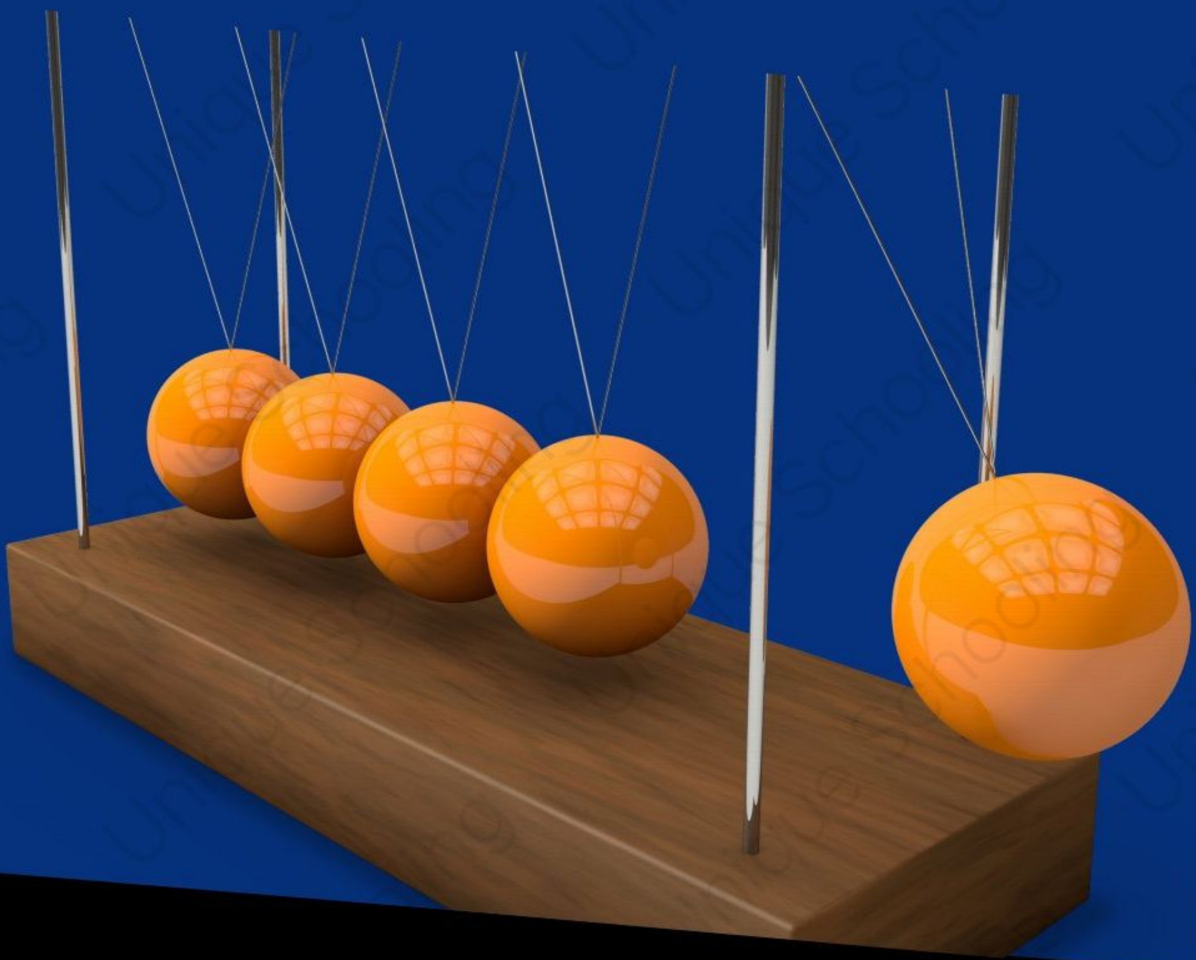
UNIQUE SCHOOLING



Physics Notebook

EDI. 1

BY MD. JAHIRUL ISLAM FAHIM

NEW FEATURING NOTEBOOK

- Easy Explanation of Theory
- 50+ Questions & Answers

ENGINEERING PHYSICS NOTEBOOK

By Unique Schooling

PHYSICAL OPTICS

- Interference of light
- Theory of Interference
- Young's double slit experiment
- Fresnel Bi-prism
- Interference by multiple reflection: constant & varying thickness films
- Newton's rings & its spectra

DIFFRACTION OF LIGHT

- Fresnel & Fraunhofer diffraction
- Fraunhofer diffraction by single slit & double slit
- Plane diffraction grating

POLARIZATION

- Production & analysis of polarized light
- Brewster's law
- Malus law
- Polarization by double refraction
- Nicole prism
- Polaroid
- Optical activity
- Polarimeters

NUCLEAR PHYSICS

- Nuclear Reaction
- Binding Energy
- Fission & fusion process
- Chain reaction

MODERN PHYSICS

- Theory of Relativity Quantum effect: Photoelectric effect, compton effect, de Broghlie wave, Wave particle duality, Interpretation of Bohr's postulates

RADIOACTIVITY

- Modes of Decay
- Laws of disintegration & successive disintegration
- Half life
- Mean life
- Radioactive Equilibrium

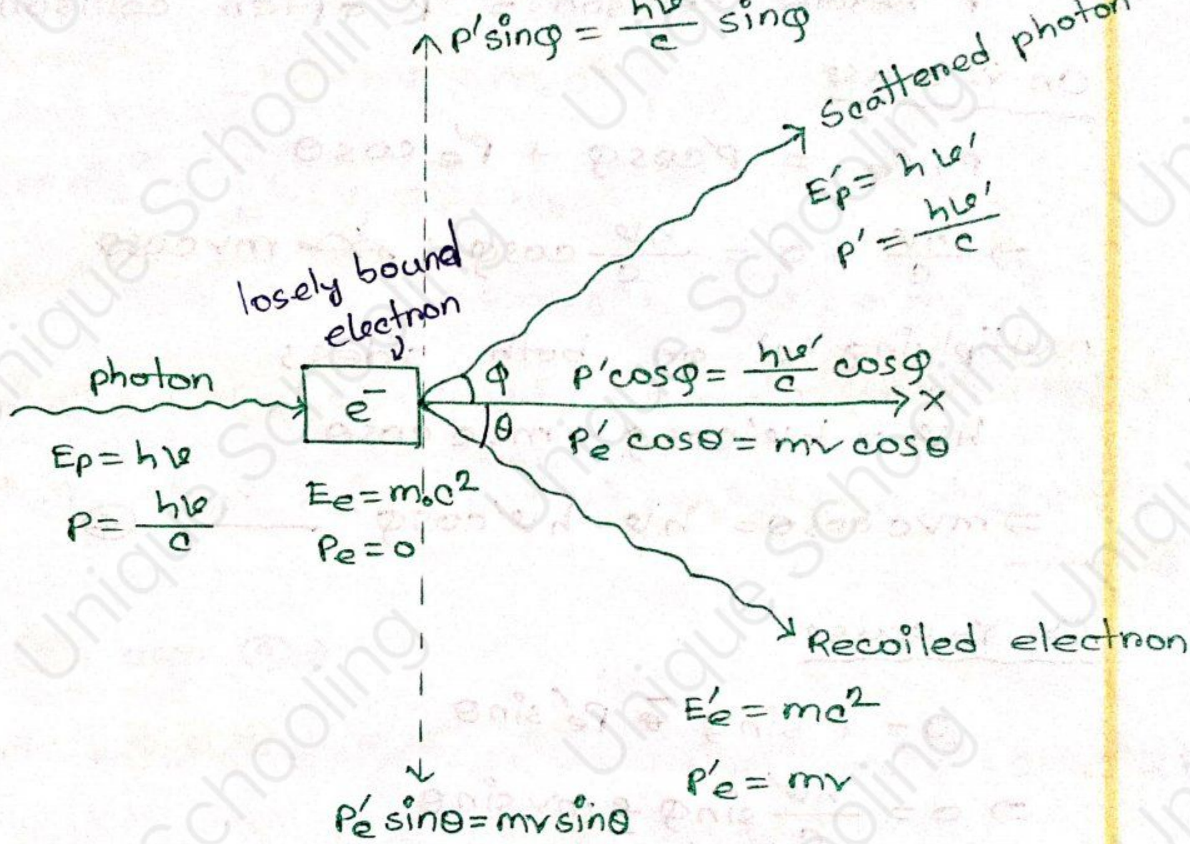
INTRODUCTION OF SOLID STATE PHYSICS

WAVES & OSCILLATIONS

LASERS & THEIR APPLICATIONS

compton effect or
compton shift

Compton Effect with derivation:



→ 1 photon interact with 1 electron.

→ φ = scattering angle and θ = recoiled angle.

→ A photon which is incident on Graphite block emit two types of photon wavelength (λ and λ') where, $\Delta\lambda = \lambda' - \lambda$; $\lambda' > \lambda$

$\Delta\lambda$ = compton shift.

→ Law of conservation of energy:

Energy before collison = Energy after colllison

$\Rightarrow E_p + E_e = E_p' + E_e'$

$\Rightarrow h\nu + m_0c^2 = h\nu' + mc^2$

$\Rightarrow h\nu - h\nu' = mc^2 - m_0c^2$

$\therefore h(\nu - \nu') = mc^2 - m_0c^2$ ——— ①

→ Law of conservation of momentum:

'p' before collison = 'p' after collison

On X-axis:

$$p + p_e = p'\cos\varphi + p_e'\cos\theta$$

$$\Rightarrow \frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\varphi + mv\cos\theta$$

multiplying 'c' on both sides;

$$h\nu = h\nu'\cos\varphi + mvc\cos\theta.$$

$$\Rightarrow mvc\cos\theta = h\nu - h\nu'\cos\varphi \quad \text{——} \quad ②$$

On Y-axis:

$$0 = p'\sin\varphi - p_e'\sin\theta$$

$$\Rightarrow 0 = \frac{h\nu'}{c}\sin\varphi - mv\sin\theta$$

multiplying 'c' on both sides;

$$h\nu'\sin\varphi - mvc\sin\theta = 0$$

$$\therefore mvc\sin\theta = h\nu'\sin\varphi \quad \text{——} \quad ③$$

$② ^2 + ③ ^2 \Rightarrow$

$$m^2v^2c^2(\cos^2\theta + \sin^2\theta) = (h\nu - h\nu'\cos\varphi)^2 + (h\nu'\sin\varphi)^2$$

$$\Rightarrow m^2v^2c^2 = h^2\nu^2 - 2h^2\nu\nu'\cos\varphi + h^2\nu'^2\cos^2\varphi + h^2\nu'^2\sin^2\varphi$$

$$\Rightarrow m^2v^2c^2 = h^2\nu^2 - 2h^2\nu\nu'\cos\varphi + h^2\nu'^2$$

$$\Rightarrow m^2v^2c^2 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\varphi$$

—— ④

Now from eqn ①;

$$h(\nu-\nu') = mc^2 - m_0c^2$$

$$\Rightarrow mc^2 = [h\nu - h\nu' + m_0c^2]$$

$$\Rightarrow (mc^2)^2 = [h(\nu-\nu') + m_0c^2]^2$$

$$\Rightarrow m^2c^4 = h^2(\nu-\nu')^2 + 2m_0c^2h(\nu-\nu') + m_0^2c^4$$

$$\Rightarrow m^2c^4 = h^2(\nu^2 - 2\nu\nu' + \nu'^2) + 2m_0c^2h(\nu-\nu') + m_0^2c^4$$

$$\Rightarrow m^2c^4 = h^2\nu^2 - 2h^2\nu\nu' + h^2\nu'^2 + m_0^2c^4 + 2m_0c^2h(\nu-\nu') \quad \text{——— ⑤}$$

eqn ⑤ - eqn ④ $\Rightarrow$

$$m^2c^4 - m^2v^2c^2 = \cancel{h^2\nu^2} - 2h^2\nu\nu' + \cancel{h^2\nu'^2} + m_0^2c^4 + 2m_0c^2h(\nu-\nu') - \cancel{h^2\nu^2} - \cancel{h^2\nu'^2} - 2h^2\nu\nu'\cos\varphi$$

$$\Rightarrow m^2c^4\left(1 - \frac{v^2}{c^2}\right) = m_0^2c^4 - 2h^2\nu\nu' + 2m_0c^2h(\nu-\nu') - 2h^2\nu\nu'\cos\varphi$$

$$\Rightarrow m^2c^4\left(1 - \frac{v^2}{c^2}\right) = m_0^2c^4 - 2h^2\nu\nu'(1-\cos\varphi) + 2m_0c^2h(\nu-\nu') \quad \text{——— ⑥}$$

We know the relativistic mass formula is;

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow m^2 = \frac{m_0^2}{1 - \frac{v^2}{c^2}}$$

$$\Rightarrow m^2 = \frac{m_0^2c^2}{c^2 - v^2} \Rightarrow m_0^2 = m^2\left(1 - \frac{v^2}{c^2}\right)$$

putting m_0^2 in eqn (6);

$$m^2c^4 \times \frac{m_0^2}{m^2} = -2h^2\nu\nu'(1-\cos\varphi) + m_0^2c^4 + 2h(\nu-\nu')m_0c^2$$

$$\Rightarrow m_0^2c^4 - m_0^2c^4 = 2h(\nu-\nu')m_0c^2 - 2h^2\nu\nu'(1-\cos\varphi)$$

$$\Rightarrow 2h^2\nu\nu'(1-\cos\varphi) = 2h(\nu-\nu')m_0c^2$$

$$\Rightarrow h\nu\nu'(1-\cos\varphi) = (\nu-\nu')m_0c^2$$

$$\Rightarrow h(1-\cos\varphi) = \frac{\nu-\nu'}{\nu\nu'}m_0c^2$$

$$\Rightarrow \frac{\nu-\nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1-\cos\varphi) \quad \text{——— (7)}$$

we know that, $\lambda = \frac{c}{\nu}$ $\therefore \lambda' = \frac{c}{\nu'}$

$$\Delta\lambda = \lambda'-\lambda = \frac{c}{\nu'} - \frac{c}{\nu} = c\left[\frac{1}{\nu'} - \frac{1}{\nu}\right]$$

$$\Rightarrow \frac{\lambda'-\lambda}{c} = \frac{\nu-\nu'}{\nu\nu'} \quad \text{——— (8)}$$

from (7) and (8);

$$\frac{\lambda'-\lambda}{c} = \frac{h}{m_0c^2}(1-\cos\varphi)$$

$$\therefore \Delta\lambda = \lambda'-\lambda = \frac{h}{m_0c}(1-\cos\varphi)$$

$$\therefore \Delta\lambda = \lambda'-\lambda = \lambda_c(1-\cos\varphi)$$

↳ compton wavelength

$$\lambda_c = \frac{h}{m_0c} = 0.0242$$

How does compton effect support the light of nature?

The shift of the wavelength increased with scattering angle according to the compton formula; compton explained and modeled the data by assuming a photon nature ~~of~~ for light and applying conservation of energy and conservation of momentum to the collison between the photon and the electron.

Cases:

i) $\varphi = 0^\circ$; $\cos 0^\circ = 1$

$\therefore \Delta\lambda = \lambda' - \lambda = \lambda_c(1-1) = 0$

$\therefore \boxed{\lambda' = \lambda}$

ii) $\varphi = \frac{\pi}{2}$; $\cos\frac{\pi}{2} = 0$

$\Delta\lambda = \lambda' - \lambda = \lambda_c(1-0) = \lambda_c$

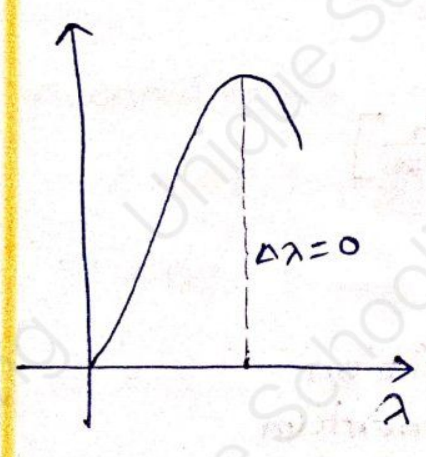
$\therefore \boxed{\Delta\lambda = 0.0242}$

iii) $\varphi = \pi$; $\cos\varphi = \cos\pi = -1$

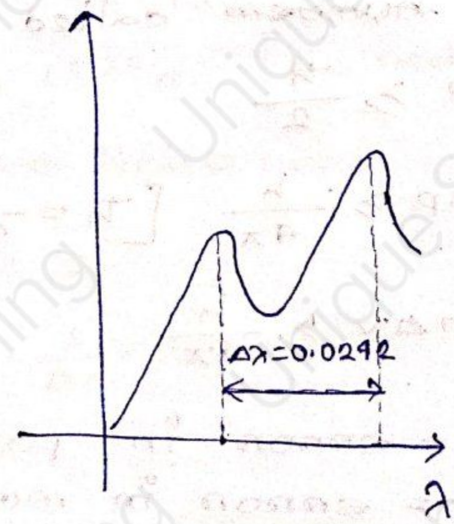
$\therefore \boxed{(\Delta\lambda)_{max} = 0.0489}$

Graph:

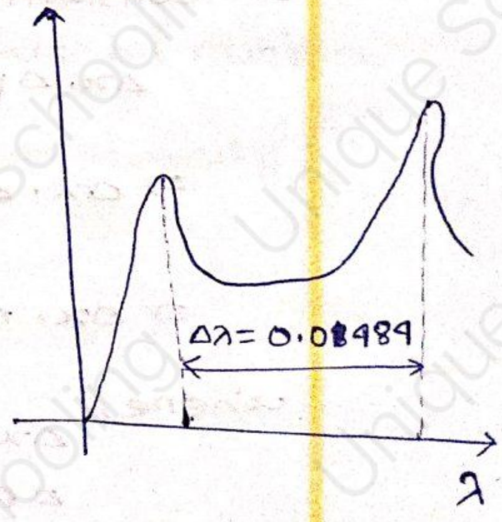
i) $\varphi = 0^\circ$



ii) $\varphi = \frac{\pi}{2}$



iii) $\varphi = \pi$



Heisenberg's Uncertainity Principle OR Uncertainity Principle:

→ It applies only on microscopic bodies.

→ It is used to calculate the probabilities for where things are and how they will behave.

Uncertainity Principle: It says that we cannot measure the position (x) and the momentum (p) of the particle simultaneously.

The more accurately we know one value, the less accurately we know the other.

→ Multiplying together the errors in measurements of position and momentum has to give a number greater than or equal to half of a constant ~~number~~ called ($\hbar$)

$$\Delta x . \Delta p \geq \frac{\hbar}{2}$$

$$\Rightarrow \Delta x . \Delta p \geq \frac{h}{4\pi} \quad \left[\hbar = \frac{h}{2\pi}\right]$$

$$\Rightarrow \Delta x . m\Delta v \geq \frac{h}{4\pi}$$

where,
Δx = error in position
Δp = error in momentum
Δv = error in velocity

example:

Imagine a car moving along a road. If you want to see exact place where the car is, you must pause time and mark its place. While you paused the time you can't know its speed.

If you unpause the time, you will know the speed of car and it is impossible to know the exact position of the car as it is changing.

Proved that, electron cannot reside inside the nucleus.

If we assume that electron resides inside the nucleus;

the maximum uncertainity of electron,

$(\Delta x)_{max} = 2r = 2 \times 5 \times 10^{-15} m$

$= 10^{-14} m$ | r = radius of nucleus

According to the Heisenberg uncertainity;

$\Delta x \Delta p \geq \cancel{\frac{h}{2}} \hbar$

$\Rightarrow \Delta P_{min} = \frac{\hbar}{\cancel{2}} \times \frac{1}{\Delta x}$

$\Rightarrow \Delta P_{min} = \frac{1.059 \times 10^{-34}}{\cancel{2\times} 10^{-14}}$

$\therefore \Delta P_{min} = 1.059 \times 10^{-20} Js\text{-}m^{-1}$

$$E_{min} = \sqrt{p_{min}^2 c^2 + m_0^2 c^4}$$

$$\Rightarrow E_{min} = \sqrt{p_{min}^2 c^2} = p_{min} c$$

$$= 1.054 \times 10^{-20} \times 3 \times 10^8$$

$$= 3.162 \times 10^{-12} \text{ J}$$

$$= 19762500 \text{ eV}$$

$$= 19.76 \text{ MeV}$$

β particle come from nucleus have maximum kinetic energy 4 MeV.

If a electron reside inside a nucleus its minimum energy must be $19.76 \approx 20$ meV but maximum energy of β particle is 4 meV. So, we can tell electron can not reside inside the nucleus.

Photoelectric effect:

When light or any other radiation of suitable wavelength falls on metal surface, electrons are emitted from the metal.

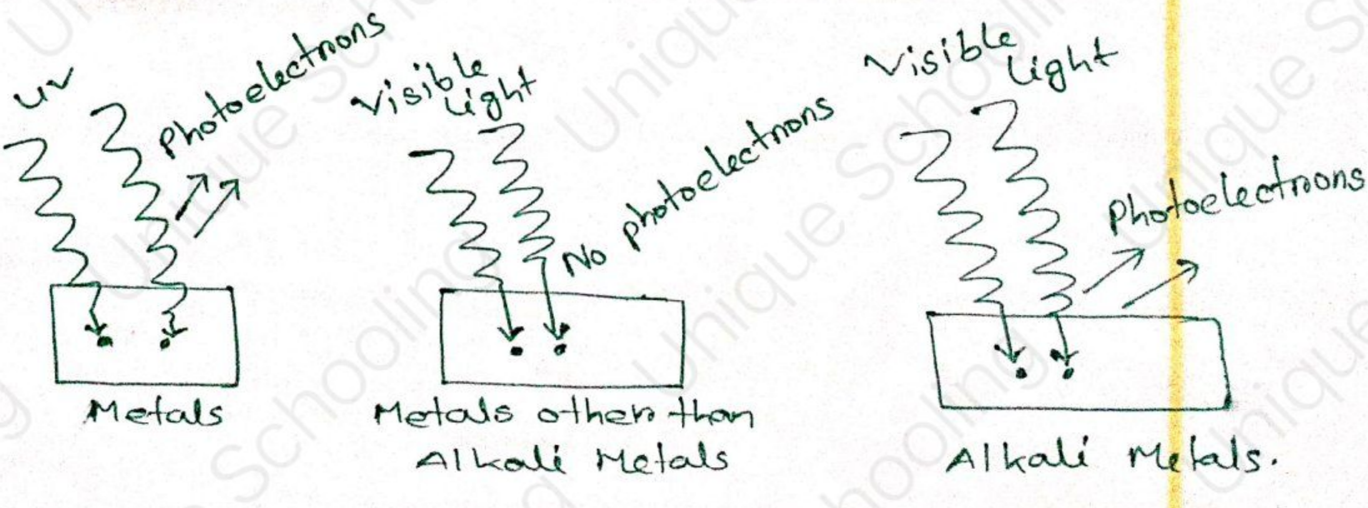
The phenomenon of emission of electrons from ~~from~~ a metal plate, when illuminated by of suitable wavelength is called photo electric effect.

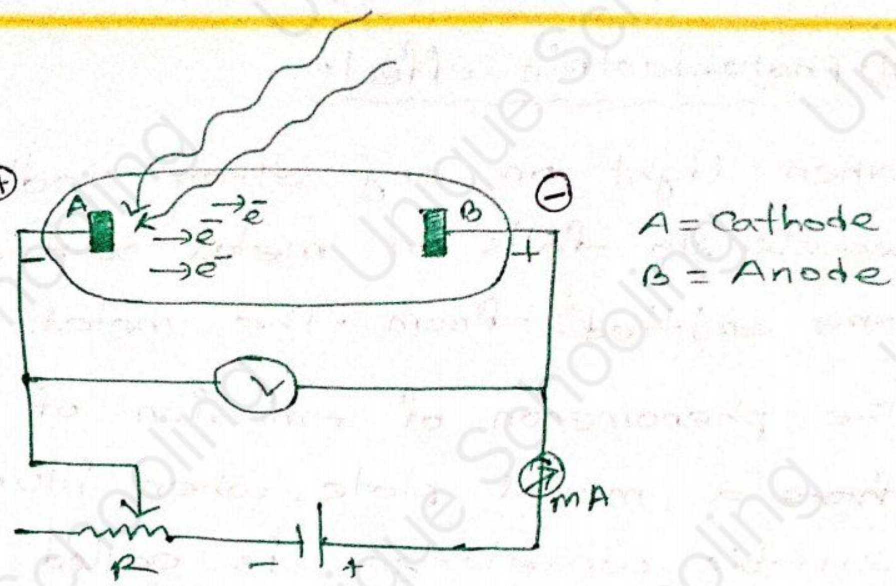
The electrons emitted by this effect are called photoelectrons.

The current constituted by photoelectrons is known as photoelectric current.

Non-metals also show photoelectric effect.

Liquids and gases also show this effect but to limited extent.





According to law of conservation of energy,

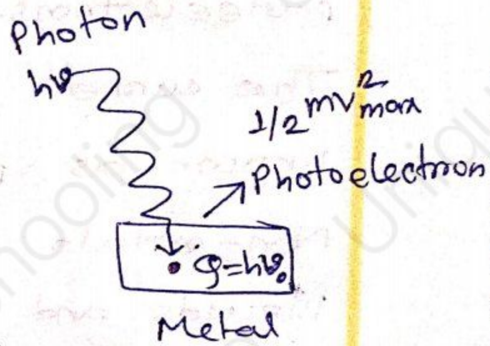
$$h\nu = \phi + \frac{1}{2}mv_{max}^2$$

$$\Rightarrow \frac{1}{2}mv_{max}^2 = h\nu - h\nu_0$$

$$= h(\nu - \nu_0)$$

$$= h\left(\frac{c}{\lambda} - \frac{c}{\lambda_0}\right)$$

$$= ch\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$



Application of photoelectric effect:

1. Automatic fire alarm.
2. Automatic burglar alarm.
3. Scanners in TV transmission.
4. Reproduction of sound in cinema film.
5. In paper industry to measure the thickness of paper.
6. In astronomy.
7. To determine opacity of solids and liquids.
8. Automatic switching of street lights.
9. To control the temperature of furnace.
10. Photometry.
11. To measure the fair complexion of skin.
12. Photoelectric sorting.
13. In cinema industry to check the light.
14. Photo counting.
15. Meterology.

What are the De-Broglie matter waves? Discuss their properties.

Matter waves are a central part of theory of quantum mechanics, being an example of wave-particle duality.
Matter waves are probability waves, amplitude of which gives the probability of existence of the particle at the point.

Matter waves like electromagnetic waves, can travel in vaccum and hence they are not mechanical waves. On the otherhand, matter waves are not electromagnetic waves because they are not produced by accelerated charges.

properties:

① Matter waves are not electromagnetic in nature.
② Represents the probability of finding a particle in space.
③ Matter waves are independent of the

① De Broglie wavelength is inversely proportional to the velocity of the particle.

② If the particle is at rest, then the Broglie wavelength is infinite. Such a wave cannot be visualized.

③ De Broglie wavelength is inversely proportional to the mass of the particle.

④ De Broglie wavelength is independent of the charge of the particle.

What is meant by velocity of De-Broglie waves?

De-Broglie hypothesized that any particle should exhibit duality.
The velocity of a particle is always equal to group velocity of the corresponding wave.
Thus, the velocity of debroglie is given by $\frac{h\nu}{mc}$.

De Broglie wavelength of electron accelerated through potential V.

Consider an $\bar{e}$ of mass m and charge $\bar{e}$. Let v be the velocity of $\bar{e}$ when accelerated from rest through a potential differental of V volt.

Kinetic energy of $\bar{e}$ $= \frac{1}{2}mv^2$

~~Potential diff~~

Work done of e^- $= eV$

We know, $KE = WD$

$$\Rightarrow \frac{1}{2}mv^2 = eV$$

$$\Rightarrow v = \sqrt{\frac{2eV}{m}}$$

Now we know that,

$$\lambda = \frac{h}{mv}$$

$$= \frac{h}{m\sqrt{\frac{2eV}{m}}}$$

$$= \frac{h}{\sqrt{2meV}}$$

$$= \frac{6.64\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times1.6\times10^{-19}\,V}}$$

$$= \frac{12.2}{\sqrt{V}}\,A^{\circ}$$

Find an expression for velocity of De-Broglie wave.

De Broglie wave equation:

Proposed that every hypothesized ~~has~~ that any particle has dual nature. One particle nature and other wave nature.

According to De-Broglie equation it is called De Broglie matter wave.

We know, if the frequency ν of a photon energy, $E = h\nu$ —— ①

By Einstein mass-energy relation;

$E = mc^2$ —— ②

From ① and ②;

$mc^2 = h\nu$

$\Rightarrow mc = \frac{h\nu}{c} = \frac{h}{\lambda}$

$$\text{mass} \times \text{velocity} = \frac{h}{\text{wavelength}}$$

$$\therefore \text{momentum} \propto \frac{1}{\text{wavelength}}.$$

Schrodinger's Time Independent equation (STIE):

We know that,

$$E = KE + PE \quad \left[KE = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m} \right]$$

$$\Rightarrow E = \frac{p^2}{2m} + V(x)$$

$$\Rightarrow \psi(x) \cdot E = \psi(x)\frac{p^2}{2m} + \psi(x) \cdot V(x) \quad \text{---} \ (1)$$

General function of $\psi(x)$ is;

$$\psi(x) = e^{i(kx - \omega t)} \quad \text{---} \ (2)$$

Differentiating eqn (2) twice w.r.t (x);

$$\frac{d}{dx}\psi(x) = ik \cdot e^{i(kx-\omega t)}$$

$$\therefore \frac{d^2}{dx^2}\psi(x) = i^2k^2 \cdot e^{i(kx-\omega t)} = i^2k^2\psi(x)$$

Here, $k = \frac{p}{\hbar}$

$$\therefore \frac{d^2}{dx^2}\psi(x) = i^2\frac{p^2}{\hbar^2}\psi(x) = -\frac{p^2}{\hbar^2}\psi(x)$$

$$\Rightarrow p^2\psi(x) = -\hbar^2\frac{d^2}{dx^2}\psi(x) \quad \text{---} \ (3)$$

Put eqn (3) in eqn (1);

$$\psi(x)E = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + \psi(x) \cdot V(x)$$

$$\Rightarrow \psi(x) \cdot E - \psi(x) \cdot V(x) + \frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) = 0$$

$$\Rightarrow \frac{2m}{\hbar^2}\psi(x)\cdot E - \frac{2m}{\hbar^2}\psi(x)\cdot V(x) + \frac{d^2}{dx^2}\psi(x) = 0$$

$$\Rightarrow \frac{d^2}{dx^2}\psi(x) + \frac{2m}{\hbar^2}\left[E\cdot\psi(x) - \psi(x)\cdot V(x)\right] = 0$$

This eqn is one dimensional (STIE)

Schrodinger's Time Dependent Equation (STDE):

→ Deals with matter wave or de-Broglie waves. ($\lambda = \frac{h}{p}$)

→ ψ = wave function that represent matter waves or de-broglie waves.

→ Eqn for particle moving freely in positive x-direction, $\psi = A\cdot e^{-i(\omega t - kx)}$ —— ①

where, $\omega = 2\pi\nu = 2\pi\frac{E}{h}$ —— ($E = h\nu$)

$k = \frac{2\pi}{\lambda} = 2\pi\frac{p}{h}$ —— ($\lambda = h/p$)

From ①; $\psi = Ae^{-i\frac{2\pi}{h}(Et - px)}$

$\therefore \psi = Ae^{-i/\hbar(Et - px)}$ $\left[\hbar = \frac{h}{2\pi}\right]$ —— ②

Differentiating eqn ② twice w.r.t x;

$$\frac{d^2}{dx^2}\psi = A\left(\frac{ip}{\hbar}\right)^2 e^{-i/\hbar(Et - px)}$$

$$= -1\left(\frac{p}{\hbar}\right)^2 \cdot Ae^{-i/\hbar(Et - px)}$$

$$\Rightarrow \frac{d^2}{dx^2}\psi = -\frac{p^2}{\hbar^2}\psi$$

$$\therefore p^2\psi = -\hbar^2\frac{d^2}{dx^2}\psi \quad \text{—— ③}$$

Now, differentiating eqn ② w.r.t 't'

$$\Rightarrow \frac{d\psi}{dt} = -\frac{i}{\hbar} E.A.e^{-i/\hbar(Et-px)}$$

$$\Rightarrow \frac{d\psi}{dt} = -\frac{i}{\hbar} E.\psi$$

$$\Rightarrow E\psi = -\frac{\hbar}{i}\frac{d\psi}{dt}$$

$$\therefore E\psi = i\hbar\frac{d\psi}{dt} \quad \text{----} \quad \left[-\frac{1}{i} = i\right]$$

⑨

Now,

$$E = KE + PE$$

$$\Rightarrow E = \frac{p^2}{2m} + V(x,t)$$

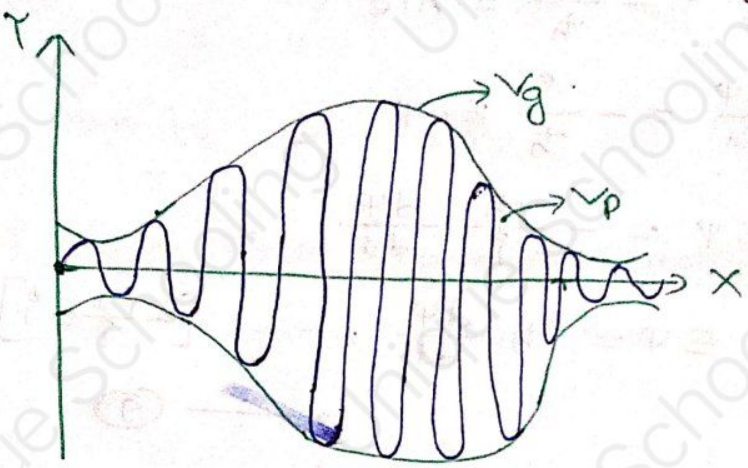
$$\Rightarrow E\psi = \frac{p^2}{2m}\psi + V(x,t)\psi$$

From eqn ③ and ④ ;

$$\frac{d\psi}{dt} = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x,t)\psi$$

↳ Equation for the one dimensional STDE.

(#) Show that, the group velocity is given by the following expression $v_g = v_p = \lambda \frac{dv_p}{d\lambda}$



Phase velocity, $v_p = \frac{\omega}{k}$ $\quad \Big| \; \omega = 2\pi n, \; k = \frac{2\pi}{\lambda}$

$$\Rightarrow \omega = v_p k \quad \text{——} \; (1)$$

group velocity, $v_g = \frac{d\omega}{dk} = \frac{d}{dk}(k, v_p)$

$$\Rightarrow v_g = v_p \frac{dk}{dk} + k \frac{dv_p}{dk} \quad \text{——} \; (2)$$

Again we know,

$$k = \frac{2\pi}{\lambda} \quad \text{and} \quad \frac{dk}{d\lambda} = -\frac{2\pi}{\lambda^2} \quad \text{——} \; (3)$$

From (2); $v_g = v_p + k \frac{dv_p}{d\lambda} \times \frac{d\lambda}{dk}$

$$= v_p + k \frac{dv_p}{d\lambda} \times \left(\frac{-\lambda^2}{2\pi}\right)$$

$$= v_p - \frac{\cancel{2\pi}}{\lambda} \times \frac{dv_p}{d\lambda} \times \frac{\lambda^2}{\cancel{2\pi}}$$

$$\therefore v_g = v_p - \lambda \frac{dv_p}{d\lambda}$$

Phase velocity: The average velocity of each individual wave of wave packet is called phase velocity. $v_p = \frac{\omega}{K}$

Group velocity: The average velocity with which the wave packet propagates in the medium is known as group velocity. $v_g = \frac{d\omega}{dk}$

Show that the energy of a plane progressive wave is given by $E = 2\pi^2 \rho n^2 a^2$.

In progressive wave, particles are executing simple harmonic motion.

The equation of a SHM is;

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) \quad \text{--- (1)}$$

$$\frac{dy}{dt} = \frac{2\pi}{\lambda} av \cos \frac{2\pi}{\lambda}(vt - x)$$

$$\therefore \frac{d^2y}{dt^2} = -\frac{4\pi^2 v^2}{\lambda^2} a \sin \frac{2\pi}{\lambda}(vt - x) \quad \text{--- (2)}$$

The (-ve) sign shows that acceleration is directed towards the mean position.

To move the particle from its mean position to a distance by, Workdone has to be done against acceleration.

$$dw = F dy$$

Let ρ be the density of the medium.

Workdone per unit volume for a displacement dy.

$$\rho\left(\frac{4\pi^2 v^2}{\lambda^2} a \sin\frac{2\pi}{\lambda}(vt-x)\right)dy$$

Total workdone for a displacement y

$$= \int_0^y \rho\left(\frac{4\pi^2 v^2}{\lambda^2} a \sin\frac{2\pi}{\lambda}(vt-x)\right)dy$$

potential energy per unit volume

$$= \left(\frac{4\pi^2 \rho v^2}{\lambda^2}\right)\int_0^y y\,dy$$

$$= \frac{4\pi^2 \rho v^2 y^2}{2\lambda^2}$$

$$P.E = \frac{2\pi^2 \rho v^2 y^2}{\lambda^2}$$

$$= \frac{2\pi^2 v^2 \rho}{\lambda^2} a^2 \sin^2\left[\frac{2\pi}{\lambda}(vt-x)\right] \quad \text{—— (3)}$$

$$K.E = \frac{1}{2}\rho\left[\frac{2\pi v}{\lambda} a \cos\frac{2\pi}{\lambda}(vt-x)\right]^2$$

$$= \frac{2\pi^2 \rho v^2}{\lambda^2} a^2 \cos^2\left[\frac{2\pi}{\lambda}(vt-x)\right] \quad \text{—— (4)}$$

$$E = PE + KE$$

$$= \frac{2\pi^2 \rho v^2 a^2}{\lambda^2}\left[\sin^2\frac{2\pi}{\lambda}(vt-x) + \cos^2\frac{2\pi}{\lambda}(vt-x)\right]$$

$$\therefore E = \frac{2\pi^2 v^2 a^2 \rho}{\lambda^2} \quad \text{—— (5)}$$

$v = n\lambda$

$\therefore E = 2\pi^2 \rho n^2 a^2$

The average KE per unit volume $= \pi^2 \rho n^2 a^2$

The average PE per unit volume $= \pi^2 \rho n^2 a^2$

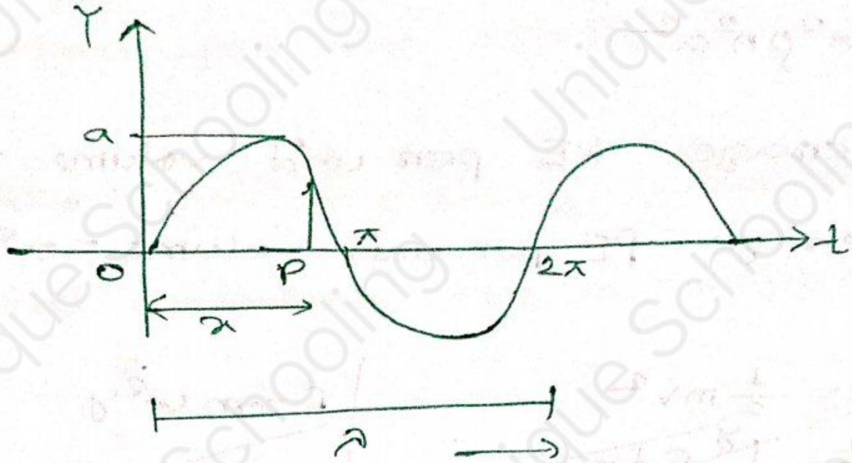
Or;

$\cancel{KE = \frac{1}{2} m v^2}$

$\cancel{PE = \int_0^y F\,dy}$

$\cancel{F = m\omega^2 y}$

Derive the differential wave $\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$



$$y = a \sin \frac{2\pi}{\lambda} (vt - x) \quad \text{---} \quad (1)$$

$$\Rightarrow \frac{dy}{dt} = \frac{2\pi}{\lambda} av \cos (vt - x) \quad \text{---} \quad (2)$$

$$\Rightarrow \frac{d^2y}{dt^2} = - \frac{4\pi^2 v^2}{\lambda^2} a \sin \frac{2\pi}{\lambda} (vt - x) \quad \text{---} \quad (3)$$

differentiating (1) w.r.t x;

$$\frac{dy}{dx} = - \frac{2\pi}{\lambda} a \cos \frac{2\pi}{\lambda} (vt - x) \quad \text{---} \quad (4)$$

$$\Rightarrow \frac{d^2y}{dx^2} = - \frac{4\pi^2 a}{\lambda^2} \sin \frac{2\pi}{\lambda} (vt - x) \quad \text{---} \quad (5)$$

From (3) and (5);

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$$

[Proved]

(#) Establish the differential eqn of SHM and solve it to obtain an expression for the displacement of a particle executing SHM.

We know that,

$$F = -ky \Rightarrow ma = -ky \qquad a = \frac{d^2y}{dt^2}$$

$$\Rightarrow m\frac{d^2y}{dt^2} = -ky \qquad \omega^2 = \frac{k}{m}$$

$$\Rightarrow \frac{d^2y}{dt^2} = -\frac{k}{m}y = -\omega^2 y$$

$$\therefore \boxed{\frac{d^2y}{dt^2} + \omega^2 y = 0}$$

Now multiplying $2\frac{dy}{dt}$ of this eqn;

$$2\frac{dy}{dt}\cdot\frac{d^2y}{dt^2} = -\omega^2 y \cdot 2\frac{dy}{dt}$$

Integratin w.r.t time;

$$\left(\frac{dy}{dt}\right)^2 = -\omega^2y^2 + c \quad \text{——— (1)}$$

$\frac{dy}{dt} = 0$; when $y = a$ (amplitude)

$$\Rightarrow \omega^2y^2 = c \Rightarrow \omega^2a^2 = c$$

From (1);

$$\left(\frac{dy}{dt}\right)^2 = -\omega^2y^2 + \omega^2a^2$$

$$\Rightarrow \frac{dy}{dt} = \pm\sqrt{\omega^2(a^2-y^2)} = \pm\omega\sqrt{a^2-y^2}$$

$$\Rightarrow \frac{dy}{\sqrt{a^2-y^2}} = \pm\omega dt \Rightarrow \int\frac{dy}{\sqrt{a^2-y^2}} = \int \omega dt$$

$$\Rightarrow \sin^{-1}\frac{y}{a} = \omega t + \varphi$$

$$\therefore \boxed{y = a\sin(\omega t + \varphi)}$$

(II) Derive the differential eqn of progressive wave : $U = -v\frac{dy}{dx}$ | U = partial velocity, v = wave velocity

We know the SHM eqn is;

$$y = a\sin\frac{2\pi}{\lambda}(vt - x) \quad \text{--- (1)}$$

$$U = \frac{dy}{dt} = \frac{2\pi v}{\lambda} a\cos\frac{2\pi}{\lambda}(vt - x) \quad \text{--- (2)}$$

For maximum partial velocity, $U = \frac{2\pi a}{\lambda}v$

where; $\cos\frac{2\pi}{\lambda}(vt - x) = 1$

differentiating (1) w.r.t x;

$$\frac{dy}{dx} = -\frac{2\pi}{\lambda} a\cos\frac{2\pi}{\lambda}(vt - x) \quad \text{--- (3)}$$

$$U = \frac{dy}{dt} = v\frac{2\pi}{\lambda} a\cos\frac{2\pi}{\lambda}(vt - x)$$

$$\Rightarrow \cancel{\frac{dy}{dt}}\ U = -v\frac{dy}{dx}$$

$$\therefore \boxed{U = -v\frac{dy}{dx}}$$

(III) A particle performs SHM given by the eqn, $y = 20\sin(\omega t + \delta)$

if the time period is 20 sec and the particle has displacement of 10 cm at $t = 0$, find;

i) δ ;

ii) the phase angle at $t = 5$ sec

iii) the phase difference between two positions of the particle 15 sec apart.

$y = 20 \sin(\omega t + \delta)$

$T = 20$ sec ; $\omega = \frac{2\pi}{T} = \frac{\pi}{10}$ rad s^{-1}

(i) at $t = 0$; $y = 10$ cm

$10 = 20 \sin\left(\frac{\pi}{10} \times 0 + \delta\right)$

$\Rightarrow \sin\delta = \frac{1}{2}$

$\therefore \delta = \frac{\pi}{6}$ rad

(ii) $t = 5$ sec; the phase angle, $\theta = (\omega t + \delta)$

$\theta_1 = \left(\frac{\pi}{10} \times 5 + \frac{\pi}{6}\right) = \cancel{\frac{\pi}{6}}\ \frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$

(iii) at $t = 15$ sec ;

$\theta_2 = \left(\frac{\pi}{10} \times 15 + \frac{\pi}{6}\right) = 3\pi + \frac{\pi}{6} = \frac{10\pi}{6} = \frac{5\pi}{3}$

$\therefore$ the phase diffenence $= \theta_2 - \theta_1$

$= \frac{5\pi}{3} - \frac{2\pi}{3}$

$= \frac{3\pi}{3} = \pi$

⑪ Find the resultant of two SHM of equal periods when they act at right angle to one another.

Or,

Composition of SH vibration at right angles to each other having equal frequency but differing in phases and amplitudes.

$$x = a\sin(\omega t + \delta) \quad \text{—— (1)}$$

$$y = b\sin\omega t \quad \text{—— (2)}$$

$$\Rightarrow \sin\omega t = \frac{y}{b}$$

From (1); $\frac{x}{a} = \sin\omega t \cdot \cos\delta + \cos\omega t \sin\delta$

$$\Rightarrow \frac{x}{a} = \frac{y}{b}\cos\delta + \left(\sqrt{1-\frac{y^2}{b^2}}\right)\sin\delta$$

$$\Rightarrow \left(\frac{x}{a} - \frac{y}{b}\cos\delta\right)^2 = \left(1 - \frac{y^2}{b^2}\right)\sin^2\delta$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\delta - 2\frac{xy}{ab}\cos\delta = \sin^2\delta - \frac{y^2}{b^2}\sin^2\delta$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\delta - 2\frac{x}{a}\frac{y}{b}\cos\delta + \frac{y^2}{b^2}\sin^2\delta = \sin^2\delta$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{x}{a}\frac{y}{b}\cos\delta = \sin^2\delta \quad \text{—— (3)}$$

Case-1: $\delta = 0°$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{x}{a}\frac{y}{b} = 0$$

$$\Rightarrow \left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0 \Rightarrow \frac{x}{a} = \frac{y}{b}$$

$$\therefore x = \frac{a}{b}y$$

case-II: $\delta = \frac{\pi}{4}$;

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{x}{a}\frac{y}{b}\frac{1}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} - \sqrt{2}\frac{x}{a}\frac{y}{b} = \frac{1}{2}$$

case-III: $\delta = \frac{\pi}{2}$;

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{x}{a}\frac{y}{b} \times 0 = 1^2$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

if $a=b$; $\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$

$$\Rightarrow x^2 + y^2 = a^2$$

case-IV: $\delta = \frac{3\pi}{4}$;

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{x}{a}\frac{y}{b}\left(-\frac{1}{\sqrt{2}}\right) = \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} + \sqrt{2}\frac{x}{a}\frac{y}{b} = \frac{1}{2}$$

case-V: $\delta = \pi$;

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{x}{a}\frac{y}{b}\cos\pi = (\sin\pi)^2$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} + 2\frac{x}{a}\frac{y}{b} = 0$$

$$\Rightarrow \left(\frac{x}{a} + \frac{y}{b}\right)^2 = 0$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = 0$$

$$\therefore x = -\frac{a}{b}y$$

(#) Show that the principle of conservation of energy is obeyed by a harmonic oscillation.

$$KE = \frac{1}{2} mv^2$$

$$PE = \int_0^y F dy$$

$$y = a \sin(\omega t + \delta)$$

$$\Rightarrow \frac{d^2 y}{dt^2} = -\omega^2 y$$

$$= \int_0^y m\omega^2 y \, dy$$

$$= \frac{1}{2} [m\omega^2 y^2]_0^y$$

$$= \frac{1}{2} m\omega^2 y^2$$

$$= \frac{1}{2} m\omega^2 a^2 \sin^2(\omega t + \delta)$$

$$KE = \frac{1}{2} mv^2 = \frac{1}{2} m \left(\frac{dy}{dt}\right)^2 = \frac{1}{2} m\omega^2 a^2 \cos^2(\omega t + \delta)$$

$$E = PE + KE$$

$$= \frac{1}{2} m\omega^2 a^2 \sin^2(\omega t + \delta) + \frac{1}{2} m\omega^2 a^2 \cos^2(\omega t + \delta)$$

$$= \frac{1}{2} m\omega^2 a^2$$

$$= \frac{1}{2} m \frac{K}{m} a^2$$

$$= \frac{1}{2} K a^2$$

total energy $= \frac{1}{2} K a^2 =$ constant.

Define the differential eqn of SHM:

F be the force acting on a particle
x is the displace from equilibrium position.

$F = -kx$

$\Rightarrow ma = -kx \Rightarrow a = -\frac{kx}{m}$

$\Rightarrow \frac{d^2x}{dt^2} = -\omega^2 x$

$\therefore \frac{d^2x}{dt^2} + \omega^2 x = 0$

A SHM is represent by

$y = 10 \sin(10t - \frac{\pi}{6})$

Calculate: i) frequency; ii) the time period
iii) the maximum displacement
iv) the maximum velocity
v) the maximum acceleration

i) $y = a \sin(\omega t + \delta)$

Here, $y = 10$; $\omega = 10$, $\delta = -\frac{\pi}{6}$

$\Rightarrow 2\pi f = 10$

$\Rightarrow f = \frac{10}{2\pi} = 1.6$ Hz

ii) $T = \frac{1}{f} = \frac{1}{1.6} = 0.625$ sec

iii) $a = 10$ m; iv) $v_{max} = \omega a = 10 \times 10 = 100$ ms^{-1}

v) $a_{max} = -\omega^2 a = -(10)^2 \times 10 = -1000$ ms^{-2}

Q. State the basic postulates of Einstein's special theory of ~~light~~ relativity.

In 1950, Albert Einstein drew two very important conclusions. These are known as the fundamental postulates of the special theory of relativity. These postulates are:

i) All physical laws are the same in all inertial frames of reference which are moving with constant velocity relative to each other.

ii) The speed of light in vaccum is the same in every inertial frame.

The theory based on these two postulates and applies to all inertial frames is called the special theory of relativity.

Ⓗ At what speed the mass of an object will be double of its value at rest?

First relativistic mass is known as the energy of a particle (up to a factor of c^2). The rest mass is always the same, regardless of however fast a particle is going.

The relativistic mass of, $E = mc^2$

$$\Rightarrow E = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow mc^2 = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow 2m_0 = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow \sqrt{1-\frac{v^2}{c^2}} = \frac{1}{2}$$

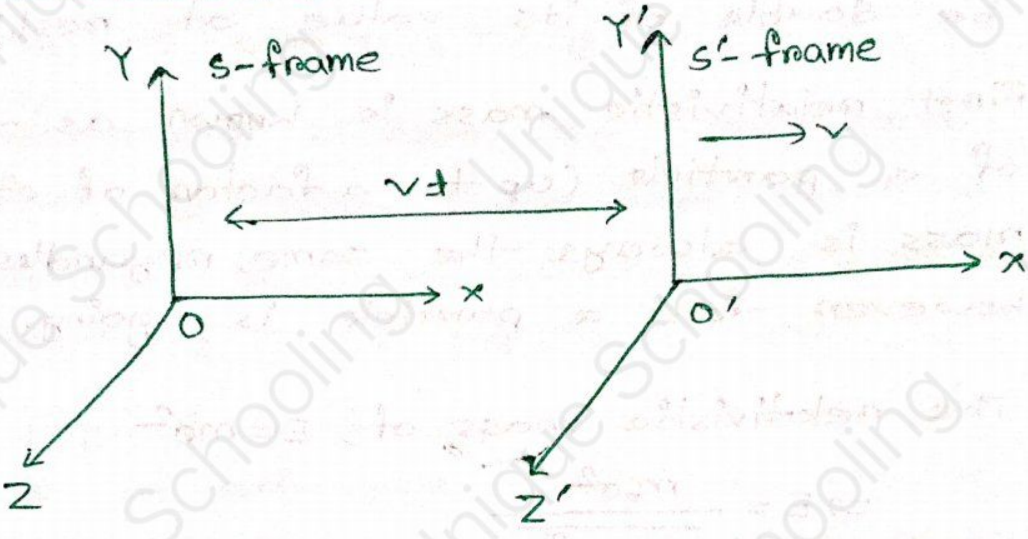
$$\Rightarrow 1-\frac{v^2}{c^2} = \frac{1}{4}$$

$$\Rightarrow \frac{v^2}{c^2} = \frac{3}{4}$$

$$\therefore v = \frac{\sqrt{3}}{2} c$$

Notice that this doesn't depend on the mass of the particle or any other aspect of the object being considered.

Lorentz transformation equation:



→ origin o' concides with O at $t=0$. A flash of light is emitted from a source situated at conciding origins.

→ Progress to ray of light described relative to frame s is,

$$ct = \sqrt{x^2+y^2+z^2}$$

$$\Rightarrow c^2t^2 = x^2+y^2+z^2$$

$$\Rightarrow x^2+y^2+z^2-c^2t^2 = 0 \quad \text{——— (1)}$$

Progress of ray of light described ~~light~~ relative to frame s' is,

$$x'^2+y'^2+z'^2-c^2t'^2 = 0 \quad \text{——— (2)}$$

Relative to observer in s-frame, origin o' is moving with velocity v along x-axis.

distance, $x = vt$

$\Rightarrow x - vt = 0$

relative to observer in s'- frame. Origin O' is not moving.

distance, $x' = 0$

$\therefore x' = \gamma(x - vt)$ ——— (3)

where, γ = constant

Relative to observer in s'- frame, origin O' is moving with velocity $(-v)$ along negative x'-axis.

distance, $x' = -vt'$

$\Rightarrow x' + vt' = 0$

relative to observer in s- frame, origin O is not moving. $x = 0$

distance, $x = \gamma'(x' + vt')$. ——— (4)

where, γ' = constant

Put eqn (3) in eqn (4);

$t' = \gamma\left[t + \frac{x}{v}\left(\frac{1}{\gamma\gamma'} - 1\right)\right]$ ——— (5)

there is no relative motion along Y and Z axis.

we get,

$$\left.\begin{array}{l} x' = \gamma(x - vt),\ y' = y,\ z' = z \\ t' = \gamma\left[t + \frac{x}{v}\left(\frac{1}{\gamma\gamma'} - 1\right)\right] \end{array}\right\} \text{——— (6)}$$

from eqn (2);

$$x'^2 + y'^2 + z'^2 - c^2t'^2 = 0$$

from eqn (6);

$$[\gamma(x - vt)]^2 + y^2 + z^2 - c^2\left[\gamma\left\{t + \frac{x}{v}\left(\frac{1}{\gamma\gamma'} - 1\right)\right\}\right]^2 = 0$$

$$\Rightarrow x^2\left[\gamma^2 - \frac{c^2\gamma^2}{v^2}\left(\frac{1}{\gamma\gamma'} - 1\right)^2\right] + y^2 + z^2 - 2xt\left[v\gamma^2 + \frac{c^2\gamma^2}{v}\left(\frac{1}{\gamma\gamma'} - 1\right)\right] + t^2(\gamma^2v^2 - c^2\gamma^2) = 0$$

comparing with eqn (1);

$$x^2 + y^2 + z^2 - t^2c^2 = 0$$

we get,

$$\gamma^2 - \frac{c^2\gamma^2}{v^2}\left(\frac{1}{\gamma\gamma'} - 1\right)^2 = 1 \quad \text{--- (7)}$$

$$\Rightarrow v\gamma^2 + \frac{c^2\gamma^2}{v}\left(\frac{1}{\gamma\gamma'} - 1\right) = 0 \quad \text{--- (8)}$$

$$\therefore \gamma^2v^2 - c^2\gamma^2 = -c^2 \quad \text{--- (9)}$$

$$\gamma^2(v^2 - c^2) = -c^2$$

$$\gamma^2 = \frac{-c^2}{v^2 - c^2} = \frac{c^2}{c^2 - v^2}$$

$$\therefore \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{--- (10)}$$

From (8), $\gamma' = \frac{1}{\left(1 - \frac{v^2}{c^2}\right)\gamma}$

$$\therefore \gamma' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{--- (11)}$$

$$\gamma = \gamma' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\therefore x' = \gamma\,(x - vt)$$

$$= \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$y' = y \quad \text{and} \quad z' = z$$

these are Lorentz transformation eqns for space and time co-ordinates.

From Lorentz's Transformation,

$$\text{LT}\begin{cases} x' = \dfrac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}} \\[2ex] t' = \dfrac{t - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \end{cases} \quad \text{and} \quad \left.\begin{matrix} x = \dfrac{x' + vt'}{\sqrt{1-\frac{v^2}{c^2}}} \\[2ex] t' = \dfrac{t' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \end{matrix}\right\} \text{LT-1}$$

From Galilean Transformation,

$$v_s' = v_s - v$$

$$v_s' = \frac{\Delta x'}{\Delta t'}$$

$$\Delta x' = x_2' - x_1'$$

$$x' = x - vt$$

$$\Delta x' = \Delta x - v\Delta t$$

$$t = t'$$

$$\Delta t = \Delta t'$$

Now from Lorentz Transformation,

$$\cancel{\Delta x'} = x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow \Delta x' = \frac{\Delta x - v\Delta t}{\sqrt{1-\frac{v^2}{c^2}}} \quad \begin{bmatrix} \Delta x' = x_2' - x_1' \\ \Delta t' = t_2' - t_1' \end{bmatrix}$$

and, $$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\therefore \Delta t' = \frac{\Delta t - \frac{v\Delta x}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$V_s = \frac{\Delta x}{\Delta t} \quad \text{---} \quad (i)$$

$$V_s' = \frac{\Delta x'}{\Delta t'} \quad \text{---} \quad (ii)$$

$$(i) \div (ii) \Rightarrow \frac{V_s'}{V_s} = \frac{\Delta x}{}$$

$$V_s = \frac{\Delta x}{\Delta t}$$

$$V_s' = \frac{\Delta x'}{\Delta t'}$$

$$= \frac{\Delta x - v\Delta t}{\sqrt{1-\frac{v^2}{c^2}}} \times \frac{\sqrt{1-\frac{v^2}{c^2}}}{\Delta t - \frac{v\Delta x}{c^2}}$$

$$= \frac{\Delta x - v\Delta t}{\Delta t - \frac{v\Delta x}{c^2}}$$

$$= \frac{\frac{\Delta x}{\Delta t} - v}{1 - \frac{v}{c^2}\frac{\Delta x}{\Delta t}}$$ [লব ও হরকে Δt দ্বারা ভাগ করে]

$$= \frac{V_s - v}{1 - \frac{V_s v}{c^2}}$$

$$\therefore \boxed{V_s' = \frac{V_s - v}{1 - \frac{v V_s}{c^2}}}$$

$$\Rightarrow c' = \frac{c-v}{1-\frac{vc}{c^2}} = \frac{c-v}{1-\frac{v}{c}} = \frac{c-v}{\frac{c-v}{c}}$$

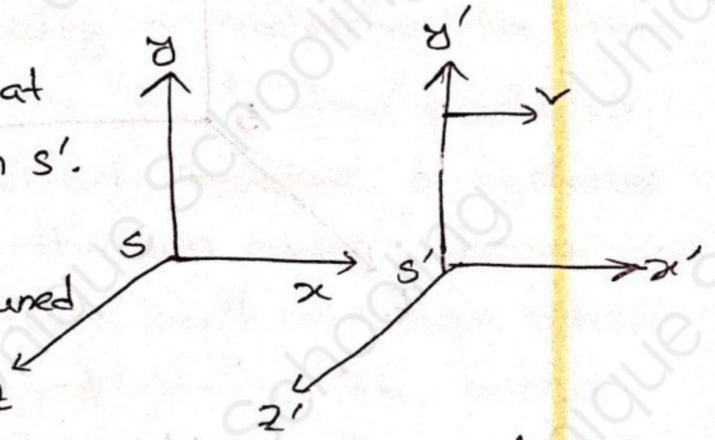
$$\therefore c' = \frac{c(c-v)}{(c-v)} = c \qquad \therefore \boxed{c' = c}$$

⊛ Relativity of Time:

GT : $t = t'$

LT ; $t' = \dfrac{t - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$

Suppose, a gun placed at the position (x', y', z') in s'. It fires two shots at times t_1' and t_2' measured with respect to s'.



Hence, $T_0 = \Delta t' = t_2' - t_1'$ is the proper time for the observer s'.

$T = \Delta t = t_2 - t_1$ represent the time interval between the two shots as measured by an observer in S.

From LT^{-1} ;

$$t_1 = \frac{t_1' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{and} \quad t_2 = \frac{t_2' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\therefore t_2 - t_1 = \frac{t_1' - t_2'}{\sqrt{1-\frac{v^2}{c^2}}}$$

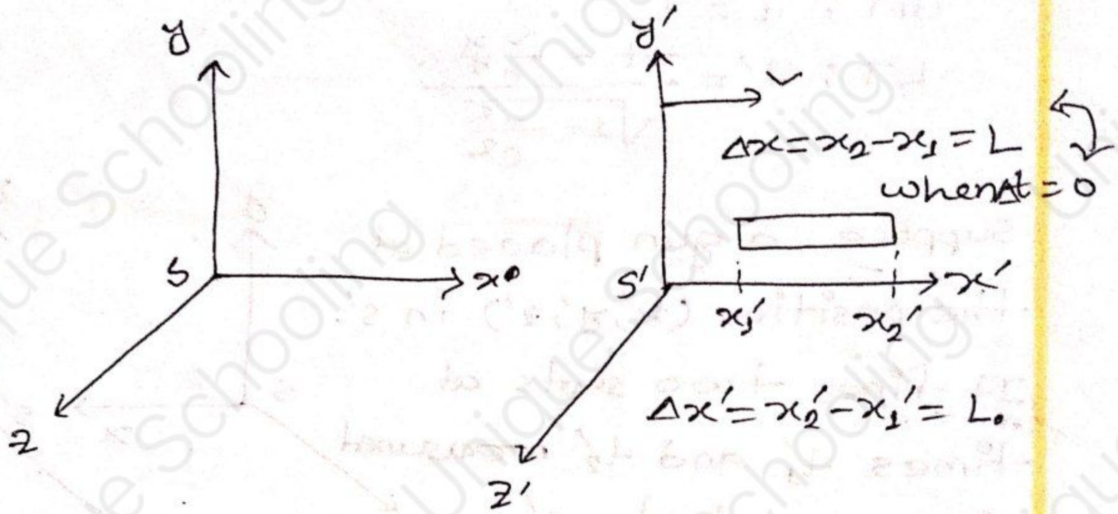
$$\Rightarrow \Delta t = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\therefore \boxed{T = \frac{T_0}{\sqrt{1-\frac{v^2}{c^2}}}} \qquad \boxed{T > T_0}$$

always হয়।

Formula of Time Dilation

Length Contraction:



$$x_1' = \frac{x_1 - vt_1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{and} \quad x_2' = \frac{x_2 - vt_2}{\sqrt{1-\frac{v^2}{c^2}}}$$

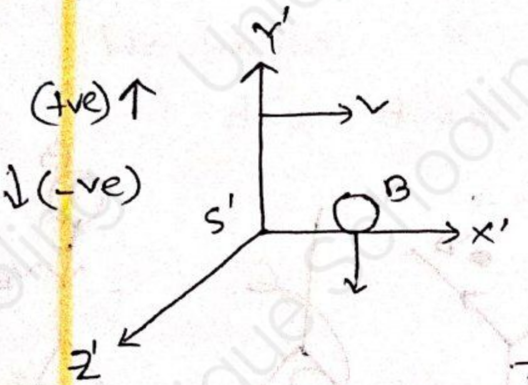
$$x_2' - x_1' = \frac{x_2 - x_1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow \Delta x' = \frac{\Delta x}{\sqrt{1-\frac{v^2}{c^2}}}$$

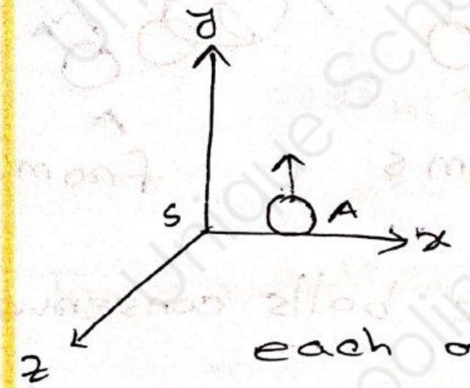
$$\Rightarrow L_0 = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\therefore L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

(*) Relativity of Mass: (airless place or space)



Consider two systems s and s'. s' is moving with a constant velocity v relative to the system s, in the positive x-direction. Suppose, s system ball is thrown over above and s' system ball thrown over down ~~at the~~ with same velocity v. The two balls ~~is~~ are same mass. They collide with each other and after collison coalesce into one body. According to the law of conservation of momentum, Momentum of ball at momentum A to B = momentum of coalesced mass.



~~From~~ Measuring from s';

mass of A ball = m_A'
mass of B ball = m_B'
Velocity " A " = v_A'
" " B " = v_B'

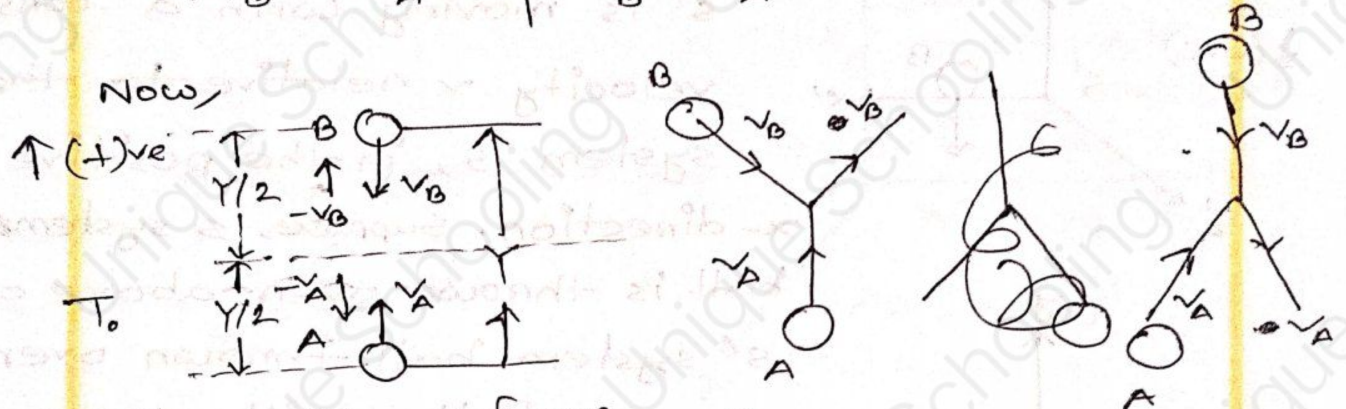
Measuring from s;

mass of A Ball = m_A
mass of B ball = m_B
velocity of A ball = v_A
" " B " = v_B

According to condition;

$$m_A = m_B' \quad \Big| \quad v_A = v_B'$$
$$m_B = m_A' \quad \Big| \quad v_B = v_A'$$

Now,



from S frame   from S   from S'

Total momentum of the balls conserved.

Now,
$$m_A v_A - m_B v_B = -m_A v_A + m_B v_B$$

$$\Rightarrow m_A v_A = m_B v_B$$

$$\Rightarrow m_A \frac{Y}{T_0} = \frac{m_B Y}{T}$$

$$\Rightarrow m_B = m_A \frac{T}{T_0}$$

$$\Rightarrow m_B = \frac{m_A}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\therefore \boxed{M = \frac{M_0}{\sqrt{1-\frac{v^2}{c^2}}}} \qquad \boxed{M > M_0}$$

$$v_A = \frac{Y/2}{T_0/2} = \frac{Y}{T_0}$$

$$v_B = \frac{Y/2}{T/2} = \frac{Y}{T}$$

$$T = \frac{T_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

⊛ Relation between Mass and Energy:

We know,

$$dE_k = dw = Fdx$$

$$= \frac{d(mv)}{dt} dx$$

$$= \left(m\frac{dv}{dt} + v\frac{dm}{dt}\right) dx$$

$$= m\frac{dv}{dt} dx + v\frac{dm}{dt} dx$$

$$\Rightarrow dE_k = mv\,dv + v^2 dm \quad \left[\frac{dx}{dt} = v\right]$$

$$\therefore dE_k = mvdv + v^2 dm \text{ --- (i)}$$

From relativity of mass,

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow m^2 = \frac{m_0^2}{1-\frac{v^2}{c^2}}$$

$$\Rightarrow m^2\left(1-\frac{v^2}{c^2}\right) = m_0^2$$

$$\Rightarrow m^2(c^2 - v^2) = m_0^2 c^2$$

$$\Rightarrow m^2c^2 - m^2v^2 = m_0^2c^2 \quad [m_0 = c_0 = \text{constant}]$$

$$\Rightarrow 2mc^2\frac{dm}{dt} - \left(v^2 . 2m\frac{dm}{dt} + m^2 . 2v\frac{dv}{dt}\right) = 0$$

$$\Rightarrow 2mc^2\frac{dm}{dt} - 2mv^2\frac{dm}{dt} - 2m^2v\frac{dv}{dt} = 0$$

$$\Rightarrow 2mdmc^2 = 2mdmv^2 + 2m^2vdv$$

$$\Rightarrow c^2dm = v^2dm + mvdv$$

$$\text{--- (ii)}$$

From (i) and (ii);

static state	kinetic state
$(E_k)_1 = 0$	E_k
m_0	m

$$\boxed{dE_k = c^2 dm}$$

$$\Rightarrow \int_0^{E_k} dE_k = \int_{m_0}^{m} c^2 dm$$

$$\Rightarrow E_k = c^2(m - m_0)$$

$$\Rightarrow \boxed{E_k = mc^2 - \underbrace{m_0c^2}_{\text{Rest energy, } E_0}}$$

→ স্থির অবস্থায় গতিশক্তি/ গতিশক্তি

Now, kinetic situation,

$$E = E_0 + E_k$$

$$= m_0c^2 + mc^2 - m_0c^2$$

$$\therefore \boxed{E = mc^2} \text{ (Total energy)}$$

(#) What do you mean by defects in crystal? How many types of defects can exist in a crystal? Discuss different types of point defects.

The term imperfection in the regular geometrical arrangement of the atoms in a crystalline solid.

There are two types of defects in crystal.

① Point defects.
which is zero dimensional defects.
- → Vacancies
- → Interstitialcies
- → Impurities
- → Electronic defects.

② Line defects.
which is one dimensional defects.
- → Edge dislocation
- → Screw dislocation

Vacancies: When an atom is missing from its lattice site in a crystal structure of a metal, it is called a vacancy. It may be indicating by a square symbol.

Interstitialcies: When an atom of the metal occupies an interstitial site, it is called interstitialcies. The size of an interstitial site is very small, The number of interstitialcies in an ordinary metal.

Impurities: There are two types of solid solutions, an impurity atom if present on the lattice by substituting the lattice site atom. The sizes of their atoms are larger than the interstitial site they occupy.

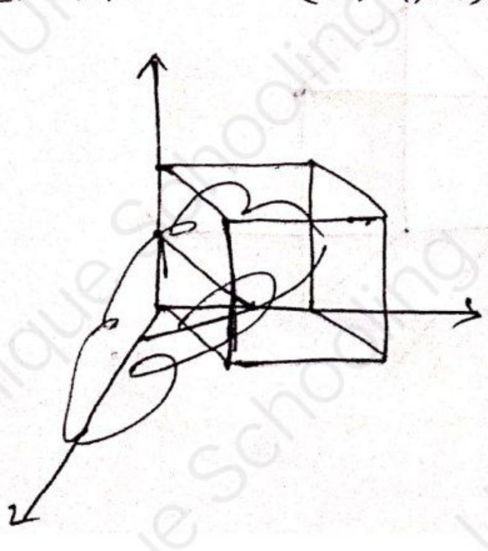
Electronic: This defect in ionic crystal is due to the electrons. When temperature increases some of the electrons may occupy higher energy states. The bonds from which electrons are removed become electron deficient. Electronic defect is due to holes and free electrons in crystal.

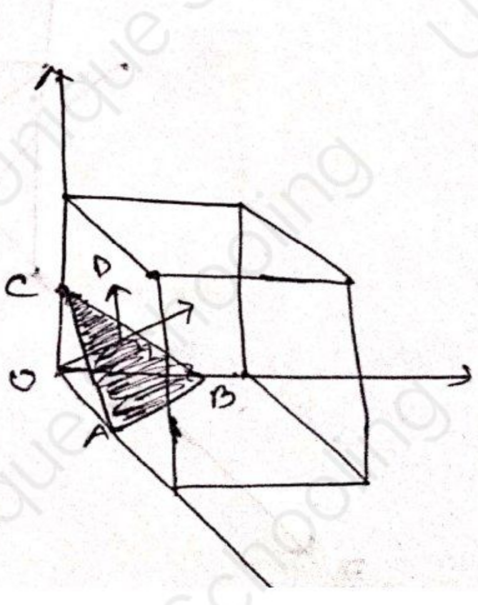
Show that in a crystal of cubic structure the distance between the planes with Miller indices h, k, l is equal to : $d = \frac{a}{\sqrt{h^2+k^2+l^2}}$, where a is the lattice parameter.

lattice parameter = a

~~Interplaner~~ Interplaner distance, $d(h,k,l)$

Miller indices (h,k,l)





Here, $OA = \frac{a}{h}$

$OB = \frac{a}{k}$

$OC = \frac{a}{l}$

$\cos\alpha' = \frac{d}{OA} = \frac{dh}{a}$

$\cos\beta' = \frac{d}{OB} = \frac{dk}{a}$

$\cos\gamma' = \frac{d}{OC} = \frac{dl}{a}$

we know,

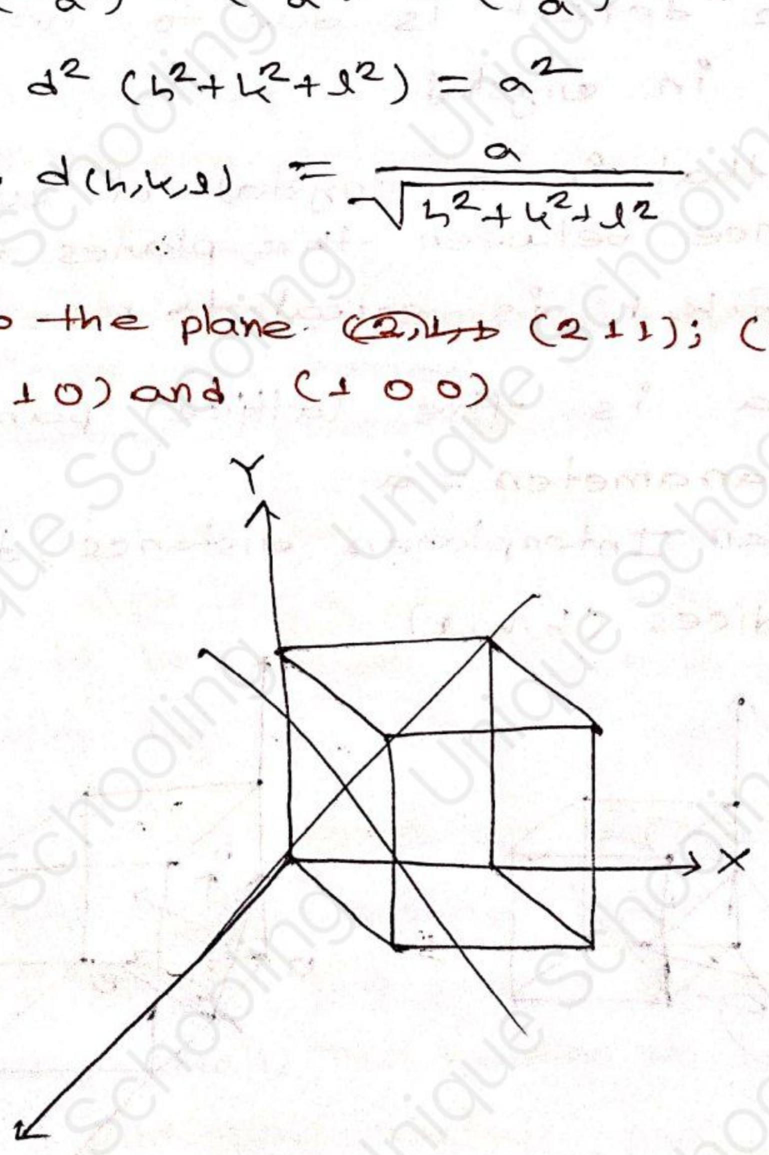
$\cos^2\alpha' + \cos^2\beta' + \cos^2\gamma' = 1$

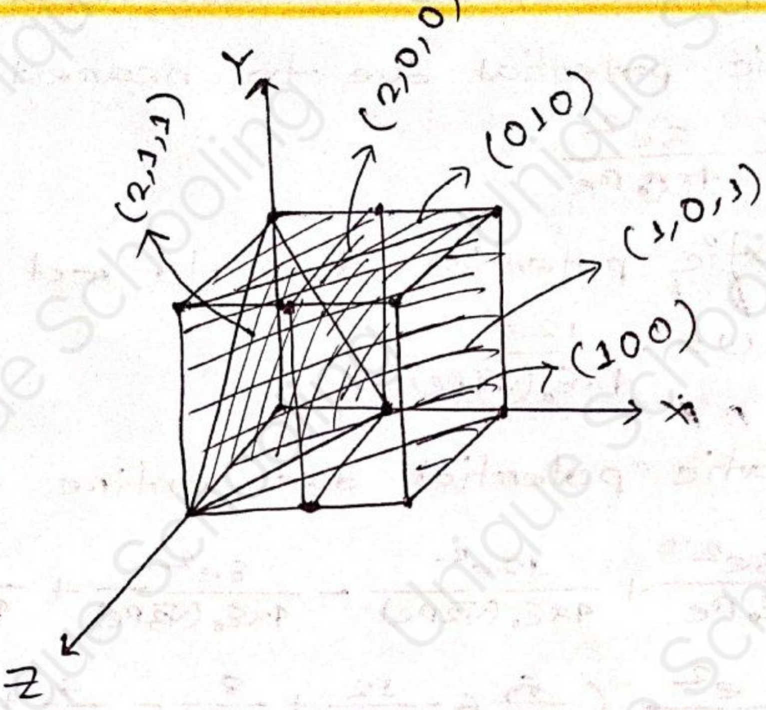
$\Rightarrow \left(\frac{dh}{a}\right)^2 + \left(\frac{dk}{a}\right)^2 + \left(\frac{dl}{a}\right)^2 = 1$

$\Rightarrow d^2(h^2+k^2+l^2) = a^2$

$\therefore d(h,k,l) = \frac{a}{\sqrt{h^2+k^2+l^2}}$

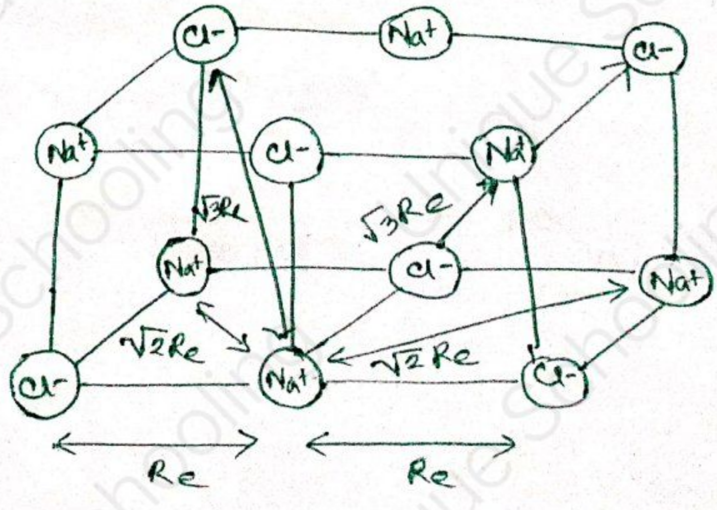
Draw the plane ~~(2),1,1~~ (2 1 1); (2 0 0); (1 0 1); (0 1 0) and (1 0 0)





(#) What is Madelung constant? Calculate it for NaCl.

The Madelung constant is used in determining the electrostatic potential of a single ion in a crystal by approximating the ions by point charges.



Electrostatic potential due to nearest ions

$$U = -\frac{6e^2}{4\pi\varepsilon_0 r_e}$$

Electrostatic potential due to next nearest ions, $U = \frac{12e^2}{4\pi\varepsilon_0(\sqrt{2}r_e)}$

Electrostatic potential due entire crystal,

$$U = -\frac{6e^2}{4\pi\varepsilon_0 r_e} + \frac{12e^2}{4\pi\varepsilon_0(\sqrt{2}r_e)} - \frac{8e^2}{4\pi\varepsilon_0(\sqrt{3}r_e)} + \frac{6e^2}{4\pi\varepsilon_0(\sqrt{4}r_e)} - \cdots$$

$$= -\frac{e^2}{4\pi\varepsilon_0 r_e}\left(6 - \frac{12}{\sqrt{2}} + \frac{8}{\sqrt{3}} - \frac{6}{2} + \cdots\right)$$

$$\because V = -\frac{\alpha e^2}{4\pi\varepsilon_0 r_e}$$

$$\alpha = 6 - \frac{12}{\sqrt{2}} + \frac{8}{\sqrt{3}} - \frac{6}{2} + \cdots$$

$= 1.748$ (approximate)

(#) Find the energy of the neutron in units of electron-volt whose de-Broglie wavelength is 1 A°.

$$\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{10^{-10}} = 6.63\times10^{-24}\ kgms^{-1}$$

$$\text{energy, } E = \frac{p^2}{2m} = \frac{(6.63\times10^{-24})^2}{2\times1.67\times10^{-27}}$$

$$= 1.32\times10^{-20}\ J$$

$$= 0.082\ eV$$

(#) How many orders will be visible if the wavelength of the incident radiation is 5000A° and the number of lines on the grating is 2540 in one inch?

Maximum angle of diffraction can be 90°.

$\theta = 90°, \lambda = 5000\times10^{-8}\ cm$

$$\text{grating element} = a+b = \frac{2.54}{2540} = 10^{-3}$$

we know,

$$(a+b)\sin\theta = n\lambda$$

$$\Rightarrow n = \frac{(a+b)\sin 90°}{\lambda}$$

$$= \frac{10^{-3}}{5000\times10^{-8}} = 20$$

Ans:

#In a double slit experiment the separation between the slits is 2.5 mm and the distance of the screen from the slits is 50 mm. If the arrangement is illuminated with Na light of wavelength 5890A°. Calculate
i) the angular position of the first maxima
ii) the linear separation between two adjacent minima.

(i) $d = 2.5\,mm$; $D = 50\,mm$; $\lambda = 5890 \times 10^{-7}\,mm$

angular position, $\theta = \frac{n\lambda}{d}$

$$= \frac{1 \times 5890 \times 10^{-7}}{2.5}$$

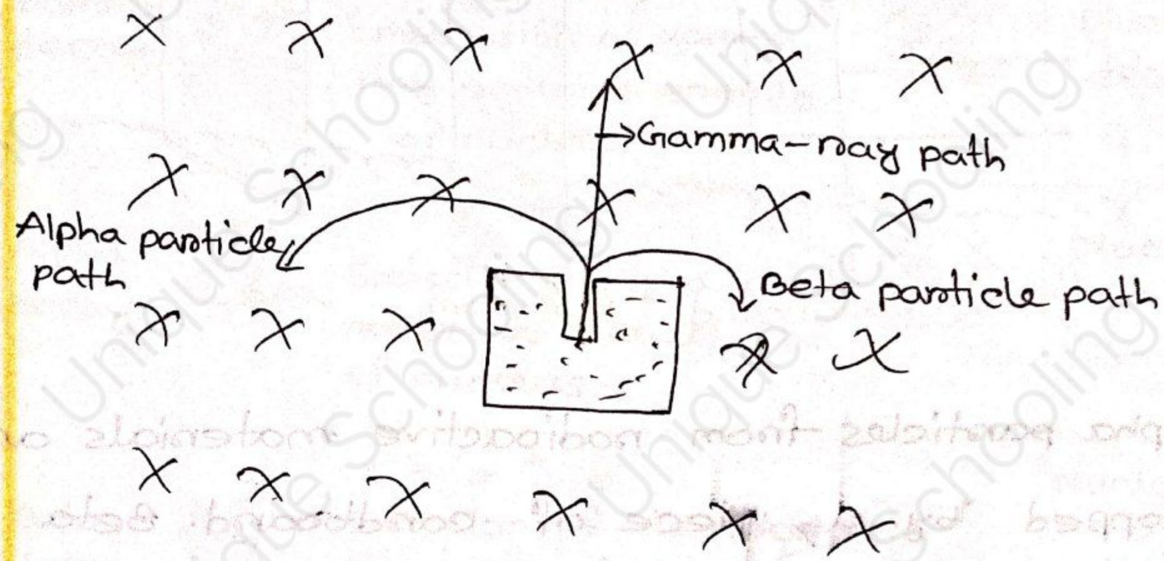
$$= 2.356 \times 10^{-9}$$

(ii) $\beta = \frac{D\lambda}{d} = \frac{5890 \times 10^{-7} \times 50}{2.5}$

$$= 0.01178\ mm$$

Ans:

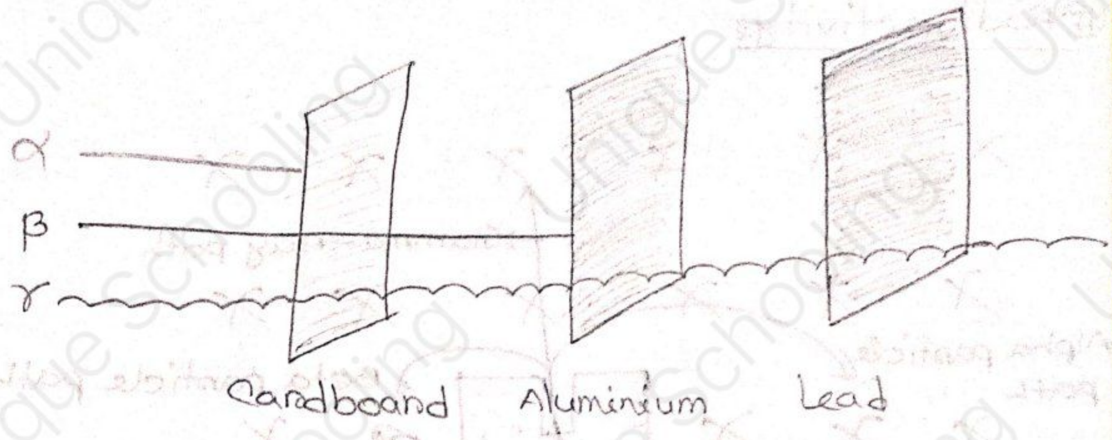
Radioactivity:



Radioactive Decay:

Decay	Transformation	Example
Alpha Decay	${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$	${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$
Beta decay	${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^-$	${}^{14}_{6}C \rightarrow {}^{14}_{7}N + e^-$
Positron ~~de~~ emission	${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^+$	${}^{69}_{29}Cu \rightarrow {}^{69}_{28}Cu + e^+$
Electron capture	${}^{A}_{Z}X + e^- \rightarrow {}^{A}_{Z-1}Y$	${}^{69}_{29}Cu + e^- \rightarrow {}^{69}_{28}Ni$
Gamma decay	${}^{A}_{Z}X^* \rightarrow {}^{A}_{Z}X + \gamma$	${}^{87}_{38}Sr^* \rightarrow {}^{87}_{38}Sr + \gamma$

The * denotes an excited nuclear state and γ denotes a gamma ray photon.



Alpha particles from radioactive materials are stopped by a piece of cardboard. Beta particles penetrate the cardboard but stopped by a sheet of aluminium. Even a thick ~~of~~ slab of lead may not stop all the gamma rays.

	Original nucleus	Decay event	Final nucleus	Reason for instability
Gamma decay		Emission of gamma ray reduces energy of nucleus		Nucleus has excess energy
Alpha decay		Emission of alpha particle ~~nucleus~~ reduces size of nucleus		Nucleus too large
Beta decay		Emission of electron by neutron in nucleus changes the proton to a neutron		Nucleus has too many neutrons relative to number of protons
Electron capture		Capture of electron by proton in nucleus changes the proton to a neutron		Nucleus has too many protons relative to number of neutrons
positron emission		Emission of positron by proton in nucleus changes the proton to a neutron.		Nucleus has too many protons relative to number of neutrons

- Proton (charge = +e)
- Neutron (charge = 0)
- Electron (charge = −e)
- Positron (charge = +e)

(#) The helium isotope $^{6}_{2}He$ is unstable. What kind of decay would you except it to undergo?

Soln:

The most stable helium nucleus is $^{4}_{2}He$, all of whose neutrons and protons are in the lowest possible energy levels. Since $^{6}_{2}He$ has four neutrons whereas $^{4}_{2}He$ has only two, the instability of $^{6}_{2}He$ must be due to an excess of neutrons. This suggests that $^{6}_{2}He$ undergoes negative beta decay to become the lithium isotope $^{6}_{3}Li$ whose neutron/proton ratio is more consistent with stability.

$$^{6}_{2}He \longrightarrow {}^{6}_{3}Li + e^{-}$$

Fundamental Laws of Radioactivity:

① In all known radioactive transformations either an α or a β-particle (never both or more than one of each kind) is emitted by the atom.

② When a radioactive atom emits an α-particle, a new atom is formed whose mass number is less by four units and atomic number less by two units than those of the parent atom.

$$^{238}_{92}U \xrightarrow{-\alpha} {}^{234}_{90}Th$$

③ When a radioactive atom emits a β-particle, the new atom formed has the same mass number but the atomic number is increased by one unit.

$$^{234}_{90}Th \xrightarrow{-{}_{-1}e^{0}} {}^{234}_{91}Pa$$

Law of Radioactive Disintegration:

$-\frac{dN}{dt} = \lambda N \qquad N = N_0 e^{-\lambda t} \qquad T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.6931}{\lambda}$

The Mean Life, $\bar{T} = \frac{1}{\lambda}$

Law of Successive Disintegration:

$A \rightarrow B \rightarrow C \rightarrow \ldots X$ (stable)

At time $t=0$, the number of initial atoms in $A = N_0$ and the number of initial atoms in $B = 0$

At time t, let the number of atoms in $A = N_1$ and the number of atoms in $B = N_2$

Let λ_1 and λ_2 be the decay constants of A and B respectively. Every time an atom of A disappears, an atom of B is produced.

$\therefore$ Rate of formation of daughter $B = \lambda_1 N_1$

The rate at which B decays $= \lambda_2 N_2$

$\therefore$ The net increase in the number of B atoms, $\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$

Hence, $\frac{dN_2}{dt} = \lambda_1 N_0 e^{-\lambda_1 t} - \lambda_2 N_2$

$\therefore \frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_0 e^{-\lambda_1 t}$

Multiplying both sides by the integrating factor $e^{\lambda_2 t}$,

$$\cancel{\frac{dN_2}{dt}e^{\lambda_2 t} - \lambda_2 N_2 e^{(\lambda_2-\lambda_1)t}}$$

$$\frac{dN_2}{dt}e^{\lambda_2 t} + \lambda_2 N_2 e^{\lambda_2 t} = \lambda_1 N_0 e^{(\lambda_2-\lambda_1)t}$$

$$\therefore N_2 = \frac{\lambda_1 N_0}{\lambda_2-\lambda_1}\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right]$$

⊕ A carbon specimen found in a cave contained $\frac{1}{8}$ as much C^{14} as an amount of carbon in living matter. Calculate the approximate age of specimen. Half life period of C^{14} is 5568 years.

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} \Rightarrow \lambda = \frac{\ln 2}{5568}$$

We know that,

$$N = N_0 e^{-\lambda t}$$

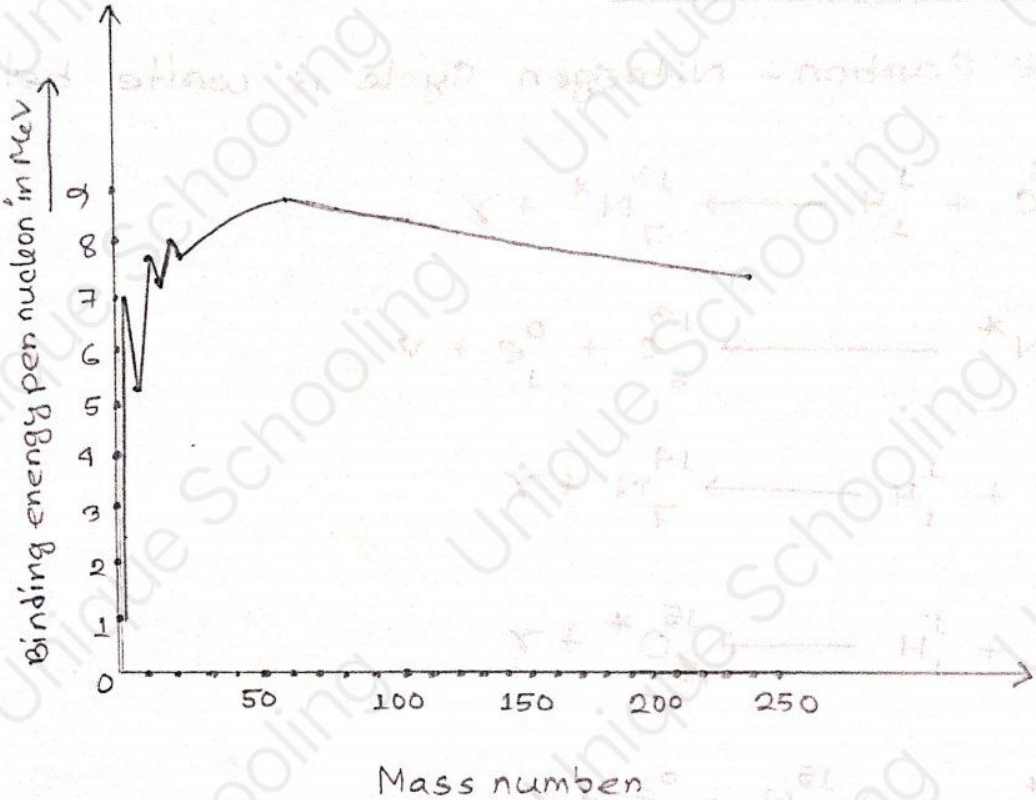
$$\Rightarrow \frac{1}{8}N_0 = N_0 e^{-\lambda t}$$

$$\Rightarrow -\lambda t = \ln\left(\frac{1}{8}\right)$$

$$\Rightarrow t = \frac{-\ln\left(\frac{1}{8}\right)}{\frac{\ln 2}{5568}}$$

$$\therefore t = 16704 \text{ years.}$$

Ans to the Q.N: 02



Stability of Nucleus and Bindging energ.

$$\text{B.E per nucleon} = \frac{\text{Total B.E of a nucleus}}{\text{The numbers of nucleons it contains}}$$

The curve rises steeply at first and then more gradually until it reaches a maximum of 8.79 MeV. The curve then drop slowly to about 7.6 MeV at the highest mass numbers.

Ans to the Q.No: 03

	Decay event	Reason
Gamma decay →	Emission of gamma ray reduces energy of nucleus	Nucleus has excess energy
Alpha decay →	Emission of alpha particle reduces size of nucleus	Nucleus too large
Beta decay →	Emission of electron by neutron in nucleus changes the neutron to a proton	Nucleus has too many neutrons relative to number of protons
Electron capture →	Capture of electron by proton in nucleus changes the proton to a neutron	Nucleus has too many protons relative to number of neutrons
positron emission →	Emission of positron by proton in nucleus changes the proton to a neutron	Nucleus has too many protons relative to number of neutrons.

*** State of matters:

Solids: Atoms and molecules are strongly bound and maintain definite volume and shape unless changed by applying external forces.

Solids state physics: The study of the physical properties of the atomic, molecular matter in the solid state.
Such as → crystal structure, bonding, thermal electrical, magnetic properties.

Solids
- → Crystalline solids
- → Non-crystalline solids / Amorphous solids / Pseudo solids / Super cooled liquid

Crystalline solids: Crystalline solids are defined as an ordered periodic arrangement of atoms, molecules or ions.

উপাদানগুলো

Non-crystalline solids: In which constituents (atoms, ions, molecules) are arranged in irregular or random manners. Non-crystalline solids are defined as an irregular arrangement of atoms, molecules or ions.

Properties of crystalline solids →

i) Arrangement of atom, molecules are periodically.

ii) Sharp melting point.

iii) Anisotropic → Physical properties such as refractive index, conductivity, thermal ~~propertyies~~ property, magnetic properties, electrical properties are different in different direction.

প্রতিসরাঙ্ক

iv) Uniform chemical composition in whole crystal.

<u>ex:</u> Fe, Ag, Cu, Au, Salt, Sugar.

<u>compounds:</u> NaCl, $C_{12}H_{22}O_{11}$, $CuSO_4 \cdot 5H_2O$, Urea (NH_2CONH_2)

Non-crystalline solids properties →

i) Arrangement random (irregular)

ii) Do not have sharp melting point.

iii) Isotropic nature.

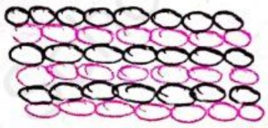
<u>ex:</u> Glass, wood, rubber, wax, pitch, tar, plastics.

Crystalline solids

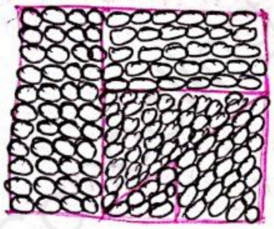
↓ Single crystalline solids

↓ Polycrystalline solids

Single crystalline: A single crystall has an atomic structure that repeats periodically across its whole volume.



Poly crystalline solids: When a large number of small crystal section of various shape and size are packed to one another separated by the boundaries.



Q: Why are solid rigid?

Ans: Rigid means unbending ~~and~~ or inflexible. In a solid, the constituents particles are very closely packed. Hence therefore of attraction among these particles are very strong.

Q: Why urea has sharp melting point whereas glass does not have?

Ans: crystalline solids geometry is definite. Every crystalline solid has fixed melting point. So, urea has sharp melting point but glass is non-crystalline solid. And every non-crystalline product has no sharp melting point. so, glass do not have melting point.

Q: Why are amorphous solids considered as supercooled liquids?

Ans: Because like liquids, amorphous solids (glass) have tendency to flow slowly.

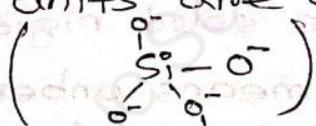
Q: Why is the window glass of old building thick at the bottom?

Ans: Glass is very high viscous liquid which flows down very slowly and make bottom thick.

Q: What is the difference between Quartz and glass through both contain SiO_4^{4-} units under what condition Quartz can be converted into glass?

Ans:

Quartz: i) Composed of crystalline ~~area~~ silica.

ii) SiO_4^{4-} (tetrahedra) units are arranged in regular manner. 

iii) They have long range order and have sharp melting point.

Glass: i) Composed of amorphous silica.

ii) SiO_4^{4-} (tetrahedra) units are joined randomly.

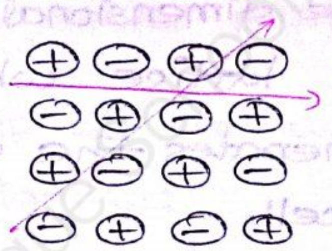
iii) They have short range order and have sharp melting point.

$$\text{Quartz} \xrightarrow[\text{cool rapidly}]{\text{High temperature}} \text{Glass}$$

Q: Crystalline solids are anisotropic in nature. Why does this statement mean?

Ans:

Anisotropic: Crystalline solids are anisotropic. This implies that physical properties, such as electrical conductivity, refractive index, thermal expansion etc. are different in different direction.



Q: Amorphous solids are isotropic in nature. Why does this statement mean?

Ans:

Isotropic: Amorphous solids are isotropic. This implies that physical properties. Such as electrical conductivity, refractive index, thermal expansion etc. are same in all directions.

Q: What is photovoltic cell?

Ans:

Photovoltic cell: The material that converts sunlight into electricity is called photovoltic cell.

ex: Amorphous silica.

Q: How can a material be made amorphous?

Ans: By melting the material and cool it rapidly.

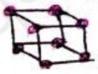
Crystal Lattice / Space Lattice:

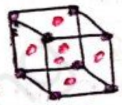
It is regular arrangement of the constituents particles (atoms, molecules, ions) of a crystal in a three dimensional space.

Unit cell:

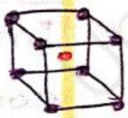
It is the smallest three dimensional portion of the complete space lattice which when repeated in space generates the crystal lattice is called unit cell.

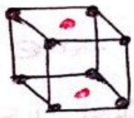
Types of Unit cell:

① Primitive / Simple Unit Cell 



② Non primitive → Face-centred, Body-centred, End-centred





Corners → 8

Faces → 6

Body centred → 1

Edges → 12

Alternate edges → 4

Body diagonals → 4

⊕ Contribution by particles present at different position in a unit cell:

1) Corner particles $= \frac{1}{8}$

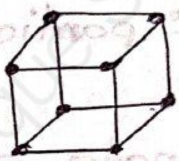
2) Face centred particles $= \frac{1}{2}$

3) Body centred particles $= 1$

4) Edge centred particles $= \frac{1}{4}$

⊕ Number of particles per unit cell of a cubic cubic lattice:

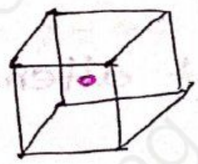
i) Simple ~~Ea~~ cube:-



$\frac{1}{8} \times 8 = 1$

∴ 1 particle / unit cell.

ii) Body centred cubic cell:-

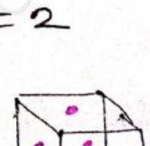


At corner; $\frac{1}{8} \times 8 = 1$

At body corner $= 1$

∴ No. of particles / unit cell $= 1 + 1 = 2$

iii) Face centred cubic cell:-

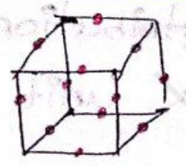


At corner $= \frac{1}{8} \times 8 = 1$

At face ~~centred~~ corner $= \frac{1}{2} \times 6 = 3$

∴ Number of particles / Unit cell $= 1 + 3 = 4$

iv) <u>Edge-centred cubic cell:-</u>

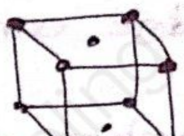


At corner $= \frac{1}{8} \times 8 = 1$

Edge corner $= \frac{1}{4} \times 12 = 3$

$\therefore$ No. of particles / unit cell $= 1 + 3 = 4$

v) <u>End-centred cubic cell:-</u>

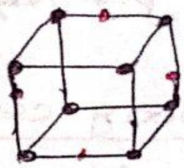


At corner $= \frac{1}{8} \times 8 = 1$

At alternate face $= \frac{1}{2} \times 2 = 1$

$\therefore$ No. of particles / unit cell $= 1 + 1 = 2$

vi) <u>Alternate edge centred cubic cell:-</u>

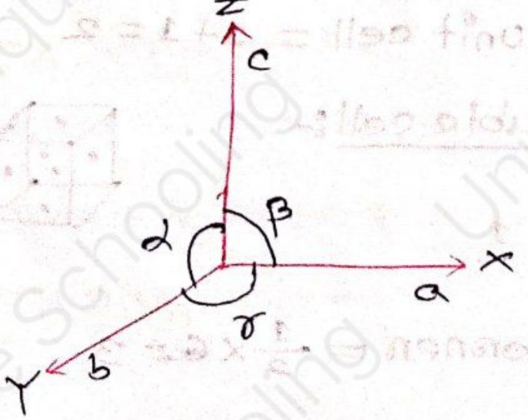


At corner $= \frac{1}{8} \times 8 = 1$

At alternate edge $= \frac{1}{4} \times 4 = 1$

$\therefore$ No of particles / unit cell $= 1 + 1 = 2$

(***)

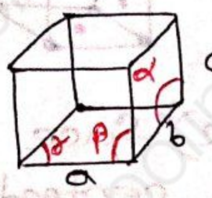


7 crystal system in 14 Bravais lattices.

French name came from Mathematician Auguste Bravais.

~~Acoo~~

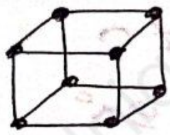
i) Cubic:



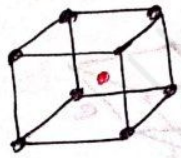
$a = b = c$

$\alpha = \beta = \gamma = 90^\circ$

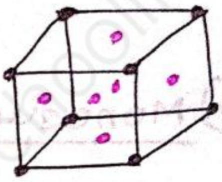
position of element:



a) primitiv

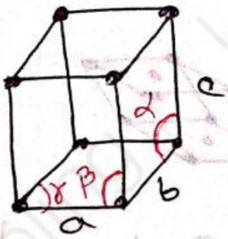


b) Body centred (BC)



c) face centred (fc)

ii) Tetragonal:

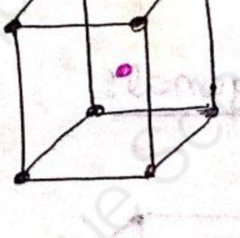


$a = b \neq c$

$\alpha = \beta = \gamma = 90^\circ$

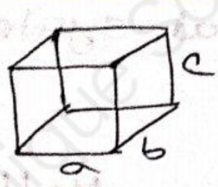
position of element:

a) primitive



b) Body centred

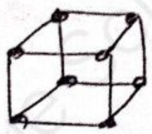
iii) Orthorhombic:



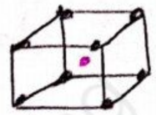
$a \neq b \neq c$

$\alpha = \beta = \gamma = 90^\circ$

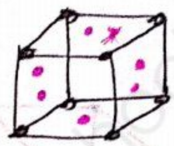
position of element:



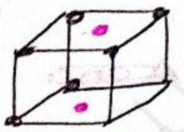
a) primitive



b) Body centred

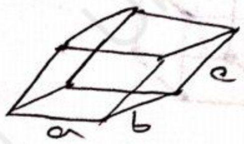


c) face centred



d) End centred

iv) Monoclinic:

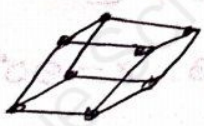


$a \neq b \neq c$

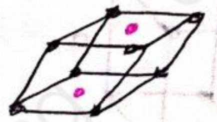
$\alpha = \gamma = 90^\circ$

$\beta \neq 90^\circ$

position of element:

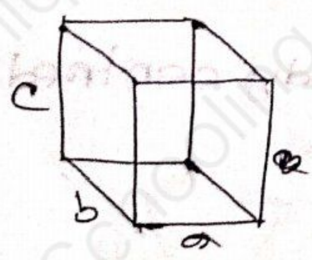


a) primitive



b) End centred

v) Hexagonal:



$a = b = c$

$\alpha = \beta = 90^\circ$

$\gamma = 120^\circ$

a) primitive

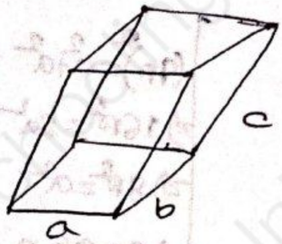
vi) Rhombohedral: (Trigonal)

$a=b=c$

$\alpha=\beta=\gamma\neq 90°$

a) primitive

vii) Triclinic:



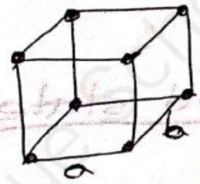
$a\neq b\neq c$

$\alpha\neq\beta\neq\gamma$

a) primitive

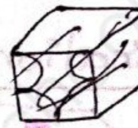
Simple cubic structure and Atomic Packing Factor:

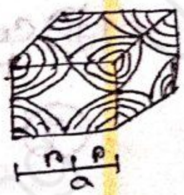
No of atom is $\frac{1\times 8}{8}=1$



$a=b=c$

$\therefore V=a^3$





$$A.P.F=\frac{\text{No. of atoms}\times\text{Volume of 1 atom}}{\text{Volume of unit cell}}$$

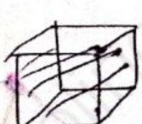
$a=2r$

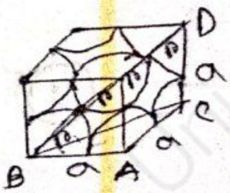
$$=\frac{1\times\frac{4}{3}\pi r^3}{a^3}=\frac{4\pi \cancel{r^3}}{3\times 8\cancel{r^3}}=\frac{4\pi}{24}=0.52$$ Ans:

Body centred cubic structure and Atomic Packing Factor:

$V=a^3$

No. of atom $=1+1=2$





$$A.P.F=\frac{2\times\frac{4}{3}\pi r^3}{a^3}=\frac{8\pi \cancel{r^3}\left(\frac{\sqrt{3}}{4}a\right)^3}{3\times a^3}$$

$$=\frac{8\pi\times 3\sqrt{3}\,\cancel{a^3}}{\underset{8}{\cancel{64}}\times 3\times\cancel{a^3}}=0.6802$$

$BC^2=AB^2+AC^2$

$BD^2=BC^2+CD^2$

$=AB^2+AC^2+CD^2$

or, $(4r)^2=a^2+a^2+a^2$

$\therefore r=\frac{\sqrt{3}}{4}a$

Face centred cubic structure and A.P.F:

No. of atom $= \left(\frac{1}{8}\times 8\right) + \left(\frac{1}{2}\times 6\right)$

$= 1 + 3 = 4$

$$A.P.F = \frac{4\times\frac{4}{3}\pi r^3}{a^3}$$

$(4r)^2 = a^2 + a^2$

$\Rightarrow 16r^2 = 2a^2$

$\Rightarrow 8r^2 = a^2$

$\therefore a = 2\sqrt{2}r$

$4r^2 = a^2 + a^2 + a^2$

$\Rightarrow 4r^2 = 3a^2$

$\Rightarrow 2r^2 = a^2$

$\therefore a = \sqrt{2}r$

$$= \frac{16\pi r^3}{3\times(2\sqrt{2}r)^3}$$

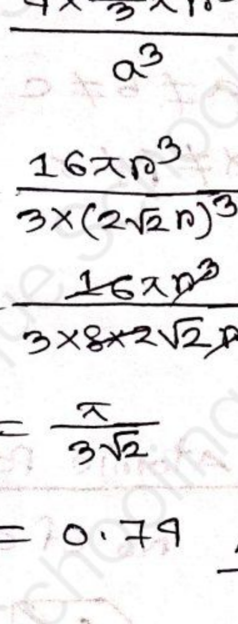
$$= \frac{\cancel{16}\pi \cancel{r^3}}{3\times \cancel{8}\times 2\sqrt{2}\cancel{r^3}}$$

$$= \frac{\pi}{3\sqrt{2}}$$

$= 0.74$ Ans:

Coordination number of solid state:

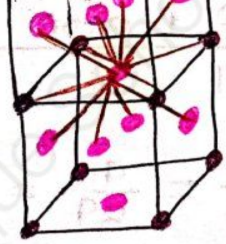
(i)



(FCC)

coordination number = 12

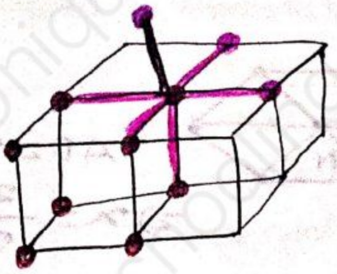
(ii)



(BCC)

coordination number = 8

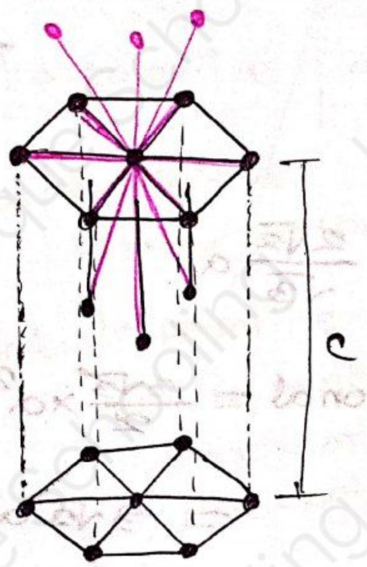
(iii)



Coordination number = 6

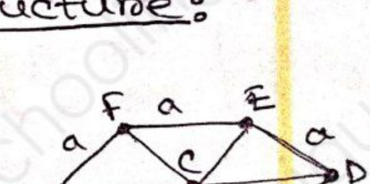
(SC)

(iv)



Coordination number = 12

(III) Hexagonal closed packed structure:



$AB = a = 2r$

$a = b \neq c$; $\alpha = \beta = 90°$, $\gamma = 120°$

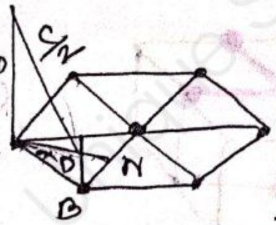
Area of Hexagon $= 6 \times \Delta ABC$

$= 6 \times \frac{1}{2} \times a \times a \times \sin 60°$

$= 3a^2 \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2} a^2$

Volume of Hexagonal cell $= \frac{3\sqrt{3}}{2} a^2 c$

$\frac{c}{a}$ ratio:



$$a^2 = \left(\frac{a}{\sqrt{3}}\right)^2 + \left(\frac{c}{2}\right)^2$$

$$\Rightarrow a^2 = \frac{a^2}{3} + \frac{c^2}{4}$$

$$\Rightarrow a^2 = \frac{4a^2+3c^2}{12}$$

$$\Rightarrow 12a^2 - 4a^2 = 3c^2$$

$$\Rightarrow 3c^2 = 8a^2$$

$$\Rightarrow \frac{c^2}{a^2} = \frac{8}{3}$$

$$\therefore \frac{c}{a} = \frac{2\sqrt{2}}{\sqrt{3}} \quad \therefore c = \frac{2\sqrt{2}}{\sqrt{3}}a$$

$AD = x$

$$\Rightarrow x = \frac{2}{3}AN$$

$$= \frac{2}{3} \times a \sin 60^\circ$$

$$= \frac{\cancel{2}}{3} \times a \times \frac{\sqrt{3}}{\cancel{2}}$$

$$= \frac{a}{\sqrt{3}}$$

$$\therefore \text{Voloume of Hexagonal} = \frac{3\sqrt{3}}{2} \times a^2 \times \frac{2\sqrt{2}}{\sqrt{3}}a$$

$$= 3\sqrt{2}a^3$$

$$\therefore A.P.F = \frac{6 \times \frac{4}{3}\pi r^3}{3\sqrt{2}a^3}$$

$$= \frac{8\pi r^3}{3\sqrt{2} \times 8r^3}$$

$$= 0.74$$

Ans:

No. of atom:

$\left(\frac{1}{6} \times 6\right) \times 2 = 2$

At face corner =

$\frac{1}{2} \times 2 = 1$

মাঝখানে আছে আর 3টি।

Total atom $= 2+1+3$

$= 6$

Name of crystal system	Bravicis lattice	Relation between primitives	Interfacial angles	Examples
① Cubic	a) primitive b) Body centred c) Face centred	$a=b=c$	$\alpha=\beta=\gamma=90^\circ$	$NaCl$
② Tetragonal	a) primitive b) Body centred	$a=b\neq c$	$\alpha=\beta=\gamma=90^\circ$	$NiSO_4$
③ Orthorhombic	a) primitive b) Body centred c) Face centred d) End centred	$a\neq b\neq c$	$\alpha=\beta=\gamma=90^\circ$	KNO_3
④ Monoclinic	a) primitive b) End centred	$a\neq b\neq c$	$\alpha=\beta=90^\circ\neq\gamma$	$FeSO_4$
⑤ Hexagonal	a) primitive	$a=b\neq c$	$\alpha=\beta=90^\circ$, $\gamma=120^\circ$	SiO_2
⑥ Trigonal	a) primitive	$a=b=c$	$\cancel{\alpha=\beta\neq\gamma=9}$ $\alpha=\beta=\gamma\neq 90^\circ$	$CaSO_4$
⑦ Triclinic	a) primitive	$a\neq b\neq c$	$\alpha\neq\beta\neq\gamma\neq 90^\circ$	$CuSO_4$

What is the significance of coordination number?

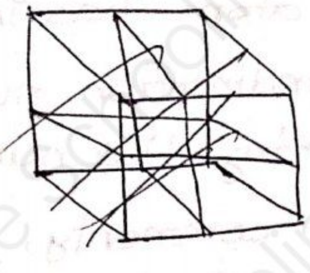
Ans: Coordination number is the number of nearest neighbour atoms that an atom has in the given crystal structure. The coordination number in crystals is found by counting the number of neighbouring atoms. Most commonly, the coordination number looks at an atom in the interior of a lattice, with neighbours extending in all directions. Which atoms coordination number is high these kind of atom packing factor is high. So, atom bound each other tightly.

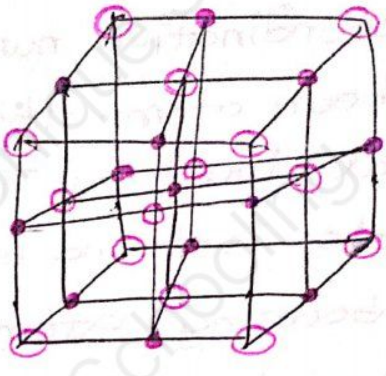
Hexagonal closed packed এবং faced centred cell কে closed packed বলা হয় কেন?

Ans: Atomic packing factor is the ratio of the volume of atoms in the unit cell to the volume of the unit cell. HCP and FCC both are closed packed. Because of their value is 74%. No other crystal can contain this kind of APF value. 74% is the highest value of crystal APF.

Structure of NaCl:

O — Cl^-
• — Na^+



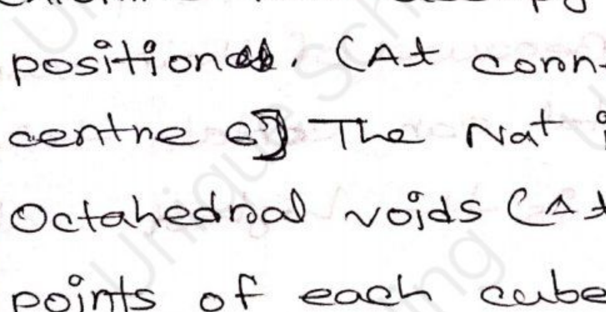


Cl^- are present at face centred cubic position. (At the centre of each face + 8 corner) $(8+6=14)$

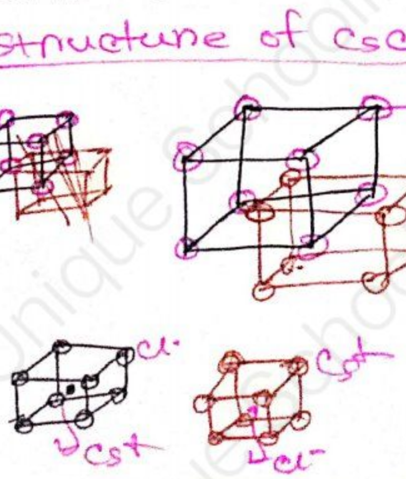
Na^+ are present at octahedral voids. (At body centred + At mid points of each cube edge) $(1+12=13)$

⇒ The lattice of NaCl is face centred cubic with basis of one Na^+ ion and one Cl^- ion. A unit cell of NaCl is shown in fig large chlorine ions accupy face centred cubic positioned. (At corners 8 + at each face centre 6) The Na^+ ions are present at Octahedral voids (At body centred + At mid points of each cube edge $(1+12=13)$

Structure of CsCl:



8 Cl^- rounded by one Cs^+ and 8 Cs^+ are rounded by one Cl^-.



Atomic Packing Fraction (APF)

Atomic packing factor is the ratio of the volume of atoms in the unit cell to the volume of the unit cell.

$$APF = \frac{\text{Volume of atom per unit cell}}{\text{Volume of unit cell}}$$

$$= \frac{n_{eff} \times \text{Volume of each atom}}{\text{Volume of unit cell}}$$

"n_{eff}"

SC, CsCl	$\longrightarrow$ 1
BCC	$\longrightarrow$ 2
FCC, ZnS, NaCl, Graphite	$\longrightarrow$ 4
HCP, Graphene	$\longrightarrow$ 6
Diamond	$\longrightarrow$ 8

$n_{eff} = n_i + \frac{1}{2} n_f + \frac{1}{8} n_c$

n_i = completely inside the cell

n_f = at ~~force~~ face

n_c = at corner

Radius:

SC $\longrightarrow 2r = a$

BCC $\longrightarrow 4r = \sqrt{3}\,a$

FCC $\longrightarrow 4r = \sqrt{2}\,a$

Diamond $\rightarrow 8r = \sqrt{3}\,a$

Graphene $\rightarrow 2r = \frac{a}{\sqrt{3}}$

NaCl $\longrightarrow 2(r_{Na} + r_{Cl}) = a$

CsCl $\longrightarrow 2(r_{Cs} + r_{Cl}) = \sqrt{3}\,a$

ZnS $\longrightarrow 4(r_{Zn} + r_{S}) = \sqrt{3}\,a$

SC $\rightarrow \frac{\pi}{6} \rightarrow$ 52%

BCC $\rightarrow \frac{\sqrt{3}\pi}{8} \rightarrow$ 68%

FCC $\rightarrow \frac{\sqrt{2}\pi}{6} \rightarrow$ 74%

HCP $\rightarrow \frac{\pi}{3\sqrt{2}} \rightarrow$ 74%

Diamond $\rightarrow \frac{\sqrt{3}\pi}{16} \rightarrow$ 34%

Graphene $\rightarrow \frac{\pi}{3\sqrt{3}} \rightarrow$ 60%

Lecture 1-3

Also lecture 9

⊕ What do you mean by coherent sources? Derive a relation between phase difference and path difference.

⇒ Two wave sources are perfectly coherent if,

i) A constant phase difference or in phase.

ii) The same frequency.

iii) The same wavelength.

iv) Two vertical source formed from a single source can be coherence.

Derive a relation between phase difference and path difference:-

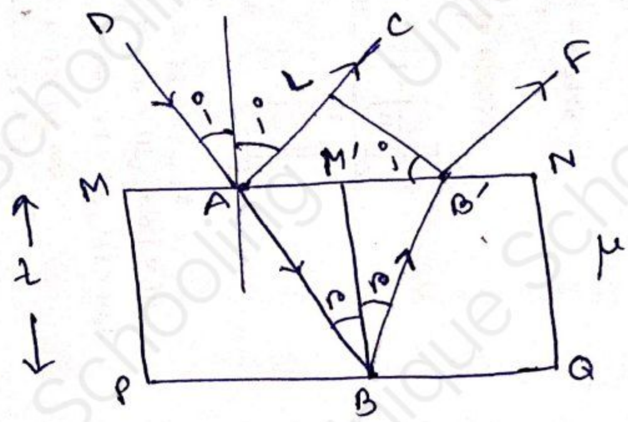
Phase difference: Difference expressed in degrees or radians between two wave.

Path difference: Difference between length of two path of two difference wave having same frequency and travelling at same velocity.

path difference, $\Delta x = \frac{\lambda}{2\pi} \times \Delta \phi$

phase difference

(#) Establish the expression of phase difference for both constructive and distructive interference due to reflected light from a plane parallel thin film.



Let, MN and PQ be the two parallel surface of thin film.

thickness → t

refractive index → μ

DA → monochromatic light in incident on the upper surface, at the point A.

AC → Reflected rays and AB → Refracted rays

$AC \perp B'L$. path difference between AC and B'F be x.

Optical path difference, $x = \mu(AB + BB') - AL$ ——— (1)

At $\triangle ABM' \rightarrow AB = \frac{t}{\cos r}$; $\triangle BB'M' \rightarrow BB' = \frac{t}{\cos r}$

$B'L \perp AL$; so, $\sin i = \frac{AL}{AB'}$ [at $\triangle AB'L$]

$\Rightarrow AL = AB' \sin i = 2AM' \sin i$ ——— (2)

From $\triangle AM'B \rightarrow \tan r = \frac{AM'}{BM'} = \frac{AM'}{t} \Rightarrow \frac{AL}{2t \sin i}$

$\Rightarrow AL = 2t \frac{\sin r}{\cos r} \times \sin i$

$= 2t \times \frac{\sin r}{\cos r} \times \frac{\sin i}{\sin r} \times \sin r$

$= 2t \frac{\sin^2 r}{\cos r} \mu$ $\left[\mu = \frac{\sin i}{\sin r}\right]$

From eqn ①;

$$x = \mu (AB + BB') - AL$$

$$= \mu \left(\frac{t}{\cos r} + \frac{t}{\cos r} \right) - 2t \frac{\sin^2 r}{\cos r} \mu$$

$$= \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r} = \frac{2\mu t}{\cos r} (1 - \sin^2 r)$$

$$= \frac{2\mu t}{\cos r} \times \cos^2 r = 2\mu t \cos r$$

Correct path difference, $\Delta x = 2\mu t \cos r \pm \frac{\lambda}{2}$

For constructive interference;

$$2\mu t \cos r \pm \frac{\lambda}{2} = n\lambda$$

$$\Rightarrow 2\mu t \cos r = (2n \pm 1) \frac{\lambda}{2}$$

$$\left.\begin{aligned} \Delta x &= 2\mu t \cos r \pm \frac{\lambda}{2} \\ &= 2\mu t \cos r - \frac{\lambda}{2} \end{aligned}\right\}$$

both are correct

phase difference, $\Delta \phi = \frac{\cancel{2}\pi}{\cancel{\lambda}} \times (2n \pm 1) \frac{\cancel{\lambda}}{\cancel{2}}$

$$= (2n \pm 1)\pi$$

For destructive interference;

$$2\mu t \cos r \pm \frac{\lambda}{2} = (2n \pm 1) \frac{\lambda}{2}$$

$$\Rightarrow 2\mu t \cos r = n\lambda$$

phase difference, $\Delta \phi = \frac{2\pi}{\cancel{\lambda}} \times n\cancel{\lambda}$

$$= 2n\pi$$

~~The optical path difference is~~

Stokes treatment (principle of reversibility): According to the principle of reversibility, in the absence of any absorbtion, a light ray that is reflected or refracted will retrace its original path if its direction is reversed.

Real example of Interference:

i) Interference is demonstrated by the light reflected from a film of oil floating on water.

ii) The thin film of a soap bubble, which reflects a spectrum of beautiful colours when illuminated by natural or artificial light sources.

Applications of interference:

i) Optical Testing: Generally interference is used in testing surface qualities like: flat surface, spherical surface, roughness of surface etc.
Sunglasses, lenses for binoculars or cameras.

ii) Space Applications:

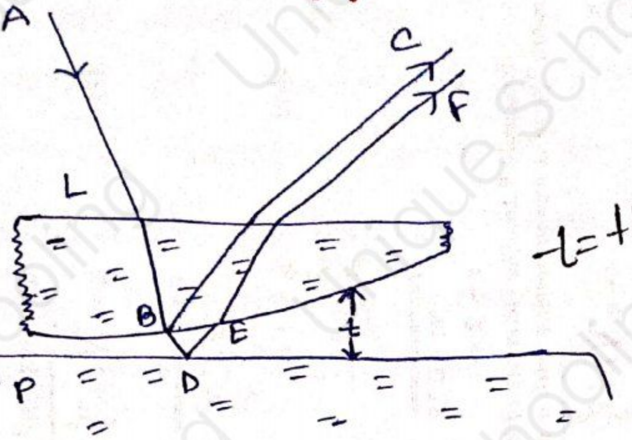
a) Radio astronomy
b) Measuring light intensity
c) In retrieving images from the telescopes.

Why can't two sources behave as coherent sources?

Two different sources can never produce waves of same phase because each source of light contains infinite number of atoms and the waves which are emitted by them will not be in phase. The atoms after absorbing energy go to excited states and emit radiations when fall back to ground state.

06

How could you produce coherent light source at laboratory?



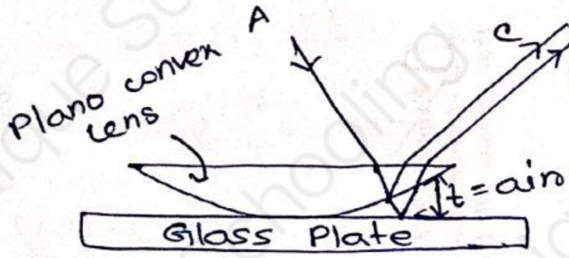
A is the parental light or primary light.

C, F → Daughter light or secondary light.

BC comes from reflection and DFF comes from transmition.

Here we neglect absorbtion because here we use transparent materials. So, absorbtion = 0.

How are the newton rings formed?



When a plano-convex surface is placed on a flat glass plate, an air film of gradually increasing thickness is formed between them. Then a monochromatic light is allowed to fall normally on the film and viewed in reflection mode, alternate dark and bright rings are observed. These are newton rings.

Why do rings get closer as their order increases?

The diameter of dark ring is proportional to the square root of natural number while bright rings are proportional to the square root of odd natural numbers hence the rings get closer with increasing order.
Additionally, they don't increase at the same rate.

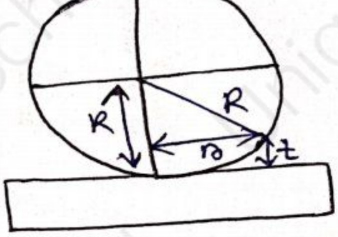
Expressed newton's rings due to reflected light.

$$R^2 = (R-t)^2 + r^2 = R^2 - 2Rt + t^2 + r^2$$

$$\Rightarrow 2Rt = t^2 + r^2 \quad [2Rt >>>> t^2]$$

$$\Rightarrow 2Rt = r^2$$

$$\therefore t = \frac{r^2}{2R}$$



For the bright fringe;

$$2\mu t \cos\theta = (2n-1)\frac{\lambda}{2}$$

$$\Rightarrow 2t = (2n-1)\frac{\lambda}{2} \quad [\text{when } \mu = 1 \text{ and } \cos\theta = 1]$$

$$\Rightarrow \cancel{2}\frac{r^2}{\cancel{2}R} = (2n-1)\frac{\lambda}{2}$$

$$\Rightarrow r^2 = (2n-1)\frac{\lambda R}{2}$$

$$\Rightarrow r = \sqrt{(2n-1)\frac{\lambda R}{2}}$$; the radius of the bright fringe.

Ans

$$\therefore r \propto \sqrt{2n-1}$$

For the dark fringe;

$$2\mu t \cos\theta = n\lambda \Rightarrow 2t = n\lambda \quad [\text{when } \mu = 1 = \cos\theta]$$

$$\Rightarrow \frac{r^2}{R} = n\lambda \Rightarrow r^2 = n\lambda R$$

$$\therefore r = \sqrt{n\lambda R}$$; the radius of the dark fringe.

$$\therefore r \propto \sqrt{n}$$

When $n=0$; the radius of the dark ring is zero and the radius of the bright ~~fringe~~ ring is $\sqrt{\frac{\lambda R}{2}}$. Therefore, the centre is dark. Alternately, dark and bright rings are produced.

Determine the wavelength of monochromatic ray using newton's ring.

the radius of the nth dark ring;

$$r_n^2 = n\lambda R$$

$$\therefore D_n^2 = 4n\lambda R \quad \text{--- (i)}$$

the diameter of the $(n+m)$th dark ring is,

$$D_{n+m}^2 = 4(n+m)\lambda R \quad \text{--- (ii)}$$

(ii) $-$ (i) $\Rightarrow$

$$D_{n+m}^2 - D_n^2 = 4m\lambda R$$

$$\therefore \lambda = \frac{D_{n+m}^2 - D_n^2}{4mR}$$

Determine the refractive index of a liquid.

For the dark rings due to reflected light,

$$2\mu t\cos\theta = n\lambda$$

$$\Rightarrow 2\mu t = n\lambda \quad [\text{when } \cos\theta = 1]$$

$$\Rightarrow \cancel{2}\mu \frac{r^2}{\cancel{2}R} = n\lambda \Rightarrow r^2 = \frac{n\lambda R}{\mu}$$

$$\therefore r_n^2 = \frac{n\lambda R}{\mu} \cdot \therefore D_n^2 = \frac{4n\lambda R}{\mu}$$

If D_n' is the diameter of the nth ring and D_{n+m}' is the diameter of th $(n+m)$th ring then,

$$(D_{n+m}')^2 = \frac{4(n+m)\lambda R}{\mu} \quad \text{and} \quad (D_n')^2 = \frac{4n\lambda R}{\mu}$$

$$\Rightarrow (D'_{n+m})^2 - (D'_n)^2 = \frac{4m\lambda R}{\mu} \quad \text{(i)}$$

wh. we know that,

$$D^2_{n+m} - D^2_n = 4m\lambda R \quad \text{(ii)}$$

From (i) $\Rightarrow$

$$\mu = \frac{4m\lambda R}{(D'_{m+n})^2 - (D'_n)^2}$$

$$\mu = \frac{(D_{n+m})^2 - (D_n)^2}{(D'_{n+m})^2 - (D'_n)^2}$$

What will happen if a transparent liquid is introduced between the lens and plate?

$\Rightarrow$ The fringes will contract means the diameter of the rings will be reduced by a factor of $(\sqrt{\mu})$ which is the refractive index of that transparent liquid.

Why are the rings circular?

Since the locus of constant thickness of air film about the point of contact is circle so the rings are circular.

In a newton's rings experiment, the diameter of the 5th ring was 0.336 cm and the diameter of the 15th ring was 0.590 cm. Find the radius of curvature of the plano-convex lens, if the wavelength of light used is 5890A°.

$D_5 = 0.336$ cm; $D_{15} = 0.590$ cm; $\lambda = 5890 \times 10^{-8}$ cm

$m = n+m-n = 15-5 = 10$

$$\lambda = \frac{D_{n+m}^2 - D_n^2}{4mR} \Rightarrow R = \frac{D_{15}^2 - D_5^2}{4m\lambda}$$

$$\therefore R = \frac{(0.59)^2 - (0.336)^2}{4 \times 10 \times 5890 \times 10^{-8}} = 99.83 \text{ cm}$$

In a newton's ring experiment, the diameter of the 12th ring changes from 1.50 cm to 1.35 cm when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid.

Here $n = 12$; $D_1^2 = \frac{4n\lambda R}{\mu}$ [For liquid]

$D_2^2 = 4n\lambda R$ [For air]

Given, $D_1 = 1.35$ cm; $D_2 = 1.5$ cm

$$\mu = \left(\frac{D_2}{D_1}\right)^2 = \left(\frac{1.5}{1.35}\right)^2 = 1.22 \text{ cm}$$

Ans:

In newton's ring experiment the diameter of 5th dark ring is reduced to half to its value after placing a liquid between glass plate and convex surface. Calculate the refractive index of liquid.

$D_n^2 = 4n\lambda R$

$\therefore (D_n')^2 = \frac{4n\lambda R}{\mu}$

$$\mu = \left(\frac{D_n}{D_n'}\right)^2 = \left(\frac{D_n}{D_n/2}\right)^2 = 4$$

Ans:

(#) Newton's ring are seen in reflected light of wave length 5896A°. Radius of curvature of lens is 1m. A liquid is introduced between plate and lens. Find ~~diametre~~ refractive index of liquid, if diameter of 16th dark ring is 5.1 mm.

$\lambda = 5896\times10^{-10}\,m$; $R = 1\,m$; $D_{16}^{2} = 5.1\times10^{-3}\,m$

$$D_n^2 = \frac{4n\lambda R}{\mu} \Rightarrow \mu = \frac{4\times16\times5896\times10^{-10}\times1}{(5.1\times10^{-3})^2}$$

$$= 1.45$$

Ans:

(#) In newton's ring experiment the diameters of 4th and 12th rings are 0.4 cm and 0.7 cm respectively. Find diameter of 20th dark ring?

$D_4 = 0.4\,cm$; $D_{12} = 0.7\,cm$; $n+m-n \Rightarrow m = 8$

$$\lambda = \frac{D_{12}^2 - D_4^2}{4mR} \Rightarrow 4\times8\times R\times\lambda = (0.7)^2 - (0.4)^2 \quad \text{——— (i)}$$

$$D_{20}^2 - D_{12}^2 = 4\times8\times R\times\lambda \quad \text{——— (ii)}$$

$$D_{20}^2 - D_{12}^2 = (0.7)^2 - (0.4)^2$$

$$\Rightarrow D_{20}^{2} = 0.904\,cm$$

Ans:

(#) Newton's rings are observed normally in reflected light of wavelength 6000A°. The diameter of the 10th dark ring is 0.5 cm. Find the radius of curvature of lens and thickness of film.

$\lambda = 6000\times10^{-8}\,cm$; $D_{10} = 0.5\,cm$

$$D_n^2 = 4n\lambda R \Rightarrow R = \frac{D_{10}^2}{4\times10\times\lambda} = \frac{(0.5)^2}{40\times6\times10^{-5}}$$

$\therefore R = 104.17\,cm$

$$t = \frac{r^2}{2R} = \frac{D^2}{8R} = \frac{(0.5)^2}{(104.17)\times8} = 0.0002999909\,cm$$

(#) The lower surface of a lens resting on a plane glass plate has a radius of curvature of 400 cm. When illuminated by monochromatic light, the arrangement produces newton's ring and 15th bright ring has a diameter of 1.16 cm. Calculate wavelength of light.

$R = 400\text{ cm}$, $D_{15} = 1.16\text{ cm}$, $n = 15$

$$\lambda = \frac{D_{15}^2}{4 \times 15 \times 400} = \frac{(1.16)^2}{60 \times 400} = 5.6 \times 10^{5}\text{ cm}$$

(#) Light containing two wavelengths λ_1 and λ_2 falls normally on a convex lens of radius of curvature R, resting on a glass plate. Now, if nth dark ring due to λ_1 concides with (n+1)th dark ring due to λ_2, then prove that the radius of the nth dark ring due to λ_1 is $\sqrt{\frac{\lambda_1 \lambda_2 R}{(\lambda_1 - \lambda_2)\mu}}$

Radius of nth dark ring of λ_1 is given by,

$$r_n = \sqrt{\frac{nR\lambda_1}{\mu}} \quad \text{--- (i)}$$

Radius of (n+1)th dark ring of λ_2 is given by,

$$r_{n+1} = \sqrt{\frac{(n+1)R\lambda_2}{\mu}} \quad \text{--- (ii)}$$

nth dark ring of λ_1 and (n+1)th dark ring of λ_2 concide.

$$r_n = r_{n+1}$$

$$\Rightarrow nR\lambda_1 = (n+1)R\lambda_2 \Rightarrow n\lambda_1 = n\lambda_2 + \lambda_2$$

$$\Rightarrow n(\lambda_1 - \lambda_2) = \lambda_2 \quad \therefore n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

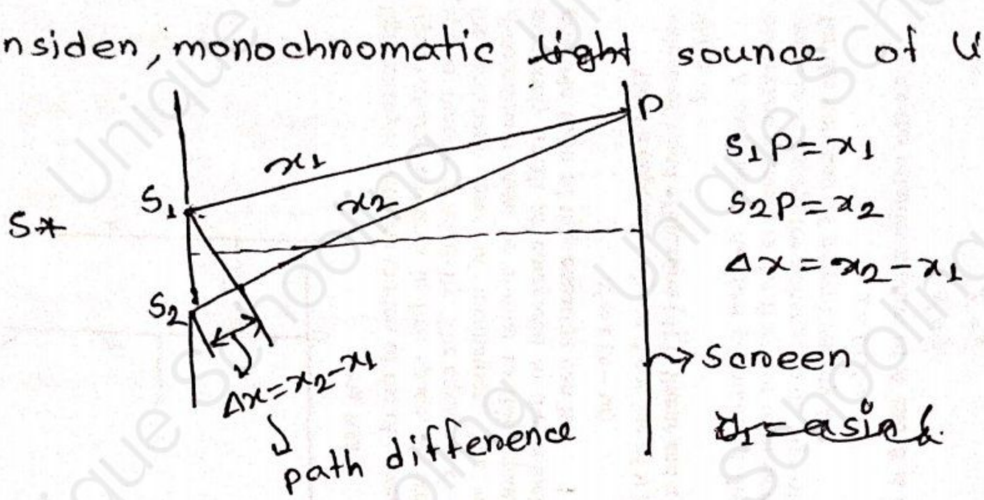
$$\therefore r_n = \sqrt{\frac{\lambda_2}{\lambda_1 - \lambda_2} \cdot \frac{R\lambda_1}{\mu}}$$

$$= \sqrt{\frac{\lambda_1 \lambda_2 R}{(\lambda_1 - \lambda_2)\mu}}$$

[showed]

⊕ Expressed interference of two light waves & by analytical treatment.

consider, monochromatic ~~light~~ source of light s.



~~$y_1 = a\sin\delta$~~

$$y_1 = a\sin\omega\left(t - \frac{x_1}{v}\right)$$

$$y_2 = a\sin\omega\left(t - \frac{x_2}{v}\right)$$

According to the superposition principle:

$$y = y_1 + y_2$$

$$= a\sin\omega\left(t - \frac{x_1}{v}\right) + a\sin\omega\left(t - \frac{x_2}{v}\right)$$

~~$= a[2\sin\omega$~~

$$= a\sin\left(\omega t - \frac{\omega x_1}{v}\right) + a\sin\left(\omega t - \frac{\omega x_2}{v}\right)$$

$$= a\left[2\sin\left(\omega t - \omega\frac{x_2 + x_1}{2v}\right)\cdot\cos\left(\frac{\omega(x_2 - x_1)}{2v}\right)\right]$$

$$\frac{\omega}{2v} = \frac{\pi}{\lambda}$$

$$= a\left[2\sin\omega\left(t - \frac{x_2 + x_1}{2v}\right)\cos\frac{\pi}{\lambda}(x_2 - x_1)\right]$$

$$= \underbrace{2a\cos\frac{\pi}{\lambda}(x_2 - x_1)}_{\text{amplitude}}\sin\omega\left(t - \frac{x_2 + x_1}{2v}\right)$$

For constructive interference:

amplitude $= \pm 2a$

Intensity $= 4a^2$

path difference, $\Delta x = n\lambda = 2n\frac{\lambda}{2}$

phase difference, $\delta = \frac{2\pi}{\lambda}\times\Delta x = \frac{2\pi}{\lambda}\times n\lambda = 2n\pi$

For destructive interference:

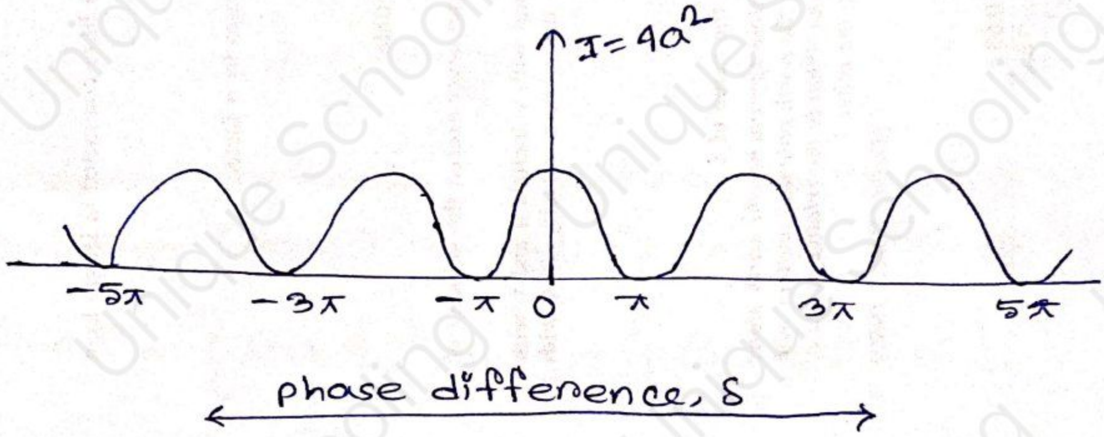
amplitude, $A = 0$

Intensity, $I = 0$

path difference, $\Delta x = (2n-1)\frac{\lambda}{2}$

phase difference, $\delta = (2n-1)\pi$

Proof: The interference of light obeys law of conservation of energy.



It is seen from graph that the energy is destroyed at point of minima.

It should be told that the destructive of light energy occurs in the phenomenon of interference of light. Since, light energy is not transformation any other form in interference of light.

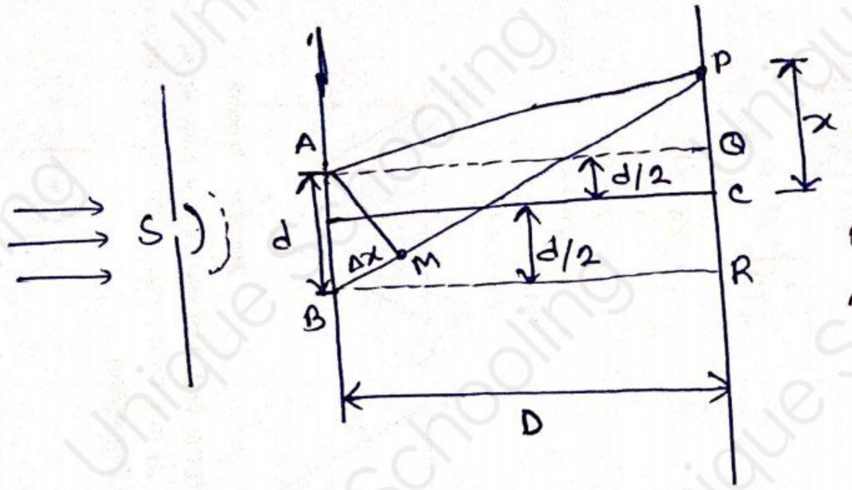
According to law of conservation of energy the total amount of energy must be constant.

Redistribution of energy: The average value of the energy ($2a^2$) over any number of fringes is the same as if the interference effects were same $2a^2$.

Hence it can be said that the interference of light obeys law of conservation.

(#) What do you mean by fringe width? Derive a

472 relation on among fringe width, wavelength, distance between screen and slit and distance between two slits.



$PQ = x - \frac{d}{2}$

$PR = x + \frac{d}{2}$

$BP^2 = PR^2 + BR^2$

$AP^2 = PQ^2 + AQ^2$

The distance between any two constructive (bright) or destructive (dark) fringe is called fringe width.

$$BP^2 - AP^2 = \left\{ \left(x + \frac{d}{2}\right)^2 + D^2 \right\} - \left\{ \left(x - \frac{d}{2}\right)^2 + D^2 \right\}$$

$$= \cancel{x^2} + dx + \cancel{\frac{d^2}{4}} - \cancel{x^2} + dx - \cancel{\frac{d^2}{4}}$$

$$= 2xd$$

$$\Rightarrow \cancel{(BP+AP)}\, BP - AP = \frac{2xd}{BP + AP} = \frac{2xd}{2D} \qquad \Big|\ BP = AP = D \text{ (Approximately)}$$

$$\therefore BP - AP = \Delta x = \frac{xd}{D}$$

path difference, $\Delta x = \frac{xd}{D}$

phase difference, $\delta = \frac{2\pi}{\lambda}\left(\frac{xd}{D}\right)$

For bright fringe,

$$\Delta x = n\lambda$$

$$\Rightarrow \frac{xd}{D} = n\lambda$$

For nth bright fringe, $x_n = \frac{Dn\lambda}{d}$

↳ n তম উজ্জ্বল ডোরার দূরত্ব

(*) Murugeshan (7-21)

470 ① Michelson-morley experiment mathematical calculation.

② Lorentz transformation with example.

③ Length contraction math example very much imp. (**)

④ Time Dilation with example

⑤ Variation of mass with velocity (***)

(*) Experimental set-up to photoelectric effect (***)

For dark fringe,

$$\Delta x = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow \frac{xd}{D} = (2n+1)\frac{\lambda}{2}$$

nth dark fringe, $x_n = (2n+1)\frac{D\lambda}{2d}$

↳ n তম অন্ধকার ডোরার দূরত্ব

The distance between any two constructive or two destructive fringe;

$$x_2 - x_1 = \frac{2\lambda D}{d} - \frac{\lambda D}{d} = \frac{D\lambda}{d} \quad [\text{bright}]$$

$$x_2 - x_1 = \frac{5\lambda D}{2d} - \frac{3\lambda D}{2d} = \frac{\lambda D}{d} \quad [\text{dark}]$$

$\therefore$ fringe width, $\beta = \frac{D\lambda}{d}$

Theory of Interference Fringes

Problem 1:

Green light of wavelength $\underbrace{}^{\lambda}$5100 Å from a narrow slit is incident on a double slit. If the overall separation of $\underbrace{10}_{n}$ fringes on a screen $\underbrace{200\text{ cm}}_{x_n}$ away is $\underbrace{2\text{ cm}}_{D}$. Find the slit separation.

$$x_n = \frac{Dn\lambda}{d} \Rightarrow d = \frac{Dn\lambda}{x_n} = \frac{2\times10\times5100\times10^{-8}}{200} = 5.1\times10^{-6}\text{ cm}$$

Problem 2:

In Young's double slit experiment the separation of the slit is 1.9 mm and the fringe spacing is $\underbrace{0.31\text{ mm}}_{\beta}$ at a distance of 1 metre from the slits. Calculate the wavelength of light.

$d = \cancel{1.9\times10}\ 0.19\text{ cm}$,

$$\frac{D\lambda}{d} = 0.031 \Rightarrow \lambda = \frac{0.031\times0.19}{100} = 5.89\times10^{-5}\text{ cm}$$

Problem 3: Two straight and narrow parallel slits $\overbrace{1\text{ mm}}^{\beta}$ apart are illuminated by monochromatic light. Fringes formed on the screen held at a distance of $\underbrace{100\text{ cm}}_{D}$ from the slits are $\underbrace{0.5\text{ mm}}_{d}$ apart. What is the $\underbrace{\text{wavelength}}_{\lambda}$ of light?

$\beta = 0.1\text{ cm}$

$D = 100\text{ cm}$

$d = 0.05\text{ cm}$

$$\beta = \frac{D\lambda}{d} \quad \therefore \lambda = \frac{\beta d}{D} = \frac{0.1\times0.05}{100} = 5\times10^{-5}\text{ cm}$$

Problem 4: Light from a sodium vapour lamp (λ = 589 nm) forms an interference pattern on a screen 0.8 m from a pair of slits. The bright fringes in the pattern are 0.35 cm apart. What is the $\underbrace{\text{slit separation}}_{d}$?

$\lambda = 589\times10^{-7}\text{ cm}$

$D = 80\text{ cm}$

$x_n = 0.35\text{ cm}$

$$x_n = \frac{Dn\lambda}{d}$$

$$\Rightarrow d = \frac{Dn\lambda}{x_n} = \frac{80\times1\times589\times10^{-7}}{0.35} = 0.01346\text{ cm}$$

What is the physical significance of Young's double slit experiment?

The double slit experiment is a demonstration that light and matter can display characteristics of both classically defined waves and particle.

(11) A thin equiconvex lens of focal length 4m and reflective index 1.5 rests on and in contact with an optical flat and using light of wavelength 5460A°. Newton's ring are viewed normally by reflection what is the diameter of the bright ray?

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5-1)\left(\frac{1}{R} + \frac{1}{R}\right)$$

$$\Rightarrow 4 = 0.5 \times \frac{2}{R} \quad \therefore R = 4$$

$$\therefore r_n^2 = (2n-1)\frac{\lambda}{2} R$$

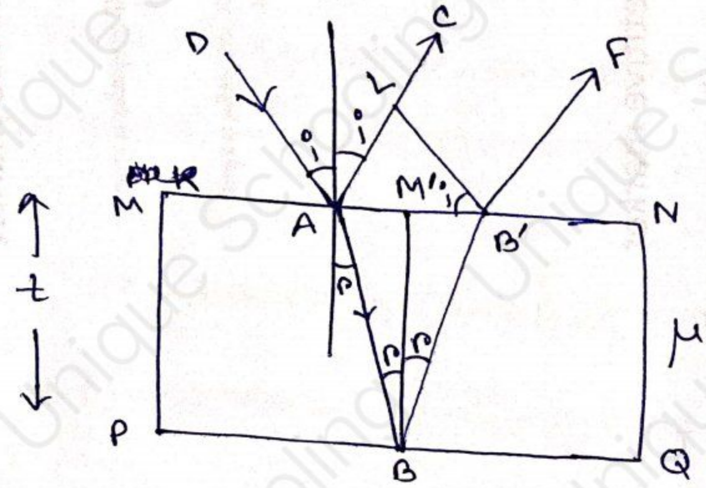
$$\therefore r = \sqrt{(2\cdot 10-1)\frac{5460\times10^{-10}}{2}\times 4}$$

$$= 3.13 \times 10^{-3}\ m$$

$$\therefore D = 6.26 \times 10^{-3}\ m$$

Ans

(#) Prove that the resultant amplitude is equal to the amplitude of the initial amplitude of the reflected light.



Let, MN and PQ be the two parallel surface of thin film.

t = thickness

μ = refractive index

DA → monochromatic light in incident on the upper surface, at the point A.

AC → Reflected rays and AB → Refracted rays

$B'L \perp AC$. Path difference between AC and B'F be x.

Optical path difference, $x = \mu(AB + BB') - AL$ ——— (1)

From $\triangle ABM' \rightarrow \cos r = \frac{BM'}{AB} \Rightarrow AB = \frac{t}{\cos r}$

From $\triangle BB'M' \rightarrow BB' = \frac{t}{\cos r}$

$B'L \perp AL$; so, $\sin i = \frac{AL}{AB'}$ $\therefore AL = 2AM' \sin i$

From $\triangle AM'B \rightarrow \tan r = \frac{AM'}{BM'} = \frac{AM'}{t} = \frac{AL}{2t \sin i}$

$\Rightarrow AL = 2t \sin i \frac{\sin r}{\cos r}$

$= 2t \times \frac{\sin i}{\sin r} \times \frac{\sin^2 r}{\cos r}$

$= 2\mu t \frac{\sin^2 r}{\cos r}$

From eqn (1);

$$x = \mu(AB + BB') - AL$$

$$= \mu\left(\frac{t}{\cos r} + \frac{t}{\cos r}\right) - 2\mu t \frac{\sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r}(1 - \sin^2 r) = \frac{2\mu t}{\cos r}\cos^2 r$$

$$= 2\mu t \cos r$$

Correct path difference, $\Delta x = 2\mu t \cos r \pm \frac{\lambda}{2}$

For constructive interference;

$$2\mu t \cos r \pm \frac{\lambda}{2} = n\lambda$$

$$\therefore 2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$$

phase difference, $\delta = \frac{2\pi}{\cancel{\lambda}} \times (2n \pm 1)\frac{\cancel{\lambda}}{\cancel{2}}$

$$= (2n \pm 1)\pi$$

For destructive interference;

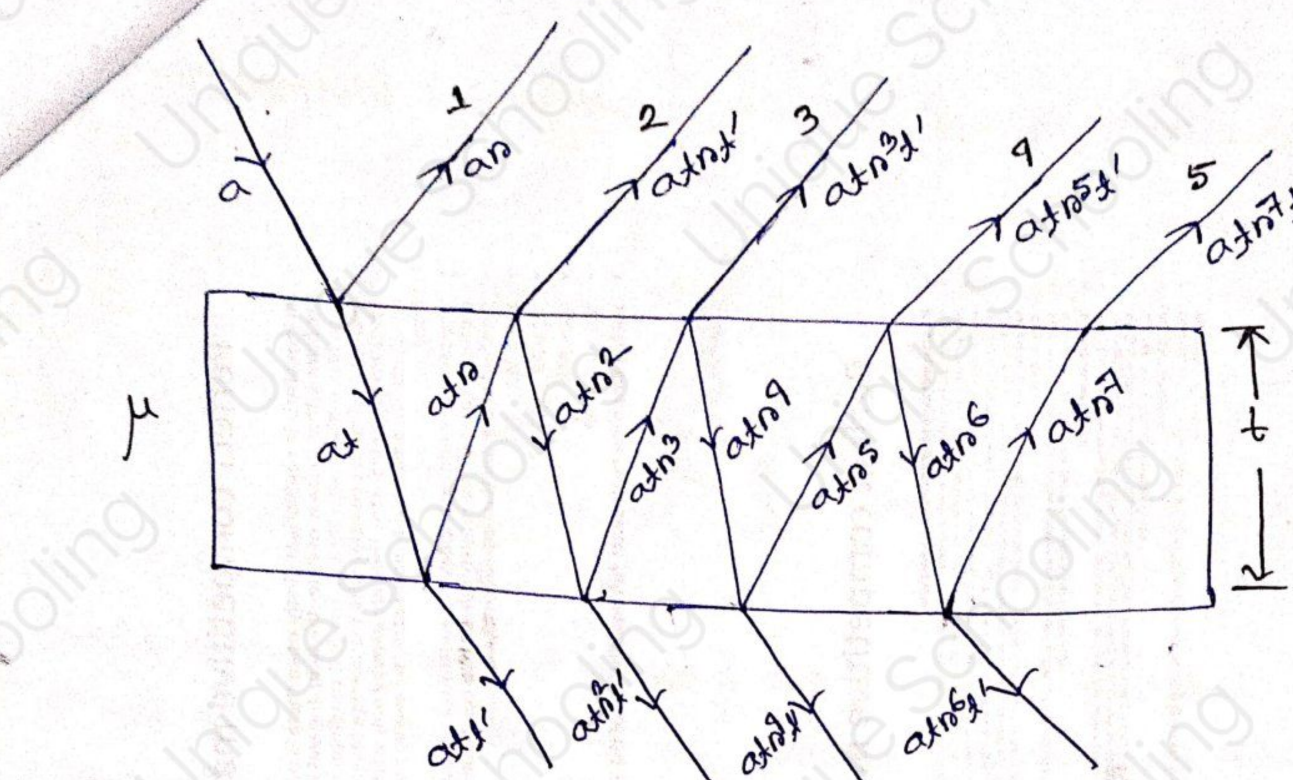
path difference, $\Delta x = 2\mu t \cos r \pm \frac{\lambda}{2}$

$$2\mu t \cos r \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$$

$$\therefore 2\mu t \cos r = n\lambda$$

phase difference, $\delta = \frac{2\pi}{\lambda} \times n\lambda$

$$= 2n\pi$$



where, r = reflection coefficient

t = transmission coefficient from rarer to denser medium.

t' = transmission coefficient from denser to rarer medium.

The resultant amplitude of 2, 3, 4 etc. is given by;

$$A = atrt' + atr^3t' + atr^5t' + \cdots$$

$$= atrt'(1 + r^2 + r^4 + \cdots) = atrt'\,(1/(1-r^2))$$

According to the principle of reversibility;

$$tt' = 1 - r^2$$

$$\therefore A = ar \times \frac{1-r^2}{1-r^2}$$

$$\boxed{A = ar}$$

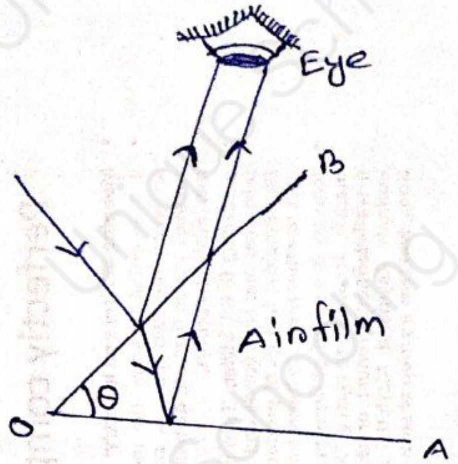
The resultant amplitude is equal to the amplitude of the reflected light 1, Therefore, the minima of the reflected system will be of zero intensity.

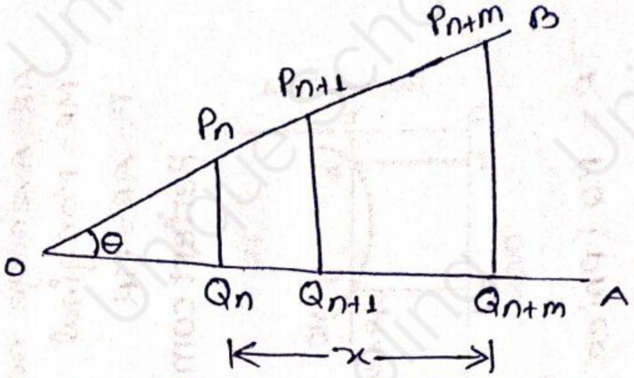
minima → outer phase

maxima → In phase

~~Equilibrium point = P~~
~~EQ per unit price OP~~
~~EQ per unit cost = OQ~~
~~TR = OQ·OP = OQEP~~
~~TC = OQ·OP = OQEP~~
~~Profit = TR − TC = 0~~
~~Normal profit, zero profit~~

(1) Express interference due to wedge shape thinfilm.





Consider two plane surfaces OA and OB inclined at an angle θ and enclosing a wedge shaped air film. The thickness of the air film increases O to A.

Suppose, the nth bright fringe occurs at P_n. The thickness of the airfilm ~~of~~ at $P_n = P_nQ_n$. ~~As~~ As the angle of ~~Frocn~~ incidence is small. So, $\cos r = 1$

For a bright fringe, $2\mu t \cos r = (2n+1)\frac{\lambda}{2}$

$\Rightarrow 2t = (2n+1)\frac{\lambda}{2}$ [when $\cos r = 1$, $\mu = 1$]

$\Rightarrow 2P_nQ_n = (2n+1)\frac{\lambda}{2}$ ——— (i)

The next bright fringe (n+1) will occur at P_{n+1};

$$2P_{n+1}Q_{n+1} = [2(n+1)+1]\frac{\lambda}{2} = (2n+2+1)\frac{\lambda}{2} = (2n+3)\frac{\lambda}{2}$$ ——— (ii)

(ii) − (i) $\Rightarrow$

$$2[P_{n+1}Q_{n+1} - P_nQ_n] = \frac{\lambda}{2}(2n+3-2n-1)$$

$$\therefore P_{n+1}Q_{n+1} - P_nQ_n = \frac{\lambda}{2}$$

এর মানে প্রত্যেক $\frac{\lambda}{2}$ দূরত্ব পর পর bright fringe পাওয়া যাবে।

Now, the bright fringe (n+m) will occur at P_{n+m};

$$P_{n+m}Q_{n+m} - P_nQ_n = \frac{m\lambda}{2}$$

From figure we see that,

$$Q_n Q_{n+m} = x$$

$$\therefore \theta = \frac{P_{n+m}Q_{n+m} - P_n Q_n}{Q_n Q_{n+m}} = \frac{m\lambda}{2x}$$

$$\therefore x = \frac{m\lambda}{2\theta}$$

Therefore, the angle inclination between OA and OB can be known. Here x is the distance corresponding to m fringes. The fringe width,

$$\boxed{\beta = \frac{x}{m} = \frac{\lambda}{2\theta}}$$

A soap film 5×10^{-5} cm thick is viewed at an angle $35°$ ~~to~~ to the normal. Find the wavelengths of light in the visible spectrum which will be absent from the reflected light. ($\mu = 1.33$)

$\mu = \frac{\sin i}{\sin r} \Rightarrow \sin r = \frac{\sin 35°}{1.33}$ $\therefore r = 25.5°$ and $\cos r = 0.9$

The condition for destructive interference, $2\mu t \cos r = n\lambda$

$n=1,\ \lambda_1 = 2\times1.33\times5\times10^{-5}\times0.9 = 12\times10^{-5}$ cm $= 120\,\mu$m

$n=2,\ \lambda_2 = \frac{2\times1.33\times5\times10^{-5}\times0.9}{2} = 6\times10^{-5}$ cm $= 6000$ A°

$n=3,\ \lambda_3 = \frac{2\times1.33\times5\times10^{-5}\times0.9}{3} = 4\times10^{-5}$ cm $= 4000$ A°

$n=4,\ \lambda_4 = \frac{2\times1.33\times5\times10^{-5}\times0.9}{4} = 3\times10^{-5}$ cm $= 3000$ A°

Out of the above wavelengths, $\lambda_2 = 6000$ A° and $\lambda_3 = 4000$ A° lie in the visible regions. Therefore, these two wavelengths are absent in the reflected light.

(#) A glass wedge of angle 0.01^c is illuminated by monochromatic light of wavelength 6000A° falling normally on it. At what distance from the edge of the wedge will ~~be~~ the 10th fringe be observed by reflected light?

$\theta = 0.01^c$; $m = 10$; $\lambda = 6000A° = 6000 \times 10^{-8}$ cm

$$x = \frac{m\lambda}{2\theta} = \frac{10 \times 6000 \times 10^{-8}}{2 \times 0.01} = 0.03 \text{ cm}$$ Ans:

(#) A beam of monochromatic light of wavelength 5.82×10^{-7} m falls normally on a glass wedge with the wedge angle of 20 sec of an arc. Find the number of dark fringes per cm of the wedge length. [$\mu = 1.5$]

$\theta = 20'' = \frac{20\pi}{60 \times 60 \times 180}$ rad; $\lambda = 5.82 \times 10^{-7}$ m, $\mu = 1.5$

$$\beta = \frac{\lambda}{2\theta\mu} = \frac{5.82 \times 10^{-7} \times 60 \times 60 \times 180}{2 \times 20 \times \pi \times 1.5} = 0.2 \text{ cm}$$

$\therefore$ Number of fringes per cm $= \frac{1}{0.2} = 5$ per cm Ans:

(#) Newton's rings are observed in reflected light of $\lambda = 5.9 \times 10^{-5}$ cm. The diameter of the 10th dark ring is 0.5 cm. Find the radius of curvature of the lens of the thickness of the airfilm.

$\lambda = 5.9 \times 10^{-5}$ cm; $m = 10$;

The radius of mth dark ring is, $r^2 = mR\lambda$

$r = \frac{0.5}{2} = 0.25$ cm

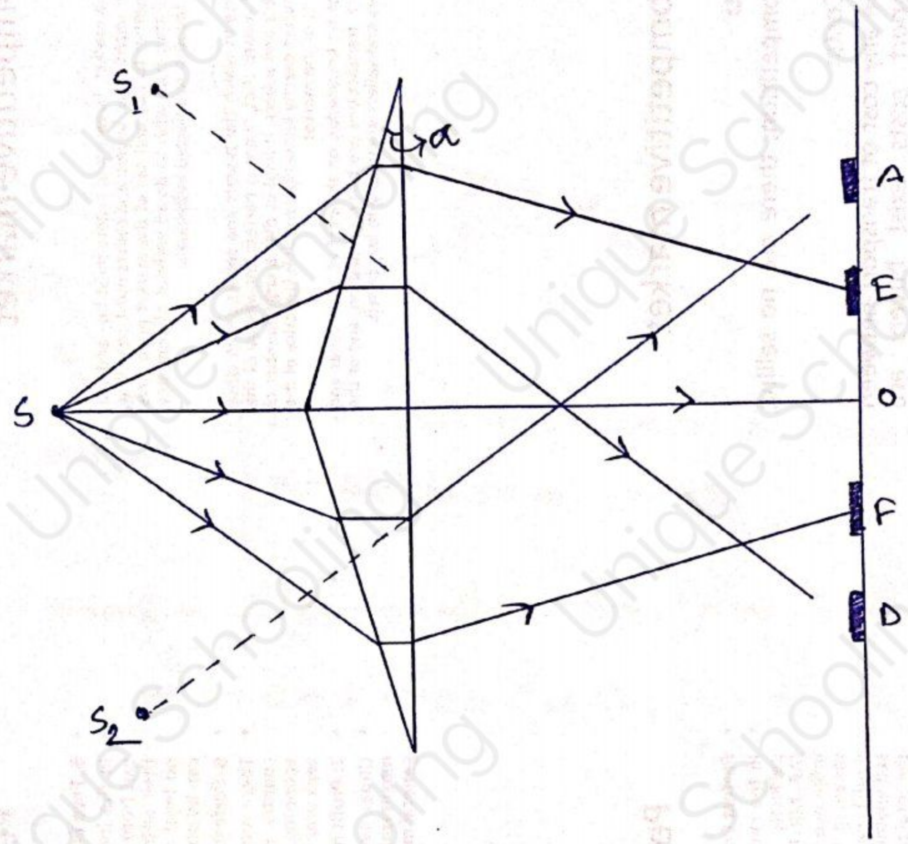
$$\therefore R = \frac{r^2}{m\lambda} = \frac{(0.25)^2}{10 \times 5.9 \times 10^{-5}} = 106 \text{ cm}$$

$$t = \frac{m\lambda}{2} = \frac{10 \times 5.9 \times 10^{-5}}{2} = 2.95\ \mu\text{m}$$

Ans:

(11) Describe the Fresnel's bi-prism method of producting interference fringes and determine the wavelength of light.

Two prism which are place base to base usually constructed from a single plate through grinding and polishing means bi-prism.

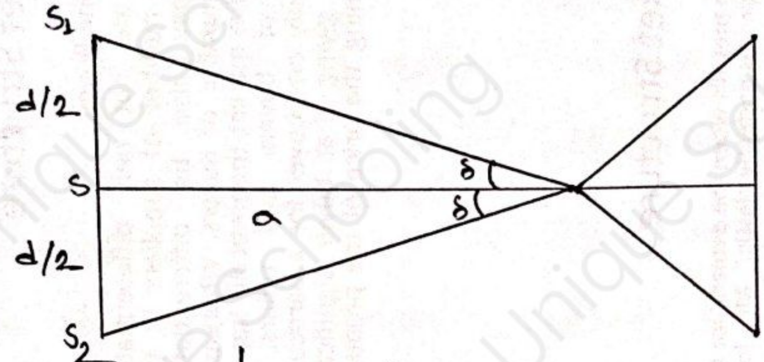


At the starting, the slits illuminated by a monochromatic light. Then the slit acting as narrow linear light source passes light which incident on a prism. After this happening, the ray get refracted downwards after deviated with a short angle. The prism which are down to the upper prism, passes light getting refracted upwards crossing the ray from other prism, such a way interference fringes form up. Through a micrometer eyepiece, fringes are observed.

Determination of the wavelength:

$\lambda = \frac{dB}{D}$ | B = fring width, d = slit
D = distance between the plane of observation and the slit acting as a source

$B = \frac{x_N - x_0}{N}$ | x_N = passing the number of fringes N, ~~said~~
x_0 = first position denoting.



$\delta = (\mu - 1)\alpha$ | μ = refractive index of prism/material.
α = refractive angle.

$\Delta SOS_1 \rightarrow \tan\delta = \frac{d/2}{a} \Rightarrow \frac{d}{2} = \delta a \; [\tan\delta \approx \delta]$

$\Rightarrow d = 2a\delta = 2a(\mu-1)\alpha$ | a = distance between refractive edge and slit line

$\therefore \lambda = \frac{dB}{D}$

$= \frac{2a(\mu-1)\alpha}{D} \times \frac{(x_N - x_0)}{N}$

λ is the wavelength of monochromatic light used in this experiment.

Difference between Interference and diffraction:

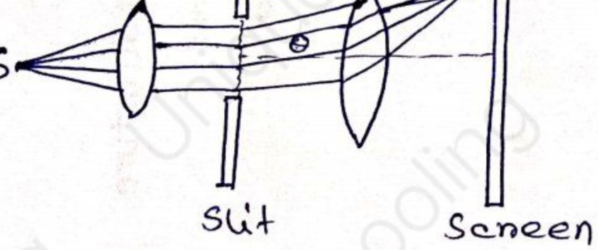
Interference	Diffraction
① It is the result of superposition of secondary waves starting from two different wavefronts originating from two coherent sources.	① It is the result of superposition of secondary waves starting from different parts of the same wavefronts.
② All bright and dark fringes are equal width.	② All fringes are not equal width.
③ All bright fringes are of same intensity.	③ Intensity fall rapidly.
④ Regions of dark fringes are perfectly dark.	④ Are not perfectly dark.
⑤ All fringes are clear.	⑤ All fringes are not clear without the central bright fringe.
⑥ Two types: i) constructive ii) destructive	⑥ Two types: i) Fresnel's diffraction ii) Fraunhofer diffraction.

What is meant by diffraction of light? Distinguish between Fresnel and Fraunhofer classes of direction.

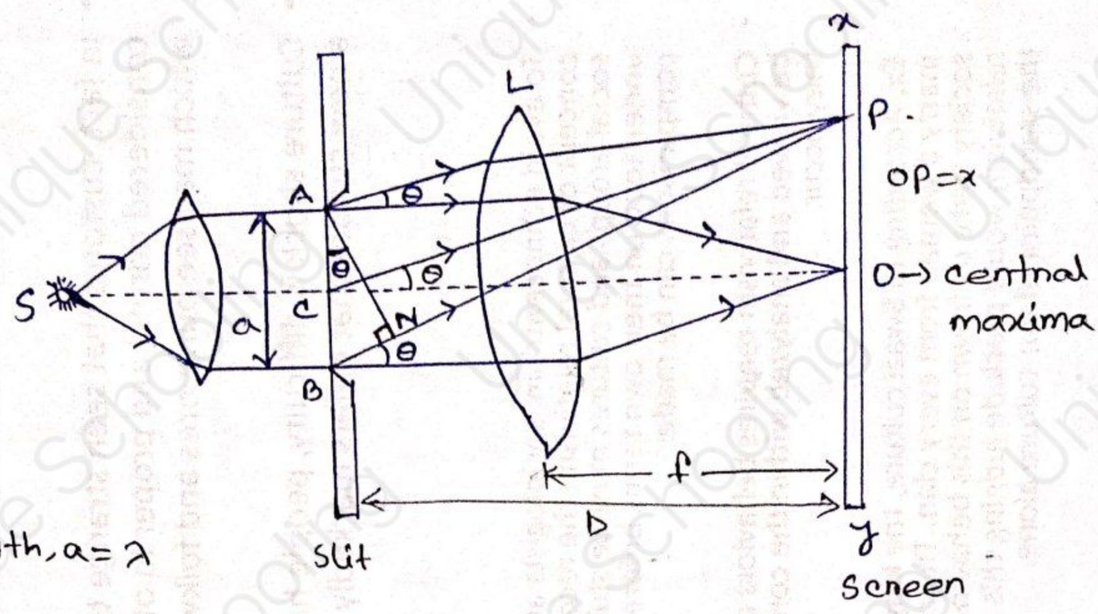
The rhythmic variations in intensity and the bending of light around the corners of an obstacle or the encroachment of light into the region of geometrical shadow constitute a class of phenomena known as the diffraction of light.

Fraunhofer	Fresnel
i) Planer wavefronts.	i) cylindrical wavefronts.
ii) Observation distance is infinite. In practice, often at focal point of lens.	ii) Source of screen at finite distance from the obstacle.
iii) Fixed in position.	iii) Move in a way that directly corresponds with any shift in the object.
iv) Patterns on spherical surfaces.	iv) Patterns on flat surfaces.
~~v) Pattern stay constant.~~ v) Shape and intensity of a fraunhofer diffraction pattern stay constant.	v) Change as we propagate them further downstream of the source of scattering.
vi) maxima and minima are well defined.	vi) maxima and minima are not well defined.

(#) Describe and Explain the fraunhofer diffraction pattern produced by a single slit illuminated by monochromatic light and find the conditions for minima and maxima.



the source and the screen are effectively at finite distance from the obstacle. The incident light is diffracted in various directions and that diffracted in a particular direction is focused on a screen by means of a convex lens.



Here, slit width, $a = \lambda$

⇒ We will investigate the resultant intensity at point O and P on the screen.

⇒ At point O, we get bright central fringe on image because secondary wavelets from point is equidistant from C.

⇒ Above and below O, there is alternate maxima and minima.

⇒ Intensity of alternate maxima decreases and alternate minima goes to zero.

⇒ Here, AB is wavefront, BN is path difference.

⇒ In ΔBAN; BN = path difference = Δx

$$\sin\theta_n = \frac{BN}{AB}$$

$$= \frac{\Delta x}{a} \Rightarrow \Delta x = a\sin\theta_n \quad \text{--- (1)}$$

phase difference, $\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi}{\lambda} a\sin\theta_n$ --- (2)

⇒ Now, AB is divided into two parts,

If $AB = a$, $AC = CB = \frac{a}{2} = \frac{\lambda}{2}$

For minimum intensity at P [Secondary minima]

path difference = even multiple of $\frac{\lambda}{2}$

$\therefore BN = a \sin\theta_n = 2n\frac{\lambda}{2}$

$\Rightarrow \sin\theta_n = \frac{2n\lambda}{2a}$ $\{n = \pm 1, 2, 3, \ldots\}$

$\Rightarrow \boxed{a \sin\theta_n = \pm n\lambda}$

$\therefore \boxed{\theta_n = \pm \frac{n}{a}\lambda}$ [If θ is small then $\sin\theta \approx \theta$]

For maximum intensity at P [secondary maxima]

path difference, BN = odd multiple of $\frac{\lambda}{2}$

$\therefore BN = a\sin\theta_n = (2n+1)\frac{\lambda}{2}$

$\Rightarrow \sin\theta_n = (2n+1)\frac{\lambda}{2a}$ $[n = \pm 1, 2, 3, \ldots]$

$\Rightarrow a\sin\theta_n = \pm (2n+1)\frac{\lambda}{2}$

$\therefore \theta_n = \pm (2n+1)\frac{\lambda}{2a}$ $[\sin\theta \approx \theta]$

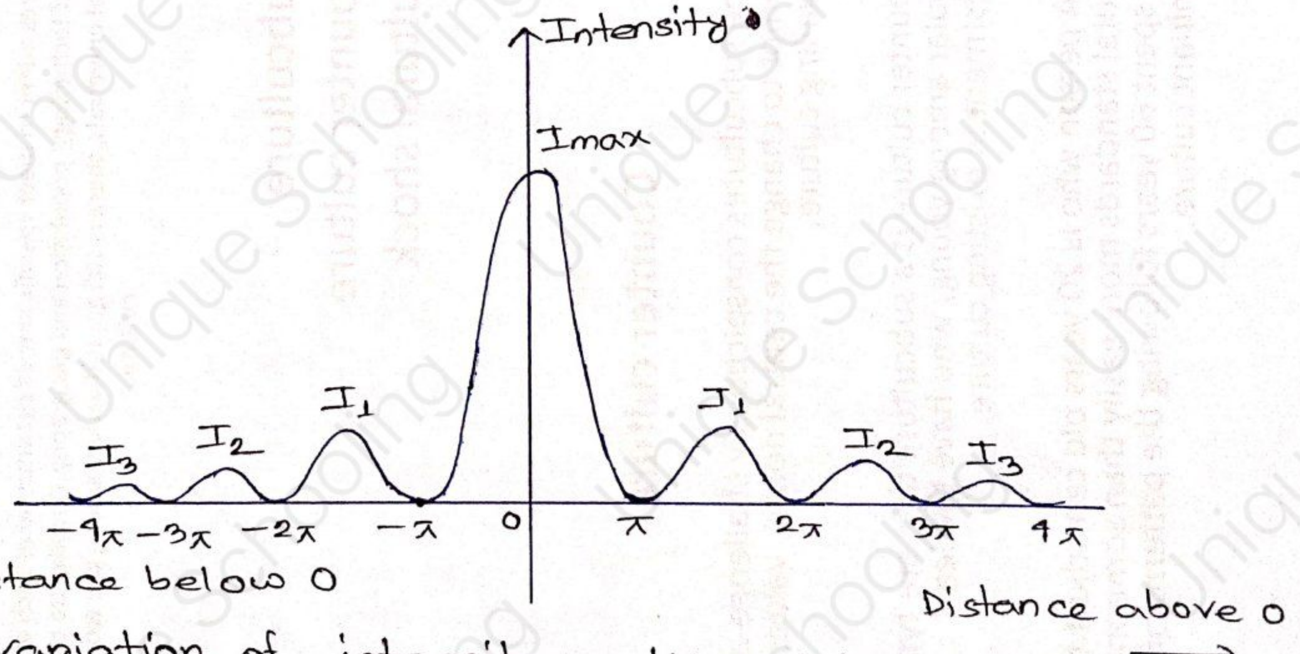


Fig: variation of intensity with distance for Fraunhofer's direction at a single slit.

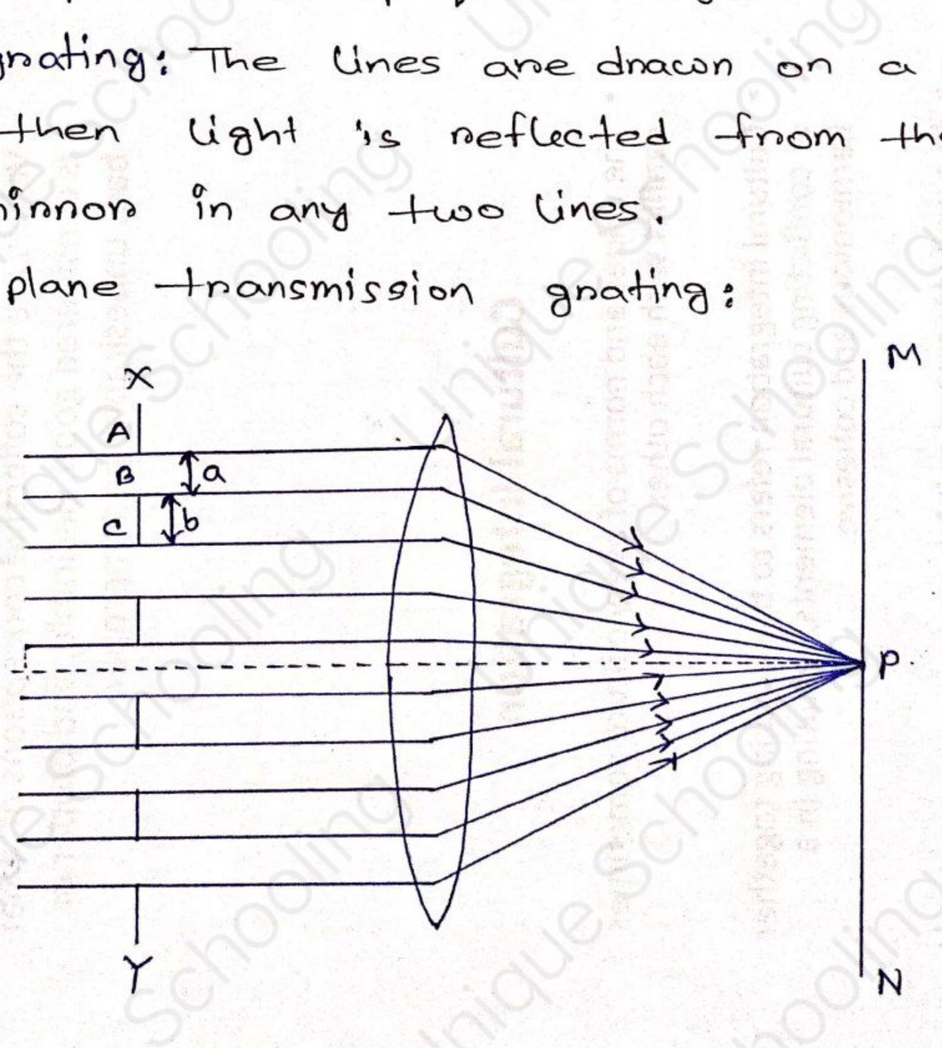
(#) What is a diffraction grating? Give the theory of the formation of spectra with a plane transmission grating and show how you would use it to find the wavelength of light.

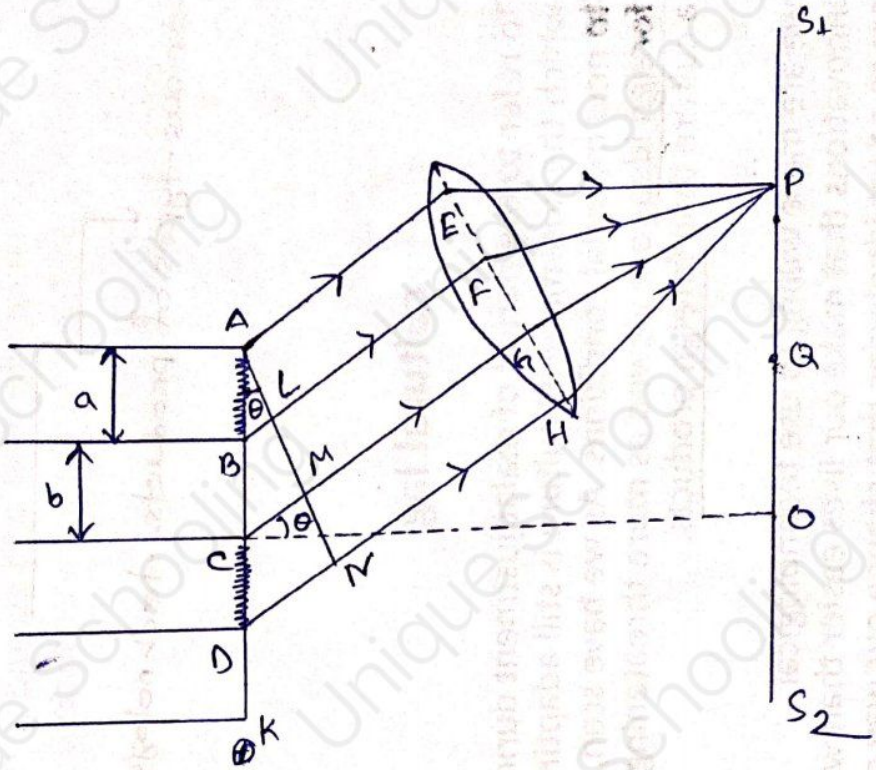
A diffraction grating is an extremely useful device and in one of its forms it consists of a very large number of narrow slits slide by slide. The slits are separated by opaque spaces. When a wave front is incident on a grating surface, light is transmitted through the slits and obstructed by the opaque portions.

Transmission grating: Gratings are prepared by ruling equidistant parallel lines on a glass surface. The lines are drawn with a find diamond point. The space in between any two lines transparent to light and the lined portion is opaque to light.

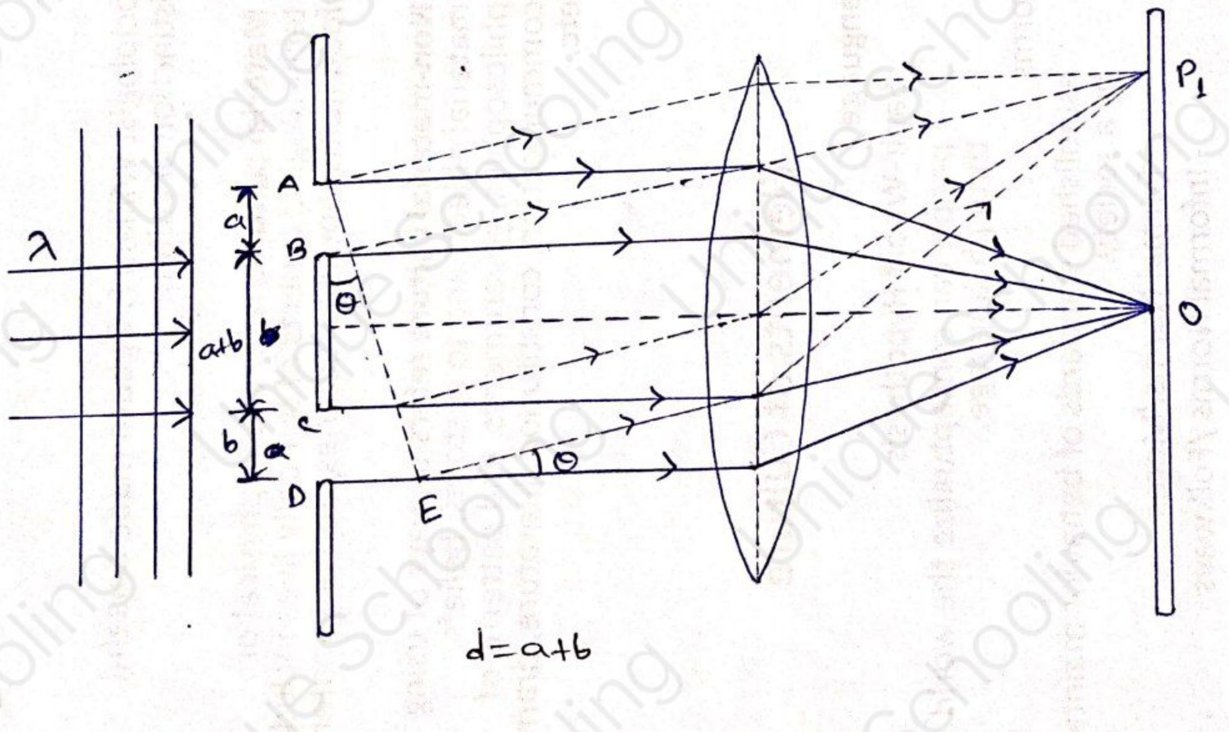
Reflection grating: The lines are drawn on a silvered surface then light is reflected from the positions of the mirror in any two lines.

Theory of plane transmission grating:





Fraunhofer diffraction due to double slit:



$d = a + b$

Diffraction pattern has 2 parts,

i) Diffraction pattern:

(due to each individual slits)

path diff, $\Delta x = a\sin\theta$

secondary max: $a\sin\theta = (2n+1)\frac{\lambda}{2}$

" min: $a\sin\theta = n\lambda$

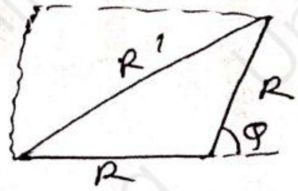
ii) Interference pattern:

(due to two slits)

path diff, $\Delta x = d\sin\theta$

max: $d\sin\theta = n\lambda$

min: $d\sin\theta = (2n+1)\frac{\lambda}{2}$



$(R')^2 = R^2 + R^2 + 2RR\cos\varphi$

$= 4R^2\cos^2\beta$

The resultant Intensity, $I = 4A^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$

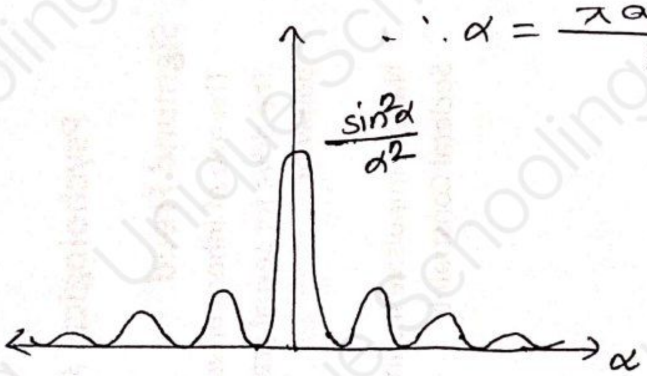
$$R^2 = A^2\frac{\sin^2\alpha}{\alpha^2}$$

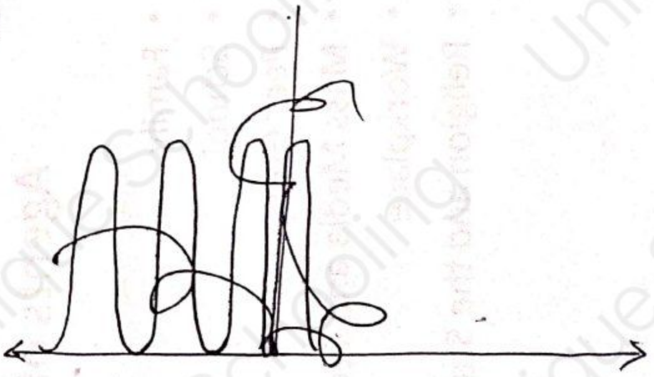
$$\beta = \frac{\varphi}{2}$$

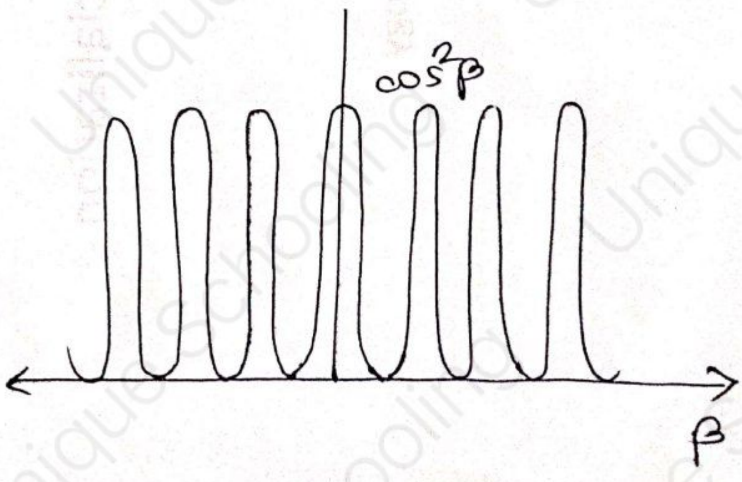
$$= \frac{1}{2}\frac{2\pi}{\lambda}(d\sin\theta)$$

$$= \frac{\pi}{\lambda}d\sin\theta$$

$$\therefore \alpha = \frac{\pi a\sin\theta}{\lambda}$$







Dependence of Intensity $\left(I = 4A^2 \frac{\sin^2\alpha}{\alpha^2} \cos^2\beta\right)$ on $\cos^2\beta$

Intensity maximum

$I = max \Rightarrow \cos^2\beta = 1$

$\therefore \beta = 0, \pm\pi, \pm 2\pi, ----$

$\beta = \pm n\pi$, $n = 0, 1, ----$

$\frac{\pi d \sin\theta}{\lambda} = \pm n\pi$

$\therefore \boxed{d \sin\theta = \pm n\lambda}$

$n = 0$, $\theta = 0 \Rightarrow$ centre max.

Intensity minimum

$I = min \Rightarrow \cos^2\beta = 0$

$\Rightarrow \beta = \pm(2n+1)\frac{\pi}{2}$

$\Rightarrow \frac{\pi d \sin\theta}{\lambda} = \pm(2n+1)\frac{\pi}{2}$

$\Rightarrow d \sin\theta = \pm(2n+1)\frac{\lambda}{2}$

where, $n = 0, 1, 2, ---$

(ii) In double slit Fraunhofer diffraction calculate the fringe spacing on a screen 50 cm away from the slits, if they are illuminated with blue light ($\lambda = 4800 A^\circ$) [slit separation, $b = 0.1$ mm] slit width, $a = 0.02$ mm.

Fringe spacing on the screen $= \sin\theta_2 - \sin\theta_1$

$$\Rightarrow \theta_2 - \theta_1 = \frac{3\lambda}{2(a+b)} - \frac{\lambda}{2(a+b)}$$

$$\Rightarrow \theta = \frac{\lambda}{(a+b)} = \frac{x}{D}$$

$$\therefore x = \frac{D\lambda}{a+b} = \frac{50 \times 4800 \times 10^{-8}}{0.002 + 0.01}$$

$$= 0.2 \text{ cm}$$

the angular seperation between the central maximum and first minima is,

$$\sin\theta_1 = \theta = \frac{\lambda}{2(a+b)} = \frac{x}{D}$$

$$\therefore x = \frac{\lambda D}{2(a+b)} = \frac{0.2}{2} = 0.1 \text{ cm}$$

(#) Interference fringes are observed with a bi-prism of refracting angle $1°$ and refractive index 1.5 on a screen 80 cm away from it. If the distance between the source and the bi-prism is 20 cm. Calculate the fringe width the wavelength of light used (i) $6900 A°$ and (ii) $5890 A°$.

(i) $\alpha = 1° = \frac{\pi}{180}^c$; ~~Or 80cm~~ $D = u + v = 20 + 80 = 100\,cm$

$\mu = 1.5$ $\qquad\qquad\qquad\qquad\qquad = 1\,m$

$$\lambda = \frac{d\beta}{D} \Rightarrow \beta = D\lambda \times \frac{1}{2a(\mu-1)\alpha} \qquad a = 20\,cm = 0.2\,m$$

$$\therefore \beta = \frac{1 \times 6900 \times 10^{-10}}{2 \times 0.2\,(1.5-1) \times \pi/180}$$

$$= 1.9767 \times 10^{-9}\,m$$

(ii) $$\beta = \frac{1 \times 5890 \times 10^{-10}}{2 \times 0.2 \times 0.5 \times \frac{\pi}{180}} = 1.6874 \times 10^{-9}\,m$$

(#) Deduce the missing orders for a double slit Fraunhofer diffraction pattern if the widths are 8.8×10^{-3} cm and they are 4.4×10^{-2} cm apart.

$(a+b)\sin\theta = n\lambda$ (maxima) —— (i)

$a \sin\theta = p\lambda$ (minima) —— (ii)

(i) ÷ (ii) $\Rightarrow \frac{a+b}{a} = \frac{n}{p}$

$$\Rightarrow \frac{n}{p} = \cancel{aa}\ 1 + \frac{b}{a}$$

$$= 1 + \frac{4.4 \times 10^{-2}}{8.8 \times 10^{-3}} = 6$$

$\therefore n = 6p$

$p = 1, 2, 3, \ldots = 6, 12, 18, \ldots$ ~~or 6p~~

⊕ A diffraction grating used at normal incidence gives a line, $\lambda_2 = 6000 A^\circ$ in a certain order superimposed on another line $\lambda_1 = 4500 A^\circ$ of the next higher order. If the angle of diffraction is 30°. How many lines are ~~there~~ three in a cm in grating?

$(a+b)\sin\theta = n\lambda_2$

$\Rightarrow (a+b)\sin 30^\circ = 3\times 6000\times 10^{-10}$

$\therefore (a+b) = 2\times 3\times 6000\times 10^{-10}$

$= 3.6\times 10^{-6}$ m

$= 3.6\times 10^{-4}$ cm

the number of line per cm $= \frac{1}{a+b}$

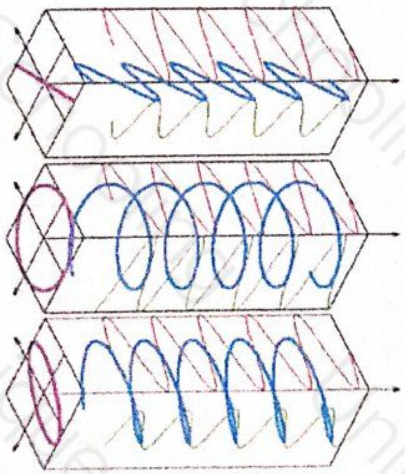
$= \frac{1}{3.6\times 10^{-4}}$

$= 2777.78$

Welcome
To
My class

TYPES OF POLARIZATION

1. Linear Polarization
2. Circular Polarization
3. Elliptical Polarization



Polarization

Polarization of light:

The process by which light waves vibrating in different planes can be made to vibrate in a particular plane is called polarization of light.

Polarization of Light Waves

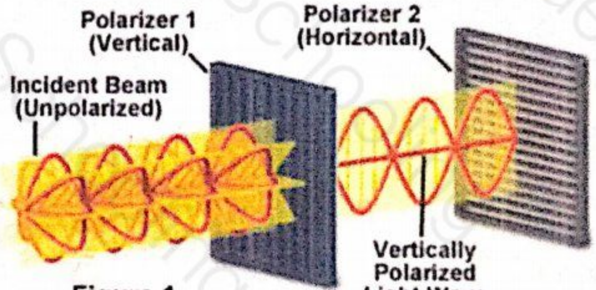


Figure 1

METHODS OF ACHIEVING POLARIZATION

1. Reflection
2. Scattering
3. Refraction
4. Double/Bi refraction

⊛ Why light is transverse wave and sound is longitudinal wave?

Ans: A transverse wave is a wave that oscillates perpendicular to its direction of propagation. Light is a wave in which an electric field propagates in vaccum or inside a medium. Light is the visible part of the electromagnetic spectrum, which comprises a range of different wavelengths of electromagnetic waves, including light, are transverse wave because they vibrates energy in a direction perpendicular to the direction in which the wave is travelling.

A sound wave is called a longitudinal wave because compressions and refractions in the air produce it. The air particles vibrates parallel to the direction of propagation.

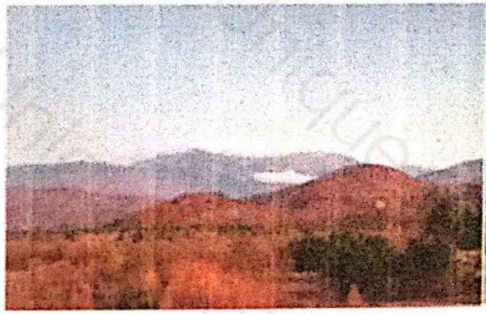
⊛ Many common light sources such as sunlight, halogen lighting, LED spotlight and incandescent bulbs produce unpolarized light.

Applications of Polarization

Polarization Photography

Reduce Sun Glare
Reduce Reflections
Darkens Sky
Increase Color Saturation
Reduce Haze

Polarization Photography

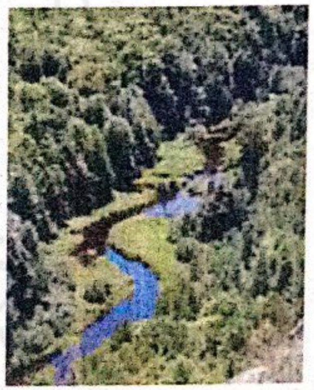


Without Polarizer

With Polarizer

- Provides better Color Saturation
- Darkens the sky

Polarization Photography



Without Polarizer

With Polarizer

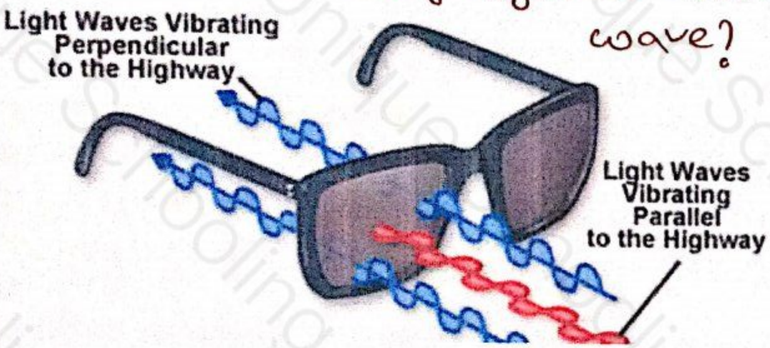
Polarization Photography : Scattering

Haze

De-hazed

Polaroid Sunglasses

why light is transvense wave?



$J_P = I$ = analyzen intensity

Polarization

Polarization of transverse wave:

I_0 = polanized intensity.

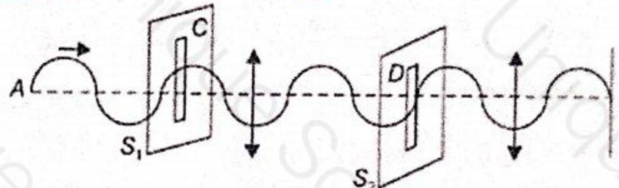


Fig. 1 (a)

$\frac{I_P}{I_0} = \cos^2\theta$

$= \cos^2 0 = 1$

$\cancel{I = I_0 \cos^2\theta}$

$=$

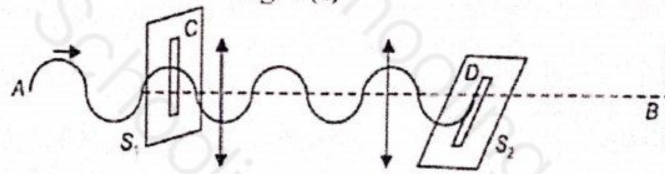


Fig. 1 (b)

$\frac{I_P}{I_0} = \cos^2 90^\circ$

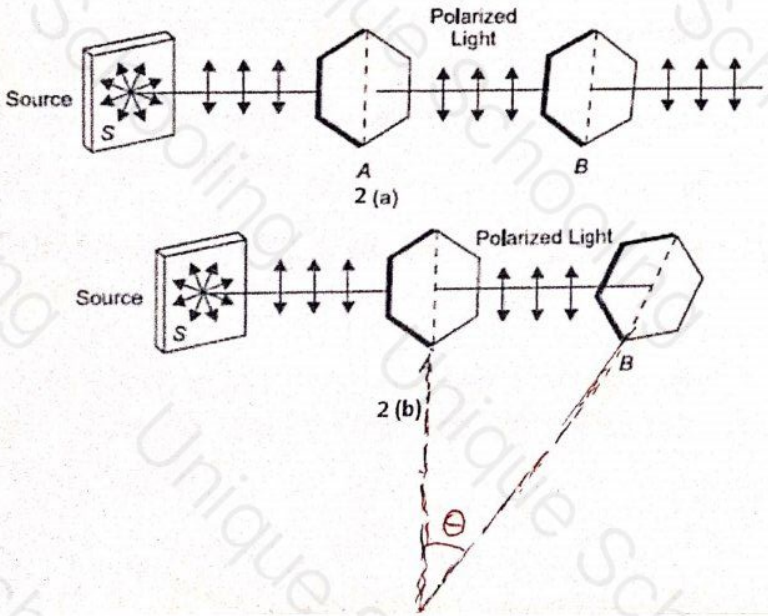
$= 0$

I_0 = অপোলারাইজড/ Analyzen আলোর তীব্রতা

I_P = পোলারাইজড

Polarization

Polarization of transverse wave:



Polarization

The crystal A *will act as the slit S1. The light is slightly coloured due to the natural colour of the crystal. On rotating the crystal A, no remarkable change is noticed. Now place the crystal B parallel to A.*

1. *Rotate both the crystals together so that their axes are always parallel. No change is observed in the light coming out of B Fig. 2(a).*
2. *Keep the crystal A fixed and rotate the crystal B. The light transmitted through B becomes dimmer and dimmer. When B is at right angles to A, no light emerges out of B Fig. 2 (b).*

If the crystal *B is further rotated, the intensity of light coming out of it gradually increases and is maximum again when the two crystals are parallel. This experiment shows conclusively that* light is not propagated as longitudinal or compressional waves.

⊕ What is plane of polarization and plane of vibration?

Ans: The plane of polarization is defined as the plane containing the direction of propagation of a polarized wave.

The plane of vibration is the plane which contains vibrations and which is along the optic axis of the wave. It is always perpendicular to the polarization.

Polarization

If we consider the propagation of light as a longitudinal wave motion then no extinction of light should occur when the crystal *B is rotated.*

It is clear that after passing through the crystal A, the light waves vibrate only in one direction. Therefore light coming out of the crystal A is said to be polarized because it has acquired the property of one sidedness with regard to the direction of the rays.

This experiment proves that light waves are transverse waves; otherwise light coming out of B could never be extinguished by simply rotating the crystal B.

Polarization

Malus law:

The intensity of the polarized light transmitted through the analyzer varies as the square of the cosine of the angle between the plane of transmission of the analyzer and the plane of the polarizer.

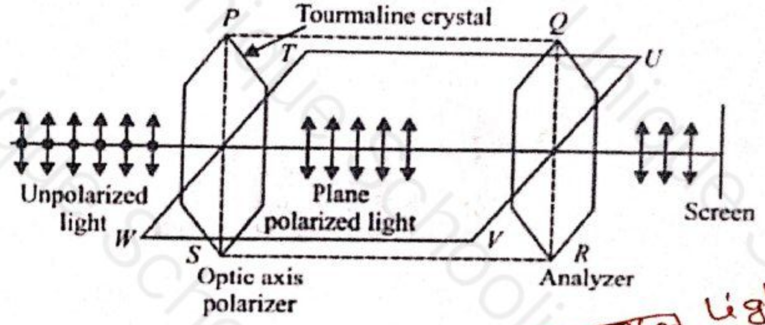
The intensity I_1 of the polarized light transmitted through the analyzer is given by the Malus Law

$I_1 = I\cos^2\theta$ *where, I is the original intensity and ϑ is the angle between the planes of the polarizer and the analyzer.*

The term plane of polarization refers to the direction of polarization of linearly polarized light on other electromagnetic radiation.

Polarization

Plane of vibration and Plane of polarization:



➢ *PQRS is the plane of vibration.* → যে তলে light vibrate করে।

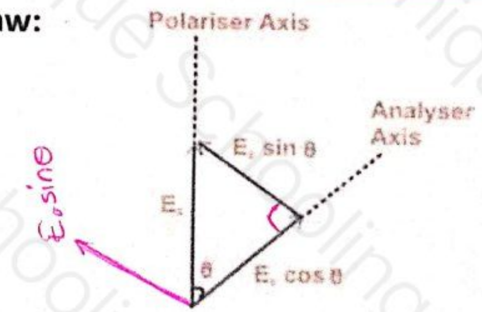
➢ *TUVW is the plane of polarization.*

The plane of polarization is always right angle to the plane of vibration.

যে তলে light polarized করা হয় সেই তলকে বলে

Polarization

Malus law:



If E_0 is the amplitude of the vibrations transmitted by the polarizer, then intensity I_0 of the light incident on the analyzer is $I_0 \propto E_0^2$

The plane of vibration is the plane which contains vibrations and which is along the optic ~~axes~~ axis of the wave.

Polarization

Malus law:

The amplitude E_0 can be resolved into two rectangular components i.e $E_0 \cos\theta$ and $E_0 \sin\theta$. The analyzer will transmit only the component (i.e $E_0 \cos\theta$) which is parallel to its transmission axis. However, the component $E_0\sin\theta$ will be absorbed by the analyser which is right angles to it.. Therefore, the intensity I of light transmitted by the analyzer is,

$I \propto (E_0 \cos\theta)^2$

$I / I_0 = (E_0 \cos\theta)^2 / E_0^2 = \cos^2\theta$ $\quad [I_0 = E_0^2]$

$I = I_0 \cos^2\theta$

Therefore, $I \propto \cos^2\theta$. This proves law of malus.

When $\theta = 0°$ (or 180°), $I = I_0 \cos^2 0° = I_0$. That is the intensity of light transmitted by the analyzer is maximum when the transmission axes of the analyzer and the polarizer are parallel.

When $\theta = 90°$, $I = I_0 \cos^2 90° = 0$. That is the intensity of light transmitted by the analyzer is minimum when the transmission axes of the analyzer and polarizer are perpendicular to each other.

How would you obtain plane polarized light by reflection?

Polarization

Polarization by reflection:

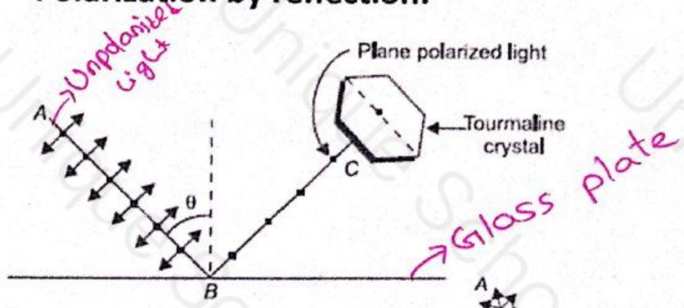
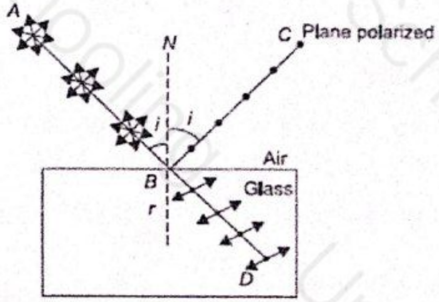


Fig.



Polarization

Polarization of light by reflection from the surface of glass was discovered by Malus in 1808.

He found that polarized light is obtained when ordinary light is reflected by a plane sheet of glass. Consider the light incident along the path *AB on the glass surface Fig.* .

Light is reflected along BC. In the path of BC, place a tourmaline crystal and rotate it slowly. It will be observed that ***light is completely extinguished only at one particular angle of incidence. This angle of incidence is equal to 57.5° for a glass surface and is known as the polarizing angle.***

Polarization

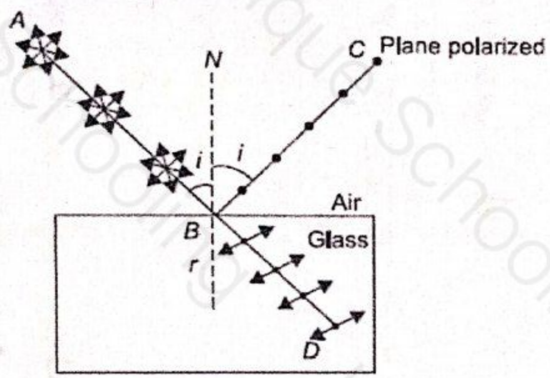
Similarly polarized light by reflection can be produced from water surface also. The production of polarized light by glass is explained as follows. The vibrations of the incident light can be resolved into components parallel to the glass surface and perpendicular to the glass surface.

Light due to the components parallel to the glass surface is reflected whereas light due to the components perpendicular to the glass surface is transmitted. Thus, the light reflected by glass is plane polarized and can be detected by a tourmaline crystal.

Polarization

Brewster's law:

The tangent of the angle of polarization is numerically equal to the refractive index of the medium.



Polarization by double refraction

Light passing through a calcite crystal is split into two rays. This process, first reported by Erasmus Bartholinus in 1669, is called double refraction. The two rays of light are each plane polarized by the calcite such that the planes of polarization are mutually perpendicular.

For normal incidence, the two planes of polarization are also perpendicular to the plane of incidence. These two beams in general have different velocities in medium.

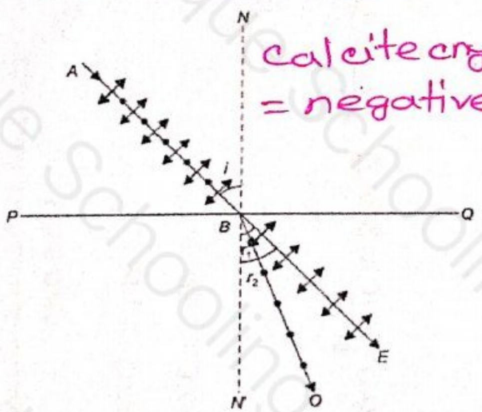


Fig. 1

calcite crystal
= negative crystal

Polarization

Brewster's law:

Suppose, unpolarized light is incident at an angle equal to the polarizing angle on the glass surface. It is reflected along BC and refracted along BD Fig.5.

From Snell's law $\quad \mu = \frac{\sin i}{\sin r}$ ……………(i)

From Brewster's law $\mu = \tan i = \frac{\sin i}{\cos i}$ ……………(ii)

Comparing (i) and (ii) $\cos i = \sin r = \cos\left(\frac{\pi}{2} - r\right)$

$\therefore \quad i = \left(\frac{\pi}{2} - r\right)$ or As $i + r = \frac{\pi}{2}$

As $i + r = \frac{\pi}{2}$, $\quad < CBD$ *is also equal to* $\frac{\pi}{2}$

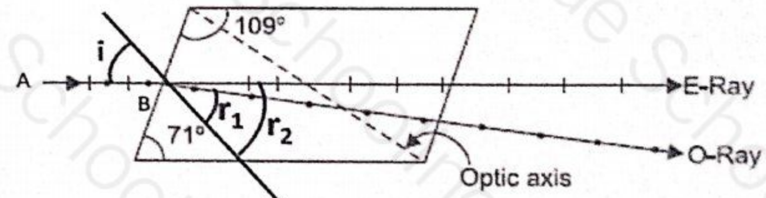
Therefore, the reflected and the refracted rays are right angles to each other.

From Brewster's law, it is clear that for crown glass of refractive index 1.52, the value of i is given by $i = tan^{-1}(1.52)$ or $i = 56.7°$

What is meant by double refraction?

Polarization by double refraction

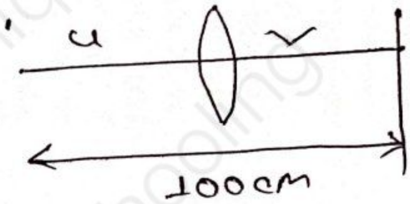
Polarization by double refraction:



(a)

When a ray of light *AB is incident on the calcite crystal making an angle of incidence = i, as shown in Fig. 1. The incident ray AB breaks up into two refracted rays BO and BE making an angle of refraction* r_1 *and* r_2*, respectively. BO is plane polarized in one plane, BE is also plane polarized but in a perpendicular plane. There are obviously two refractive indices,* $\mu_1 = \sin i / \sin r_1$, $\mu_2 = \sin i / \sin r_2$

(#) In a bi-prism fringes 0.196 cm in width are observed at a distance of 100 cm from the slit. A convex lens is the put at 30 cm from the slit. So, as to give two image of the source 0.70 cm apart. Calculate the wavelength?



$u = 30$ cm

$v = 100 - 30 = 70$ cm

$m = \frac{v}{u} = \frac{70}{30} = \frac{d_1}{d}$

$d_1 = 0.70$cm

$D = 100$cm

$\Rightarrow d = \frac{0.7 \times 3\cancel{0}}{7\cancel{0}} = 0.3$ cm

$x = 0.196$ cm $= \beta$

$\lambda = \frac{\beta d}{D} = \frac{0.196 \times 0.3}{100} = \cancel{5880 A^\circ}\ 5.88 \times 10^{-4}$ cm

Polarization by double refraction

If the angle of incidence i is varies, $sini/sin\,r$ = constant holds for one of the rays e.g., BO only. For the other ray this law does not generally hold. The ray BO which follows the laws of refraction is called ordinary (O) and for it refractive index is constant, say equal to μ_0.

The other ray BE is called extra ordinary (E) and for it refractive index μ_e varies with the angle of incidence and it is not fixed. In directions more and more inclined to the optic axis, the difference of velocities for the ordinary and the extra-ordinary becomes larger and larger.

This difference is thus maximum in a plane perpendicular to the optic axis. Crystals in which the extra-ordinary ray travels faster than the ordinary are called negative crystal, reverse is the case for positive crystal

Thank

So m

Ⓠ How will you orient the polarizer and the analyzer so that a beam of natural light is reduced to (i) 0.125 (ii) 0.25 (iii) 0.5 and (iv) 0.75 of its original intensity.

From malus law, $I = I_0 \cos^2\theta$

$$\Rightarrow \cos^2\theta = \frac{I}{I_0} \quad \therefore \theta = \cos^{-1}\sqrt{\frac{I}{I_0}}$$

i) $\theta = \cos^{-1}\sqrt{0.125} = 69.295°$

ii) $\theta = \cos^{-1}\sqrt{0.25} = 60°$

iii) $\theta = \cos^{-1}\sqrt{0.5} = 45°$

iv) $\theta = \cos^{-1}\sqrt{0.75} = 30$

Ⓠ Calculate the specific rotation of sugar if the plane of vibration is turned through 26.4° transversing 20 cm length if 20% sugar solution.

Specific rotation $= S$

sugar solution, $C = 20\%$
$= 0.2\ gm/cm^3$

length, $l = 20$ cm ; $\theta = 26.4°$

$$S = \frac{10\theta}{lC}$$

$$= \frac{10 \times 26.4}{20 \times 0.2} = 66°$$

Welcome

To

My class

Polarization

Liquid containing an optically active substance e.g., sugar solution, camphor in alcohol, tartaric acid etc. rotate the plane of the linearly polarized light.

The angle through which the plane polarized light is rotated depends upon

(1) the thickness of the medium

(2) concentration of the solution or density of the active substance in the solvent

(3) Wavelength of light and

(4) temperature

Polarization

The specific rotation is defined as the rotation produced by a decimeter (10 cm) long column of the liquid containing 1 gram of the active substance in one cc of the solution.

Therefore $S_\lambda^t = 10\,\theta/lc$

Where S_λ^t, represents the specific rotation at temperature t°C for a wavelength λ, *ϑ is the angle of rotation, l is the length of the solution in cm through which the plane polarized light passes and c is the concentration of the active substance in g/cc in the solution.*

Polarization

Laurent's half shade polarimeter :

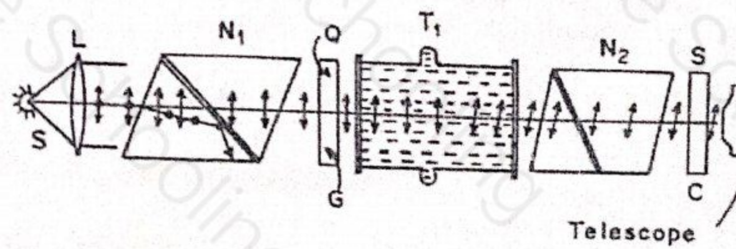


Fig.-01

Fig.-01 shows the basic components of a polarimeter. Here S- Source of sodium light, L-Collimating lens, N_1- Polarizing nicol, QG-Half shade plate, T_1- Tube containing solution, N_2-Analyzing nicol, SC- Circular scale

Polarization

Laurent's half shade Polarimeter :

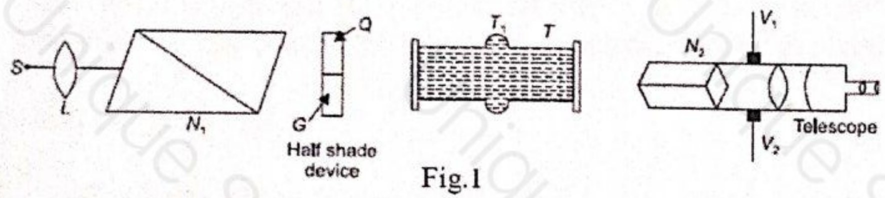


Fig.1

Suppose the plane of vibration of the plane polarized light incident on the half shade plate is along *AB. Here AB makes an angle ϑ with YY' (Fig. 2).*

The vibration of the beam emerging out of quartz will be along *CD whereas the vibrations of the beam emerging out of the glass plate will be along AB.*

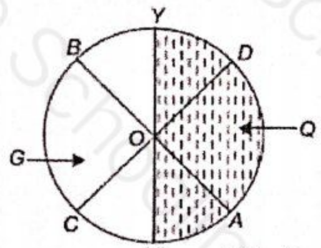
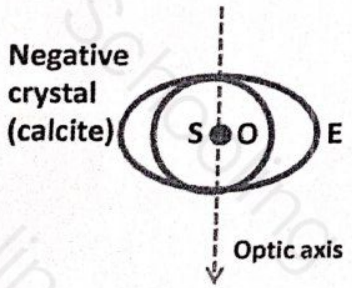
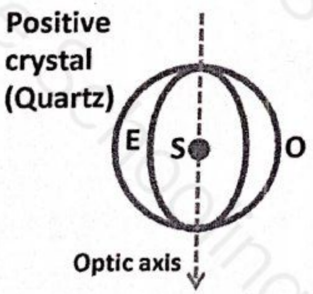


Fig.2

Huygens explanation of double refraction in uniaxial crystals:

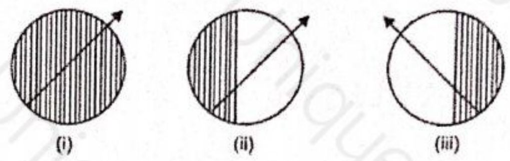




For a –ve crystal , $\mu_0 > \mu_e$

For a +ve crystal , $\mu_e > \mu_0$

Polarization



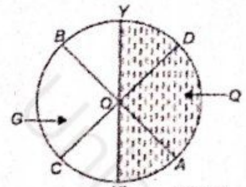


Fig.2

If the analyzer N_2 *has its principal plane or section along YY'' i.e., along the direction which bisects the angle AOC, the amplitudes of light incident on the analyzer N_2 from both the halves (i.e., quartz half and glass half) will be equal. Therefore, the field of view will be equally bright (Fig. (i)).*

If the analyzer N_2 is rotated to the right of YY' , then the right half will be brighter as compared to the left half (Fig. (ii))

on the other hand, if the analyzer N_2 is rotated to the left of YY' , the left half is brighter as compared to the right half (Fig. (iii)).

Quarter-wave plate

Quarter-wave plate: If the thickness of the crystal plate is such that a path difference of $\lambda/4$ (or a phase difference of $\pi/2$) is introduced between the two waves, then the plate is called a quarter-wave plate.

For a –ve crystal , $\mu_0 > \mu_e$

Thus the path difference= $(\mu_0 - \mu_e)t = \lambda/4$

For a +ve crystal , $\mu_e > \mu_0$

Thus the path difference= $(\mu_e - \mu_o)t = \lambda/4$

In particular, a plate whose thickness satisfy the eq.

$(\mu_o \sim \mu_e)t = (4n+1)\lambda/4$ where n = 0, 1, 2.....

Half-wave plate

Half-wave plate: If the thickness of the crystal plate is such that a path difference of $\lambda/2$ (or a phase difference of π) is introduced between the two waves, then the plate is called a Half-wave plate.

For a –ve crystal , $\mu_0 > \mu_e$

Thus the path difference= $(\mu_0 - \mu_e)t = \lambda/2$

For a +ve crystal , $\mu_e > \mu_0$

Thus the path difference= $(\mu_e - \mu_o)t = \lambda/2$

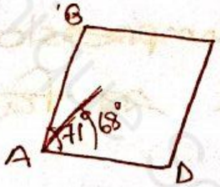
In particular, a plate whose thickness satisfy the eq.

$(\mu_o \sim \mu_e)t = (2n+1)\lambda/2$ where n = 0, 1, 2.....

Welcome

To

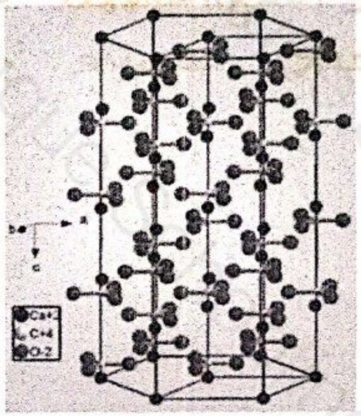
My class

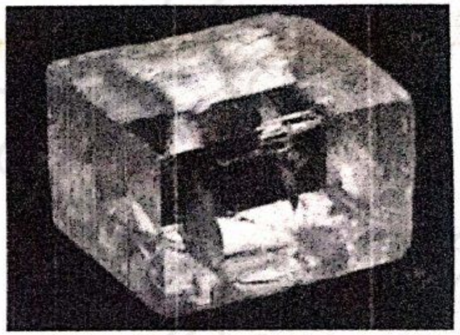


যদি কোনো light optic axis বরাবর transmit করে তখন double reflection পাওয়া যাবে না।

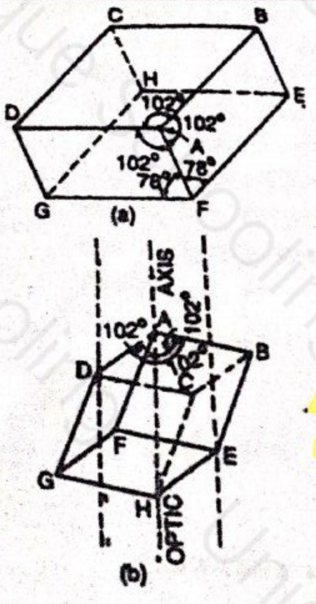
Polarization

Calcite: $CaCO_3$





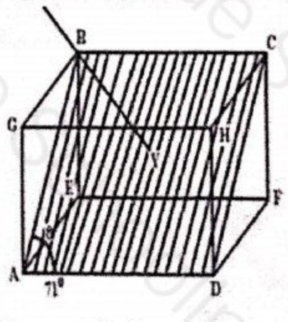
Polarization

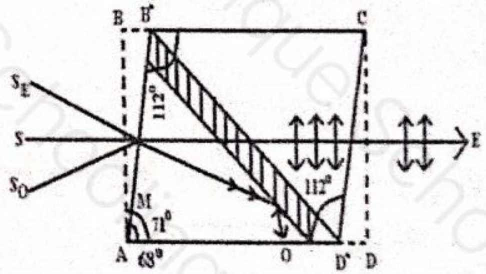


Optic axis: The optics axis of a crystal is a direction within the crystal parallel to the straight line through either of the blunt corners and equally inclined to the edges meeting there. The corners where the obtuse angles meet are known as blunt corners. If the edges are all equal, then the straight line joining two blunt corners gives the direction of the optics axis. It may be emphasized that the optics axis is a direction and not a particular line. Crystals having one optics axis are called uniaxial crystals (e.g. quartz & calcite) and those having two optics axes are called biaxial crystals (e.g mica)

Polarization by double refraction

Nicol prism (Construction):

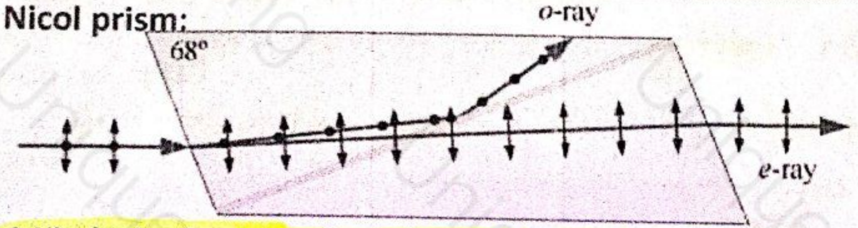




A calcite crystal's length is three times its breadth. Let *ADFEGBCH be such a crystal having ABCD as a principle section of the crystal with BAD = 71°. The end faces of the crystal are cut in such a way that they make angles of 68° and 112° in the principle section instead of 71° and 109°. The crystal is then cut into two pieces from one blunt corner to the other .The two cut surfaces are ground, polished optically flat and then cemented together with Canada balsam, a transparent glue so that the crystal is just as transparent as it was previously to its having been sliced.*

Polarization by double refraction

Nicol prism:



A Nicol prism is a type of polarizer, an optical device used to produce a polarized beam of light from an unpolarized beam. It is made in such a way that it eliminates one of the rays by Total Internal Reflection i.e, the O-ray is eliminated and only the E-ray is transmitted through the prism. It was the first type of polarizing prism to be invented, in 1828 by William Nicol (1770–1851) of Edinburgh. It consists of a rhombohedral crystal of Iceland spar (a variety of calcite) that has been cut at an angle of 68° with respect to the crystal axis, cut again diagonally, and then rejoined as shown using, as a glue, a layer of transparent Canada balsam.

The faces *A'BCD* and *EFG'H* are ground in such a way that the angle ACG becomes = 68° instead of 71°. The crystal is then cut along the plane *AKGL* as shown in Fig. 10.15. The two cut surfaces are grounded and polished optically flat and then cemented together by Canada balsam whose refractive index lies between the refractive indices for the ordinary and the extraordinary rays for calcite.

Refractive index for the ordinary

$$\mu_o = 1.658$$

Refractive index for Canada balsam

$$\mu_B = 1.55$$

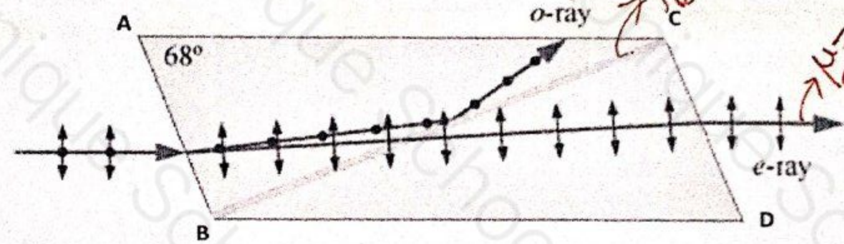
Refractive index for the extraordinary $\mu_E = 1.486$

Nicol prism (Working principle):

$\mu_o = 1.658$

$\mu_B = 1.55$

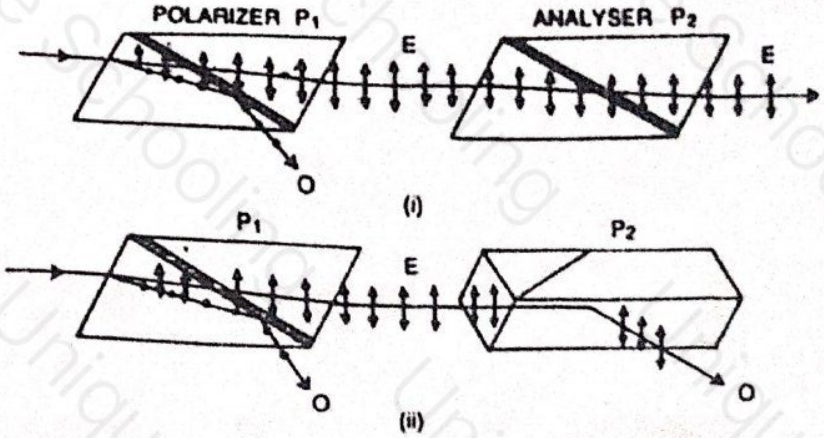
$\mu_e = 1.486$



critical angle (ক্রান্তি সংকট কোণ) = 69°

Polarization

The Nicol's prism can be used both as a polariser and an analyser.



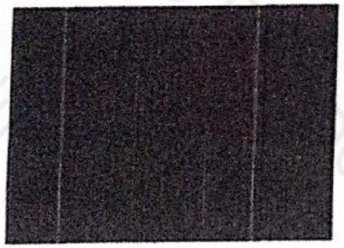
Polarization

Nicol prism can be used for the production and detection of plane-polarizer light.

When two nicol prisms P_1 and P_2 are placed adjacent to each other as shown in Fig. 10.17 (*i*), one of them acts as a polarizer and the other acts as an analyser. Fig. 10.17 (*i*) shows the position of two parallel nicols and only the extraordinary ray passes through both the prisms.

If the second prism P_2 is gradually rotated, the intensity of the extraordinary ray decreases in accordance with Malus Law and when

the two prisms are crossed [*i.e.*, when they are at right angles to each other, Fig. 10.16 (*ii*), then no light comes out of the second prism P_2. It means that light coming out of P_1 is plane polarized. When the polarized extraordinary ray enters the prism P_2 in this positions it acts as



Polarization

an ordinary ray and is totally internally reflected by the Canada balsam layer and so no light comes out of P_2. Therefore, the prism P_1 produces plane-polarized light and the prism P_2 detects it.

Hence P_1 and P_2 are called the polarizer and the analyser respectively. The combination of P_1 and P_2 is called a polariscope.