 8abr\cos^7\varphi\sin^7\varphi\,dr=8ab\int_0^1 dr\int_0^{\pi/2}\sin^7\varphi\cos^7\varphi\,d\varphi=$$

$$=4ab\cdot I_{\pi/2}$$

$$I=\frac{1}{2^7}\int_0^{\pi/2}\sin^7 2\varphi\,d\varphi\underset{2\varphi=t}{=}\frac{1}{2^8}\int_0^{\pi}\sin^7 t\,dt\underset{u=\cos t}{=\!=\!=}\frac{1}{2^8}\int_{-1}^{1}(1-u^2)^3\,du=$$

$$=\frac{1}{2^7}\int_0^1(1-3u+3u^4-u^6)\,du=\frac{1}{2^7}\cdot\left(\frac{3}{5}-\frac{1}{7}\right)=\frac{1}{280}\Rightarrow \boxed{S=\frac{ab}{70}}$$

№ 10

$(a_1x+b_1y+c_1)^2+(a_2x+b_2y+c_2)^2=1$, $\Delta=a_1b_2-b_2a_1\neq 0$

$u=a_1x+b_1y+c_1$

$v=a_2x+b_2y+c_2$

$$\frac{D(u,v)}{D(x,y)}=\begin{vmatrix}a_1 & b_1\\ a_2 & b_2\end{vmatrix}\neq 0$$

$$\left.\begin{aligned}u&=r\cos\varphi\\ v&=r\sin\varphi\end{aligned}\right\}\Rightarrow G=\left\{\begin{matrix}0<r<1\\ 0<\varphi<2\pi\end{matrix}\right\}$$

$$S=\iint_G dx\,dy=\frac{1}{|\Delta|}\iint_{G_1}du\,dv=\frac{1}{|\Delta|}\iint_{G^*}r\,dr\,d\varphi=\frac{1}{|\Delta|}\int_0^{2\pi}d\varphi\int_0^1 r\,dr=\boxed{\frac{\pi}{|\Delta|}}$$

№ 6.13(5)

$z=x^2+y^2$ $\quad z=x+y$

$$\left.\begin{aligned}x&=r\cos\varphi\\ y&=r\sin\varphi\\ z&=h\end{aligned}\right|\Rightarrow G=\left\{r^2<h<r(\cos\varphi+\sin\varphi),\ 0<r<\cos\varphi+\sin\varphi,\ \varphi\in\left[-\frac{\pi}{4},\frac{3\pi}{4}\right]\right\}$$

$$V=\iint_G r\cdot(ra-r^2)\,dr\,d\varphi=\int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}d\varphi\underbrace{\int_0^a r^2(a-r)\,dr}_{I_a}\quad\Big|\quad I_a=a\int_0^a r^2dr-\int_0^a r^3dr=$$

$$=\frac{1}{3}a^4-\frac{1}{4}a^4=\frac{1}{12}a^4$$

$$V=\frac{1}{12}\int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}(\cos\varphi+\sin\varphi)^4\,d\varphi=\boxed{\frac{\pi}{8}}$$

№9.16(3)

$(x^2+y^2+z^2)^2 = a^3 x, \ a>0$

$x = r\cos\varphi\cos\psi$ $\qquad r^4 = a^3 r\cos\varphi\cos\psi \Rightarrow r = \sqrt[3]{a^3\cos\varphi\cos\psi}$

$y = r\sin\varphi\cos\psi$ $\qquad \Rightarrow \cos\varphi\cos\psi \geq 0$

$z = r\sin\psi$ $\qquad G = \{0 < r < \sqrt[3]{a^3\cos\varphi\cos\psi}, \ -\frac{\pi}{2} < \psi < \frac{\pi}{2}, \ -\frac{\pi}{2} < \varphi < \frac{\pi}{2}\}$

$$V = \iiint_G r^2\cos\psi \, d\varphi \, d\psi \, dr = \int_{-\pi/2}^{\pi/2} d\varphi \int_{-\pi/2}^{\pi/2} d\psi \int_0^{\sqrt[3]{a^3\cos\varphi\cos\psi}} r^2\cos\psi \, dr = \frac{a^3}{3}\int_{-\pi/2}^{\pi/2}\cos\varphi \, d\varphi \int_{-\pi/2}^{\pi/2}\cos^2\psi \, d\psi = \frac{2}{3}a^3 I_a$$

$$I_a = \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cancel{\cos 2\psi}^{\,0} \, d\psi + \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\psi = \frac{\pi}{2} \Rightarrow \boxed{V = \frac{\pi}{3}a^3}$$

№9.21

$a_1x+b_1y+c_1z = \pm d_1, \ a_2x+b_2y+c_2z = \pm d_2, \ a_3x+b_3y+c_3z = \pm d_3 \quad \Delta \neq 0$

$$\left.\begin{aligned} u &= a_1x+b_1y+c_1z \\ v &= a_2x+b_2y+c_2z \\ w &= a_3x+b_3y+c_3z \end{aligned}\right| \Rightarrow \frac{D(u,v,w)}{D(x,y,z)} = \Delta \neq 0$$

$G = \{-d_1 < u < d_2, \ -d_2 < v < d_2, \ -d_3 < w < d_3\}$

$$V = \iiint_G \frac{1}{|\Delta|} du \, dv \, dw = \frac{1}{|\Delta|}\int_{-d_1}^{d_1} du \int_{-d_2}^{d_2} dv \int_{-d_3}^{d_3} dw = \boxed{8\frac{d_1d_2d_3}{|\Delta|}}$$

№9.63(4)

$\sqrt{x^2+y^2} \leq z \leq h, \ \rho = \rho_0 z^2$

$$\left.\begin{aligned} x &= r\cos\varphi \\ y &= r\sin\varphi \\ z &= \lambda \end{aligned}\right| \Rightarrow \begin{aligned} &G = \{0 < r < h, \ r < \lambda < h, \ 0 < \varphi < 2\pi\} \\ &\rho = \rho_0\lambda^2 \end{aligned}$$

$$M_x = \iiint_G \rho \, x \, dx \, dy \, dz = \rho_0 \iiint_G \lambda^2 r^2\cos\varphi \, dr \, d\varphi \, d\lambda = \rho_0 \int_0^{2\pi}\cos\varphi \, d\varphi \int_0^h r^2 dr \int_r^h \lambda^2 d\lambda = 0$$

$M_y = M_x$ (очев)

$$M_z = \rho_0 \iiint_G \lambda^3 r \, dr \, d\varphi \, d\lambda = \rho_0 \int_0^{2\pi} d\varphi \int_0^h r \, dr \int_r^h \lambda^3 d\lambda = \frac{2\pi}{4}\rho_0 \int_0^h (h^4 - r^4) r \, dr =$$

$$= \frac{1}{2}\pi\rho_0 \left(\frac{1}{2} h^4 r^2 \Big|_0^h - \frac{1}{6} h^6 \Big|_0^h \right) = \frac{1}{6}\pi\rho_0 h^6$$

$$M = \rho_0 \iiint_G \lambda^2 r\,dr\,d\varphi\,d\lambda = \rho_0 \int_0^{2\pi} d\varphi \int_0^h r\,dr \int_r^h \lambda^2 d\lambda = \frac{2}{3}\pi\rho_0 \int_0^h (h^3 - r^3) r\,dr =$$

$$= \frac{2}{3}\pi\rho_0 \left(\frac{1}{2} h^3 r^2 \Big|_0^h - \frac{1}{5} r^5 \Big|_0^h \right) = \frac{1}{5}\pi\rho_0 h^5$$

$$\boxed{z_c = \frac{M_z}{M} = \frac{5}{6} h, \quad x_c = y_c = 0}$$