Exam Date: 25/01/2023

CHITTAGONG UNIVERSITY OF ENGINEERING AND TECHNOLOGY
B.Sc. ENGINEERING LEVEL-1 TERM-II (20 Batch) EXAMINATION '2021

DEPARTMENT	: ELECTRONICS AND TELECOMMUNICATION ENGINEERING
FULL TITLE OF PAPER	: Vector Analysis and Operational Calculus
COURSE NO.	: MATH 185
FULL MARKS	: 210
TIME	: 3 HOURS

The figures in the right margin indicate full marks. Answer any <u>*THREE*</u> *questions from each section. Use separate script for each section.*

Section-A

Q.1(a) Define vector triple product. Find the volume of the parallelepiped whose edges are represented by: **11**
$\vec{A} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$, $\vec{B} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$, $\vec{C} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$

(b) Find the velocity and acceleration of a particle which moves along the curve $x = 2\sin 3t$, $y = 2\cos 3t$, $z = 8t$ at any time $t>0$. Find the magnitude of the velocity and acceleration. **12**

(c) Find the angle between the surface $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2,-1,2). **12**

Q.2(a) Define Directional derivative. Find the directional derivative of $P = 4e^{2x-y-z}$ at the point (1,1,-1) in the direction toward the point (-3,5,6). **12**

(b) Show that, $\vec{A} = (6xy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\hat{k}$ is irrotational. Find φ such that $\vec{A} = \nabla\varphi$. **13**

(c) Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\hat{\imath} - 5z\hat{\jmath} + 10x\hat{k}$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$. **10**

Q.3(a) Evaluate $\iint_S \vec{A}.\hat{n}ds$, where $\vec{A} = 18z\hat{\imath} - 12\hat{\jmath} + 3y\hat{k}$ and S is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant. **18**

(b) State the Green's theorem in the plane. Verify the theorem in the plane for $\oint_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where c is the boundary of the region defined by $y = \sqrt{x}$, $y = x^2$. **17**

Q.4(a) By using divergence theorem, evaluate $\iint_S \vec{F}.\hat{n}ds$ where $\vec{F} = 2xy\hat{\imath} + yz^2\hat{\jmath} + xz\hat{k}$ and S is the surface of the region bounded by $x = 0, y = 0, y = 3, z = 0$ and $x + 2z = b$. **20**

(b) Evaluate $\iiint_v (2x + y)dv$ where v is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0, y = 0, y = 2$ and $z = 0$. **15**

Section-B

Q.5(a) Define Fourier series of a function f(x) in $[-\frac{T}{2}, \frac{T}{2}]$. Represent the following function in Fourier series: **22**

$$f(x) = \begin{cases} 0 \ ; \text{for } -\pi < x < 0 \\ \pi \ ; \text{for } 0 < x < \pi \end{cases}$$

And hence deduce that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\cdots\cdots$$

(b) Find a half range *sine* or *cosine* series for $f(x) = x;\ 0 < x < \pi$. **13**

Q.6(a) Find the Fourier sine integral for $f(x) = e^{-\beta x}$, Hence show that **17**

$$\frac{\pi}{2}e^{-\beta x} = \int_0^\infty \frac{\lambda \sin \lambda x}{\beta^2 + \lambda^2}\, d\lambda$$

(b) Find the Fourier transform of F(x) defined by, **18**

$$F(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

And hence evaluate $\int_{-\infty}^{\infty} \frac{\sin pa \cos px}{p}\, dx$.

Q.7(a) Define Laplace transformation. Find the Laplace transform of the following function: **21**

i. $2 + \sqrt{t} + \frac{1}{\sqrt{t}}$

ii. $te^{-t}\cos ht$

iii. $\int_0^\infty \frac{e^{-t} - e^{-3t}}{t}\, dt$

(b) Let f(t) be a periodic function with period T, then **14**

$$L[f(t)] = \frac{\int_0^T e^{-st} f(t)\, dt}{1 - e^{-sT}}$$

Q.8(a) State the convolution theorem. Use the theorem to evaluate **20**

i. $L^{-1}\{\frac{s}{(s^2 + a^2)^2}\}$

ii. $L^{-1}\{\frac{1}{(s^2(s+1)^2}\}$

(b) Solve the differential equation $Y''(t) + 9Y(t) = \cos 2t, , Y(0) = 1, , Y\left(\frac{\pi}{2}\right) = -1$ by using Laplace Transform. **15**

THE END

Exam Date: 14/03/2022

CHITTAGONG UNIVERSITY OF ENGINEERING AND TECHNOLOGY
B.Sc. ENGINEERING LEVEL-1TERM-II(19 Batch) EXAMINATION '2020

DEPARTMENT : ELECTRONICS AND TELECOMMUNICATION ENGINEERING
FULL TITLE OF PAPER : Vector Analysis and Operational Calculus
COURSE NO. : MATH 185
FULL MARKS : 210
TIME : 3 HOURS

The figures in the right margin indicate full marks. Answer any THREE questions from each section. Use separate script for each section.

Section-A

Q.1(a) Define vector and scalar with example. Find a unit vector parallel to the resultant of vectors $\vec{r_1} = 2\hat{\imath} + 3\hat{\jmath} + 8\hat{k}, \vec{r_2} = -5\hat{\imath} + 11\hat{\jmath} - 2\hat{k}$. 10

(b) Determine whether the vectors $\vec{a} = \hat{\imath} - \hat{\jmath},\ \vec{b} = \hat{\jmath} - \hat{k},\ \vec{c} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$ are linearly independent or linearly dependant. 13

(c) A particle moves along the curve $x = 2t^2, y = t^2 - 4t$ and $z = 3t - 5$, where t is the time. Find the components of its velocity and acceleration at time $t = 1$, in the direction $\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$. 12

Q.2(a) Find the directional derivative of $\varphi = 4xz^3 - 3x^2y^2z$ at $(2,-1,2)$ in the direction $2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$. 10

(b) Find the value of n for which the vector $r^n\underline{r}$ is solenoidal, where $\underline{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$. 11

(c) If $\vec{\nabla}.\vec{E} = 0, \vec{\nabla}.\vec{H} = 0, \vec{\nabla} \times \vec{E} = -\frac{\partial H}{\partial t}, \vec{\nabla} \times \vec{H} = \frac{\partial E}{\partial t}$, show that $\vec{E}$ and $\vec{H}$ satisfy $\vec{\nabla}^2 u = \frac{\partial^2 u}{\partial t^2}$ 14

Q.3(a) Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\hat{\imath} - 5z\hat{\jmath} + 10x\hat{k}$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$. 15

(b) Evaluate $\iint_S \vec{A}.\hat{n}dS$, where $\vec{A} = 18z\hat{\imath} - 12\hat{\jmath} + 3y\hat{k}$ and S is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant. 20

Q.4(a) Verify Green's theorem in the plane for $\oint_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where c is the boundary of the region defined by $x = 0, y = 0, x + y = 1$. 20

(b) Evaluate $\iiint_v (2x + y)dv$ where v is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0, y = 0, y = 2$ and $z = 0$. 15

Section-B

Q.5(a) Define Fourier series in the interval (a,b). Obtain a series of *Sines* and *Cosines* of multiples of x which will represent $f(x)$ in the interval $-\pi < x < \pi$, when 20

$$f(x) = \begin{cases} 0 & ; -\pi < x < 0 \\ \frac{1}{4}\pi x & ; 0 < x < \pi \end{cases}$$

And hence deduce that

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots\ldots\ldots$$

(b) Define Fourier Cosine series. Prove that $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{Cosnx}{n^2}, -\pi \le x \le \pi$ 15

Q.6(a) Define Fourier *Sine* and *Cosine* transform. Find Fourier *Sine* and *Cosine* transform of $\frac{1}{\sqrt{x}}$. 15

(b) State and prove the convolution theorem for Fourier transforms. 20

Q.7(a) Define Laplace transform of a function $f(t)$. Prove that the following shifting rules: 20

i) $L\{e^{at}f(t)\} = F(s-a)$

ii) $L\{f(t-a)\} = e^{-as}F(s)$ 15

(b) Find Laplace transform of the following

$$f(t) = \begin{cases} Cos(t - \frac{2\pi}{3}), & t > \frac{2\pi}{3} \\ 0, & t < \frac{2\pi}{3} \end{cases}$$

15

Q.8(a) Obtain Laplace transform of rectangular wave given by

$$L\{f(t)\} = \frac{\int_0^T e^{-st} f(t)dt}{1 - e^{-sT}}$$

20

(b) Solve by the method of Laplace transforms, the equations

i) $\frac{d^2x}{dt^2} + 9x = Cos2t$, if $x(0) = 1, x\left(\frac{\pi}{2}\right) = -1$

ii) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x}Sinx$ where $y(0) = 0, y'(0) = 1$

THE END

CHITTAGONGUNIVERSITY OF ENGINEERING AND TECHNOLOGY
B.Sc. ENGINEERING LEVEL-IISELF STUDYEXAMINATION '2019

DEPARTMENT	: ECTRONICS AND TELECOMMUNICATION ENGINEERING
FULL TITLE OF PAPER	: Vector Analysis and Operational Calculus
COURSE NO.	: MATH 185
FULL MARKS	: 210
TIME	: 3 HOURS

The figures in the right margin indicate full marks. Answer any THREE *questions from each section. Use separate script for each section.*

Section-A

Q.1(a) A particle moves along the curve $x = 2t^2, y = t^2 - 4t$ and $z = 3t - 5$ where t is the time. Find the components of its velocity and acceleration at time t=1, in the direction $\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$. 12

(b) Find the angle between the tangent planes to the surfaces $xlogz = y^2 - 1$ and$x^2y = 2 - z$ at the point (1,1,1). 12

(c) Find the value of the constant λ such that the vector field defined by $\underline{F} = (2x^2y^2 + z^2)\underline{i} + (3xy^3 - x^2z)\underline{j} + (\lambda xy^2z + xy)\underline{k}$ is solenoidal. 11

Q.2(a) Define divergence and curl of a vector function. Prove that the angular velocity at any point is equal to half the curl of linear velocity a any point of the body. 12

(b) Show that $E = (2xy + z^3)\hat{\imath} + x^2\hat{\jmath} + 3xz^2\hat{k}$ is a conservative forcefield. Also find the scalar potential. 12

(c) If $\underline{a}$ is a constant vector and $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$ show that $div(\underline{a} \times \underline{r}) = 0$ and $curl(\underline{a} \times \underline{r}) = 2\underline{a}$ 11

Q.3(a) State Green's theorem. Verify Green's theorem in the plane for $\oint_C \{(2xy - x^2)dx + (x + y^2)dy\}$, where C is the flossed curve of the region bounded by $y = x^2$and$y^2 = x$. 18

(b) Verify divergence theorem, given that $E = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$ and S is the surface of the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. 17

Q.4(a) Obtain a series of sines and cosines of multiples of x which will represent $f(x)$ in the interval $-\pi < x < \pi$, when $\begin{cases} 0; \ -\pi < x < 0 \\ \frac{1}{4}\pi x; \ 0 < x < \pi \end{cases}$ 20

(b) Define even and odd function. Prove that $\pi - x = 2\left[sinx + \frac{sin2x}{2} + \frac{sin8x}{3} + \cdots\right]$ in the interval $0 < x < 2\pi$. 15

Section-B

Q.5(a) Define Fourier sine and cosine transform. Find Fourier sine and cosine transform of 20
(a) x^{n-1} (b) $1/\sqrt{x}$

(b) State the Parseval's identity for Fourier transforms. If $f(x) = \begin{cases} 1, |x| > a \\ 0, |x| > a \end{cases}$ 15

and $F(u) = \frac{2sinau}{u}, u \neq 0$, then using the Parseval's identity for Fourier transform. Prove that $\oint_0^\infty \frac{sin^2 ax}{x^2} dx = \frac{\pi a}{2}$

Q.6(a) Define Laplace transform. Find the Laplace transform of a function f defined by 15
$f(t) = \begin{cases} sint \text{ for } 0 < t < \pi \\ 0 \text{ for } t > \pi \end{cases}$

(b) Evaluate the following (any two): 12
(i) $L\{t^3 e^{-2t}\}$
(ii) $L\{t^2 e^{-2t} cost\}$
(iii) $L\left\{\frac{1}{t}(cosat - cosbt)\right\}$

(c) Evaluate $\int_0^t te^{-2t} sin^3 tdt$ 08

Q.7(a) Evaluate any two of the following: 17
(i) $L^{-1}\{\frac{3s+2}{(s-1)^5}\}$
(ii) $L^{-1}\{\frac{2s^2-6s+5}{s^3-6s^2+11s-6}\}$
(iii) $L^{-1}\{\frac{1}{s^2(s^2+4)}\}$

(b) State the convolution theorem. Obtain the inverse Laplace transform by convolution: 18
(i) $\frac{s^2}{(s^2+a^2)^2}$
(ii) $\frac{1}{(s+1)(s^2+1)}$
(iii)

Q.8(a) Solve any two of the following: 35
i) $[tD^2 + (1-2t)D - 2]y = 0; y(0) = 1, y'(0) = 2$
ii) $(D^2 + 2D + 1)y = 3te^{-t}$, $t > 0$ subject to the conditions $y = 4, Dy = 2$ when $t = 0$
iii) $\frac{d^2y}{dt^2} - 6\frac{dy}{dx} + ay = t^2 e^{3t}$, $y(0) = 2, y'(0) = 6$

THE END

2

CHITTAGONG UNIVERSITY OF ENGINEERING AND TECHNOLOGY
B. Sc. ENGINEERING LEVEL-I TERM-II (17 BATCH) EXAMINATION '2018

DEPARTMENT	: ELECTRONICS AND TELECOMMUNICATION ENGINEERING
FULL TITLE OF PAPER	: Vector Analysis and Operational Calculus
COURSE NO.	: MATH 185
FULL MARKS	: 210
TIME	: 3 HOURS

The figures in the right margin indicate full marks. Answer any *__THREE__* *questions from each section. Use separate script for each section.*

Section-A

Q.1(a) Define linearly dependent and independent vectors. If $\vec{r}_1=(5,0,3)$, $\vec{r}_2=(2,-7,8)$ and $\vec{r}_3=(-11,12,1)$. Determine whether the vectors are linear dependent or independent. 15

(b) Define directional derivative. Find the directional derivative of $\Phi=x^2yz+4xz^2$ at $(1,-2,-1)$ in the direction $2\bar{i}-\bar{j}-2\bar{k}$. 10

(c) Define solenoidal vector. Determine the constant 'a' so that the vector $\bar{V}=(x+3y)\bar{i}+(y-2z)\bar{j}+(x+az)\bar{k}$ is solenoidal. 10

Q.2(a) Define divergence and curl of a vector field. Interpret curl physically. 15

(b) Show that $\bar{A}=(6xy+z^3)\hat{i}+(3x^2-z)\hat{j}+(3xz^2-y)\hat{k}$ is irrotational. Find Φ such that $\bar{A}=\bar{\nabla}\Phi$. 20

Q.3(a) If $\bar{F}=(2xz^3+6y)\hat{i}+(6x-2yz)\hat{j}+(3x^2z^2-y^2)\hat{k}$, find the work done by it along the path C from (1, -1, 1) to (2, 1, -1). 10

(b) Evaluate $\iint_S \bar{A}.\hat{\eta}ds$, where $\bar{A}=18z\hat{i}-12\hat{j}+3y\hat{k}$ and S is that part of the plane $2x+3y+6z=12$, which is located in the first octant. 15

(c) Let $\bar{F}=2xz\hat{i}-x\hat{j}+y^2\hat{k}$, evaluate $\iiint_V \bar{F}dv$, where V is the region bounded by the surfaces $x=0, y=0, y=6, z=x^2, z=4$. 10

Q.4(a) State Green's theorem. Verify Green's theorem in the plane for $\oint_C (xy+y^2)dx+x^2dy$ where C is the closed curve of the region bounded by $y=x$ and $y=x^2$. 20

(b) State Gauss divergence theorem. Using Gauss divergence theorem, evaluate the surface integral $\iint_S \vec{F}.\hat{\eta}ds$ where $\vec{F}=4x\bar{i}+y^2\bar{j}+z^2\bar{k}$ and S is $x^2+y^2=4; z=0$, z=2. 15

Section-B

Q.5(a) Define Laplace transform of a function $f(\mathrm{t})$. Find the Laplace transform of the wave from $f(\mathrm{t}) = \left(\frac{2t}{3}\right)$; $0 \le t \le 3$ 11

(b) Find out the following (any two): 16

$(i)\ L\left\{t^2 \cos at\right\}$

$(ii)\ L\left\{te^{-t} \sin 3t\right\}$

$(iii)\ L\left\{\frac{1-e^t}{t}\right\}$

(c) Evaluate $\int_0^\infty \frac{e^{-t} \sin t}{t} dt$ 08

Q.6 (a) State the Convolution theorem. Apply the theorem, find: 20

i) $L^{-1}\left\{\frac{s^2}{(s^2+4)^2}\right\}$ ii) $L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}$

(b) Applying Laplace transform technique, solve: $t.y'' + (1-2\mathrm{t})\mathrm{y}' - 2\mathrm{y} = 0;\ y(0) = 1,$ $y'(0) = 2.$ 15

Q.7(a) Define Fourier series in the interval (a, b). Find the Fourier series for 20

$$f(\mathrm{x}) = \begin{cases} -\pi; & -\pi < x < 0 \\ x; & 0 < x < \pi \end{cases}.$$

Hence deduce $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + ...$

(b) Define even function and odd function. Obtain a Fourier series for a function defined as 15

$$f(\mathrm{x}) = \begin{cases} \cos x; & 0 \le x \le \pi \\ -\cos x; & -\pi \le x \le 0 \end{cases}$$

Q.8 (a) Express the function $f(\mathrm{x}) = \begin{cases} 1 & if\,|x| \le 1 \\ 0 & if\,|x| > 1 \end{cases}$ as a Fourier integral, hence evaluate 15

$\int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda \cdot$

(b) Define finite Fourier sine and cosine transform. Using finite Fourier transforms to solve 20

$\frac{\delta u}{\delta t} = \frac{\delta^2 u}{\delta x^2}$; where $0 < x < 4,\ t > 0$ subject to the conditions $U(0, \mathrm{t}) = 0$; $U(4, \mathrm{t}) = 0$; $U(\mathrm{x}, 0) = 2\mathrm{x}$

THE END

CHITTAGONG UNIVERSITY OF ENGINEERING AND TECHNOLOGY
B.Sc. ENGINEERING LEVEL-II TERM-I EXAMINATION '2017

DEPARTMENT	: ECTRONICS AND TELECOMMUNICATION ENGINEERING
FULL TITLE OF PAPER	: Vector Analysis and Operational Calculus
COURSE NO.	: MATH 281 185
FULL MARKS	: 210
TIME	: 3 HOURS

The figures in the right margin indicate full marks. Answer any <u>*THREE*</u> *questions from each section. Use separate script for each section.*

Section-A

Q.1(a) Define Linear dependence of vectors. Determine the values of λ such that the vectors (1, 2, 1), (-1, 0, 1), (2, λ, 4) are linearly dependent. 12

(b) A particle moves along the curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$ where t is the time. Find the components of its velocity and acceleration at time t=1 in the direction $\underline{i}$-3$\underline{j}$+2$\underline{k}$. 13

$v = 2t^2 \hat{i} + (t^2-4t)\hat{j} + (3t-5)\hat{k}$

(c) If the direction of $\underline{a}$ vector function $\underline{a}$(t) is constant, show that $\underline{a} \times (\frac{d\underline{a}}{dt}) = 0$ 10

Q.2(a) Define directional derivative .Find the directional derivative of $\varphi = 4xz^3 - 3x^2y^2z$ at (2,-1,2) in the direction $2\underline{i}-3\underline{j}+\underline{k}$ 12

(b) Show that $\underline{\nabla}\phi$ is a vector at right angles to the surface whose equation is $\varphi(x,y,z)=c$, where c is constant. 11

(c) Show that, $\underline{A}=(6xy+z^3)\underline{i}+(3x^2-z)\underline{j}+(3xz^2-y)\underline{k}$ is irrotational. Find φ such that $\underline{A}=\underline{\nabla}\varphi$. 12

Q.3(a) Define line integral. Evaluate $\int_c \underline{F}.d\underline{r}$ where $\underline{F}=(x-3y)\underline{i}+(y-2x)\underline{j}$ and C is the closed curve in the xy plane, x=2cost, y= 3sint from t=0 to t=2π. 12

(b) If $\underline{F}=2z\underline{i}-x\underline{j}+y\underline{k}$, evaluate $\iiint_v \underline{F}dv$ where v is the region by the surfaces $x=0, y=0, x=2, y=4, z=x^2, z=2$ 11

(c) Prove that, a spherical coordinate system is orthogonal. 12

Q.4(a) State Green's theorem. Verify the theorem in the plane for $\oint_c (3x^2-8y^2)dx+(4y-6xy)dy$ where c is the boundary of the region defined by $y=\sqrt{x}, y=x^2$ 17

(b) Verify Stoke's theorem for $\underline{A}=(zx-y)\underline{i}- yz^2\underline{j}-y^2z\underline{k}$ where s is the upper half surface of the sphere $x^2+y^2+z^2=1$ and c is its boundary. 18

$\frac{1}{2}$

Section-B

Q.5(a) Define Fourier's series in the interval (a,b).Find the fourier series for *f(x)* if 20

$$f(x)=\begin{cases}0; & -\pi<x<0\\ 1; & 0<x<\pi\end{cases}$$

Verify your answer assuming complex form of fourier series.

(b) Obtain the half range cosine series for $f(x) = x^2$ in $0 < x < \pi$. 15

Q.6 (a) Define Fourier sine and cosine transform. Find the Fourier transform of 18

$$f(x)=\begin{cases}1 & for\ |x|<1\\ 0 & for\ |x|>1\end{cases}$$

Hence evaluate $\int_0^\infty \frac{\sin x}{x}dx$

(b) Use finite Fourier transform to solve $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$, u(0,t),u(4,t)=0,u(x,0)=2x where 0<x<4,t>0 and give a physical interpretation . 17

Q.7(a) Define Laplace transform of a function. Find the laplace transforms of the following (any two): 15

$(i)\left(\sqrt{t}-\frac{1}{\sqrt{t}}\right)^3$

$(ii) t^3e^{-3t}$

$(iii)\frac{1-e^t}{t}$

(b) Evaluate $\int_0^\infty te^{-2t}\sin t\,dt$ 8

(c) Find the inverse laplace transform of the following (any two): 12

(i) $\frac{s-2}{6s^2+20}$

(ii) $\frac{3s+7}{s^2-2s-3}$

(iii) $\tan^{-1}\left(\frac{1}{s}\right)$

Q.8(a) Prove that $L\{y''(t)\}=s^2f(s)-sF(0)-F'(0)$ 15

(b) Applying Laplace transform technique solve $\frac{d^2x}{dt^2}-2\frac{dx}{dt}+x=e^t$ With x=2, $\frac{dx}{dt}=-1$ at t=0 20

THE END

CHITTAGONG UNIVERSITY OF ENGINEERING AND TECHNOLOGY
B.Sc. ENGINEERING LEVEL-II TERM-I EXAMINATION '2016

DEPARTMENT	: ECTRONICS AND TELECOMMUNICATION ENGINEERING
FULL TITLE OF PAPER	: Vector Analysis and Operational Calculus
COURSE NO.	: MATH ~~281~~ 185
FULL MARKS	: 210
TIME	: 3 HOURS

The figures in the right margin indicate full marks. Answer any ***THREE*** *questions from each section. Use separate script for each section.*

Section-A

Q.1(a) Define Directional derivative. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1,-2,-1) in the direction $2\bar{i} - \bar{j} - 2\bar{k}$. 10

(b) Define Solenoidal vector. Determine the constant a so that the vector $\bar{V} = (x+3y)\bar{i} + (y-2z)\bar{j} + (x+az)\bar{k}$ is Solenoidal. 10

(c) Show that, the set of vectors $\bar{r}_1 = \bar{j} - 2\bar{k}, \bar{r}_2 = \bar{i} - \bar{j} + \bar{k}, \bar{r}_3 = \bar{i} + 2\bar{j} + \bar{k}$ is linearly independent. 15

Q.2(a) Define divergence and curl of a vector field. Interpret curl physically. 18

(b) Find constants a, b,c so that $\bar{V} = (x+2y+az)\bar{i} + (bx-3y-z)\bar{j} + (4x+cy+2z)\bar{k}$ is irrotational. Show that $\bar{V}$ can be expressed as the gradient of a scalar function. 17

Q.3(a) Find the work done in moving a particle in the force field $\bar{F} = 3x^2\bar{i} + (2xz - y)\bar{j} + z\bar{k}$ along the (i) space curve $x = 2t^2, y = t, z = 4t^2 - t$ *from* $t = 0$ *to* $t = 1$. 15

(ii) curve defined by $x^2 = 4y,\ 3x^3 = 8z$ *from* $x = 0$ *to* $x = 2$.

(b) Define line and surface integral if $\bar{A} = (3x^2 + 6y)\bar{i} - 14yz\bar{j} + 20xz^2\bar{k}$, evaluate $\int_C \bar{A}.d\bar{r}$ where C is given by $x = t, y = t^2, z = t^3$ *and t varies from* 0 *to* 1. 20

Q.4(a) State the Green's theorem in the plane. Verify the theorem for $\oint_C [(3x - 8y^2)dx + (4y - 6xy)dy]$ where c is the boundary of the region bounded by *x=0, y=0* and *x+y=1*. 18

(b) State the Gauss Divergence theorem. Using Divergence theorem, evaluate the surface integral $\iint_s \bar{F}.\hat{\eta}\, ds$ *where* $\bar{F} = 4x\bar{i} + y^2\bar{j} + z^2\bar{k}$ *and s is* $x^2 + y^2 = 4; z = 0, z = 2$ 17

Section-B

Q.5(a) Define Fourier's series of a function *f(x)* over $[0, 2\pi]$. Find the Fourier series for $f(x) = x + x^2$ *for* $-\pi \le x \le \pi$. Hence deduce that, 20

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \text{------------}$$

(b) If 15

$$f(x) = \sin x \quad for\ 0 \le x \le \frac{\pi}{4}$$
$$= \cos x \quad for\ \frac{\pi}{4} \le x \le \frac{\pi}{2}$$

Expand *f(x)* in a series of sines.

Q.6 (a) Express the function 15

$$f(x)=\begin{Bmatrix} 1\ for & |x|\le 1 \\ 0\ for & |x|\ge 1 \end{Bmatrix}$$

as a Fourier integral. Hence evaluate $\int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda}d\lambda$

(b) Using the fourier sine transform of e^{-ax} $(a>0)$, *show that* $\int_0^\infty \frac{x\sin kx}{a^2+x^2}dx=\frac{\pi}{2}e^{-ak}\,(k>0)$ 20

Hence obtain the Fourier sine transform of $\frac{x}{(x^2+a^2)}$.

Q.7 (a) Define Laplace transform of a function *f(t)*. Find out the following 20

(i) $L\{t^2e^t\sin 4t\}$

(ii) $L\{e^{-2t}U_\pi(t)\}$, where

$$U_\pi(t)=\begin{cases} 0\,;\ t<\pi \\ 1;\ t>\pi \end{cases}$$

(b) Using Laplace Transform evaluate the followings: 15

$i)\int_0^\infty te^{-3t}\sin t\,dt$

$ii)\int_0^\infty \left(\frac{e^{-2t}-e^{-4t}}{t}\right)dt$

Q.8(a) State the Convolution theorem. Using the theorem evaluate $L^{-1}\left\{\frac{s^2}{(s^2+4)(s^2+9)}\right\}$ 15

(b) Using Laplace transform technique, solve 20

(i) $Y''+9Y=18t,\ Y(0)=0,\ Y(\frac{\pi}{2})=0.$

(ii) $tY''+2Y'+tY=0,\ Y(0)=-1.$

THE END

CHITTAGONG UNIVERSITY OF ENGINEERING AND TECHNOLOGY
B.Sc. ENGINEERING LEVEL-II TERM-I EXAMINATION '2015

DEPARTMENT : ECTRONICS AND TELECOMMUNICATION ENGINEERING
FULL TITLE OF PAPER : Vector Analysis and Operational Calculus
COURSE NO. : MATH~~281~~ 185
FULL MARKS : 210
TIME : 3 HOURS

The figures in the right margin indicate full marks. Answer any THREE questions from each section. Use separate script for each section.

Section-A

Q.1(a) Define divergence and curl of a vector field. Interpret divergence physically. 20

(b) Find constants a, b, c so that $\overline{V} = (x+2y+az)\hat{i} + (bx-3y-z)\hat{j} + (4x+cy+2z)\hat{k}$ is irrotational. Show that $\overline{V}$ can be expressed as the gradient of a scalar function. 15

Q.2(a) State the Green's theorem in the plane. Verify Green's theorem in the plane for 20

$$\oint_C (x^2 - 2xy)dx + (x^2y + 3)dy$$

Where C is the boundary of the region defined by $y^2=8x$ and $x=2$.

(b) Evaluate $\iint_S \overline{A}.\overline{n}ds$ where $\overline{A} = z\hat{i} + x\hat{j} - 3y^2z\hat{k}$ and S is the surface of the cylinder $x^2+y^2=16$ included in the first octant between z = 0 and z = 5. 15

Q.3(a) State the divergence theorem. Applying the theorem evaluate $\iint_S \overline{A}.\overline{n}ds$ where $\overline{A} = 2x^2y\overline{i} - y^2\overline{j} + 4xz^2\overline{k}$ and S is the region bounded by $y^2+z^2=9$, x=2 (By volume integral) 17

(b) Let S be a closed surface and let $\overline{r}$ denote the position vector of any point (x, y, z) measured from the origin 0. Prove that 18

$$\iint_S \frac{\overline{n}.\overline{r}}{r^3} ds = 4\pi, \quad \textit{0 inside S}$$

$$= 0, \quad \textit{0 outside S}$$

Q.4(a) If $\overline{f}(u,v,w)$ be a vector point function of orthogonal curvilinear co-ordinates u, v, w ; prove that 15

$$\text{Divergence } \overline{f} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u}(h_2h_3f_1) + \frac{\partial}{\partial v}(h_3h_1f_2) + \frac{\partial}{\partial \omega}(h_1h_2f_3)\right]$$

(b) Define linear dependent and independent vectors. Prove that the vectors $\overline{A} = \overline{i} - 3\overline{j} + 2\overline{k}$; $\overline{B} = 2\overline{i} - 4\overline{j} - \overline{k}$ and $\overline{C} = 3\overline{i} + 2\overline{j} - \overline{k}$ are linearly independent. 15

(c) Find the unit vector normal to the surface $x^3+y^3+3xyz=3$ at the point (1, 2, -1) 05

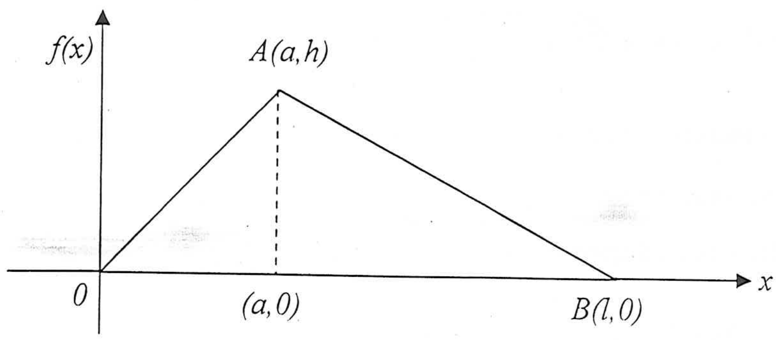
Section-B

Q.5(a) Define Fourier's series for a function *f(x)* with period T=2π. Given that $f(x)=x+x^2$ for $-\pi<x<\pi$, find the Fourier expression of *f(x)*. 20

Deduce that

$$\frac{\pi^2}{6}=1+\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+------------$$

(b) For the function defined by the graph OAB in the following figure, find the half-range Fourier sine series. 15



Q.6 (a) Define Fourier integral, Fourier sine integral and Fourier cosine integral. Using Fourier integral, show that 20

$$(i)\int_0^\infty \frac{\cos\omega x}{1+\omega^2}d\omega=\frac{\pi}{2}e^{-x}, x \ngtr 0$$

$$(ii)\int_0^\infty \frac{\sin\omega\cos x\omega}{\omega}d\omega=\frac{\pi}{2},(0\le x\le 1)$$

(b) Using Fourier transform technique, 15

Solve

$$\frac{\partial V}{\partial t}=K\frac{\partial^2 V}{\partial x^2},\quad V(x,0)=0,\ V_x(0,t)=-\mu \text{ and } V(x,t) \text{ is bounded.}$$

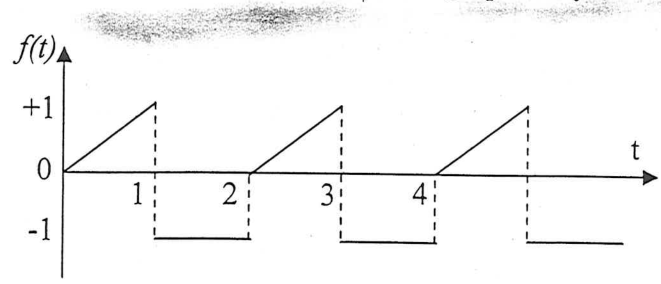
Q.7 (a) Define Laplace transform of a function *f(t)*. Find out the following 15

$$(i)\ L\left\{\frac{1-\cos t}{t^2}\right\}$$

$(ii)\ L\{e^{-2t}U_\pi(t)\}$, where

$$U_\pi(t)=\begin{cases}0\ ;\ t<\pi\\ 1;\ t>\pi\end{cases}$$

(b) Find the Laplace transform of the function shown graphically below: 12



(c) Evaluate: $\int_0^\infty te^{-3t}\sin t\,dt$ 08

Q.8(a) State and prove the convention theorem of Laplace transform. 15

(b) Using Laplace transform technique, solve 20

$$(i)\ Y''(t)+Y(t)=8\cos t,\ Y(0)=1,\ Y'(0)=-1.$$
$$(ii)\ tY''(t)+(t-1)Y'(t)-Y(t)=0,\ Y(0)=5,\ Y(\infty)=0.$$

THE END

CHITTAGONG UNIVERSITY OF ENGINEERING AND TECHNOLOGY
B.Sc. ENGINEERING LEVEL-II TERM-I EXAMINATION '2014

DEPARTMENT	: ECTRONICS AND TELECOMMUNICATION ENGINEERING
FULL TITLE OF PAPER	: Vector Analysis and Operational Calculus
COURSE NO.	: MATH ~~281~~ 185
FULL MARKS	: 210
TIME	: 3 HOURS

The figures in the right margin indicate full marks. Answer any <u>*THREE*</u> *questions from each section. Use separate script for each section.*

Section-A

Q.1(a) Define unit vector. Determine the angles α, β and γ that the vector $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$ makes with the positive directions of the co-ordinate axes and show that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ 15

(b) Find the unit vector parallel to the xy-plane and perpendicular to the vector $\underline{r} = 4\underline{i} - 3\underline{j} + \underline{k}$. 10

(c) Suppose a particle moves along the curve $x = 2\sin 3t$, $y = 2\cos 3t$, $z = 8t$ at any time $t > 0$. Find the velocity and acceleration of the particle. 10

Q.2(a) Define divergence and curl of a vector. Show that, 15

$$\underline{A} = \left(6xy + z^3\right)\underline{i} + \left(3x^2 - z\right)\underline{j} + \left(3xz^2 - y\right)\underline{k}$$ is irrotational. Find φ such that $\underline{A} = \underline{\nabla}\varphi$.

(b) Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$ where n is constant. 10

(c) Define irrotational and solenoidal for a vector $\underline{B}$. Suppose $\underline{A}$ and $\underline{B}$ are irrotational. Prove that $\left(\underline{A} \times \underline{B}\right)$ is solenoidal. 10

Q.3(a) If $\underline{F} = \left(5xy - x^2\right)\underline{i} + \left(2y - 4x\right)\underline{j}$ find $\int_c \underline{F}.d\underline{r}$ along the curve C given by $y = x^3$ in xy plane from the point (1, 1) to (2, 4). 15

(b) Find the angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point $(1, 1, 1)$. 10

(c) Prove that if a rigid body is in motion, the curl of its linear velocity at any point gives twice its angular velocity. 10

Q.4(a) State and prove the Green's theorem in xy plane. 17

(b) Verify Stoke's theorem for $\underline{A} = (2x - y)\underline{i} - yz^2\,\underline{j} - y^2z\underline{k}$ where s is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and c is the boundary. Let R be the projection of s on the xy plane. 18

Section-B

Q.5(a) Define Fourier's series for a function $f(x)$ over $\left[-\frac{T}{2}, \frac{T}{2}\right]$. Express the function $f(x)$ in Fourier's series where 20

$$f(x) = \begin{cases} 0, & for\ -\pi \le x \le 0 \\ \pi, & for\ 0 \le x \le \pi \end{cases}$$

And hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + - - - - - - - - - - - -$

(b) State the condition of convergence of Fourier series. Find a half range sine or cosine series for $f(x) = x$, $0 < x < \pi$. 15

Q.6 (a) Define Fourier transform, Fourier sine transformer and Fourier cosine transformer. Find the Fourier transform of the following function: 18

$$f(x) = \begin{cases} |x|; & -1 < x < 1 \\ 0, & otherwise \end{cases}$$

(b) Obtain the Fourier complex transform of f(x) defined by 17

$$f(x) = \begin{cases} 1; & |x| < a \\ 0; & |x| > a \end{cases}$$

And hence evaluate

$$\int_{-\infty}^{\infty} \frac{\sin \omega a \cos \omega x}{\omega} d\omega \quad and \quad \int_{0}^{\infty} \frac{\sin x}{x} dx$$

Q.7 (a) Define Laplace transform of a function $f(t)$. 20

$(i)\ L\{\sin(\omega t + \delta)\}$

$(ii)\ L\left\{\int_0^{\infty} \frac{\sin t}{t} dt\right\}$

$(iii)\ L\{f(t-a)u(t-a)\}$, where

$$u(t-a) = \begin{cases} 0; & t < a \\ 1; & t > a \end{cases}$$

(b) Evaluate: 15

$(i)\ L^{-1}\left\{\frac{s+4}{s^2 - s - 6}\right\}$

$(ii)\ L^{-1}\left\{\frac{2s^2 - s}{(s^2+4)^2}\right\}$

Q.8(a) State and prove the convolution theorem for Laplace transform. 17

(b) Use Laplace transform to solve 18

$x''(t) + 4x' + 4x(t) = 4e^{-2t}$; $x(0) = -1$, $x'(0) = 4$. Hence verify that your solution satisfies the differential equation and the boundary conditions.

THE END