Laplace Transform

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1 Definition, Existence, and Basic Properties of the Laplace Transform

1.1 Definition and Existence

Definition 1.1.1: Laplace Transform

Let F be a real-valued function of the real variable t, defined for $t > 0$. Let s be a variable that we shall assume to be real, and consider the function f defined by

$$f(s) = \int_0^\infty e^{-st} F(t)\, dt \tag{1}$$

for all values of s for which this integral exists. The function f defined by the integral (**??**) is called the Laplace Transform of the function F. We shall denote the Laplace transform of F by $\mathscr{L}\{F(t)\}$.
Thus the Laplace transform of a function f is given by

$$\mathscr{L}\{F(t)\} = f(s) = \int_0^\infty e^{-st} F(t)\, dt = \lim_{R\to\infty} \int_0^R e^{-st} F(t)\, dt \tag{2}$$

Some ways to write Laplace transforms:

$$\mathscr{L}F(t) = f(s) = \int_0^\infty e^{-st} F(t)\, dt$$

$$\mathscr{L}G(t) = g(s)$$

$$\mathscr{L}u(t) = \tilde{u}(s)$$

Table 1: Functions and their Laplace Transform

$F(t)$	$\mathscr{L}\{F(t)\} = f(s)$	$F(t)$	$\mathscr{L}\{F(t)\} = f(s)$
1	$\frac{1}{s}$	n	$\frac{n}{s}$
t	$\frac{1}{s^2}$	t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$	e^{-at}	$\frac{1}{s+a}$
$\sin at$	$\frac{a}{s^2+a^2}$	$\sinh at$	$\frac{a}{s^2-a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$	$\cosh at$	$\frac{s}{s^2-a^2}$