Homework: Quantum Field Theory #7

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1. LIPS for N-body:

$$\mathrm{d}\Phi_{N_\beta}=\prod_{f=1}^{N_\beta}\frac{\mathrm{d}^3p_f}{(2\pi)^3 2E_{\mathbf{p}}}\delta^4(\sum_{N_\alpha}p_\alpha-\sum_{N_\beta}p_\beta)$$

For two-body scenario:

i). $m_1=m_2=0.\,(p_1^\mu+p_2^\mu=\Big(M,\mathbf{0}\Big)$, center-of-mass frame)

$$\begin{aligned}\mathrm{d}\Phi&=\frac{\mathrm{d}^3p_1\mathrm{d}^3p_2}{(2\pi)^6 2E_{\mathbf{p_1}}2E_{\mathbf{p_2}}}\delta^4(\sum_{N_\alpha}p_\alpha-\sum_{N_\beta}p_\beta)\\&=\frac{p_1^2\mathrm{d}p_1\mathrm{d}\Omega}{(2\pi)^6 2E_1 2E_2}\delta(M-E_1-E_2)\end{aligned}$$

and $\delta(M-E_1-E_2)=\left|\frac{\mathrm{d}(M-E_1-E_2)}{\mathrm{d}p_1}\right|^{-1}_{p_1=p_0}\delta(p_1-p_0)=\frac{E_1E_2}{p_0(E_1+E_2)}\delta(p_1-p_0)$ where p_0 is the solution of $f(p_1)=M-E_1-E_2=0$

$$\begin{aligned}&=\frac{p_0^2\mathrm{d}\Omega}{(2\pi)^6 2E_1 2E_2}\frac{E_1E_2}{p_0(E_1+E_2)}\\&=\frac{p_0}{(2\pi)^3 4M}\mathrm{d}\Omega\end{aligned}$$

note that $p_0=\sqrt{\frac{M^2}{4}-m^2}=\frac{M}{2}$

$$=\frac{1}{8(2\pi)^6}\mathrm{d}\Omega$$

ii). $m_1=m_2=m$, $M>2m$.

We only need to apply the result of the last one (before we apply the massless condition) and we can get

$$\mathrm{d}\Phi=\frac{\sqrt{M^2-4m^2}}{(2\pi)^6 8M}\mathrm{d}\Omega$$