§ Intuitionistic Logic (Natural Deduction without RAA)

Under these rules we introduce the **Kripke model**, which includes:

- A set of worlds with partial order $(W, \leq)$
- Evaluation $v : W \times \text{atomic prop} \to \{0, 1\}$
- $w_1 \geq w_0$ implies $v(w_1, p) \geq v(w_0, p)$ for any atomic prop p
- $v(w, \bot) = 0$

We further define $v(w, \psi \to \varphi)$ iff $v(w', \varphi) \geq v(w', \psi)$ for all $w' \geq w$.

A simple example: we set $w_0 \leq w_1$, and $v(w_0, p) = 0, v(w_1, p) = 1$.

- In w_1 everything is normal, with $v(\neg p) = 0, v(\neg\neg p) = 1$.
- In w_0 things change: $v(w_0, \neg p) \leq v(w_1, \neg p) = 0$, and $v(\neg\neg p) = 1$ using natural deduction (note that we didn't use RAA in this proof!). Hence $v(w_0, (\neg\neg p \to p)) = 0$!