# Derivation of Energy Equation with turbulent kinetic

# Classical Equations

The classical momentum and energy conservation is given by:

Momentum:



Energy



Here,

 is the viscous stress tensor, and , that means, equation in a bit more detailed form looks as



From equation , we find a formulation for the kinetic energy



and, finally, subtraction of equation from gives



The terms have different signification:

 comes from the density change due to compression or expansion

 is the heating of the material due to compression

 is the dissipation function and leads to heating due to viscous friction

# Derive a generalized energy equation containing turbulent sources terms

Now let us have a look at the complete stress tensor



where  is the turbulent addition to the stress tensor. We have to consider here only the deviatoric part of the stress tensor (hydrostatic part is the pressure , i.e.



In general, the formulation for the momentum as well as energy do not change, as they come from a global balance consideration. In fact, for momentum we have



and for the energy, we have to take into account that the kinetic turbulent energy now also takes part in the global balance in the sense



## Formulation for kinetic energy

From equation , we can derive a formulation for the mean kinetic energy (not the turbulent kinetic energy!!!!) in the following way.

First, scalar multiplication of the equation with the mean velocity



and by adding the value of zero, we find



Simplification yields



We would like to eliminate the term , and equation allows an easy formulation by



plugging into gives



which, through devision by 2, yields



## Generalized, assembled energy equation

The term is now ready to be plugged into the energy equation . First, let us write a bit more detailed in the sense



which shows the total energy in all of its details:

 = internal (thermal) energy

 = kinetic energy of the mean velocity

 = kinetic energy of the turbulent fluctuations

## Separation into turbulent energy part

In order to find out a separate formulation for turbulent energy, we have to subtract equation from . We obtain



In order to find out the precise formulation for , we have to decouple equation furthermore. We can simply use equation however we have to extend it by the additional heating that comes from the dissipation of turbulent kinetic energy, i.e.



Finally, subtracting equation from yields a proper formulation for the turbulent kinetic energy by



This equation does not take into account effects by turbulent buoyancy, which in the case of airbag deployment do not play a role. We see clearly the production term of turbulent kinetic energy to be



We have now a collection of the change of all three mayor parts of energy:

- Mean kinetic energy given by the mean velocity, see equation



- turbulent kinetic energy, see equation



- thermal energy, see equation



If we put all equations together, we find back the global energy conservation equation .

# Detailed representation of the turbulent production term

We refer to equation :



The chain rule leads to



Detailed explication of  yields



where  is the divergence of the velocity, and the term  makes the stress tensor trace free.

We find that the production rate now is



simplification results in



and furthermore



Another way of simplification consists in



which yields



which finally yields



which nicely demonstrates the positivity of the production term.