at (0,0)Constants child node at (-1,0)Fixed child node at (1,0)Arbitrary ;

Function
A rule associating a unique output to a given input.
Limit

Limit of variable
A constant a is said to be the limit of the variable x, if

where δ is a pre-assigned positive quantity as small as we please.
Limit of a Sequence
Sequence
A *sequence* is a function whose domain is N.
Given a function $f : N \to R$, $f(n)$ is the nth term on the list. The notation for sequences reinforces this familiar Unders
Each of the following are common ways to describe a sequence.
[(i)] $\left(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots\right)$ $\left(\frac{1+n}{n}\right)_{n=1}^{\infty} = \left(\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \cdots\right)$ (a_n), where $a_n = 2^n$ for each $n \in N$ (x_n), where $x_1 = 2$ and $x_{n+1} = \frac{x_n+1}{2}$
Convergence of a Sequence
A sequence (a_n) *converges* to a real number a if

To indicate that (a_n) converges to a, we write either $\lim a_n = a$ or $(a_n) \to a$.
ϵ-neighbourhood and δ-neighbourhood
ϵ-neighbourhood:

δ-neighbourhood:

Convergence of a Sequence: Topological Version
A sequence (a_n) converges to a if, given any ϵ-neighbourhood $U_\epsilon(a)$ of a, there exists a point in the sequence after

The natural number N in the original version of the definition is the point where the sequence (a_n) enters $U_\epsilon(a)$, n
Prove that

Let's consider an arbitrary $\epsilon > 0$. Choose $N \in N$ with $N > \frac{1}{\epsilon}$. To verify that the choice of N is appropriate, let n

To prove the previous example, let's work in the backward direction. To prove the convergence, we have to make su

So, we have to find out when or where $n > \frac{1}{\epsilon}$ is true. If we choose $N > \frac{1}{\epsilon}$, our statement holds, and thus we prove the c
Verify, using the definition of convergence of a sequence, that the following sequences converge to the proposed limi
[(a)] $\lim \frac{1}{6n^2+1} = 0$ $\lim \frac{3n+1}{2n+5} = \frac{3}{2}$ $\lim \frac{2}{\sqrt{n+3}} = 0$
(a) Let's consider $\varepsilon > 0$. Choose $N \in N$ with $N > \frac{1}{\sqrt{6\varepsilon}}$. For any $n \geq N$

which finally gives

(b) For $\varepsilon > 0$, let's choose $N \in N$ such that $N > \frac{13}{4\epsilon}$. Now, let there be $n \in N$ and $n \geq N$. Hence, n ¿ 13 $\overline{4\varepsilon or, \varepsilon > \frac{13}{4n}}$

(c) For $\varepsilon > 0$, let's choose $N > \frac{4}{\varepsilon^2}$. Let there be $n \geq N$. Hence, n ¿ 4 $\overline{\varepsilon^2 \varepsilon^2 > \frac{4}{n} \varepsilon > \frac{2}{\sqrt{n}}}$

[Scratch]
Scratch works for solving previous examples
(a) $\left|\frac{1}{6n^2+1} - 0\right| < \varepsilon$
$6n^2 + 1 > \frac{1}{\varepsilon}$
$n^2 > \frac{1}{6\varepsilon} - \frac{1}{6}$
$n > \frac{1}{\sqrt{6\varepsilon}}$
Bounded Sequence
A sequence (x_n) is bounded if