ETE-205: Digital Logic Design

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Contents

1 Digital Systems

Definition 1.0.1: Digital Systems

A digital system is a system that processes digital signals. A digital signal is a signal that can take on only a discrete set of values.

2 Number Systems

Definition 2.0.1: Binary Numbers

A binary number is a number expressed in the binary numeral system, which uses only two symbols: typically 0 (zero) and 1 (one).

2.1 Number Base Conversions

Example 2.1: Convert $(7956.6875)_{10}$ to binary

Integer Part:

	Integer Quotient		Remainder	Coefficient
$7956/2 =$	3978	+	0	$a_0 = 0$
$3978/2 =$	1989	+	0	$a_1 = 0$
$1989/2 =$	994	+	1	$a_2 = 1$
$994/2 =$	497	+	0	$a_3 = 0$
$497/2 =$	248	+	1	$a_4 = 1$
$248/2 =$	124	+	0	$a_5 = 0$
$124/2 =$	62	+	0	$a_6 = 0$
$62/2 =$	31	+	0	$a_7 = 0$
$31/2 =$	15	+	1	$a_8 = 1$
$15/2 =$	7	+	1	$a_9 = 1$
$7/2 =$	3	+	1	$a_{10} = 1$
$3/2 =$	1	+	1	$a_{11} = 1$
$1/2 =$	0	+	1	$a_{12} = 1$

$$\therefore (7956)_{10} = (a_{12}a_{11}\dots a_1a_0)_2 = (1111100010100)_2$$

Fractional Part:

	Fractional Quotient		Product	Coefficient
$0.6875 \times 2 =$	1.375	=	1	$b_0 = 1$
$0.375 \times 2 =$	0.75	=	0	$b_1 = 0$
$0.75 \times 2 =$	1.5	=	1	$b_2 = 1$
$0.5 \times 2 =$	1.0	=	1	$b_3 = 1$

$$\therefore (0.6875)_{10} = (b_0b_1b_2b_3)_2 = (1011)_2$$

$$\therefore (7956.6875)_{10} = (1111100010100.1011)_2$$

Example 2.2: Convert $(AF66.9BC)_16$ to binary

$$
\begin{aligned}
(A)_{16} &= (1010)_2 \\
(F)_{16} &= (1111)_2 \\
(6)_{16} &= (0110)_2 \\
(9)_{16} &= (1001)_2 \\
(B)_{16} &= (1011)_2 \\
(C)_{16} &= (1100)_2
\end{aligned}
$$

$$\therefore (AF66.9BC)_{16} = (1010111101100110.100110111100)_2$$

Example 2.3: Convert (10101101.101011) to decimal

$$
\begin{aligned}
(10101101)_2 &= 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\
&= 128 + 0 + 32 + 0 + 8 + 4 + 0 + 1 \\
&= 173
\end{aligned}
$$

$$
\begin{aligned}
(0.101011)_2 &= 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 0 \times 2^{-4} + 1 \times 2^{-5} + 1 \times 2^{-6} \\
&= 0.5 + 0 + 0.125 + 0 + 0.03125 + 0.015625 \\
&= 0.671875
\end{aligned}
$$

$$\therefore (10101101.101011)_2 = (173.671875)_{10}$$

Example 2.4: Convert $(0.513)_{10}$ to octal

	Fractional Quotient		Product	Coefficient
$0.513 \times 8 =$	4.104	=	4	$b_0 = 4$
$0.104 \times 8 =$	0.832	=	0	$b_1 = 0$
$0.832 \times 8 =$	6.656	=	6	$b_2 = 6$
$0.656 \times 8 =$	5.248	=	5	$b_3 = 5$
$0.248 \times 8 =$	1.984	=	1	$b_4 = 1$
$0.984 \times 8 =$	7.872	=	7	$b_5 = 7$

$$\therefore (0.513)_{10} = (0.406517\ldots)_8$$

2.2 Complements of Numbers

2.2.1 Diminished Radix Complement

Definition 2.2.1: Diminished Radix Complement

Given a number N in base r having n digits, the $(r-1)$'s complement of N, i.e., its diminished radix complement, is defined as $(r^n - 1) - N$.

Example 2.5: 9's complement

The 9's complement of 546700 is

$$10^6 - 1 - 546700 = 999999 - 546700 = 453299$$

The 9's complement of 012398 is

$$999999 - 012398 = 987601$$

Example 2.6: 1's complement

The 1's complement of 1011000 is

$$(2^7 - 1)_2 - 1011000 = 1111111 - 1011000 = 0100111$$

The 1's complement of 0101101 is

$$1111111 - 0101101 = 1010010$$

2.2.2 Radix Complement

Definition 2.2.2: Radix Complement

The r's complement of an n-digit number N in base r is defined as $r^n - N$ for $N \neq 0$ and as 0 for $N = 0$. Comparing with the $(r-1)$'s complement, we note that the r's complement is obtained by adding 1 to the $(r-1)$'s complement, since $r^n - N = [(r^n - 1) - N] + 1$.

Example 2.7: 10's complement

The 10's complement of 012398 is

$$10^6 - 012398 = 1000000 - 012398 = 987602$$

The 10's complement of 246700 is

$$1000000 - 246700 = 753300$$

Example 2.8: 2's complement

The 2's complement of 1101100 is

$$(2^7)_2 - 1101100 = 10000000 - 1101100 = 0010100$$

The 2's complement of 0110111 is

$$10000000 - 0110111 = 1001001$$

2.2.3 Subtraction with Complements

Example 2.9: Using 10's complement, subtract $72532 - 3250$

Let $M = 72532$, and $N = 3250$
10's complement of N:

$$100000 - 3250 = 96750$$

Now,

$$\begin{aligned} M &= 72532 \\ \text{10's complement of } N &= +96750 \\ \text{Sum} &= 169282 \\ \text{Discard end carry } 10^5 &= -100000 \\ \text{Answer} &= 69282 \end{aligned}$$

Example 2.10: Subtract $1010100 - 1000011$ **and** $1000011 - 1010100$ **using 2's complement**

Let $X = 1010100$, and $Y = 1000011$
2's complement of Y:

$$10000000 - 1000011 = 0111101$$

Now, $X - Y$

$$\begin{aligned} X &= 1010100 \\ \text{2's complement of } Y &= +0111101 \\ \text{Sum} &= 10010001 \\ \text{Discard end carry } 2^7 &= -10000000 \\ \text{Answer} &= 0010001 \end{aligned}$$

$$\therefore X - Y = 0010001$$

Again, 2's complement of X:

$$10000000 - 1010100 = 0101100$$

Now, $Y - X$

$$\begin{aligned} Y &= 1000011 \\ \text{2's complement of } X &= +0101100 \\ \text{Sum} &= 1101111 \end{aligned}$$

There is no end carry. So, 2's complement of the sum:

$$10000000 - 1101111 = 0010001$$

$$\therefore Y - X = -(\text{2's complement of Sum}) = -0010001$$

2.3 Binary Coded Decimal Codes (BCD)

Decimal Symbol	BCD Digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

Example 2.11

$$(185)_{10} = (0001\ 1000\ 0101)_{BCD} = (10111001)_2$$

2.3.1 BCD Addition

Example 2.12

Add $6 = (0110)_2$ to the binary sum when it's greater than or equal to $10 = (1010)_2$

4	0100	4	0100	8	1000
+5	+0101	+8	+1000	+9	+1001
9	1001	12	1100	17	10001
			+0110		+0110
			10010		10111

Example 2.13: 184 + 576 = 760 in BCD

BCD	1	1		
	0001	1000	0100	184
	+0101	0111	0110	+576
Binary sum	0111	10000	1010	
Add 6		0110	0110	
BCD sum	0111	0110	0000	760

2.4 Gray Code

Definition 2.4.1: Gray Code

Gray code is a binary numeral system where two successive values differ in only one bit. It is commonly used in Analog-to-Digital converters.

Gray Code	Decimal Equivalent
0000	0
0001	1
0011	2
0010	3
0110	4
0111	5
0101	6
0100	7
1100	8
1101	9
1111	10
1110	11
1010	12
1011	13
1001	14
1000	15

2.5 Error-Detecting Code

Definition 2.5.1: Parity bit

A parity bit is an extra bit included with a message to make the total number of 1's either even or odd.

ASCII Code	With even parity	With odd parity
A = 1000001	01000001	11000001
T = 1010100	11010100	01010100