# FINITE POINTSET METHOD (FPM) FOR COMPRESSIBLE FLUID FLOW

*ABSTRACT*

This paper sums up the upwind ideas in FPM.

# General remarks

We want to solve the system of conservation equations of gasdynamics



where

 is the density,

 is the velocity,

 is the pressure,

 is the total energy,

 is the temperature (Kelvin),

 is the specific heat capacity.

We have to complete the set of equations by the equation of state



with  being the specific gas constant.

The sound speed can be derived by



where

 is the isentropic exponent.

For the numerical integration, we introduce the so called upwind velocity  and upwind pressure , leading to a stabilization of the scheme. Numerically, we solve the modified conservation equations



The following sections now provide different ways of approaching  and .

# Classical upwind formulation

The classical upwind formulation is given in [ 1], section 6.2. There, we use the upwind pressure and velocity of the form



where  is the upwind direction given by the pressure gradient. Here, we evaluate , and , using FPM’s approximation tools (east squares approximation) at the upstream location



where  is the upwind parameter that defines the relative distance (compared to smoothing length ) to go upstream in order to evaluate the upwind quantities.

The derivatives of the upwind quantities are consequently



# Simplified, fast upwind formulation

The upwind scheme of section 2 has a major drawback: the evaluation of physical quantities at the locations  and . Depending on the upwind direction , these locations might be outside of the flow domain. The idea of the simplified scheme is to approximate the upwind values by first order Tylor series expansion.



where

 is the upwind step size,

 are mixed terms we assume to be of minor importance and therefore neglect them.

The derivatives of the upwind quantities are then



# More simplified, fast upwind scheme

In equations ff. , the pressure formulation is difficult. However, one could further simplify



With this we are on the safe side, the damping in would only be smaller by magnitude, never bigger.

The derivatives of the upwind quantities are now



where  is simply a shortcut for more compact writing.

With this scheme we are able to prove/show the existence of mathematical damping in the upwind schemes. It is obvious that with the terms  and  we have mathematical damping for velocity and pressure, which act as stabilization of the scheme.

# Quasi-implicit scheme

The fascinating thing comes now. We will show that with a semi-implicit formulation we find the same upwind quantities as in section 4.

From section 1, we have (after slight modifications) the total time derivatives of velocity and pressure by



In order to come up with a quasi-implicit scheme, we look slightly into the future, for example we could establish a first order (in time) scheme by



We might not want to use  and  , but the values after a certain, intermediate time step size  , i.e.



with  and . This gives rise to the general scheme



If we base the implicit time shift  on the sound speed and the smoothing length with the proportionality factor  , then we have  and obtain



Please compare with section 4. We see that  and  are similar, as well as  and  are similar.

# Generalization

There are two principle upwind methods:

## Method 1 (original method)



With   
 the upwind step length

 the upwind direction

## Method 2 (simplified)



With

 the upwind step length (needed in the Tylor series expansion)

## Combined method

We can bring both methods together into one



How are the upwind lengths chosen?

FPM1 (original)



FPM1 (simplified – implemented in VPS)



FPM3



# Conclusions

* The upwind formulation induces proper mathematical damping terms.
* The quasi-implicit formulation is similar to the upwind formulation  the upwind formulation is similar to the quasi-implicit formulation. Both formulations can be combined into a generalized upwind function.

# Literature

[ 1 ] Jörg Kuhnert, „General Smoothed Particle Hydrodynamics“, dissertation, University of Kaiserslautern, 1999