

Quantum Physics

Recitation 6: Tunneling and Alpha Decay

1 Geiger-Nuttall Law

1. Using the data from Perlman et. al., Phys. Rev. 77, 26 (1950), show the Geiger-Nuttall law

$$\log_{10} \tau = a_1 \frac{Z}{\sqrt{E}} + a_2,$$

where τ is the lifetime, and is related to half-life $t_{1/2}$ by $\tau \ln 2 = t_{1/2}$, is valid for the even-even species¹. Find a_1 and a_2 by fitting the data.

2. If you re-plot the data in problem 1 including the even-odd and odd-even species, you will find deviation from the Geiger-Nuttall law. What are the possible reasons for the deviation?

2 Alpha Decay

An α particle is emitted by the parent nucleus $^{212}_{84}\text{Po}$. Estimate the Coulomb potential it feels at the nuclear surface, and then use this information to estimate the lifetime of $^{212}_{84}\text{Po}$.

1. Compute the alpha decay energy $Q_\alpha = [M_{Z,A} - (M_{Z-2,A-4} + M_{2,4})]c^2$, where $M_{Z,A}$ is the mass of the parent nucleus, $M_{Z-2,A-4}$ is the mass of the daughter nucleus, and $M_{2,4}$ is the mass of the Helium nucleus. Use this to explain why U^{238} can not reduce its energy by spontaneously emitting a proton, i.e., is stable for proton emission.
2. Consider the recoil of the daughter nucleus, and estimate the kinetic energy of the α -particle.
3. We approximate the daughter nucleus and the α -particle as uniformly charged spheres, estimate the Coulomb repulsion potential energy when they are just touching. The density of the nuclear matter is relatively constant, and the radius of the nucleus is given approximately by $r \approx 1.07 A^{1/3} \text{ fm}$, where $1 \text{ fm} = 10^{-15} \text{ m}$.
4. Estimate the lifetime of the $^{212}_{84}\text{Po}$ using a square potential barrier, and compare it with the experimental observation $\tau = 0.5 \mu\text{s}$.
5. Calculate the lifetimes of U^{238} and Po^{212} using Gamow's model.

¹This set of data is provided in `AlphaDecay.ipynb`