№ 10.1

Γ - отр. $(0;-2)$ — $(4;0)$

4) $\int\limits_\Gamma \frac{ds}{y-x} \ominus$

$\Gamma: y=\frac{1}{2}x-2 \Leftrightarrow x=2y+4,\ -2\le y\le 0$

$\ominus \int\limits_{-2}^{0} \frac{dy}{y-2y-4}\cdot\sqrt{1+4} = -\sqrt{5}\int\limits_{-2}^{0}\frac{dy}{y+4} = \boxed{-\sqrt{5}\,\ln 2}$ / ОТВЕТ

№ 10.9

$\int\limits_\Gamma (x^{\frac{4}{3}}+y^{\frac{4}{3}})\,ds \quad \Gamma: x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$

$x=a\cos^3 t \qquad x'_t=-3a\cos^2 t\sin t$

$y=a\sin^3 t \qquad y'_t=3a\sin^2 t\cos t$

$0<t<2\pi$

$\sqrt{x_t'^2+y_t'^2}=\ldots=\frac{3}{2}a\sin 2t,\ 0<t<\frac{\pi}{2}$

$x^{\frac{4}{3}}+y^{\frac{4}{3}}=a^{\frac{4}{3}}(\cos^4 t+\sin^4 t)=a^{\frac{4}{3}}\left(1-\frac{\sin^2 2t}{2}\right)=\frac{1}{2}a^{\frac{4}{3}}(1+\cos^2 2t)$

$\int\limits_\Gamma x^{\frac{4}{3}}+y^{\frac{4}{3}}ds = 4\int\limits_{\Gamma_{\frac{1}{4}}}(x^{\frac{4}{3}}+y^{\frac{4}{3}})ds = 3a^{\frac{7}{3}}\int\limits_0 (1+\cos^2 2t)\sin 2t\,dt =$

$\Gamma_{\frac{1}{4}}$ – часть циклоиды в I квадр.

$= \left| \begin{array}{l} u=\cos 2t \\ dt=\frac{du}{-2\sin 2t} \end{array} \right| = \frac{3}{2}a^{\frac{7}{3}}\int\limits_{-1}^{1}(1+u^2)du = 3a^{\frac{7}{3}}\int\limits_0^1(1+u^2)du = \boxed{4a^{\frac{7}{3}}}$

№ 10.29

4) $\int\limits_\Gamma (x^2+y^2)dx+(x^2-y^2)dy$

Γ - Δ $(0,0)$ $(1,0)$ $(0,1)$, обход не часовой)

$\int\limits_\Gamma = \underbrace{I_1+I_2+I_3}_{\text{на кажд. отр.}}$

$I_1: x=0,\ 0\le y\le 1 \Rightarrow I_1=\int\limits_0^1 y^2dy=\frac{1}{3}$

$I_2: y=0,\ 0\le x\le 1 \Rightarrow I_2=\int\limits_0^1 x^2dx=\frac{1}{3}$

I_3: $y=1-x$, $0\le x\le 1 \Rightarrow I_3=\int_0^1 \left(x^2+(1-x)^2 - x^2+(1-x)^2\right)dx = -\frac{2}{3}$ $\Big| \Rightarrow$

$\Rightarrow \int_\Gamma = \frac{1}{3}+\frac{1}{3}-\frac{2}{3}=0$ /ОТВЕТ

№10.85

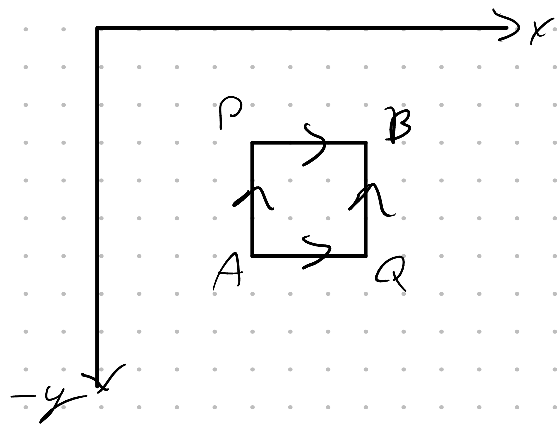
4) $\Gamma: y^2=x$, $A(1,1)$ $B(4,2)$ $\rho(x,y)=y$; $m-?$

$$m=\int_\Gamma \rho(x,y)\,ds=\int_1^2 y\sqrt{1+(2y)^2}\,dy=\frac{1}{2}\int_1^2\sqrt{1+4y^2}\,dy^2=$$

$$=\frac{1}{12}\left(1+4y^2\right)^{\frac{3}{2}}\Big|_1^2=\frac{1}{12}\left(17\sqrt{17}-5\sqrt{5}\right)$$ /ОТВЕТ

№10.110

$F=(4x-5y,\ 2x+y)$ $A(1,-9)$ $B(3,-3)$	1) $\Gamma=APB$, $P(1,-3)$ 2) AQB, $Q(3,-9)$



1) AP: $x=1$, $-9\le y\le -3$

$A_1=\int_{-9}^{-3}(2+y)\,dy=-24$

$A_2=\int_1^3(4x+15)\,dx=46$

$\Rightarrow A=22$

/ОТВЕТ

2) AQ: $y=-9$, $1\le x\le 3$

$A_1=\int_1^3(4x+45)\,dx=106$

$A_2=\int_{-9}^{-3}(6+y)\,dy=0$

$\Rightarrow A=106$

№10.46

$\int_\Gamma (2xy-y)\,dx+x^2\,dy$ $\quad \Gamma: \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$Q = x^2 \qquad \dfrac{\partial Q}{\partial x} = 2x$

$P = 2xy - y \qquad \dfrac{\partial P}{\partial y} = 2x - 1$

$\Rightarrow \dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y} = 1 \Rightarrow \int\limits_{\Gamma} = \iint\limits_{S} dx\,dy = \boxed{\pi a b - S_{\text{эллипса}}}$ / ОТВЕТ

№10.100

Пусть $I = \int\limits_{\partial G^+} \overbrace{(Ax + By)}^{P}dx + \overbrace{(Cx + Dy)}^{Q}dy$

$\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y} = C - B \Rightarrow I = (C - B)\iint\limits_{G} dx\,dy = (C - B)\cdot S \Rightarrow S = \dfrac{I}{C - B} \Rightarrow$

$\Rightarrow A = B = D = 0,\ C = 1: \quad S = \int\limits_{\partial G^+} x\,dy$

$A = C = D = 0,\ B = 1: \quad S = -\int\limits_{\partial G^+} y\,dx$

$\Rightarrow S = \dfrac{1}{2}\int\limits_{\partial G^+} x\,dy - y\,dx$ / ч.т.д.

№10.39

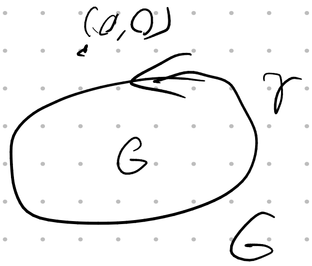
$\int\limits_{\Gamma} 2xy\,dx + x^2dy, \quad A(0,0)\ B(-2,-1), \quad \Gamma: AB$

$\int\limits_{\Gamma} 2xy\,dx + x^2dy = \int\limits_{\Gamma} d(\underbrace{x^2y}_{u}) = u(B) - u(A) = \boxed{-4}$ / ОТВЕТ

Т1

$\int\limits_{\gamma} \dfrac{x\,dy - y\,dx}{x^2 + y^2}$, γ — прост. гл. не проход. через $(0,0)$ против. час.

$\partial G = \gamma$



$P = -\dfrac{y}{x^2+y^2} \Rightarrow \dfrac{\partial P}{\partial y} = \dfrac{y^2 - x^2}{(x^2+y^2)^2}$

$Q = \dfrac{x}{x^2+y^2} \Rightarrow \dfrac{\partial Q}{\partial x} = \dfrac{y^2 - x^2}{(x^2+y^2)^2}$

$\Rightarrow \dfrac{\partial P}{\partial y} - \dfrac{\partial Q}{\partial x} = 0 \Rightarrow$

$\Rightarrow \boxed{\int\limits_{\gamma} = 0}$ по ф-ле Грина

G' малая обл.

$G_1 = G \setminus G' \qquad \int\limits_{\gamma} + \int\limits_{\partial G'\downarrow} = \int\limits_{\partial G_1^+} \Rightarrow$

$$\Rightarrow \int_{\gamma_{\text{вн}}} = \int_{\partial G'_{\text{вн}}} \Rightarrow \begin{matrix} x = r\cos t \\ y = r\sin t \end{matrix} \Rightarrow \int_{\gamma} = \int_0^{2\pi} \frac{r^2\cos^2 t + r^2\sin^2 t}{r^2}\, dt = \boxed{2\pi}$$

N 4.37

$z = \sqrt{x^2+y^2}$ $x^2+y^2=2x$

$$S = \iint_G \sqrt{1+\left(\frac{df}{dx}\right)^2+\left(\frac{df}{dy}\right)^2}\, dx\, dy$$

$$G = \{(x-1)^2+y^2 \leq 1\}$$

$$\frac{\partial z}{\partial x} = \frac{x}{\sqrt{x^2+y^2}} \qquad \frac{\partial z}{\partial y} = \frac{y}{\sqrt{x^2+y^2}}$$

$$S = \iint_G \sqrt{2}\, dx\, dy = \boxed{\pi\sqrt{2}}$$ /ОТВЕТ

N 4.57

$x = (b + a\cos\psi)\cos\varphi$

$y = (b + a\cos\psi)\sin\varphi$ $0 < a < b$ $S - ?$

$z = a\sin\psi$

$$[\overline{r}'_\varphi, \overline{r}'_\psi] = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ -(b+a\cos\psi)\sin\varphi & (b+a\cos\psi)\cos\varphi & 0 \\ -a\sin\psi\cos\varphi & -a\sin\psi\sin\varphi & a\cos\psi \end{vmatrix} =$$

$$= \begin{pmatrix} a(b+a\cos\psi)\cos\varphi\cos\psi \\ a(b+a\cos\psi)\sin\varphi\cos\psi \\ a(b+a\cos\psi)\sin\psi \end{pmatrix}$$

$$([r'_\varphi, r'_\psi])^2 = a^2(b+a\cos\psi)^2$$

$$S = \int_0^{2\pi} d\varphi \int_0^{2\pi} a(b+a\cos\psi)\, d\psi = \boxed{4\pi^2 ab}$$ /ОТВЕТ

T.2

$x = R\cos\varphi\cos\psi$

$y = R\sin\varphi\cos\psi$

$z = R\sin\psi$

$R\sin\psi_1 = c$

$R\sin\psi_2 = c+h$

$\Rightarrow$ $\varphi \in [0, 2\pi]$

$\psi \in [\psi_1, \psi_2]$

$$S=\int_0^{2\pi}d\varphi\int_{\psi_1}^{\psi_2}R^2\cos\psi\,d\psi=2\pi R^2\int_{\psi_1}^{\psi_2}\cos\psi\,d\psi=2\pi R^2\left(\frac{c+h}{R}-\frac{c}{R}\right)=\boxed{2\pi Rh}$$

/ОТВЕТ

№11.1(1)

$$\iint_S(x+y+z)\,dS\quad S\text{ – часть }\ x+2y+4z=4\quad \begin{matrix}x\geq 0\\ y\geq 0\\ z\geq 0\end{matrix}$$

$$\begin{matrix}x=4-2y-4z\\ \frac{\partial x}{\partial y}=-2\quad \frac{\partial x}{\partial z}=-4\end{matrix}\ \Bigg|\Rightarrow\iint_S(x+y+z)\,dS=\iint_G(4-2y)-4z+y+z)\cdot$$

$$\cdot\sqrt{1+4+16}\,dy\,dz=\sqrt{21}\iint_G(4-y-3z)\,dy\,dz=\sqrt{21}\int_0^2dy\cdot$$

$$\cdot\int_0^{1-\frac{1}{2}y}(4-y-3z)\,dz=\sqrt{21}\int_0^2\left(4-2y-y+\tfrac{1}{2}y^2+\tfrac{3}{2}(1-\tfrac{1}{2}y)^2\right)dy=$$

$$=\frac{\sqrt{21}}{8}\int_0^2(20-(2y+y^2))\,dy=\frac{\sqrt{21}}{8}\left(40-24+\frac{8}{3}\right)=\boxed{\frac{7}{3}\sqrt{21}}$$ /ОТВЕТ

№11.18(1)

$$x^2+y^2+z^2=R^2\quad x,y,z\geq 0$$

$$\begin{matrix}x=r\cos\varphi\cos\psi\\ y=R\sin\varphi\cos\psi\\ z=R\sin\psi\end{matrix}\ \Bigg|\Rightarrow G:\ \begin{matrix}\varphi\in[0,\frac{\pi}{2}]\\ \psi\in[0;\frac{\pi}{2}]\end{matrix}$$

$$M=\iint_S dS=\iint_G r^2\cos\psi\,d\varphi\,d\psi=r^2\int_0^{\frac{\pi}{2}}d\varphi\int_0^{\frac{\pi}{2}}\cos\psi\,d\psi=\frac{\pi R^2}{2}$$

$$x_C=y_C=z_C=\frac{1}{M}\iint_S x\,dS=\frac{r^3}{M}\int_0^{\pi/2}\cos\varphi\,d\varphi\int_0^{\pi/2}\cos^2\psi\,d\psi=\frac{2}{\pi r^2}\cdot\frac{\pi r^3}{4}=\frac{r}{2}\Rightarrow$$

(т.к. симм.)

$$\Rightarrow\boxed{r_c=\left(\frac{r}{2};\frac{r}{2};\frac{r}{2}\right)^T}$$ /ОТВЕТ

№11.37(1)

$$\iint_S yz\,dz\,dx\quad S\text{ - внешн. стор. }\ \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1,\ z\geq 0$$

$$x=a\cos\varphi\cos\psi$$
$$y=b\sin\varphi\cos\psi$$
$$z=c\sin\psi$$

$$\begin{vmatrix} 0 & bc\sin\varphi\cos\varphi\cos\psi & 0 \\ -a\sin\varphi\cos\psi & b\cos\varphi\cos\psi & 0 \\ -a\cos\varphi\sin\psi & -b\sin\varphi\sin\psi & c\cos\psi \end{vmatrix} = abc^2\sin^2\varphi\cos^3\psi$$

$$\iint_S yz\,dz\,dx = abc^2\int_0^{2\pi}\sin^2\varphi\,d\varphi\int_0^{\pi/2}\sin\psi\cos^3\psi\,d\psi = abc^2\pi\int_0^{\pi/2}\cos^3\psi\cdot d\cos\psi =$$

$$= \boxed{\frac{1}{4}\pi abc^2}$$ /ОТВЕТ

N11.38

$\iint_S (2x^2+y^2+z^2)\,dy\,dz$; S - внеш. ст. $\sqrt{y^2+z^2} \leq x \leq H$

$x^2 = y^2+z^2$

$x=z$
$y=x \Rightarrow z^2 = x^2+y^2$
$z=y$

$x = r\cos\varphi \quad \varphi\in[0,2\pi]$
$y = r\sin\varphi \quad r\in[0,H]$

$$\iint_S = -\iint_G (2(x^2+y^2)+x^2+y^2)\,dx\,dy = -3\int_0^{2\pi}d\varphi\int_0^H r^3\,dr = \boxed{-\frac{3}{2}\pi H^4}$$ /ОТВЕТ

N11.42

$\iint_S x^6\,dy\,dz + y^4\,dz\,dx + z^2\,dx\,dy$; S-внеш. ст. $z = x^2+y^2$, $z\leq 1$

$x = r\cos\varphi \quad \varphi\in[0;2\pi]$
$y = r\sin\varphi \quad r\in[0;1]$

$$\iint_S = -\iint_G \begin{vmatrix} x^6 & y^4 & (x^2+y^2)^2 \\ 1 & 0 & 2x \\ 0 & 1 & 2y \end{vmatrix} dx\,dy = -\iint_G \left[x^6\cdot(-2x) + y^4(-2y) + (x^2+y^2)^2\right]dx\,dy =$$

$$= -\int_0^{2\pi}d\varphi\int_0^1(-2r^7\cancel{\cos^7\varphi}^{\,0} - 2r^6\cancel{\sin^6\varphi}^{\,0} + r^4)r\,dr = -2\pi\int_0^1 r^5\,dr = \boxed{-\frac{\pi}{3}}$$ /ОТВЕТ

N3.44(1)

$f = 3x^4+y^3+xy \quad M(1;2) \quad \alpha = 135° = \widehat{\bar{a}, Ox}$ (обр. вект. луча)

$\operatorname{grad} f = (12x^3+y,\ 3y^2+x)^T = \operatorname{grad} f(M) = (14;13)^T$

$\bar a = \frac{\overline{AM}}{AM} = \left(-\frac{1}{\sqrt{2}} ; \frac{1}{\sqrt{2}}\right)^T$

$\frac{df}{da} = (\operatorname{grad} f(M), \bar a) = -\frac{14}{\sqrt{2}} + \frac{13}{\sqrt{2}} = \boxed{-\frac{1}{\sqrt{2}}}$ / ОТВЕТ

N 3.48(1)

$f = xy^2 - 3x^4y^5 \quad M(1;1) \quad \max \frac{df}{de}(M) - ?$

$\operatorname{grad} f = (y^2 - 12x^3y^5, 2xy - 15x^4y^4)^T$

$\operatorname{grad} f(M) = (-11; -13)^T$

$\bar e = (\cos\alpha, \sin\alpha)^T \Rightarrow \frac{df}{de} = (\bar e, \operatorname{grad} f) \le |\operatorname{grad} f|$

$|\operatorname{grad} f(M)| = \sqrt{11^2 + 13^2} = \boxed{\sqrt{290}}$ / ОТВЕТ

N 12.13

u - дифф. поле $f(t)$ - дифф. функц. $t \in R$

доказ: $\operatorname{grad} f(u) = f'(u) \operatorname{grad} u$

$\operatorname{grad} f(u) = \left(\frac{\partial f(u)}{\partial x} ; \frac{\partial f(u)}{\partial y} ; \frac{\partial f(u)}{\partial z}\right) = \left(f'(u)\frac{du}{dx} ; f'(u)\frac{du}{dy} ; f'(u)\frac{du}{dz}\right)^T = \boxed{f'(u) \operatorname{grad} u \quad \text{ч.т.д}}$

N 12.19

$f(r)$ - дифф. ; $\bar r = \bar i x + \bar j y + \bar k z$; $r = |\bar r|$

Док-ть: $\nabla f(r) = f'(r) \frac{\bar r}{|\bar r|}$

$\nabla f(r) = \operatorname{grad} f(r) = f'(r) \cdot \operatorname{grad} r = \frac{f'(r)}{\sqrt{x^2+y^2+z^2}} (\bar i x + \bar j y + \bar k z) = \boxed{f'(r)\frac{\bar r}{r}}$

N 12.15

$r = \bar i x + \bar j y + \bar k z$

3) $u = \frac{1}{r}$: $\operatorname{grad} u = -\frac{1}{r^2} \cdot \frac{\bar r}{r} = -\frac{\bar r}{r^3}$

5) $u = (\bar a, \bar r)$: $u = (a_x x ; a_y y ; a_z z)^T \Rightarrow \operatorname{grad} u = \bar a$

6) $u = (a, b, r)$: $u = ([\bar a, \bar b] \, \bar r) \Rightarrow \operatorname{grad} u = [\bar a, \bar b]$

N 12.37(2)

$$\operatorname{div}(u\bar{a}) = (\nabla, u\bar{a}) = (\nabla_u, u\bar{a}) + (\nabla_a, u\bar{a}) = (\operatorname{grad} u, \bar{a}) + u \cdot \operatorname{div}\bar{a}$$

$$\operatorname{div}(u\bar{a}) = \frac{\partial(u a_x)}{\partial x} + \frac{\partial(u a_y)}{\partial y} + \frac{\partial(u a_z)}{\partial z} = a_x\frac{\partial u}{\partial x} + a_y\frac{\partial u}{\partial y} + a_z\frac{\partial u}{\partial z} +$$

$$+ u\frac{\partial a_x}{\partial x} + u\frac{\partial a_y}{\partial y} + u\frac{\partial a_z}{\partial z} = (\operatorname{grad} u, \bar{a}) + u \operatorname{div}\bar{a}$$

N 12.39

$$\operatorname{div}(\operatorname{grad} u) = (\nabla, \operatorname{grad} u) = (\nabla, \nabla u) = (\nabla, \nabla)u = \Delta u =$$

$$= \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

N 12.40

$$\operatorname{div}(u \operatorname{grad} u) = (\nabla, u\nabla u) = (\nabla_u u, \nabla u) + u(\nabla_{\nabla u}, \nabla u) = (\nabla u)^2 + u\Delta u$$

N 12.41

3) $\operatorname{div} r\bar{e} = (\nabla, r\bar{e}) = (\nabla_r r, \bar{e}) = \frac{1}{r}(\bar{r}, \bar{e})$

6) $\operatorname{div}(f(r), \bar{e}) = (\nabla_f f(r), \bar{e}) = f'(r) \cdot \frac{(\bar{r}, \bar{e})}{r}$

7) $\operatorname{div}[\bar{c}, \bar{r}] =$

$$= \frac{\partial(c_y z - c_z y)}{\partial x} + \frac{\partial(c_z x - c_x z)}{\partial y} + \frac{\partial(c_x y - c_y x)}{\partial z} = 0$$

N 12.49

3) $\operatorname{rot}(u\bar{a}) = [\nabla, u\bar{a}] = [\nabla_u u, \bar{a}] + u[\nabla_{\bar{a}}, \bar{a}] = [\operatorname{grad} u, \bar{a}] + u \operatorname{rot}\bar{a}$

5) $\operatorname{rot}[\bar{a}, \bar{b}] = [\nabla, [\bar{a}, \bar{b}]] = \bar{a} \cdot (\bar{\nabla}, \bar{b}) - \bar{b}(\bar{\nabla}, \bar{a}) = \bar{a} \operatorname{div}\bar{b} - \bar{b} \operatorname{div}\bar{a}$

6) $\operatorname{div}[\bar{a}, \bar{b}] = (\nabla, [\bar{a}, \bar{b}]) = (\nabla_{\bar{a}}, \bar{a}, \bar{b}) + (\nabla_{\bar{b}}, \bar{a}, \bar{b}) = (\bar{b}, \operatorname{rot}\bar{a}) + (\bar{a}, \operatorname{rot}\bar{b})$

N 12.59

5) $\operatorname{rot}(u(r), \bar{r}) = [\nabla, u\bar{r}] = [\nabla_u u, \bar{r}] + u[\nabla_{\bar{r}}, \bar{r}] = \frac{u'}{r}[\bar{r}, \bar{r}] + u[\nabla_{\bar{r}}, \bar{r}] = 0$

N 12.54(2)

$$\operatorname{rot}[\bar{r}, [\bar{c}, \bar{r}]] \underset{c = const}{=} \operatorname{rot}[\bar{c}\, r^2 - \bar{r}(\bar{r}, \bar{c})] = [\nabla_r r^2, \bar{c}] - [\nabla_{\bar{r}}, \bar{r}](\bar{r}, \bar{c}) -$$

$-[\bar\nabla_{(\bar r,\bar c)}(\bar r,\bar c),\bar r] = 2r[\frac{\bar r}{r},\bar c] - [\bar c,\bar r] = 3[\bar r,\bar c]$

N11.45 $\iint\limits_S z\,dx\,dy + (3x+y)\,dy\,dz$

2) $\bar a = (5x+y,0,z)^T \quad \operatorname{div}\bar a = 6 \quad \Big| \quad \frac{x^2}{4}+\frac{y^2}{9}+z^2=1 \; : S$

$x = 2r\cos\varphi\cos\psi$
$y = 3r\sin\varphi\cos\psi$
$z = r\sin\psi$
$\Rightarrow J = 6r^2\cos\psi$

$$\iint\limits_S = -6\iiint\limits_G dx\,dy\,dz = -36\int_0^{2\pi}d\varphi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\psi\,d\psi\int_0^1 r^2dr = -48\pi$$

3) S: внешн. ст. $1 < x^2+y^2+z^2 < 4$

$x = r\cos\varphi\cos\psi$
$y = r\sin\varphi\cos\psi$
$z = r\sin\psi$
$\Rightarrow J = r^2\cos\psi$, $r\in(1,2)$

$$\iint\limits_S = 6\int_0^{2\pi}d\varphi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\psi\,d\psi\int_1^2 r^2dr = 56\pi$$

N11.52(3)

S: внешн. стор. $x^2+y^2=z^2$
$z\in(0;H]$

$\bar a = (x^2,y^2,z^2)^T \quad \operatorname{div}\bar a = 2(x+y+z)$

$x = r\cos\varphi$
$y = r\sin\varphi$
$z = h$

$$\Rightarrow \iint\limits_{S_{бок}} + \iint\limits_{S_{кр}} = 2\iiint\limits_S(x+y+z)\,dx\,dy\,dz = 2\int_0^{2\pi}d\varphi\int_0^H dh\int_0^h[r(\cos\varphi+\sin\varphi)+h]\,r\,dr = 2\int_0^{2\pi}d\varphi\int_0^H\frac{1}{2}h^3dh = \frac{\pi H^4}{2}$$

$$S_{кр}: \int_0^{2\pi}d\varphi\int_0^H H^2 r\,dr = 2\pi\cdot\frac{H^4}{2} = \pi H^4$$

$$\iint\limits_{S_{бок}} = -\frac{\pi H^4}{2}$$

N11.62

ориент. полож. отн. $(0,1,0)$

$\oint\limits_L y^2dx + z^2dy + x^2dz$, L — гран. треуг. $(a,0,0)$ $(0,a,0)$ $(0,0,a)$

$$\bar{n} = \frac{1}{\sqrt{3}}(1,1,1)^T \qquad \bar{m}(0,1,0) \quad \Rightarrow \cos(\bar{n}\hat{\ }\bar{m}) = \frac{1}{\sqrt{3}} > 0$$

$$\operatorname{rot}\bar{a} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & z^2 & x^2 \end{vmatrix} = -2z\bar{i} - 2x\bar{j} - 2y\bar{k}$$

$$\int_L = \iint_S (\operatorname{rot}\bar{a}, \bar{n})\,dS = -\frac{2}{\sqrt{3}}\iint_S (x+y+z)\,dS = -\frac{2a}{\sqrt{3}}\iint_S dS = -\frac{2a}{\sqrt{3}}\cdot\frac{\sqrt{3}}{4}(\sqrt{2}a)^2 = \boxed{-a^3}$$

N11.63(2)

$$\int_L \frac{x\,dy - y\,dx}{x^2+y^2} + z\,dz \qquad L \text{ - окр} \begin{cases} x^2+y^2+z^2 = R^2 \\ x+y+z=0 \end{cases}; \text{ ориент полож отн } (0,0,1)$$

$L = \partial G$, G_0 — круг. обл. сод. т $(0,0)$, $G_1 = G \setminus G_0$

$$\iint_{\partial G_0\downarrow} + \iint_{L\uparrow} = \iint_{\partial G_1\uparrow} = \iint_S (\operatorname{rot}\bar{a}, d\bar{s})$$

$$\bar{a} = \left(-\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2}, z\right)^T \Rightarrow \operatorname{rot}\bar{a} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\frac{y}{x^2+y^2} & \frac{x}{x^2+y^2} & z \end{vmatrix} = \bar{0}$$

$$\iiint_{G_1} = 0 \Rightarrow \iint_{L\uparrow} = \iint_{\partial G_0\uparrow}$$

$x = R\cos\varphi$

$y = R\sin\varphi$

$z = -R(\cos\varphi + \sin\varphi)$

$dz = -R(\cos\varphi - \sin\varphi)$

$$\Rightarrow \int_0^{2\pi} (1 + R^2\cos 2\varphi)\,d\varphi = \boxed{2\pi}$$

T3

N12.68(4)

S $(r = x\bar{i} + y\bar{j} + z\bar{k},\ r = |\bar{r}|)$; $a = \bar{r}, r^3$, S - внеш. $x^2+y^2+z^2 = R^2$

на S $r=r_0$, $\operatorname{div}\bar r=3$

$$\iint_S(\bar a,\bar n)ds=\iint_S(\bar a,d\bar s)=\frac{1}{r_0^3}\iiint_G \operatorname{div}\bar r\,dxdydz=\frac{1}{r_0^3}\cdot 3V_G=\frac{3}{r_0^3}\cdot\frac{4}{3}\pi r_0^3=$$
$$=\boxed{4\pi}$$

N 12.94(4)

$a=y\bar i-x\bar j+z\bar k$, $\Gamma=\{x^2+y^2+z^2=4,\ x^2+y^2=z^2,\ z\geq 0\}$

$\bar a=(y,-x,z)^T$ $\operatorname{rot}\bar a=(0,0,-2)^T$

$$\oint_\Gamma(\bar a,d\bar r)=\iint_S(\operatorname{rot}\bar a,d\bar s)=-2\iint_S ds=\boxed{-4\pi\ (S\ \text{окр. с}\ r=\sqrt2)}$$

N 12.104

$M=2I\dfrac{-y\bar i+x\bar j}{x^2+y^2}$ $(x,y)\neq(0,0)$

$\operatorname{rot}\bar a=\bar 0$

1) $G=\{x>0\}$ – пов. односв. $\Rightarrow$ **да**

2) $G=\{x\neq 0, y\neq 0\}$ $\operatorname{rot}\bar a=\nabla\times\begin{pmatrix}-y\\x\\0\end{pmatrix}=(0,0,2)^T$

$$\oint_\Gamma(\bar a,d\bar r)=\frac{2I}{R^2}\iint_S(\operatorname{rot}\bar a,d\bar s)=\frac{2I}{R^2}\cdot 2\pi R^2=4\pi I\neq 0\Rightarrow \boxed{\text{нет}}$$

N 12.112(1)

$\bar r=x\bar i+y\bar j+z\bar k$ $a=\bar r/r^3$

$G=\mathbb{R}\setminus\{0\}$

1) $\operatorname{rot}\bar a=0$, G – односв. пов. $\Rightarrow$ **пот.**

2) $\operatorname{div}\bar a=\dfrac{3}{r^3}-\dfrac{3}{r^4}\cdot r=0$

2.1) $z>0$ – объем односвяз. $\Rightarrow$ **солен.**

2.2) $r>0$ – необъемн. одн.

$$\iint_S\left(\frac{\bar r}{r^3},d\bar s\right)=\frac{1}{R^3}\iint_S(\bar r,d\bar s)=\frac{1}{R^3}\iiint_G \operatorname{div}\bar r\,dxdydz=\frac{3}{R^3}\cdot\frac{4}{3}\pi R^3=$$
$$=4\pi\neq 0\Rightarrow \boxed{\text{не солен.}}$$