- Determine if each of the following equations are functions:

a. $y = x^2 + 1$

If $x = 2$, then $y = (2)^2 + 1 = 5$, there is only one output $y = 5$. The equation is a function because there is only one unique output.

b. $y^2 = x + 1$

$y = \pm\sqrt{x+1}$, is not a function because more than one value for y.

- Which functions are surjective (i.e., onto)

1. $f: \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 3n$.

It is not a surjective because it doesn't cover all integers for example it will never output 1, 2, 4, 5, etc.

2. $g: \{1, 2, 3\} \to \{a, b, c\}$ defined by $g = \left\{ \begin{matrix} 1 & 2 & 3 \\ c & a & a \end{matrix} \right\}$

$g(1) = c$, $g(2) = a$, $g(3) = a$

Not surjective because b has no input from the domain.

3. $h: \{1, 2, 3\} \to \{1, 2, 3\}$ defined as follows:

$$\begin{tikzcd}
1 \arrow[dr] & 2 \arrow[dl] & 3 \arrow[d] \\
1 & 2 & 3
\end{tikzcd}$$

$h(1) = 2$, $h(2) = 1$, $h(3) = 3$

It's surjective because every element of the codomain is mapped.

- Using the same problem as above, determine which functions are injective.

1. $f(n) = 3n$, Injective because different inputs lead to different output.

2. $g(1) = c$, $g(2) = a$, $g(3) = a$

   Not Injective, beause different inputs lead to the same output.

3. $h(1) = 2$, $h(2) = 1$, $h(3) = 3$

   Injective, because each input has a distinct output.

- If $f(x) = \frac{1}{x+2}$ and $g(x) = \frac{1}{x} - 2$, is $g = f^{-1}$?

$$f(g(x)) = f\left(\frac{1}{x} - 2\right) = \frac{1}{\left(\frac{1}{x} - 2\right) + 2} = \frac{1}{1/x} = x$$

$$g(f(x)) = g\left(\frac{1}{x+2}\right) = \frac{1}{1/x+2} - 2 = (x+2) - 2 = x$$

Since both x, so g(x) is the inverse of f(x)

- Find the inverse of the function $f(x) = 2 + \sqrt{x-4}$

$y = 2 + \sqrt{x-4}$

$x = 2 + \sqrt{y-4}$

$x - 2 = \sqrt{y-4}$

$(x-2)^2 = y - 4$

$y = (x-2)^2 + 4$

$= (x-2)(x-2) + 4$

$= x^2 - 4x + 8$

$f^{-1}(x) = x^2 - 4x + 8$

- Find a formula for the inverse function that gives F° temperature as a function of C° temperature

$C = \frac{5}{9}(F-32)$

$\frac{9}{5} C = F - 32$

$F = \frac{9}{5} C + 32$

- Find the domain and range of the following function:

$g(x) = 2\sqrt{x-4}$

$x - 4 \geq 0$

$x \geq 4$

Domain $= [4, \infty)$

$g(4) = 2\sqrt{4-4} = 0$

Range $= [0, \infty)$

- Find the domain, and range of the following function:

$h(x) = -2x^2 + 4x - 9$

Domain $= (-\infty, \infty)$ because it is quadratic function

For range, the quadratic function concave down

$$\text{Vertex} = \frac{-b}{2a} = \frac{-4}{2(-2)} = 1$$

$$h(1) = -2(1)^2 + 4(1) - 9 = -7$$

$$\text{Range} = (-\infty, 7]$$

- Find the domain of the following functions:

$$f(x) = \frac{x-4}{x^2-2x-15}$$

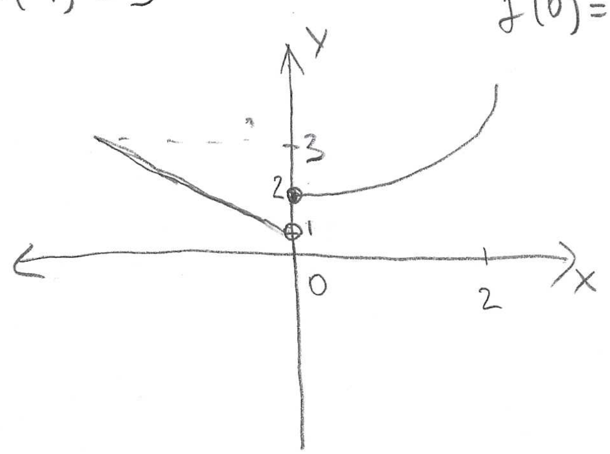
$$x^2 - 2x - 15 = (x-5)(x+3) \Rightarrow x = 5, x = -3$$

$$\text{Domain} = (-\infty -3) \cup (-3, 5) \cup (5, \infty -)$$

- Evaluate the following piecewise defined function for the given values of x, and graph the function:

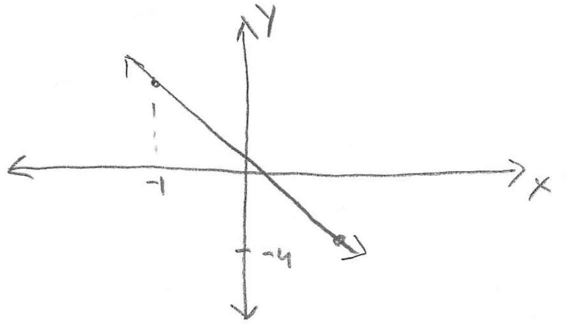
$$f(x) = \begin{cases} -2x+1 & -1 \leq x < 0 \\ x^2+x & 0 \leq x \leq 2 \end{cases}$$

$x = -1$; $f(x) = -2x+1$, $f(-1) = 3$, $x = 0$; $f(x) = x^2+2$, $f(0) = 2$, $x = 2$; $f(x) = x^2+2$, $f(2) = 6$



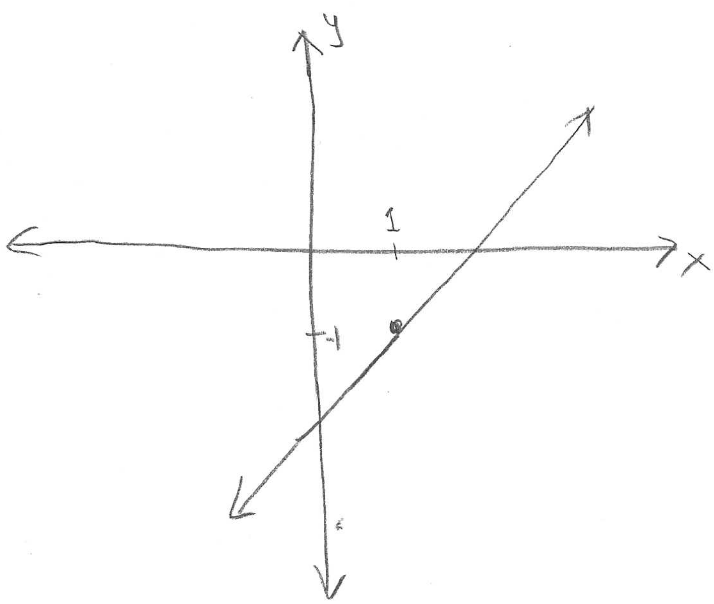
- Find the slope of the line that passes through the points $(-1,2)$ and $(3,-4)$. Plot the points and graph the line. 5

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{-4-2}{3-(-1)} = -\frac{6}{4} = -3/2$$



- Graph the line passing through the point $(1,-1)$ whose slope is $m = 3/4$.

$$y - y_1 = m(x-x_1)$$
$$y-(-1) = 3/4\,(x-1)$$
$$y = 3/4\,(x-1) - 1$$



- Compute the average rate of change of $f(x) = x^2 - \frac{1}{x}$ on the interval $[2,4]$.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{15.75 - 3.5}{4 - 2} = 6.125$$

OR

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- Given $f(x) = t^2 - t$ and $h(x) = 3x + 2$, evaluate $f(h(1))$.

$h(1) = 3(1) + 2) = 5$

$f(h(1)) = f(5) = (5)^2 - 5 = 20$

- Find the domain of $(f \cdot g)(x)$ where $f(x) = \frac{5}{x-1}$ and $g(x) = \frac{4}{3x-2}$

Domain of $g(x)$; $3x - 2 = 0$

$x = 2/3$, $(-\infty, 2/3) \cup (2/3, \infty)$

Domain of $f(g(x))$;

$$f(g(x)) = f\left(\frac{4}{3x-2}\right) = \frac{5}{(4/3x-2) - 1}$$

$\Rightarrow \frac{4}{3x-2} - 1 = 1$

$\frac{4}{3x-2} = 2$

$4 = 2(3x-2)$

$x = 4/3$

$(-\infty, 2/3) \cup (2/3, 4/3) \cup (4/3, \infty)$

- Find and simplify the functions $(g-f)(x)$ and $(g/f)(x)$, given $f(x) = x-1$ and $g(x) = x^2-1$. Are they the same function?

$$(g-f)(x) = g(x) - f(x) = (x^2-1) - (x-1)$$
$$= x^2 - 1 - x + 1$$
$$= x^2 - x$$

$$(g/f)(x) = \frac{g(x)}{f(x)} = \frac{x^2-1}{x-1}$$
$$= \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} = x+1$$

They are not the same function.

- Is the function $f(x) = x^3 + 2x$ even, odd, or neither?

$f(-x) = f(x)$ even

$f(-x) = -f(x)$ odd

$$f(-x) = (-x)^3 + 2(-x) = -x^3 - 2x$$

$f(-x) \neq f(x)$ not even

$$-f(x) = -(x^3 + 2x) = -x^3 - 2x$$

$f(-x) = -f(x)$ odd, function is odd.

- Is the function $f(s) = s^4 + 3s^2 + 7$ even, odd, or neither?

$$f(-s) = (-s)^4 + 3(-s)^2 + 7$$
$$= s^4 + 3s^2 + 7$$

$f(-s) = f(s)$ even, the function is even.

- Write the point-slope form of an equation of a line that passes through the points (5,1) and (8,7). Then rewrite it in the slope-intercept form $y = mx + b$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-1}{8-5} = 6/3 = 2$$

$$y - y_1 = m(x - x_1)$$
$$y - 1 = 2(x - 5)$$
$$y = 2x - 9 \;//$$

- If $f(x)$ is a linea function, and (3,-2) and (8,1) are points on the line, find the slope. Is this function increasing or decreasing?

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-2)}{8-3} = \frac{3}{5}$$

if $m > 0$ increasing, the function is increasing.

- Given the functions below, identify the functions whose graphs are a pair of parallel lines and a pair of perpendicular lines.

$f(x) = 2x + 3$ $\qquad$ $h(x) = -2x + 2$

$g(x) = \frac{1}{2}x - 4$ $\qquad$ $j(x) = 2x - 6$

$y = mx + b$

$f(x) \Rightarrow m = 2$ $\quad$ $h(x) \Rightarrow m = -2$

$g(x) = m = 1/2$ $\quad$ $j(x) \Rightarrow m = 2$

The slopes of $f(x)$ and $j(x)$ are both $m=2$, it means they are parallel.

The slopes of $g(x)$ and $h(x)$ are negative reciprocals, it means they are perpendicular.

- Solve $\begin{cases} 2x+y=7 \\ x-2y=6 \end{cases}$

let's solve for y; $\quad 2x+y=7$

$$y = 7-2x$$

substitute y into the $x-2y=6$;

$$x-2(7-2x)=6$$
$$x-14+4x=6$$
$$x=4$$

substitute $x=4$ into the $y=7-2x$;

$$y=7-2(4)$$
$$y=-1$$

, Solution $x=4, y=-1$

- solve $\begin{cases} 4x+2y=4 \\ 6x-y=8 \end{cases}$

Eliminate y;

$$\begin{cases} 4x+2y=4 \\ (6x-y=8)\times 2 \end{cases}$$

$$\begin{array}{r} 4x+2y=4 \\ +\ 12x-2y=16 \\ \hline 16x+0=20 \end{array}$$

$$16x=20$$
$$x=5/4$$

Substitute $x=5/4$;

$$4(5/4)+2y=4$$
$$5+2y=4$$
$$y=-1/2$$, Solution $x=5/4$, $y=-1/2$

- Find the vertex of the quadratic function $f(x)=2x^2-6x+7$.
Rewrite the quadratic in standard form (vertex form).

$$h=-\frac{b}{2a}=-\frac{-6}{2(2)}=3/2$$

$$k=f(h)=2(3/2)^2-6(3/2)+7=5/2$$

$f(x)=a(x-h)^2+k$ standard form

$$f(x)=2(x-3/2)^2+5/2$$

- Find the domain and range of $f(x)=-5x^2+9x-1$

Domain = $(-\infty, \infty)$ because it's polynomial.

Range; $f(x)$ concave down,

$$x = \frac{-b}{2a} = \frac{-9}{2(-5)} = 9/10$$

$$y = f(9/10) = -5(9/10)^2 + 9(9/10)$$

$$y = 61/20$$

$$(-\infty, 6/20]$$

- Find the y and x-intercepts of the quadratic $f(x) = 3x^2 + 5x - 2$

y-intercept, x=0; $f(0) = 3(0)^2 + 5(0) - 2$

$$(0, -2)$$

x-intercept, y=0; $3x^2 + 5x - 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{5 \pm \sqrt{5^2 - 4(3)(-2)}}{2(3)}$$

$$x = \frac{-5 \pm 7}{6} \Rightarrow x = 1/3, \; x = -2$$

y-intercept: $(0, -2)$

x-intercept: $(1/3, 0)$ and $(-2, 0)$

- Solve the inequality, graph the solution set on a number line and show the solution set in interval notation:

a. $-1 \leq 2x-5 < 7$

$-1 \leq 2x-5$

$-1+5 \leq 2x$

$x \geq 2$

$2x-5 < 7$

$2x < 12$

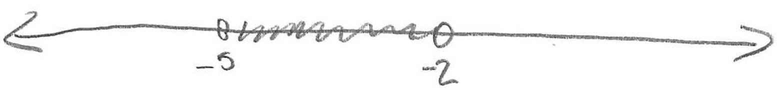
$x < 6$

$2 \leq x < 6$, $[2,6)$

b. $x^2+7x+10 < 10$

$(x+5)(x+2) < 0$

$x=-5$, $x=-2$ $(-5,-2)$



c. $-6 < x-2 < 4$

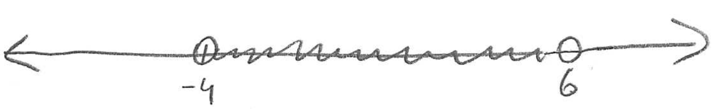
$-6+2 < x$

$x > -4$

$x-2 < 4$

$x < 6$

$-4 < x < 6$ $(-4,6)$



- Solve the inequality and graph the solution set. State the answer in both set builder notation and in interval notation.

$$10-(2y+1) \leq -4(3y+2)-3$$

$$9-2y \leq -12y-11$$

$$y \leq -2 \quad , \quad (-\infty, 2]$$

- Solve: $x(x+3)^2(x-4)<0$

$x=0, \ x=-3, \ x=4$

$(-\infty,-3), \ (-3,0), \ (0,4), \ (4,\infty)$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$+ \quad + \quad - \quad +$

Solution: $(0,4)$

- Solve $2x^4 > 3x^3+9x^2$

$$2x^4-3x^3-9x^2>0$$

$$x^2(2x^2-3x-9)>0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(3)\pm\sqrt{(-3)^2-4(2)(-9)}}{2(2)}=\frac{3\pm 9}{4}$$

$$x=3, \ x=-3/2$$

$x = 0$, $x = 3$, $x = -3/2$

$(-\infty, -3/2), (-3/2, 0), (0, 3), (3, \infty)$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$+ \quad - \quad - \quad +$

Solution: $(-\infty, -3/2) \cup (3, \infty)$

- Given the function $f(x) = -1/2\,|4x-5| + 3$, determine the x-value for which the function values are negative.

$$-\frac{1}{2}|4x-5| + 3 < 0$$

$$-\frac{1}{2}|4x-5| < -3$$

$$|4x-5| > 6$$

$4x - 5 > 6$, $4x - 5 < -6$

$4x > 11$ $\qquad$ $4x < -1$

$x > 11/4$ $\qquad$ $x < -1/4$

$$(-\infty, -1/4) \cup (11/4, \infty)$$

- Solve $13 - 2|4x-7| \leq 3$

$$|4x-7| \geq 5$$

$4x - 7 \geq 5$, $4x - 7 \leq -5$

$x \geq 3$ $\qquad$ $x \leq 1/2$

$$(-\infty, 1/2) \cup [3, \infty)$$