Logical Definitions

Modus Ponens

$$\frac{\begin{array}{c} A \longrightarrow B \\ B \end{array}}{A}$$

First click Modus Ponens - to see the mathematical definition

Here is my understanding... essentially, with Modus Ponens - we assume the case where A implies B, i.e., "if 2+2 = 4, then 4 - 2 = 2". In this scenario, if we just assume B is true, then we don't need to worry about A becuase logically if B is true, then A MUST be true and here is the proof of that.

Proof:

$\longrightarrow ((A \longrightarrow B) \wedge B) \longrightarrow A$

$\longrightarrow A \longrightarrow B \equiv \neg A \vee B$ *(By implication as disjunction)*

$\longrightarrow ((\neg A \vee B) \wedge B) \longrightarrow A$

$\longrightarrow \neg ((\neg A \vee B) \wedge B) \vee A)$ *(By implication as disjunction)*

{ *Subroutine* }:

$\neg ((\neg A \vee B) \wedge B) \equiv \neg (\neg (A) \vee B) \vee \neg B$ *(By De Morgan's Law)*

$\neg (\neg (A) \vee B) \equiv A \wedge \neg B$ *(By De Morgan's Law)*

$A \wedge \neg B \equiv \bot$ *(Since we asuumed $A \wedge B = \top$)*

$\therefore \bot \vee \neg B \equiv \bot$ *(Since we assumed $B = \top$)*

$\longrightarrow \bot \vee A \longrightarrow A$ *(By Law of Excluded Middle)*

$\therefore ((A \longrightarrow B) \wedge B) \longrightarrow A \equiv A$

□

A	⊥	B	¬ B	⊥ ∨ A	A ∨ ¬ B
T	F	T	F	T	T
T	F	F	T	T	T
F	F	T	F	F	F
F	F	F	T	F	T

Modus Tollens

$$\frac{A \to B \qquad \neg B}{\neg A}$$

First click Modus Tollens - to see the mathematical definition

However, here is my interpretation of Modus Tollens. Basically if you know anything about logic - you know that the contrapositive is the logical equivalent to implication. This is the closest relation I can think of when thinkin with respect to Modus Tollens.

Here is the proof:

Proof:

→ *Claim:* $A \to B \leftrightarrow \neg B \to \neg A$

→

A	¬ A	B	¬ B	A → B	¬ B → ¬ A
T	F	T	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	T	F	T	T	T

□

Hypothetical Syllogism

$$\frac{A \to B \qquad B \to C}{A \to C}$$

First click Hypothetical Syllogism - to see the mathematical definition

One way I look at Hypothetical Syllogism is the transitivity rule. Pretty much if you know about mathematical functions then you should know about the transitivty rule. If not here is a scenario, if you have $A \leq B$ $B \leq C$ - then you can conclude $A \leq C$.