Text Books:

Book No.	Title	Author (s)	Edition
T-1	Physics for Engineers	Dr. Giasuddin Ahmad	1st
T-2	A Text Book of Optics	N. Subrahmanyam, Brijlal	22nd
T-3	Fundamentals of Optics	Francis A. Jenkins, White	4th

Interference of light:

If two beams of light cross each other at a certain point, in the region of cross over where both the beams are acting simultaneously, according to **superposition principle** a modification in their intensity is expected. The resultant intensity will be either great or less than that which would be given by one beam alone. This modification of intensity due to superposition of two or more beams of light is known as **interference of light**.

Superposition principle:

According to Thomas Young, when a medium is disturbed simultaneously by more than one wave, the instantaneous resultant displacement of medium at every point at every instant is the algebraic sum of the displacement of the medium that would be produced at the point by the individual wave trains if each were present alone. After the superposition at the region of crossover, the wave trains emerge unimpeded as if they have not met each other at all. Each wave train retains its individual characteristics. Each wave train behaves as if others are absent.

Suppose two trains cross each other at a certain point and let y_1 be the displacement of the point produced by the first wave in the absence of second wave. If y_2 be the displacement of the same point produced by the second wave in the absence of the first wave, then the resultant displacement y of the point due to the two waves acting together is expressed by

$y = y_1 + y_2$..(1)

If the two waves cross each other in phase then Eq. (1) can be written as-

$y = y_1 + y_2$ ....(2)

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If the two waves cross each other out of phase. Then Eq. (1) can be written as-

$y = y_1 \sim y_2$ ....(3)

From equation (2) we can say that, if the two individual displacements are in the same direction, the resultant displacement will be enhanced. So, two waves reinforce each other and are said to produce constructive interference. For example, in [Fig-1.1 (a)], two waves are of the same frequency but of different amplitudes, say a and b where a > b. when they reach a certain point in phase each other, then resultant displacement or amplitude is equal to the sum of the two amplitude i.e., (a + b).

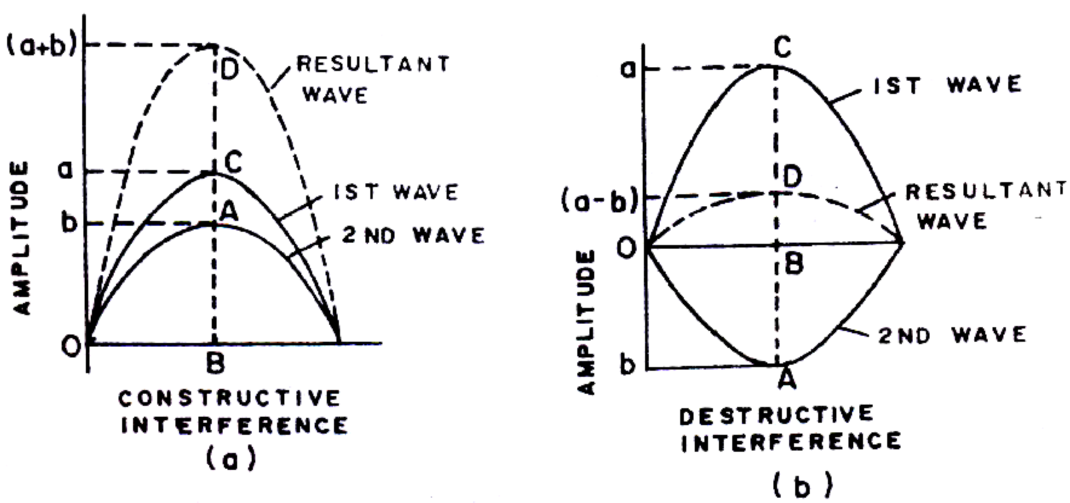

Fig.-1.1 (a) Constructive interference, (b) Destructive interference

From equation (3) we can say that, if the two individual displacements are in the opposite direction, the resultant displacement will be diminished. So, two waves neutralize each other and are said to produce **destructive interference**. For example, in Fig.-[1.1(b)], two waves are π radians or 180^0 out of phase with each other, the resultant amplitude is equal to the difference of the two amplitudes i.e., (a-b). If in addition, a = b, then the resultant amplitude is zero.

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Conditions of the interference:

a) The two beams of light which interfere must be coherent i.e., must originate from the same source of light.
b) The two interfering waves must have the some amplitude.
c) The original source must be monochromatic.
d) The fringe-width should reasonably be as large as possible and the separation between the two sources should be as small possible while the distance of the screen form the sources should be as large as possible.
e) The two interfering waves must be propagated in almost the same direction.

Uses of interference:

1. To determine the refractive index of liquids and gases.
2. To determine the width of a liquid and glass plate.
3. To determine the smoothness of a liquid and glass plate.
4. To determine the wavelength of light.
5. To measure the thickness of a very thin film.

Coherent Sources:

Two sources are said to be coherent if they emit light waves of the same frequency, nearly the same amplitude and are always in phase with each other. It means that the two sources must emit radiations of the same colour (wavelength). In actual practice it is not possible to have two independent sources which are coherent. But for experimental purposes, two virtual sources formed from a single source can act as coherent sources.

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Relation between path difference and phase difference:

If the path difference between the two waves is λ, the phase difference = 2π

Suppose for a path difference x, the phase difference is δ.

For a path difference λ, the phase difference is = 2π

According to unitary method, we can write

The path difference λ is equal to phase difference 2π

$\therefore$ 1 $\frac{2\pi}{\lambda}$

$\therefore$ x $\frac{2\pi}{\lambda}$ x

So, phase difference $\delta = \frac{2\pi}{\lambda}$ x

Or, phase difference $\delta = \frac{2\pi}{\lambda} \times \text{path difference(x)}$

Or, $\frac{\delta}{2\pi} = \frac{x}{\lambda}$.

Interference of two light waves: analytical treatment

Consider a monochromatic source of light S emitting waves of wavelength λ and two slits A and B (Fig.-1.2). A and B are equivalent from S and act as two virtual sources. Let a be the amplitude of the waves. The phase difference between the two waves reaching the point P, at any instant, is δ.

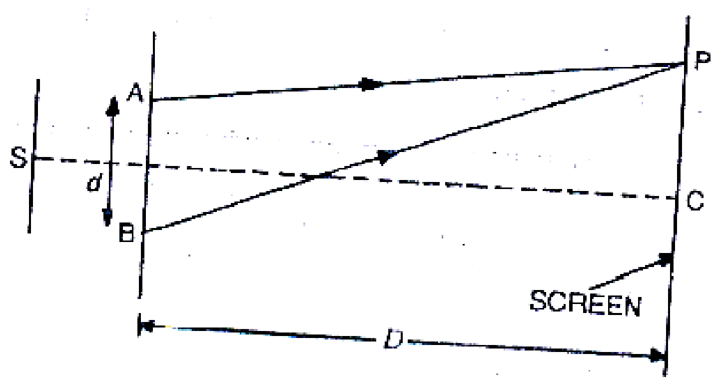

Fig.-1.2

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Let y_1 be the displacement of the particle P due to waves emanating from the source A alone (in the absence of waves coming from B) at any instant of time.

$$y_1 = a \sin \omega t \quad \text{...}(1)$$

Let y_2 be the displacement of the particle P due to waves emanating from the source B alone (in the absence of waves coming from A) at any of time.

$$y_2 = a \sin(\omega t + \delta) \quad \text{...}(2)$$

If y be the resultant displacement of the particle P at that instant of time then according to principle of superposition-

$$y_1 + y_2 = a \sin \omega t + a \sin(\omega t + \delta)$$

$$= a \sin \omega t + a \sin \omega t \cos\delta + a \cos \omega t \sin\delta$$

$$= a \sin \omega t\,(1 + \cos\delta) + a \cos \omega t \sin\delta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Taking $a\,(1 + \cos\delta) = R \cos\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$

And $a \sin\delta = R \sin\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$

$$\therefore y = R \sin \omega t \cos\theta + R \cos \omega t \sin\theta$$

$$= R \sin(\omega t + \theta)$$

which represents the equation of simple harmonic vibration of amplitude R. Squaring (4) and (5) and adding

$$R^2 \sin^2\theta + R^2 \cos^2\theta = a^2 \sin^2\delta + a^2 (1 + \cos\delta)^2$$

$$R^2 = a^2 \sin^2\delta + a^2 + a^2 \cos^2\delta + 2a^2 \cos\delta$$

$$R^2 = 2a^2 + 2a^2 \cos^2\delta = 2a^2 (1 + \cos\delta) = 2a^2.2.\cos^2 \frac{\delta}{2}$$

$$R^2 = 4a^2 \cos^2 \frac{\delta}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(6)$$

We know that, the intensity at a point is given by the square of the amplitude

$$I = R^2$$

$$I = R^2 = 4a^2 \cos^2 \frac{\delta}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Special case:

(1) When the phase difference $\delta = 0, 2\pi, 2(2\pi), 3(2\pi), \ldots\ldots\ldots n(2\pi)$ or the path difference $x = 0, \lambda, 2\lambda, \ldots\ldots\ldots n\lambda$, The intensity is

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$I = 4a^2$

Intensity is maximum when the phase difference is a whole number multiple of 2π of the path difference is a whole number multiple of wavelength.

(2) When the phase difference $\delta = \pi, 3\pi, \ldots\ldots\ldots (2n+1)\pi$ or the path difference $x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \ldots\ldots\ldots (2n+1)\frac{\lambda}{2}$, The intensity is

$I = 0$

Intensity is minimum when the phase difference is an odd number multiple of π or the path difference is an odd number multiple of half wavelength.

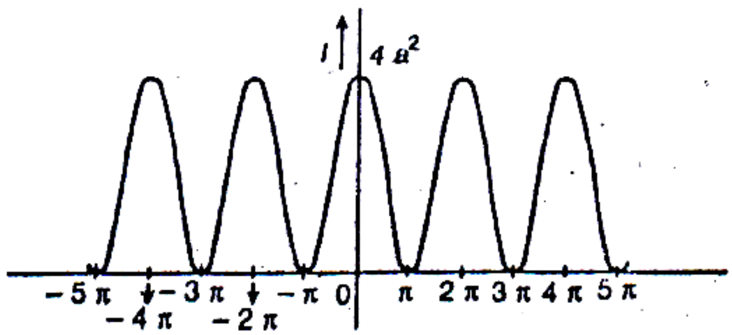

Fig.-1.3

From equation (7), it is found that the intensity at bright points is $4a^2$ and at dark points it is zero. According to the law of conservation of energy, the energy can not be destroyed. Here also the energy is not destroyed but only transferred form the points of minimum intensity to the points of maximum intensity.

Young's double-slit experiment

Historically, the phenomenon of interference of light was first demonstrated by Thomas Young in about 1801 by a simple experiment. Young allowed sunlight to pass through a pin hole S and then at some distance through two sufficiently close pin holes $S_1$and S_2 in an opaque screen. Finally the light was received on a screen on which he observed an uneven distribution of light intensity consisting of many alternate bright and dark spots. The corpuscular theory was found to be totally inadequate to explain this. On the other hand, Young was able to explain this due to superposition of two light waves. This

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experiment, described below, was regarded as crucial one at that time, since it definitely established the wave nature of light.

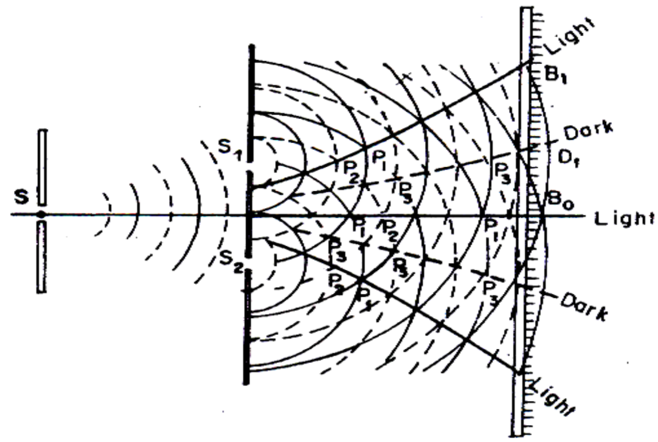

Fig.-1.3a

In accordance with the modern laboratory techniques Young's experiment is performed by illuminating a narrow parallel slit S with monochromatic light of wavelength λ. The light coming out of the slit S is then allowed to fall on two more narrow parallel equidistant slits S_1 and S_2 on a screen placed at a certain distance to the right of S (Fig.-1.3a). According to Huygen's principle, cylindrical waves spread out from slit S and reach the slits S_1 and S_2. As the slits S_1 and S_2 are equidistant from S the waves reach the slits at the same time i.e., S_1 and S_2 are on the same wavefront. A train of secondary wavelets, having the same amplitude, velocity, wavefront and precisely the same phase at the start, therefore, diverge to the right from both of these slits. Let the crest and the trough in each wave be represented by continuous and dotted circular arcs respectively. Furthermore, let the points where a crest of one wave is superposed on the crest of another wave or a trough of one wave is superposed on the trough of another wave be marked by P_1 and P_2 respectively and the points where the crest of one wave is superposed on the trough of another wave be marked by P_3. If a screen be placed at a certain distance from the slits S_1 and S_2, solid lines connecting the points marked P_1s and P_2s will intersect the screen at points B_o, B_1. Since the resultant intensity along these lines is always maximum (constructive interference), the points B_o, B_1 will appear as bright lines on the screen. Similarly D_1s, the points of intersection of lines, connecting the points marked P_3s with the screen will represent points of zero intensity (destructive interference) and consequently appear on the screen as dark lines. Thus the result of

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interference between waves coming from the slits S_1 and S_2 will appear on the screen as alternate bright and dark lines. As long as the experimental arrangement remains undisturbed, the alternate bright and dark lines on the screen remain stationary. This is known as interference pattern. That the observed pattern is truly due to interference of two waves of light can be demonstrated by covering one of the slits. Then the well defined dark and bright lines on the screen are replaced by a pattern much coarser due to diffraction of light by the uncovered single light. Thus a point on the screen, bright when only one slit is uncovered changes to dark when both the slits are uncovered. This cannot be explained on the basis of corpuscular theory of light, but can be readily explained on the basis of interference of two waves of light. The dark and bright lines are usually referred to as fringes.

Theory of interference fringes: expression for the width of a fringe.

Consider a narrow monochromatic source S and two slits A and B, equidistance form S. A and B act as two coherent sources separated by a distance d. Let a screen be placed at a distance D from the coherent sources. The point C on the screen is equidistance form A and B. Therefore, the path difference between the two waves is zero. Thus, the point C has maximum intensity.

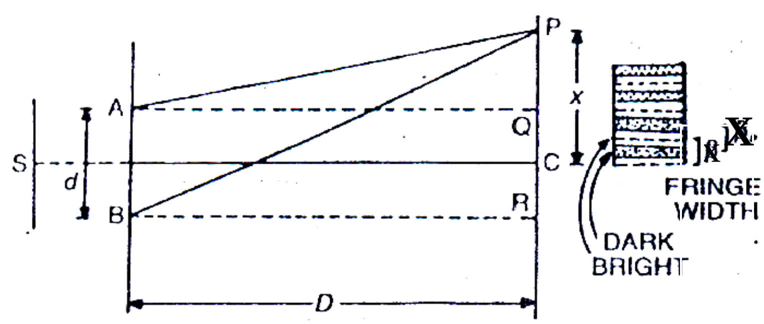

Fig.-1.4

Consider a point P at a distance x from C. The wavelength reaches at the point P from A and B.

Here, PQ = $x - \dfrac{d}{2}$, PR = $x + \dfrac{d}{2}$

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$(AP)^2 = \left[D^2 + \left(x - \frac{d}{2} \right)^2 \right]$, $\qquad (BP)^2 = \left[D^2 + \left(x + \frac{d}{2} \right)^2 \right]$

$$(BP)^2 - (AP)^2 = \left[D^2 + \left(x + \frac{d}{2} \right)^2 \right] - \left[D^2 + \left(x - \frac{d}{2} \right)^2 \right]$$

(BP-AP) (BP +AP) = 2xd

$$BP - AP = \frac{2xd}{BP + AP}$$

But BP = AP = D (approximately)

$$\text{Path difference} = \frac{xd}{D}$$

$$\text{Phase difference} = \frac{2\pi}{\lambda}\left(\frac{xd}{D}\right)$$

(1) Bright fringes:

If the path difference is a whole multiple of wavelength λ, the point P is bright.

$$\therefore \frac{xd}{D} = n\lambda$$

Where n = 0, 1, 2, 3, … … …

Or, $x_n = \frac{n\lambda D}{d}$

This equation gives the distance of the bright fringe form the point C. At C, the path difference is zero and a bright fringe is formed.

When n = 1, $x_1 = \frac{\lambda D}{d}$

n = 2, $x_2 = \frac{2\lambda D}{d}$

n = 3, $x_3 = \frac{3\lambda D}{d}$

Therefore the distance between any two consecutive bright fringes

$$x_2 - x_1 = \frac{2\lambda D}{d} - \frac{\lambda D}{d} = \frac{\lambda D}{d} \ldots\ldots\ldots\ldots\ldots(1)$$

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Thus the distance between any two consecutive bright fringes is same or all bright fringes are equally spaced.

(2) **Dark fringes:**

If the path difference is an odd number multiple of half wavelength λ, the point P is dark.

$$\therefore \frac{\text{xd}}{D} = (2n+1)\frac{\lambda}{2}$$

Where n = 0, 1, 2, 3, ...

Or, $\text{x}_\text{n} = \frac{(2n+1)\lambda D}{2d}$

This equation gives the distance of the dark fringe form the point C.

When n = 1, $x_1 = \frac{3\lambda D}{2d}$

n = 2, $x_2 = \frac{5\lambda D}{2d}$

n = 3, $x_3 = \frac{7\lambda D}{2d}$

Therefore the distance between any two consecutive bright fringes

$$x_2 - x_1 = \frac{5\lambda D}{2d} - \frac{3\lambda D}{2d} = \frac{\lambda D}{d} \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Thus the distance between any two consecutive dark fringes is same or all dark fringes are equally spaced.

Moreover, from equations (1) and (2), it is clear that the width of the bright fringe is equal to the width of the dark fringe. This distance between any two consecutive bright or dark fringes is called the **fringe-width (X).**

$$\text{X} = \frac{\lambda \text{D}}{\text{d}}$$

From this equation it can be seen that

$$\lambda = \frac{\text{Xd}}{\text{D}}$$

Thus if the fringe-width, the distance of separation of the sources and the distance of the screen from the sources are known, then the phenomenon of interference can be employed to determine the wavelength of unknown monochromatic light.

Interference in thin film:

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Everyone is familiar with the brilliant colours produced by a thin film of oil on the surface of water and a thin film of a soap bubble. The explanation of the origin of this colour phenomenon was given by Young, in 1802, in terms of the interference of light waves reflected from the upper and the lower surface of the thin film. It has been observed that interference in the case of thin film takes place due to both reflected as well as transmitted light.

Interference due to reflected light from a plane parallel (thin) film:

Consider transparent film of thickness t and refractive index μ, bounded by two parallel surfaces MN and PQ. A ray AB of monochromatic light is incident on the upper (Fig.-1.5) surface at the point B. A part of it is reflected along BC and a part is refracted along BD. The refracted beam is again partly reflected at the point D back into the medium along DE and the rest refracts into the surrounding medium along DV. The ray along DE suffers both reflection and refraction at point E on the upper surface MN. The refracted ray goes along EF. The difference in path between BC and EF can be calculated. Draw, El normal to BC and BR normal to DE. Also produce ED to meet BT produced at S.

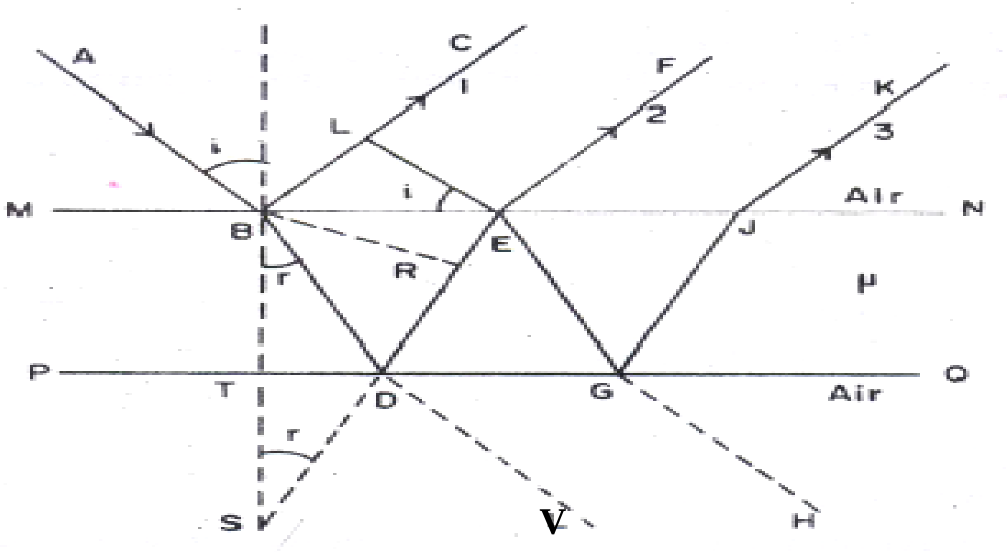

Fig.-1.5: Interference due to reflected light (thin) film

Let the angle of incidence and refraction be i and r respectively. The optical path difference

$x = \mu\,(BD + DE) - BL$

$= \mu\,(BD + DR + RE) - BL$ ....(1)

Also, BD=DE

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Here, μ = $\frac{\sin i}{\sin r} = \frac{BL/BE}{RE/BE} = \frac{BL}{RE}$

Or, BL=μRE ....(2)

$\therefore$ The path difference x = μ (RD + DS) = μRS

In the triangle BSR,

cos r = $\frac{RS}{BS}$

RS = BS cos r

= (BT +TS) cos r

=2t cos r ....(3)

Where BT = TS = t

$\therefore$ x = μ RS = 2t μ cos r ....(4)

Since an abrupt change of π (equivalent to a path difference of $\frac{\lambda}{2}$) is introduced whenever a ray is reflected from a surface backed by a denser medium, equation (4) does not represent the total path difference between BC and BDEF. Taking into account this additional path difference of $\frac{\lambda}{2}$ for reflection suffered at the point D the total path difference will be

x = 2t μcos r $\pm\frac{\lambda}{2}$

Therefore, for constructive interference or brightness

2t μcos r $\pm\frac{\lambda}{2}$ = nλ

2t μcos r = $(2n\pm1)\frac{\lambda}{2}$ ………………………………Bright

In terms of phase difference, there will be constructive interference or brightness, when the total phase difference

$$\delta = \frac{2\pi}{\lambda}(2n\pm1)\frac{\lambda}{2}$$

$= (2n\pm1)\pi$ …………………………………Bright

And for destructive interference or darkness

2t μcos r $\pm\frac{\lambda}{2}$ = $(2n\pm1)\frac{\lambda}{2}$

2t μcos r = nλ ……………………………..Dark

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In terms of phase difference, there will be destructive interference or darkness, when the total phase difference

$$\delta = \frac{2\pi}{\lambda} n\lambda = 2\pi n \text{ ....Dark}$$

Where n = 0, 1, 2, 3, … … …etc.

It should be remembered that the interference pattern will not be perfect because the amplitudes of the rays BC and EF are not same.

Interference due to reflected light from a plane of varying thickness (wedge shaped film):

Suppose the film is not parallel sided but is in the shape of thin wedge i.e., its surfaces make an angle θ with each other. Consider the film to be illuminated with monochromatic light. The incident light wave propagating along AB will give rise to two light waves- the directly reflected ray along BR and the internally reflected ray along B_1R_1. The rays BR and B_1R_1 are, therefore, coherent and capable of producing interference.

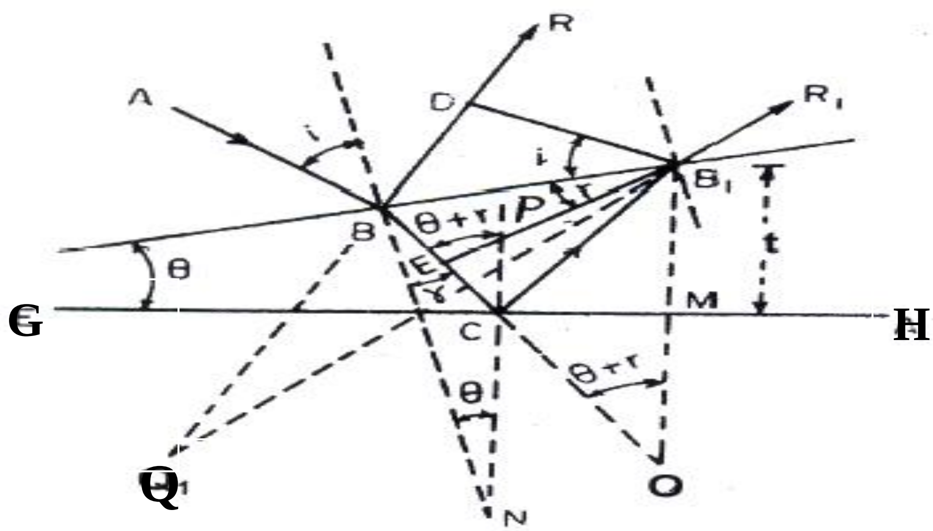

Fig.-1.6: Interference due to reflected light (thin film of varying thickness)

To derive an expression for the optical path difference between them, let us draw an arc with the optical path difference between them, let us draw an arc with the point Q as the center and QB_1 as the radius. The arc B_1D will be approximately straight and perpendicular to BR. From points B_1 and D onwards, the two waves or rays travel equal distances. Let us also draw a perpendicular B_1E from B_1 on BC. The optical path

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difference x, between the waves BR and B_1R_1 in reaching the arc B_1D from B, the point of their origination, is expressed by

$x = \mu\ (BC + CB_1) - BD$

$x = \mu\ (BE + EC + CB_1) - BD$....................(1)

Now the angle $\angle DB_1B = i$ = angle of incident and

Angle $\angle BB_1E = r$ = angle of refraction

$\sin i = \frac{BD}{BB_1}$ and $\sin r = \frac{BE}{BB_1}$

Hence $\frac{\sin i}{\sin r} = \mu = \frac{BD}{BE}$

Or, $BD = \mu\ BE$..............................(2)

Hence equation (1) becomes

$x = \mu\ (BE + EC + CB_1) - \mu BE$

$= \mu\ (EC + CB_1)$..........................(3)

Let BN and CN be the normal to the upper and lower surfaces of the film respectively. Hence we get

$\angle CNB = \theta$

$\angle PCB = \angle CBN + \angle CNB = \theta + r$ and $\angle PCB_1 = \angle PCB = \theta + r$

In the fig-1.6, B_1M is perpendicular from B_1 of the lower surface of the film and BC when produced further intersects it at O. Thus we have the relation

$\angle B_1OC = \angle PCB = \theta + r$.

Also $\angle CB_1O = \angle PCB_1 = \theta + r = \angle B_1OC$. Thus B_1OC is an isosceles triangle. Hence $CB_1 = CO$.

Equation (3), therefore reduces to

$x = \mu\ (EC+CO) = \mu\ EO = \mu B_1O \cos(\theta+r)$.............................(4)

If this thickness of the film at the point B_1 and since $B_1M = MO = t$,

$x = \mu\ 2t \cos(\theta+r) = 2t\ \mu \cos(\theta+r)$ ....(5)

The path difference x, therefore, varies both on account of changing thickness as well changing angle of incidence, provided the broad light source is at a finite distance from the film. Equation (5) does not, however, represent the total path difference between the rays. Now we have to consider a path difference of $\frac{\lambda}{2}$ introduced as a result of reflection

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at the point B which represents reflection at a surface backed by a denser medium. Thus the total path difference between the rays is

$$x = 2t\,\mu\cos(\theta+r) \pm \frac{\lambda}{2}$$

Therefore, for constructive interference or brightness

$$2t\mu\cos(\theta+r) \pm \frac{\lambda}{2} = n\lambda$$

$$2t\mu\cos(\theta+r) = (2n\pm1)\frac{\lambda}{2} \text{ ....Bright}$$

In terms of phase difference, there will be constructive interference or brightness, when the total phase difference

$$\delta = \frac{2\pi}{\lambda}(2\text{n}\pm1)\frac{\lambda}{2}$$

$$= (2\text{n}\pm1)\pi \text{ ……………………………………Bright}$$

And for destructive interference or darkness

$$2t\mu\cos(\theta+r) \pm \frac{\lambda}{2} = (2\text{n}\pm1)\frac{\lambda}{2}$$

$$2t\mu\cos(\theta+r) = \text{n}\lambda \text{ ……………………………..dark}$$

In terms of phase difference, there will be destructive interference or darkness, when the total phase difference

$$\delta = \frac{2\pi}{\lambda}\text{n}\lambda = 2\pi\text{ n} \text{ ……………………………..dark}$$

Where n = 0, 1, 2, 3, … … …etc.

Consider the wedge shaped film to be illuminated by a parallel beam of monochromatic light of wavelength λ. Then the angle of incidence i, will be constant at every point of the film and so will be r, the angle of refraction. The total optical path difference will, therefore, be only due to variation of the thickness, t, from point to point of the film. At the edge of the wedge, since t = 0, the film appears perfectly dark and the two interfering waves are π out of phase. At distances from the edge where the total path difference

$$x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \text{......... etc. the film is bright.}$$

While at distances where

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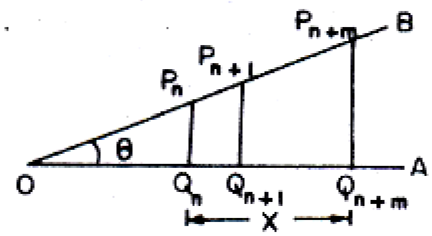
$x = \lambda, 2\lambda, 3\lambda, 5\lambda, \dots\dots\dots$ etc. the film appears to be dark.

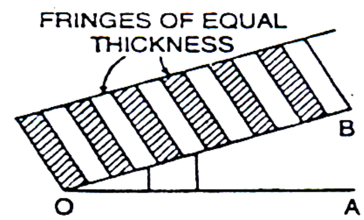
Thus as we go along the wedge in the direction of increasing thickness there will be alternate dark and bright bands parallel to the edge of the film.

To determine the fringe-width produced in a wedge shaped thin film

Consider two plane surfaces OA and OB inclined at an angle θ and enclosing a wedge shaped air film. The thickness of the air film increase from O to A (Fig-1.7). When the air film is viewed with reflected monochromatic light, a system of equidistance interference fringes is observed which are parallel to the line of intersection of the two surfaces. The effect is best observed when the angle of incidence is small.



a. Wedge shaped thin film



b. Fringes

Fig.-1.7: Fringe-width produced in a wedge shaped thin film

Suppose the rays are incident normally on the film which is in air, i.e. $\mu = 1$. Then the angle of refraction is very small and since angle θ is also very small cos (θ+r) = 1. Under this condition the n^{th} bright fringe occurs at the point P_n where the thickness of the film, $t = P_nQ_n$. (Fig-1.7a).

Applying the relation for bright fringe (for reflected light),

$$2t\mu\cos(\theta + r) = (2n \pm 1)\frac{\lambda}{2}$$

$$2P_nQ_n = (2n \pm 1)\frac{\lambda}{2} \qquad [\mu = 1, \cos(\theta+r) = 1]$$

The next bright fringe (n+1) will occur at P_{n+1} such that

$$2P_{n+1}Q_{n+1} = [2(n+1) \pm 1]\frac{\lambda}{2}$$

Subtracting

$$P_{n+1}Q_{n+1} - P_nQ_n = \frac{\lambda}{2}$$

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Thus the next bright fringe will occur at the point where the air-film thickness has increase by $\frac{\lambda}{2}$. Suppose P_{n+m} represents the position of the $(n+m)^{th}$ bright fringe. Hence, there will be m bright fringes between P_n and P_{n+m}.

$$P_{n+m}Q_{n+m} - P_nQ_n = m\frac{\lambda}{2}$$

Let the distance $Q_nQ_{n+m} = x$

Then the angle of inclination θ, between OA and OB is

$$\theta = \frac{P_{n+m}Q_{n+m} - P_nQ_n}{Q_nQ_{n+m}} = \frac{m\lambda}{2x}$$

Or, $x = \frac{m\lambda}{2\theta}$

Since x is the distance corresponding to m fringes, the fringe width

$$X = \frac{x}{m} = \frac{\lambda}{2\theta}$$

Newton's rings:

When a plano-convex or bi-convex lens of large radius of curvature is placed on a glass plate p, a thin air film of progressively increasing thickness in all directions form the point of contact between the lens and the glass plate is very easily formed (Fig.-1.9) The air film thus possesses a radial symmetry about the point of contact. When it is illuminated normally with monochromatic light, an interference pattern consisting of a series of alternate dark and bright circular rings, concentric with the point of contact is observe (Fig-1.8). The fringes are the loci of points of equal optical film thickness and gradually become narrower as their radii increase until the eye or the magnifying instrument can no longer separate them. The rings are localized in the air film. Since the phenomenon was first examined in detail by Newton, the rings are termed as Newton's rings.

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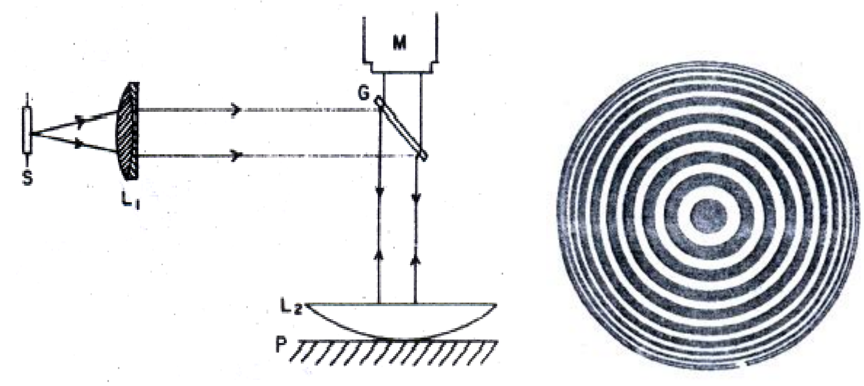

(a) **(b)**

Fig.-1.8: (a) Newton's rings experiment apparatus, (b) Newton's rings

Theory of Newton's rings:

Let us consider a ray of monochromatic light AB from an extended source to be incident at the point B on the upper surface of the film (Fig-1.9). One portion of the ray is reflected from point B on the glass-air boundary and goes upwards along BC. The other part refracts into the air film along BD. At point D, part of the light is again reflected along DEF but with an abrupt phase reversal of π (or a path difference of $\frac{\lambda}{2}$).

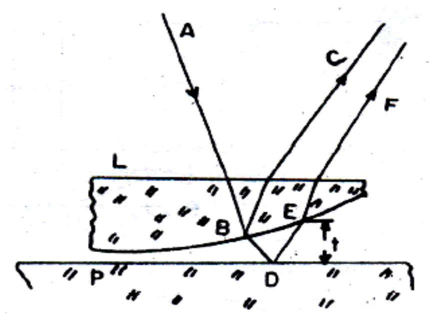

Fig.-1.9: Two virtual rays from one monochromatic ray

The two reflected rays BC and BDEF are derived from the same source and are coherent. They will produce constructive or destructive interference depending on their path difference. Let t be the thickness of the film at the point E and let the tangent to the convex surface at the point be inclined at an angle θ with the horizontal. Then the optical path difference between the two rays is given by

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$$2t\mu\cos(\theta + r) \pm \frac{\lambda}{2}$$

Where r is the angle of refraction at the point B and μ is the refractive index of the film with respect to air.

Thus the two rays will interfere constructively when

$$2t\mu\cos(\theta + r) \pm \frac{\lambda}{2} = n\lambda$$

$$2t\mu\cos(\theta + r) = (2n - 1)\frac{\lambda}{2} \text{...Bright (1)}$$

The minus sing has been chosen purposely since n con not have a value of zero for bright fringes seen in reflected light.

The rays will interfere destructively when

$$2t\mu\cos(\theta + r) \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$$

$$2t\mu\cos(\theta + r) = n\lambda \text{..Dark (2)}$$

λ is the wavelength of light in air.

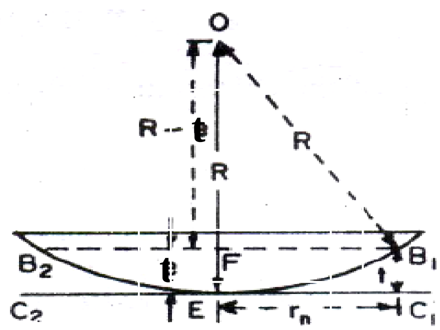

Fig.-1.10: Thickness of the air film

In practice, however, a thin lens of extremely small curvature is used in order to keep the film enclosed between the lens and the plane glass plate extremely thin. As a consequence, the angle θ becomes negligibly small as compared to r. Furthermore, the experimental arrangement is so designed (Fig-1.10) that the light is incident almost normally on the film and is viewed from nearly normal directions by reflected light, so that cos (θ+ r) = 1. Accordingly eqns. (1) and (2) reduce to

$$2t\mu = (2n - 1)\frac{\lambda}{2} \text{...............................Bright (3)}$$

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And

$2t\mu = n\lambda$ ………..……………………….Dark (4)

Let us now compute the radius of any ring. Let R be the radius of curvature of the convex surface which rests on the plane glass surface (Fig.-1.10). From the right angled triangle OFB_1, we get the relation

$$R^2 = r_n^2 + (R - t)^2$$

$$r_n^2 = 2Rt - t^2$$

Where r_n is the radius of the circular ring corresponding to the constant film thickness t. As outlined above, the condition of the experiment makes t extremely small; so to a sufficient degree of accuracy, t^2 may be neglected compare to 2Rt. Then

$$t = \frac{r_n^2}{2R}$$

Substituting the value of t in the above expressions [(3) and (4)] for bright and dark rings, we have

$$r_n^2 = (2n-1)\frac{\lambda R}{2\mu} \text{ ……………………………Bright (5)}$$

And $r_n^2 = \dfrac{n\lambda R}{\mu}$ ………………………….……Dark (6)

The square of the diameters of the bright and dark rings are, therefore, given by the expressions

$$D_n^2 = 2(2n-1)\frac{\lambda R}{\mu} \text{ ……………………………Bright (7)}$$

$$D_n^2 = \frac{4n\lambda R}{\mu} \text{ ……………………………..…Dark (8)}$$

Thus the diameters of the different bright rings may be written as

1st ring, $D_1 = \sqrt{1}\sqrt{\dfrac{2\lambda R}{\mu}}$

2nd ring, $D_2 = \sqrt{3}\sqrt{\dfrac{2\lambda R}{\mu}}$

3rd ring, $D_3 = \sqrt{5}\sqrt{\dfrac{2\lambda R}{\mu}}$ and so on.

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Hence it can be seen that the diameters (also radii) of bright rings are proportional to the square root of the odd natural numbers.

Similarly, the diameters of the different dark rings can be written as

Central ring $D_0 = 0$

1st ring, $D_1 = 2\sqrt{1}\sqrt{\frac{\lambda R}{\mu}}$

2nd ring, $D_2 = 2\sqrt{2}\sqrt{\frac{\lambda R}{\mu}}$

3rd ring, $D_3 = 2\sqrt{3}\sqrt{\frac{\lambda R}{\mu}}$ and so on.

It is obvious that the diameters (also radii) of the dark rings are proportional to the square root of the natural numbers.

At the point of contact of the lens and the glass plate, t = 0; therefore, the total phase difference between directly and internally reflected rays reduces to π. As a consequence, when Newton's rings are viewed in reflected light, the central spot appears to be dark. This central spot is surrounded alternately by a large number of bright and dark rings. This is very interesting result. How could you get bright at central spot?

If we consider the difference in diameters of the 5th and 4th dark rings, then

$$D_5 - D_4 = 2\left(\sqrt{5} - \sqrt{4}\right)\sqrt{\frac{\lambda R}{\mu}} = 0.46\sqrt{\frac{\lambda R}{\mu}}$$

And that between the 17th and 16th dark rings

$$D_{17} - D_{16} = 2\left(\sqrt{17} - \sqrt{16}\right)\sqrt{\frac{\lambda R}{\mu}} = 0.26\sqrt{\frac{\lambda R}{\mu}}$$

Thus it is clear that the alternate bright and dark rings surrounding the central dark spot in Newton's rings gradually become narrower as their radii increases.

This is also very interesting result. Do you know why it is happened?

Determination of wavelength:

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In the laboratory, the diameters of the Newton's rings can be measured with traveling microscope. Usually a little away from the centre, a bright (or dark) ring is chosen which is clearly visible and its diameter measured. Let it can be the n^{th} order ring. For an air film $\mu = 1$. Then we have

$$D_n^2 = 2(2n-1)\lambda R \text{ ……………………………Bright (9)}$$

and

$$D_n^2 = 4n\lambda R \text{ …………………………….…Dark (10)}$$

The wavelength of the monochromatic light employed to illuminate the film can be computed either of the above equations, provided R is known.

However, in actual practice, another ring, p rings from this ring onwards, is selected. The diameter of this $(n+p)^{th}$ ring is also measured. Then we have

$$D_{n+p}^2 = 2[2(n+p)-1]\lambda R$$

$$D_{n+p}^2 = 2(2n+2p-1)\lambda R \text{ ……………………………Bright (11)}$$

and

$$D_{n+p}^2 = 4(n+p)\lambda R \text{ ……………………………..…Dark (12)}$$

Subtracting either eqn. (9) from (11) or eqn. (10) from (12), we get for both the dark and bright rings the relation

$$D_{n+p}^2 - D_n^2 = 4p\lambda R$$

Or, $$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR} \text{ …………………………..(13)}$$

Thus, if the radius of curvature of the surface of the lens is known the eqn. (13) can be used to determine the wavelength of the light used.

Determination of radius of curvature of the lens:

Equation (13) can be rearranged as

$$R = \frac{D_{n+p}^2 - D_n^2}{4p\lambda} \text{ ……………………………(14)}$$

Thus, if the wavelength of the light used is known then eqn. (14) can be used to determine the radius of curvature of the surface of the lens in contact with the plane glass plate.

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Determination of refractive index of a liquid with Newton's rings:

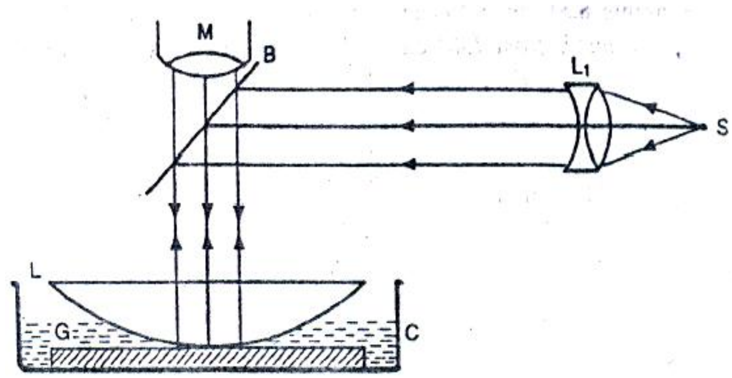

Fig.-1.11: Refractive index of a liquid with Newton's rings

It is possible to determine the refractive index of a liquid by Newton's rings method. The diameters of two particular rings, say the n^{th} and $(n + p)^{th}$, obtained in Newton's rings with an air film, are measured. The difference in diameters of the two rings is

$$(D^2_{n+p} - D^2_n)_{air} = 4\,p\lambda R \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(15)$$

Then a drop of liquid, whose refractive index is to be measured, is carefully introduced into the air film. This liquid is drawn in at the centre by the capillary action forming a liquid film between the lens and the plate. When the film is illuminated with the same monochromatic light, another set of Newton's rings is obtained. The diameters of the same two rings (n^{th} and $(n + p)^{th}$) are then measured. The difference in diameters of the two rings for the two films are

$$(D^2_{n+p} - D^2_n)_{liquid} = \frac{4\,p\lambda R}{\mu} \quad ; \qquad \mu = \frac{4\,p\lambda R}{(D^2_{n+p} - D^2_n)_{liquid}}$$

Where μ is the refractive index of the liquid.

Then,

$$\mu = \frac{(D^2_{n+p} - D^2_n)_{air}}{(D^2_{n+p} - D^2_n)_{liquid}} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(16)$$

By using equation (16) we can get refractive index of a liquid.

Mathematical problem:

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1. Two coherent sources of monochromatic light of wavelength **6000 Å** produce an interference pattern on a screen kept at a distance of **1 m** from them. The distance between two consecutive bright fringes on the screen is **0.5 mm**. Find the distance between two coherent sources.

Solution:

Here $\lambda = 6000\,\text{Å} = 6000\times10^{-10} = 6\times10^{-7}$ m.

D = 1 m, X = 0.5 mm = 5×10^{-4} m and d = ?

$$X = \frac{\lambda D}{d}$$

$$\text{or, } d = \frac{\lambda D}{X} = \frac{6\times10^{-7}\times1}{5\times10^{-4}} = 1.2\times10^{-3}\text{ m} = 1.2\text{ mm.}$$

2. Two straight and narrow parallel slits **1mm** apart are illuminated by monochromatic light. Fringes formed on the screen held at a distance of **100 cm** from the slits are **0.50 mm** apart. Calculate the wavelength of light.

Solution:

Here D = 100 cm = 1 m, X = 0.5 mm = 5×10^{-4} m and d = 1 mm = 1×10^{-3} m

$$X = \frac{\lambda D}{d}$$

$$\text{or, } \lambda = \frac{X\,d}{D} = \frac{5\times10^{-4}\times1\times10^{-3}}{1} = 5\times10^{-7}\text{ m} = 5000\,\text{Å}.$$

3. In Young's double slit experiment the separation of the slits is **1.9 mm** and the fringe spacing is **0.31 mm** at a distance of **1 m** from the slits. Calculate the wavelength of light.

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Solution:

Here D = 1 m, X = 0.31 mm = 3.1×10^{-4} m and d = 1.9 mm = 1.9×10^{-3} m

$$X = \frac{\lambda D}{d}$$

$$\text{or, } \lambda = \frac{X\,d}{D} = \frac{3.1\times10^{-4}\times1.9\times10^{-3}}{1} = 5.89\times10^{-7}\ \text{m} = 5890\,\text{Å}.$$

4. Newton’s rings are observed by keeping a spherical surface of 100 cm radius on a plane glass plate. If the diameter of the 15th bright ring is 0.590 cm and the diameter of the 5th ring is 0.336 cm, what is the wavelength of light used?

Solution:

Here D_5 = 0.336 cm, D_{15} = 0.590 cm, R = 100 cm, p = 10 and $\lambda = ?$

$$R = \frac{D^2_{n+p} - D^2_n}{4\,p\lambda}$$

$$\text{or, } \lambda = \frac{D^2_{n+p} - D^2_n}{4\,pR} = \frac{D^2_{15} - D^2_5}{4\,pR}$$

$$= \frac{(0.596)^2 - (0.336)^2}{4\times10\times100}$$

$$= 5880\times10^{-8}\,cm = 5880\,\text{Å}$$

<u>Physics for Engineers- Dr. Giasuddin Ahmad (1st Edition)</u>

Mathematical Problem:

Example: 27.1, 27.4, 27.16-27.27.

<u>A Text Book of Optics- N. Subrahmanyam, Brijlal (22nd Edition)</u>

Mathematical Problem:

Example: 8.1-8.4, 8.6, 8.7, 8.8, 8.47-8.56.

<u>Exercises</u>

1. What is mean by interference of light? Explain.
2. What do you mean by coherent sources? Derive a relation between path difference and

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Phase difference.

3. State the fundamental conditions for the production of interference fringes.

4. Discuss interference of light analytically and obtain the conditions of maximum and minimum intensities.

5. What do you mean by fringe width? Show that for bright and dark fringe, the fringe width is $X = \frac{\lambda D}{d}$, where the symbols have their usual meaning.

or,

5. Prove that the distance β between two successive bright fringes formed in Young's experiment is given by $\beta = \frac{\lambda D}{d}$ **,** where the symbols have their usual meaning.

6. For both constructive and destructive interference derive an expression of phase difference due to reflected light from a plane parallel (thin) film.

or,

6. Establish the expression of phase difference for both constructive and destructive interference due to reflected light from a plane parallel (thin) film.

7. Describe and explain the formation of Newton's rings in the reflected light. Show that **(i)** the diameters (also radii) of bright rings are proportional to the square root of the odd natural numbers and diameters (also radii) of the dark rings are proportional to the square root of the natural numbers. **(ii)** Account for the perfect blackness of the central spot in Newton's rings. **(iii)** What will happen if a little water is introduced between the lens and glass plate? (iv) Show that the alternate bright and dark rings surrounding the central dark spot in Newton's rings gradually become narrower as their radii increases.

or,

7. Explain the formation of Newton's rings. Describe with necessary theory the Newton's rings method of measuring wavelength of monochromatic light.

8. Describe the phenomena of interference due to reflected light from a plane of varying thickness (wedge shaped film).

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