

LENS

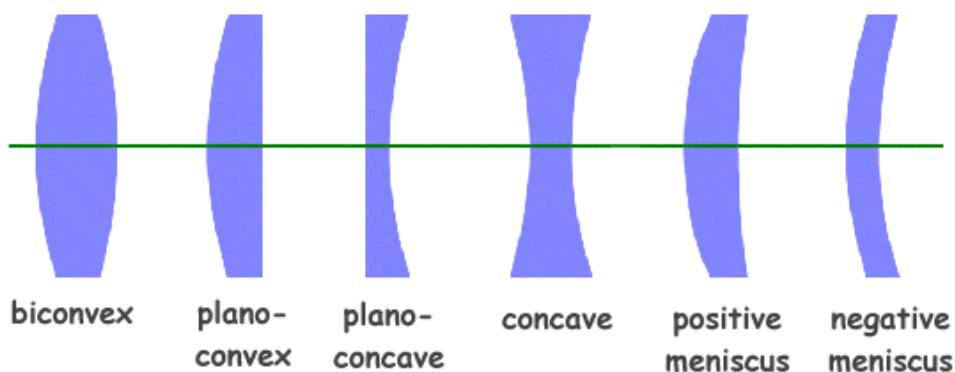
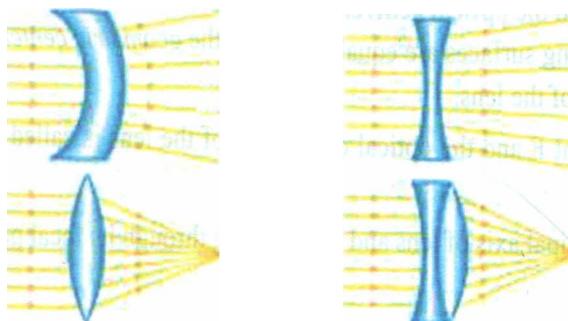
Definition

A lens is an image-forming device. It forms an image by refraction of light at its two bounding surfaces. In general, a lens is made of glass and is bounded by two regular curved surfaces; or by one spherical surface and a plan surface.

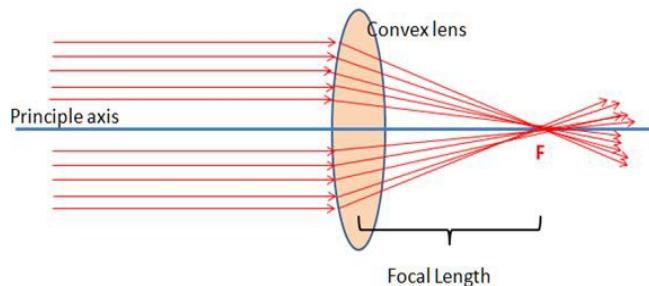
Spherical surfaces are easy to make. Therefore, most lenses are made of spherical surfaces and have a wide range of curvatures. Other transparent materials such as quartz, fused silica and plastics are also used in making lenses. A single lens with two refracting surfaces is a simple lens.

Types of Lenses:

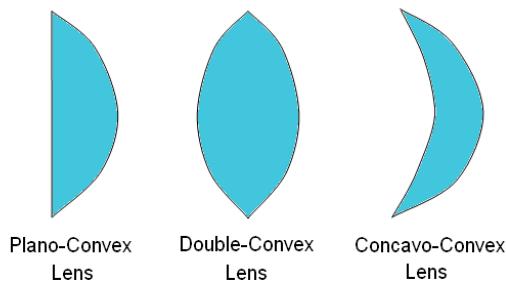
Lenses are mainly of two types- convex lens and concave lens. A convex lens is thicker at the center than at the edges while a concave lens is thinner at the center than at the edges. A convex lens is a converging lens since a parallel beam of light, after refraction, converges to a point. A concave lens is called a diverging lens since rays coming parallel to the principal axis, after refraction, diverge out seem to come from a point.



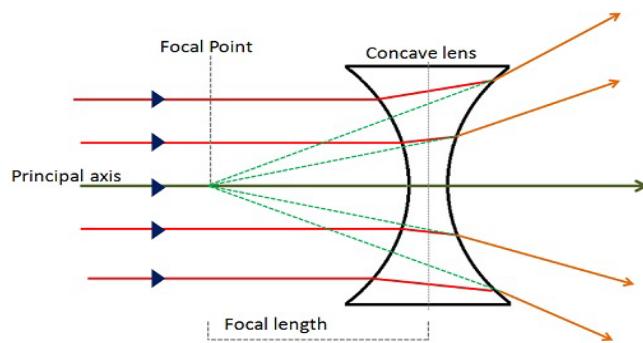
Convex Lens: A convex lens is a converging lens. When parallel rays of light pass through a convex lens the refracted rays converge at one point called the principal focus. The distance between the principal focus and the centre of the lens is called the focal length.



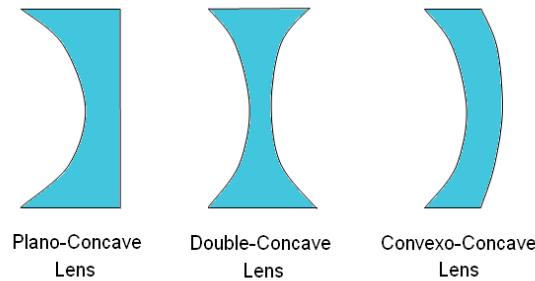
Convex lenses are known as many names, being also called positive lenses, plus lenses, converging lenses, and condensers. Convex lenses are fat in the middle and skinny on the edges. There are three main types of convex lenses: plano-convex lenses, double-convex lenses, and concavo-convex lenses.



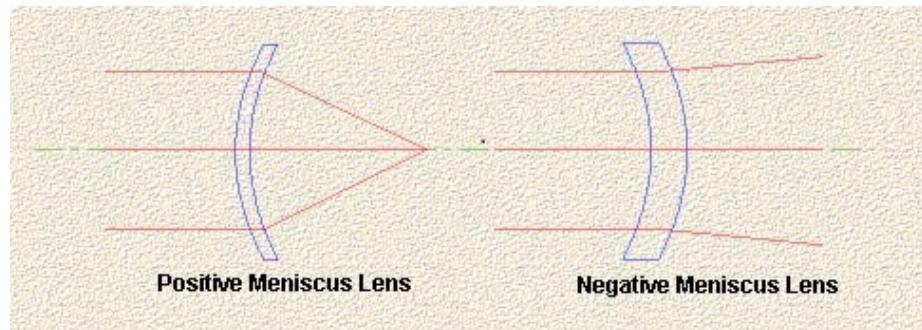
Concave Lens: A concave lens is a lens that possesses at least one surface that curves inwards. It is a diverging lens, meaning that it spreads out light rays that have been refracted through it. A concave lens is thinner at its centre than at its edges, and is used to correct short-sightedness (myopia).



Concave lenses also have many names. They include negative **lenses**, minus **lenses**, and **diverging lenses**. There are also three **types of concave lenses** that resemble their **convex** counterparts. Plano-concave lenses have a flat surface and one inward curving side.



Positive meniscus: Positive meniscus lens is a convex-concave lens thicker at the center than at the edges. It is used to minimize spherical aberration. When used in combination with another lens, it will shorten the focal length, and increase the NA (Numerical Aperture) of system.

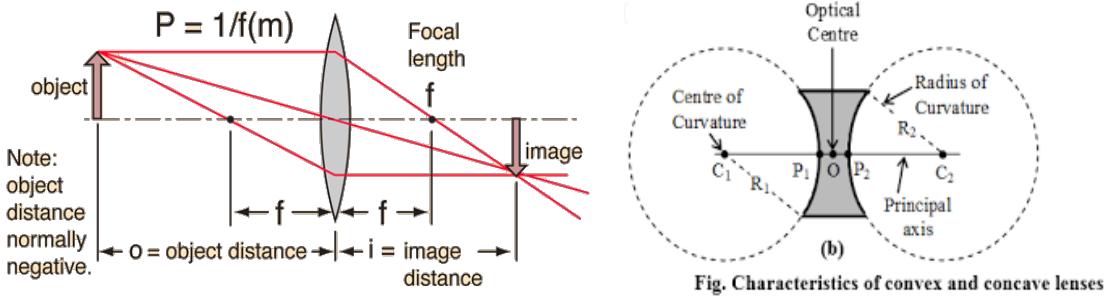


Negative Meniscus Lens. A lens having one convex and one concave surface, with the radius of curvature of the convex surface greater than that of the concave surface. Also known as diverging meniscus lens.

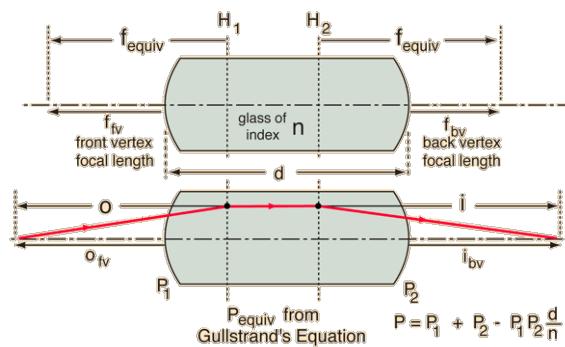
TERMINOLOGY

We first acquaint with the terminology and the sign convention associated with lenses.

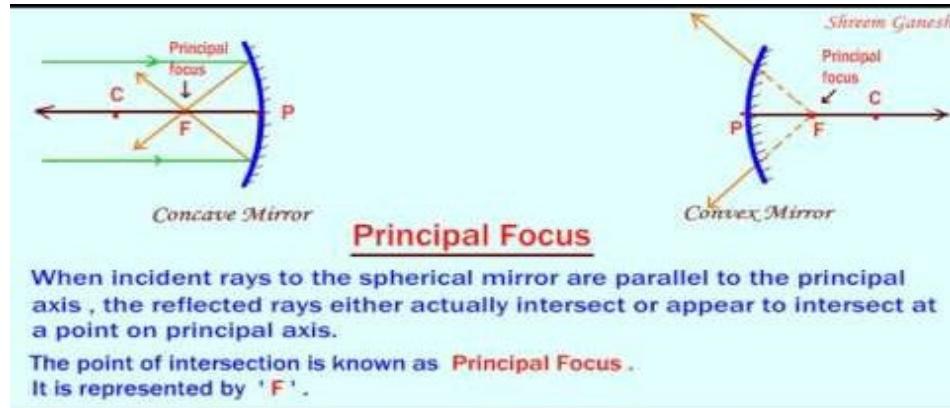
- A lens has two curved surfaces, each surface having a **curvature**.
- The length of the radius of curvature of surface is called the **radius of curvature**, R.



- The reciprocal of the length of the radius of **curvature** is known as the curvature C ($C = 1/R$). A lens has two centers of curvature and two radii of curvature, one for each refracting surface.
- For every lens, there is a point on the principal axis for which the rays passing through it emerge parallel to the incident ray. Such a point is called the **optical centre**. When the lens is thin and the radii of curvature of the two refracting surfaces are equal, then the geometric center of the lens becomes the *optical center* of the lens.
- The distance between the focal point and the optical center of the lens is called the **focal length** of the lens.
- The line joining the centers of curvature of the two curved surfaces is called the **principal axis** or simply **axis** of the lens.
- The plane perpendicular to the principal axis of lens and passing through its focal point is known as the **focal plane**.
- The points where the principal axis intersects the two refracting surfaces are called the **front vertex** and the **back vertex**.



- The point to which a set of rays parallel to the principal axis is caused to converge (in case of convex lens) or appear to diverge (in case of concave lens) is the **principal focus**.



- When a point object or a linear object is placed on one side of a convex lens beyond the focal plane, an image is formed on the opposite side. The distance from the front vertex to the object is called the **object distance, u** and the distance from the back vertex to the image is the **image distance, v**.
- The **power** of a lens is the reciprocal of its focal length.

SIGN CONVENTION

The following convention of the signs is adopted for obtaining the relation between these quantities.

- (a) The diagrams are drawn showing the incident light travelling from left to right.
- (b) The distances are measured by taking the optical centre of the lens as the origin.
- (c) The distances measured in the direction of the incident light are considered positive while those measured in the direction opposite to the incident light are taken as negative. (All quantities measured to the right of P are positive and all those to its left are negative.)

- (d) Heights measured upward and perpendicular to the principal axis are taken as positive while those measured downward are considered negative.
- (e) The angle made by a ray with the principal axis is taken to be positive if the ray has 'to be rotated anti-clockwise to become coincident with the axis, otherwise it is negative.

THIN LENS

Lenses are broadly classified into thin and thick lenses. A lens is said to be thin if the thickness of the lens can be neglected when compared to the lengths of the radii of curvature of its two refracting surfaces, and to the distances of the objects and images from it. No lens is actually a thin lens.

The power of a lens, or

Power of Convex Lens is Positive and Power of Concave lens is Negative

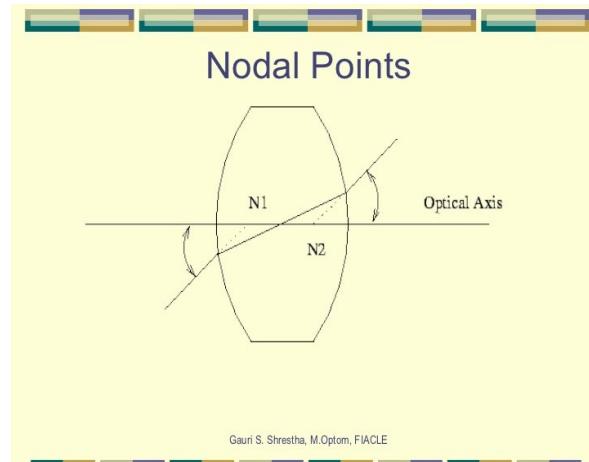
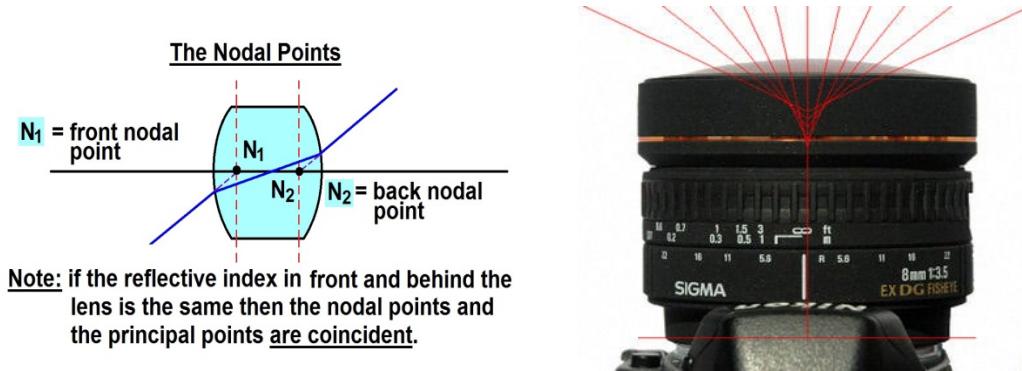
The power of a lens is the measure of its ability to produce convergence of a parallel beam of light.

The unit in which the power of a lens is measured is called diopter (D). A convex lens of focal length one meter has a power = +1 diopter and a convex lens of focal length 2 meter has a power = 0.5 diopter.

Mathematically, Power = 1 / Focal Length in meters

A convex lens of large focal length produces a small converging effect and a convex lens of small focal length produces a large converging effect. Due to this reason, the power of a convex lens is taken as positive and a convex lens of small focal length has high power. On the other hand, a concave lens produces divergence. Therefore, its power is taken as negative.

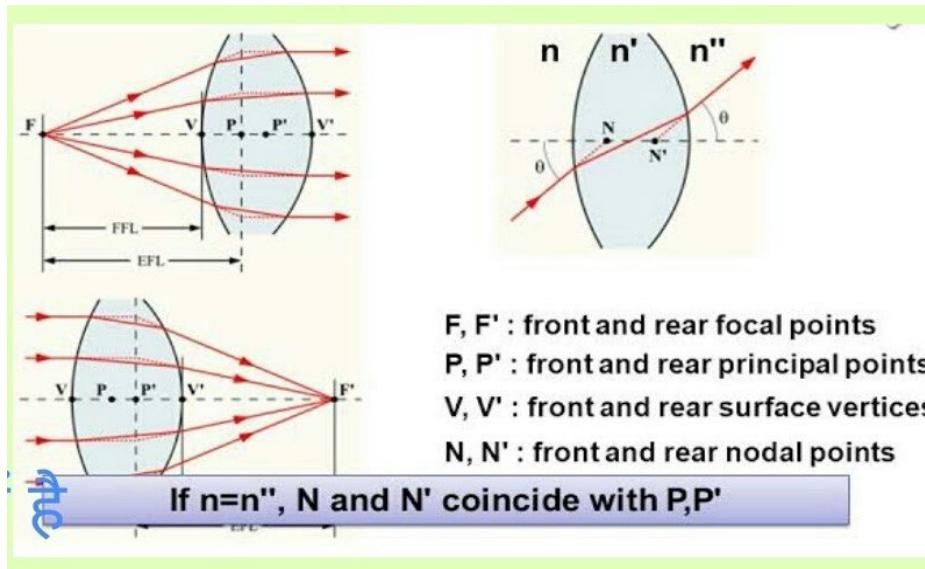
Nodal Points. Nodal points are points on the principal axis of the optical system where light rays, without refraction, intersect the optic axis. The Nodal Point of the lens (or more correctly, the entrance pupil) can be considered as the point at which the rays entering the lens converge. It can also be considered as the centre of perspective of the lens or the apparent pupil. This point can be considered as the **Front Nodal Point** as the lens also has a Rear Nodal Point and in a simple lens the two nodes converge to a single point.



Cardinal Points: For the combination of lenses (or for the thick lenses) the points of intersection of planes of those lenses with the axis are called Principal points. In fact, there are six points in all, which characterize an optical system. They are –

- (i) Two focal points
- (ii) Two Principal points
- (iii) Two Nodal points

These six points are known as Cardinal Points of an optical system.



Cardinal Planes: The planes passing through the cardinal points and perpendicular to the principal axis are known as Cardinal Planes.

LECTURE - 01

Equivalent lenses

When two lenses are arranged coaxially, the image formed by the first lens system becomes the object for the second lens system and the two system acts as a single optical system and forming the final image from an original object. Such a system acts like an equivalent lens of the two constituent lenses.

Let us consider a simple optical system that consists of two thin lenses L_1 and L_2 placed on a common axis and separated by a distance d , as shown in

figure. The lenses are separated by a distance and have

focal lengths f_1 and f_2 . We are interested to know how the combination works. We find that two lenses, separated by a finite distance can be replaced by a single lens that is the equivalent lens. The equivalent lens, when placed at a suitable fixed point will produce an image of the same size as that produced by the combination

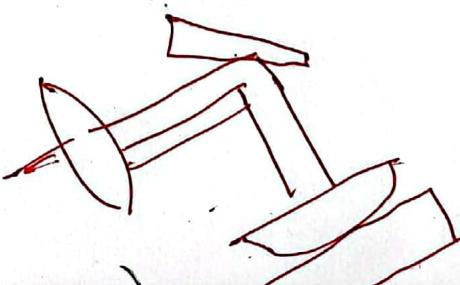
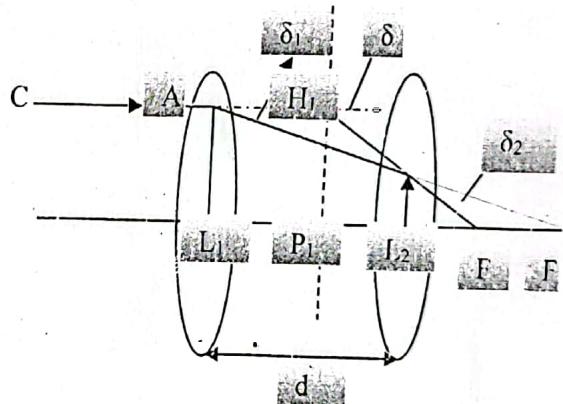
of the two lens. The focal length of the two lens is called the equivalent focal length and. We now derive the equivalent focal length, f for the combination of two thin lenses.

Let us a ray CA of monochromatic light parallel to the principle axis be incident on the first lens L_1 at a height h_1 above the axis. The ray CA is deviated through an angle δ_1 by the lens L_1 . The incident ray CA after refraction, is directed towards F_1 , which is the second principle focus of lens L_1 . Then the deviation produced by the first lens id given by

$$\delta_1 = \frac{h_1}{f_1}$$

The emergent ray, AB from the first lens is incident on the lens L_2 at a height h_2 . On the other hand the ray is deviated at and angle δ_2 by the lens L_2 and meets the principle axis at F. since the incident ray CA is parallel to the principle axis and after refraction the optical system meets the axis at F. F must be the second principle focus of the combined lens system. The deviation from the 2nd lens is given by

$$\delta_2 = \frac{h_2}{f_2}$$



If the incident ray CA is extended forward and the final emergent ray BF backward, they meet at a point H₂. It is clear that a single thin lens placed at P₂ will produce the same deviation as that produced by the two lenses put together. The lens of focal length P₂F placed at P₂ is termed as the equivalent lens, which can replace the two lenses L₁ and L₂. The deviation produced by the equivalent lens is

$$\delta = \frac{h_1}{f}$$

Focal length of the equivalent lens.

Deviation produced by the first lens L₁ is

$$\delta_1 = \frac{h_1}{f_1}$$

And deviation produced by the second lens L₂ is

$$\delta_2 = \frac{h_2}{f_2}$$

But $\delta = \delta_1 + \delta_2$

$$\text{Therefore, } \frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_2}{f_2} \quad (1)$$

The triangles AL₁F₁ and BL₂F₁ are similar.

$$\text{Therefore } \frac{AL_1}{L_1F_1} = \frac{BL_2}{L_2F_1}$$

$$\text{Or, } \frac{h_1}{f_1} = \frac{h_2}{f_1 - d}$$

$$\text{Or, } h_2 = \frac{h_1(f_1 - d)}{f_1} \quad (2)$$

Using equation (2) in (1) we get

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1(f_1 - d)}{f_1 f_2}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \quad (3)$$

Therefore the equivalent focal length is given by

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} \quad (4)$$

Decisions :

1. From equation 4 we find that if $d > f_1 + f_2$ then f is negative. It implies the negative focal length so the system is divergent.
2. If the medium between two concave lens is other than air then equation 4 becomes

$$f = \frac{f_1 f_2}{f_1 + f_2 - d/\mu}$$

Where μ is the refractive index of the medium.

Power of the equivalent lens

The power of a lens is the measure of its ability to produce converge of a parallel beam of light. A convex lens of large focal length produces a small converging effect on the rays of light and a convex lens of small focal length produces a large converging effect. Due to this reason, the power of a convex lens is taken as positive and a convex lens of small focal length has high power. On the other hand, a concave lens produces divergence. Therefore, its power is taken as negative.

The unit in which the power of a lens is measured is called diopter (D). A convex lens of focal length one meter has a power is 1 and a convex lens of focal length 2 meter has a power $\frac{1}{2}$ diopter.

Mathematically,

$$\text{Power} = 1 / \text{Focal length in meter}$$

If two lenses of focal length f_1 and f_2 are in contact

$$1/F = 1/f_1 + 1/f_2$$

$$P = P_1 + P_2$$

Where P_1 and P_2 are the powers of two lenses and P is the equivalent power.

When two thin lenses of focal lengths f_1 and f_2 are placed coaxially and separated by a distance d , the equivalent focal length f is given by

$$1/F = 1/f_1 + 1/f_2 - d/f_1 f_2$$

$$P = P_1 + P_2 - dP_2 P_1$$

Problem: two convex lenses of focal length 10 cm and 20 cm are placed at 5 cm apart in air. find the equivalent focal length.

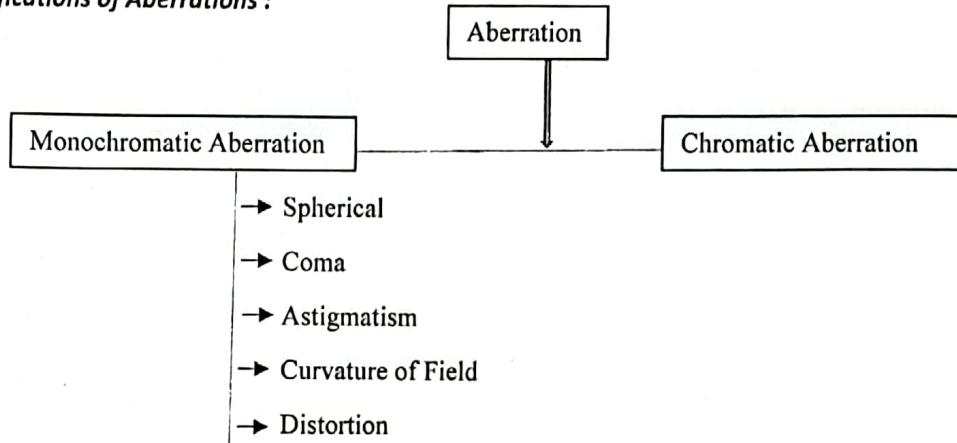
ABERRATIONS

What is aberration?: In an ideal optical system, all rays of light from a point in the object plane would converge to the same point in the image plane, forming a clear image. The influences which cause different rays to converge to different points are called **aberrations**.

The deviations of real image from ideal images in respect of the actual size, shape and position called aberration .

Due to aberration, simple formulae of lens and mirror are not precisely followed. It is a consequence of the laws of refraction of the spherical surfaces not due to defective construction.

Classifications of Aberrations :

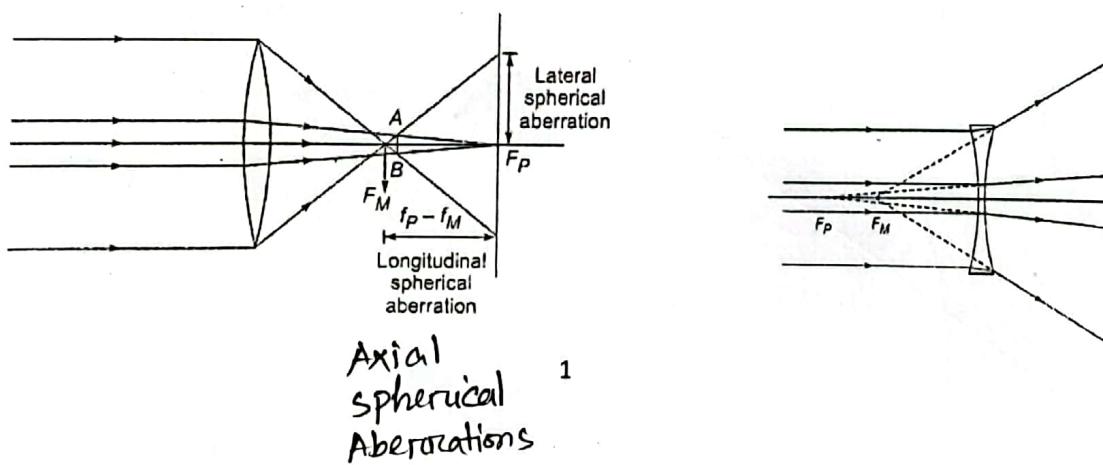


Spherical Aberration :

It is an optical problem that occurs when all incoming light rays end up focusing at different points after passing through a spherical surface.

Light rays passing through a lens near its horizontal axis are refracted less than rays closer to the edge or "periphery" of the lens and as a result, end up in different spots across the optical axis.

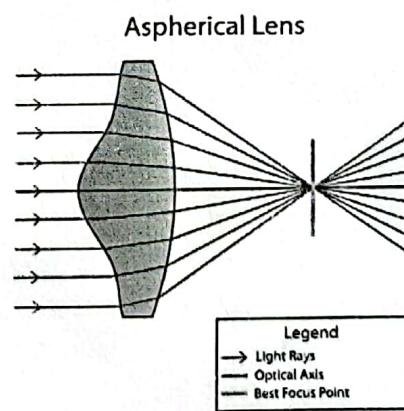
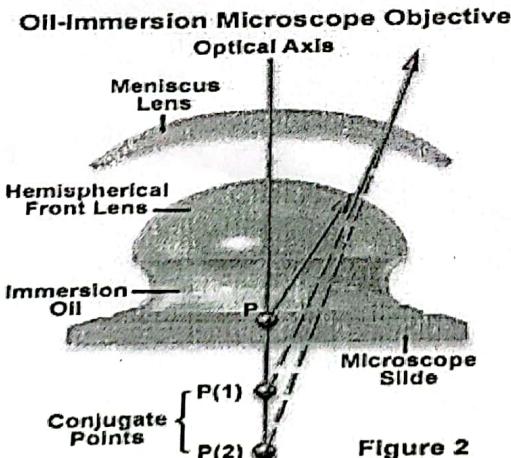
In other words, the parallel light rays of incoming light do not converge at the same point after passing through the lens. Because of this, spherical aberration can affect resolution and clarity, making it hard to obtain sharp images.



- Occurs when the marginal rays and paraxial rays are focused at different position.
- FP at which the paraxial rays strike the axis is called the paraxial focus.
- FM at which the rays near the periphery strike is called the marginal focus.
- Distance between FP and FM is termed **longitudinal spherical aberration**.
- Distance between FP and the point at which the marginal ray strikes the paraxial image plane is called the **lateral spherical aberration**.
- On a plane AB the circular patch has the least diameter. This is called **the circle of least confusion**.

Reducing Spherical Aberration:

- Can be minimized by using stops which reduce the effective aperture of the lens. The stop used can be such as to permit either the axial rays or the marginal rays of light.
- • Using the cross lens for which the radius of curvature has a ratio $R_1/R_2 = -1/6$. Since in each surface light rays have the same deviation and marginal and paraxial light rays focused at minimum spherical aberration.
- • Using a plano-convex lens. It is found that when the curved faces the incoming light then total deviation is shared between two surfaces and as a result spherical aberration becomes minimum.
- Spherical aberration can be made minimum by using two plano convex lens separated by a distance equal to the difference of their focal length.
- Spherical aberration for a convex lens is positive and negative for concave lens. So by making a suitable combination of convex and concave lenses spherical aberration can be made minimum

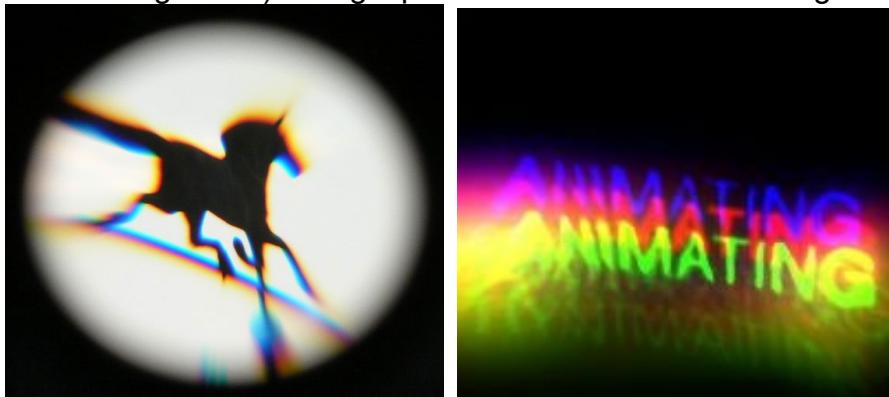


DEFECT OF IMAGES: ABERRATION

Meaning A departure from what is normal, usual, or expected, typically one that is unwelcomed.

Synonym: Anomaly, deviation

Definition. **Aberration** in optics refers to a defect in a lens such that light is not focused to a point, but is spread out over some region of space, and hence an image formed by a lens with aberration is blurred or distorted, with the nature of the distortion depending on the type of aberration. More specifically, it can be defined as a departure of the performance of an optical system from the predictions of paraxial optics. In an imaging system, it occurs when light from one point of an object does not converge into (or does not diverge from) a single point after transmission through the system.



An image-forming optical system with aberration will produce an image which is not sharp. Makers of optical instruments need to correct optical systems to compensate for aberration.

Classification of aberration: Aberration mainly of two types:

- a. Monochromatic
- b. Chromatic

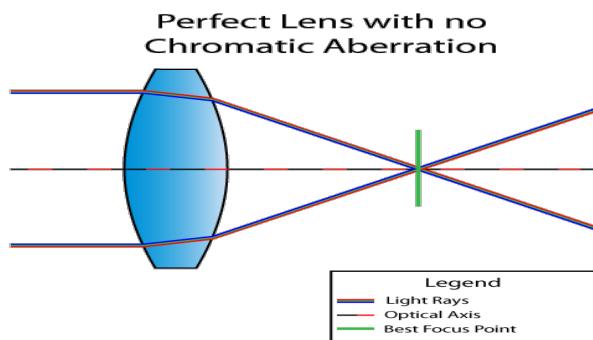
Monochromatic Aberration again subdivided into five types:

- i. Spherical Aberration
- ii. Coma
- iii. Astigmatism
- iv. Curvature
- v. Distortion

What is Chromatic Aberration?

Chromatic aberration, also known as “color fringing” or “purple fringing”, is a common optical problem that occurs when a lens is either unable to bring all wavelengths of color to the same focal plane, and/or when wavelengths of color are focused at different positions in the focal plane. Chromatic aberration is caused by lens dispersion, with different colors of light travelling at different speeds while passing through a lens. As a result, the image can look blurred or noticeable colored edges (red, green, blue, yellow, purple, magenta) can appear around objects, especially in high-contrast situations.

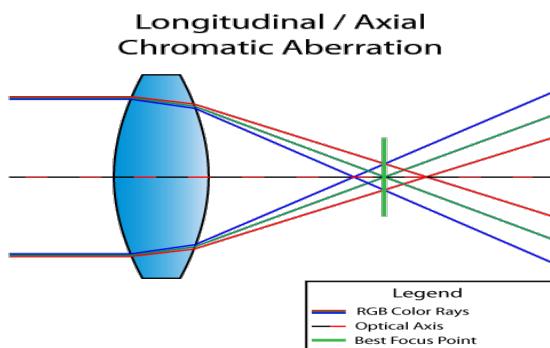
A perfect lens would focus all wavelengths into a single focal point, where the best focus with the “circle of least confusion” is located, as shown below:



In reality, the refractive index for each wavelength is different in lenses, which causes two types of Chromatic Aberration – Longitudinal Chromatic Aberration and Lateral Chromatic Aberration.

Longitudinal Chromatic Aberration

Longitudinal Chromatic Aberration, also known as “LoCA” or “bokeh fringing”, occurs when different wavelengths of color do not converge at the same point after passing through a lens, as illustrated below:



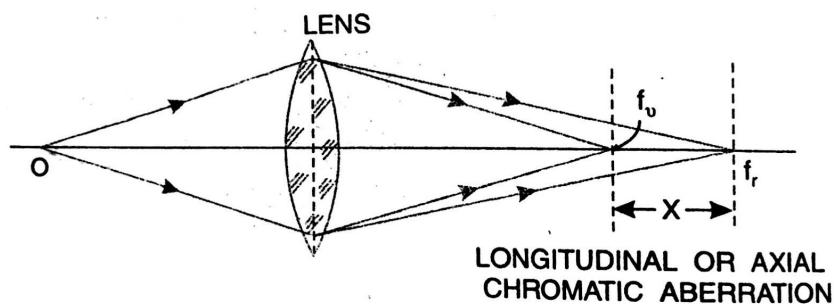
Lenses with Longitudinal Chromatic Aberration problems can show fringing around objects throughout the image, even in the center. Red, Green, Blue or a combination of these colors can appear around objects. Longitudinal Chromatic Aberration can be dramatically reduced by stopping down the lens.

Expression for Longitudinal Chromatic Aberration for an object at infinity

Or

The axial chromatic aberration for an object at infinity is equal to the product of the dispersive power of the material of the lens and mean focal length (f).

When a *parallel beam* of white light is passed through a lens, the beam gets dispersed and rays of light of different colors (wavelengths) come to focus at different points along the axis. The violet rays of light come to focus at a point nearer the lens and the red rays of light at a farther point. (Fig-1). f_v is the focus for the violet rays and f_r is the focus for the red rays. The colors in between violet and red come to focus between f_v and f_r . The distance $(f_r - f_v) = x$ is called longitudinal or axial chromatic aberration.



RESTRICTED

The focal length of a lens is given by,

Similarly,

$$\frac{1}{f_v} = \frac{(\mu_v - 1)}{(\mu - 1)f} \dots \dots \dots \quad (iv)$$

and

From equ (iii) and (v), we get,

$$\frac{1}{f_v} - \frac{1}{f_r} = \frac{1}{(\mu - 1)f} (\mu_v - 1 - \mu_r + 1)$$

$$\frac{f_r - f_v}{f_v f_r} = \frac{\mu_v - \mu_r}{(\mu - 1) f}$$

Taking $f_v f_r = f^2$ (where f is the mean focal length), we can write

$$\frac{f_r - f_v}{f^2} = \frac{\mu_v - \mu_r}{(\mu - 1)f}$$

$$f_r - f_v = \frac{(\mu_v - \mu_r)f}{(\mu - 1)}$$

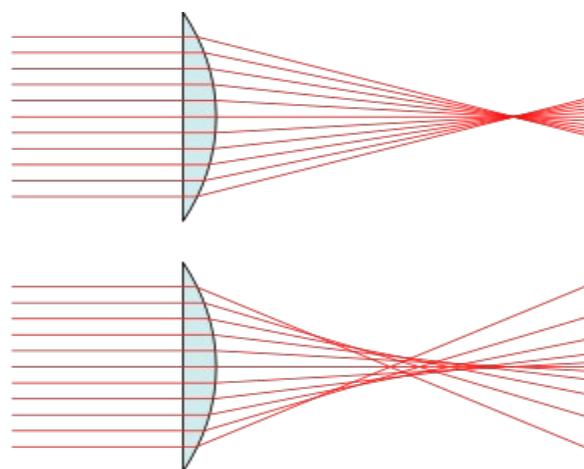
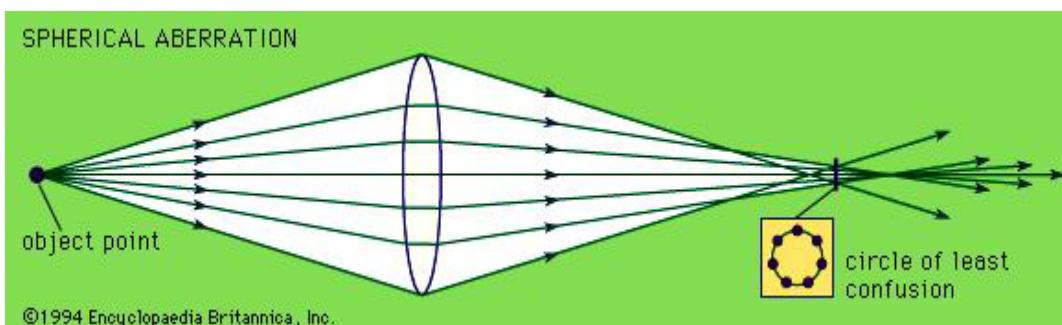
$$f_r - f_v = \omega \cdot f$$

Where, $\omega = \frac{(\mu_v - \mu_r)}{(\mu - 1)}$ is known as the dispersive power of the material.

Thus, The axial chromatic aberration for an object at infinity is equal to the product of the dispersive power of the material of the lens and mean focal length (f).

Spherical Aberration:

Spherical aberration is an optical effect observed in an optical device (lens, mirror, etc.) that occurs due to the increased refraction of light rays when they strike a lens or a reflection of light rays when they strike a mirror near its edge, in comparison with those that strike nearer the centre. It signifies a deviation of the device from the norm, i.e., it results in an imperfection of the produced image. ([Video -2](#))



On top is a depiction of a perfect lens without spherical aberration: all incoming rays are focused in the **focal point**. The bottom example depicts a real lens with spherical surfaces, which produces spherical aberration: The different rays do not meet after the lens in one focal point. The further the rays are from the **optical axis**, the closer to the lens they intersect the optical axis (positive spherical

aberration).

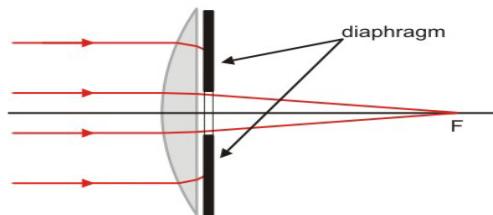
Methods of reducing Spherical Aberration: (Video -4)

- i. Using a stop
- ii. Using Crossed lens
- iii. Using Plano- Convex lens
- iv. Using two convergent lenses separated by a fixed distance.
- v. Using suitable combination of Convergent and divergent lenses of proper shape.

For a single lens, spherical aberration cannot be entirely eliminated. However, it can be reduced by following methods

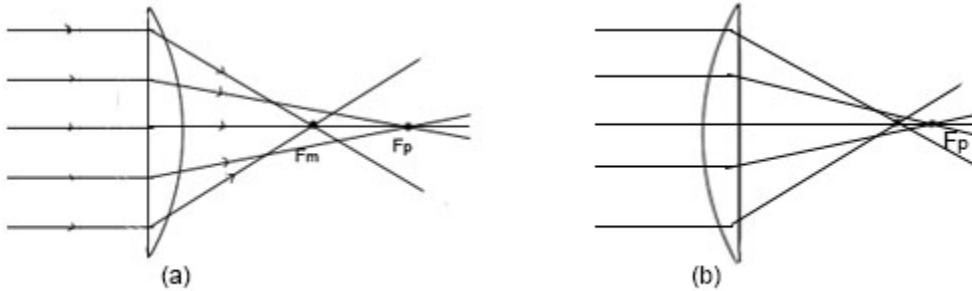
1. Using Stop :

Spherical aberration can be minimized by using stops, which reduce the effective lens aperture. The stop used can be such as to permit either the axial rays of light or the marginal rays of light. However, as the amount of light passing through the lens is reduced, corresponding the image appears less bright.



2. By using Plano-convex lenses

If parallel rays of light incident on the plane surface of the plano-convex lens, the spherical aberration will be maximum because incident rays entire deviation at the convex surface. Similarly if parallel rays of light incident on the convex surface, spherical aberration will be minimum.



Reduction of spherical aberration by using plano - convex lens in different positions.

3. By using cross lens

Crossed Lens is a double-convex lens whose ratio for a refractive index (the ratio of the velocity of light in a vacuum to its velocity in a specified medium) is 1:6. It produces minimum spherical aberration for parallel incident rays. Double-Convex **Lenses** are used in image relay applications, or for imaging objects at close conjugates.

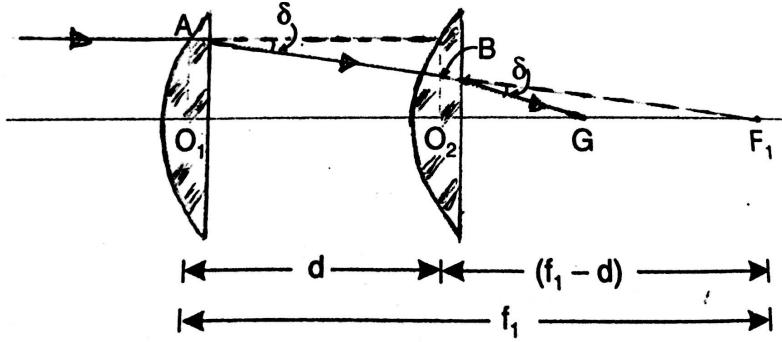
Spherical Aberration for a double convex lens (Shape factor 0.5) is minimum when the surface of smaller radius of curvature faces the incident parallel light. The spherical aberration for a plano-convex (Shape factor+1.0) when the curved surface faces the incident light is only slightly more than the double convex lens. Hence plano-convex lens are preferred.

Shape factor refers to a value that is affected by an object's **shape** but is independent of its dimensions. It may refer to one of number of values in physics, engineering, image analysis, or statistics.

4. By using two lenses separated by a distance

Spherical aberration can also be made minimum by using two plano-convex lenses separated by a distance equal to difference in their focal length. In this arrangement, the lenses equally share the total deviation and the spherical aberration is minimum. In the fig two plano-convex lenses of focal length f_1 and f_2 are separated by a distance d .

RESTRICTED



Let δ be the angle of deviation produced by each lens.

$$\text{so, } \angle BF_1G = \delta, \angle F_1BG = \delta$$

and from ΔBFG_1 , $BG = BF_1$ or, $O_2G = GF_1$ (approximately)

$$\therefore O_2 = \frac{1}{2} (O_2F_1) = \frac{1}{2} (f_1 - d)$$

For the second lens, F_1 is the virtual object and G is the real image. Substituting these values of object and image distances in the formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_2} \quad (u, v, f^2 \text{ all are + ve})$$

$$\frac{1}{O_2G} - \frac{1}{O_2F_1} = \frac{1}{f_2}$$

$$\frac{2}{(f_1 - d)} - \frac{1}{(f_1 - d)} = \frac{1}{f_2}$$

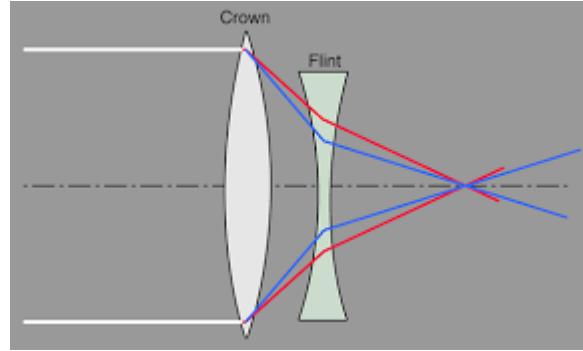
$$\frac{1}{(f_1 - d)} = \frac{1}{f_2}$$

$$f_2 = f_1 - d$$

$$f_1 - f_2 = d$$

5. By combining suitable concave and convex lenses

It is known that convex lens has positive spherical aberration and concave lens has negative spherical aberration. So by selecting suitable pair of concave and convex lens, spherical aberration can be minimized.

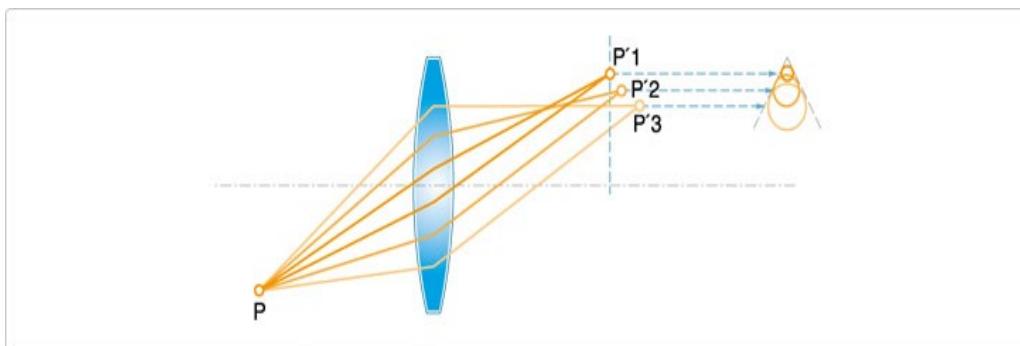


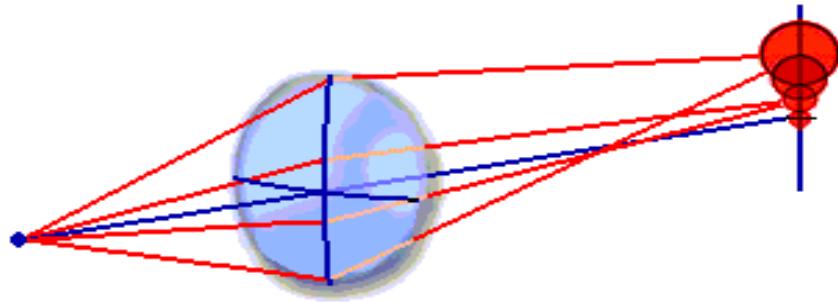
Coma:

Rays coming from an object point not situated on the axis of the lens suffers another type of aberration called coma. Comatic aberration happens due to failure of the lens to bring all rays from a object point to focus at the same point.

In optics (especially telescopes), the coma, or Comatic aberration, in an optical system refers to aberration inherent to certain optical designs or due to imperfection in the lens or other components that results in off-axis point sources such as stars appearing distorted, appearing to have a tail (coma) like a comet. ([Video- 6](#))

Coma is proportional to the distance to the central axis, so more the rays are away from the centre more the focal point change the positions and get blurry images, mainly off-axis.



**Fig: Coma****Reduction of Coma:**

More versatile correction of coma, one can use a combination of two lenses, both of which are corrected for zero coma at infinite object distance. The appropriate separation of these two lenses can correct for coma at various object distances. Coma can also be corrected by an appropriately placed stop, but the placement and size of an optimum stop also depends upon the other aberrations.

Coma of a single lens or a system of lenses can be minimized (and in some cases eliminated) by choosing the curvature of the lens surfaces to match the application. Lenses in which both spherical aberration and coma are minimized at a single wavelength are called best form or aplanatic lenses.

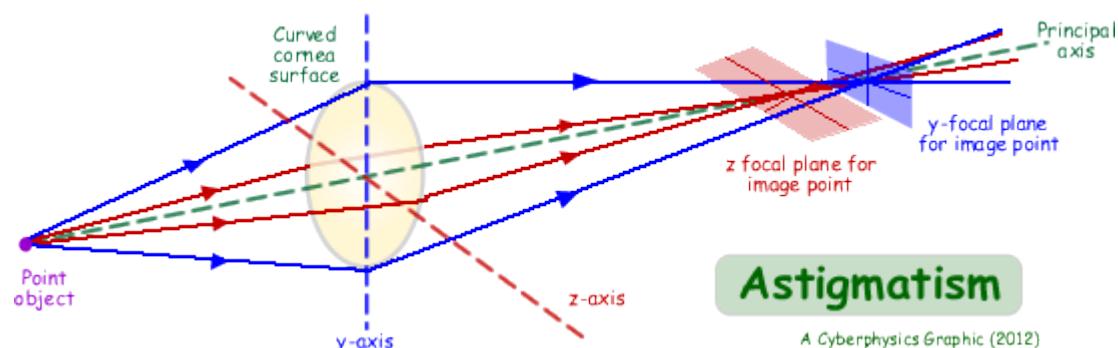
Astigmatism

Astigmatism, similar to coma, is the aberration in the image formed by a lens, of object points off the axis. The difference between astigmatism and coma, however, is that in coma the spreading of the image takes place in a plane perpendicular to the lens axis and in astigmatism the spreading takes place along the lens axis. (**Video -7 and 7.1**)

Higher amounts of astigmatism may cause blurry vision, squinting, asthenopia, fatigue, or headaches.

.

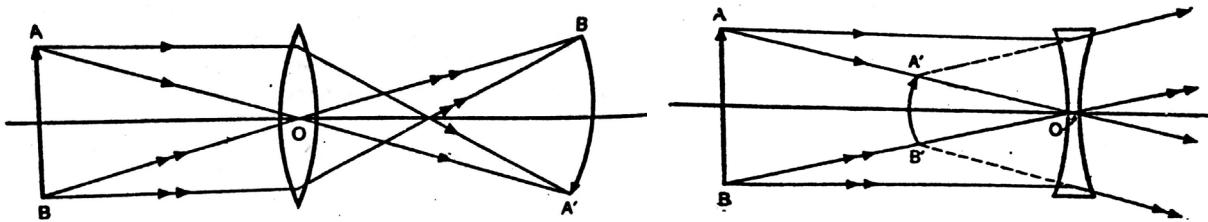
.



It is possible to minimize the astigmatic difference by using a convex and a concave lens of suitable focal lengths and separated by a distance and such a lens combination is called an **anastigmat**.

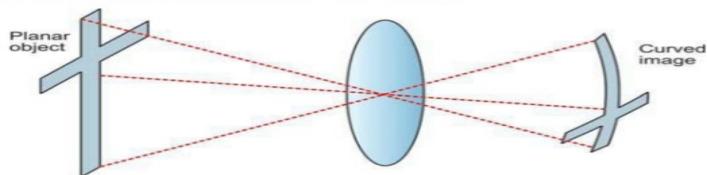
Curvature

The image of an extended plane object due to single lens is not a flat one but will be a curved surface. The central portion of the image nearer the axis is in focus but the outer regions of the image away from the axis are blurred. This defect is called the curvature of the field. This defect is due to the fact that the paraxial focal length is greater than the marginal focal length.



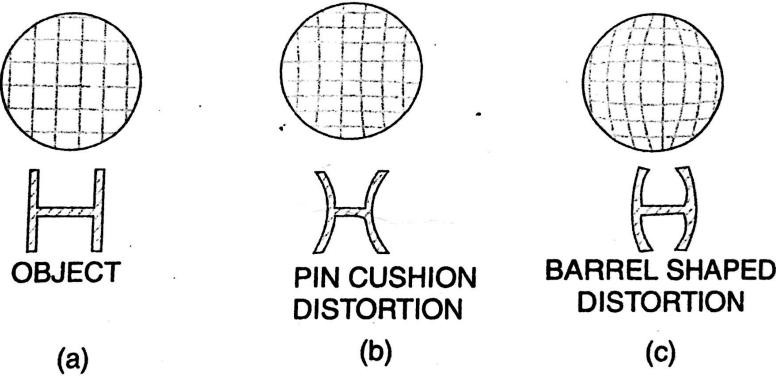
Even if the lens or the lens system is free from spherical aberration, coma and astigmatism, the image of an extended object plane is not a flat one but, in general, a curved surface. This defect is known as curvature.

CURVATURE OF FIELD



Distortion:

The variation in the magnification produced by a lens for different axial distances results in an aberration called distortion. Distortion is of two types - (a) Pin cushion distortion and (b) barrel shaped distortion.



In pin-cushion distortion, the magnification increases with increasing axial distance and the image of an object appears as shown in fig (b). On the other hand, if the magnification decreases with increasing axial distance, it results in barrel shaped distortion and the image appears as shown in fig (c).

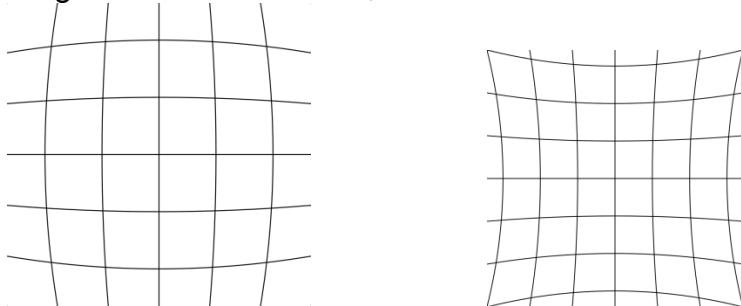
Reduction: Curvature of the field can be minimized by introducing suitable stops on the lens axis.

Barrel distortion:

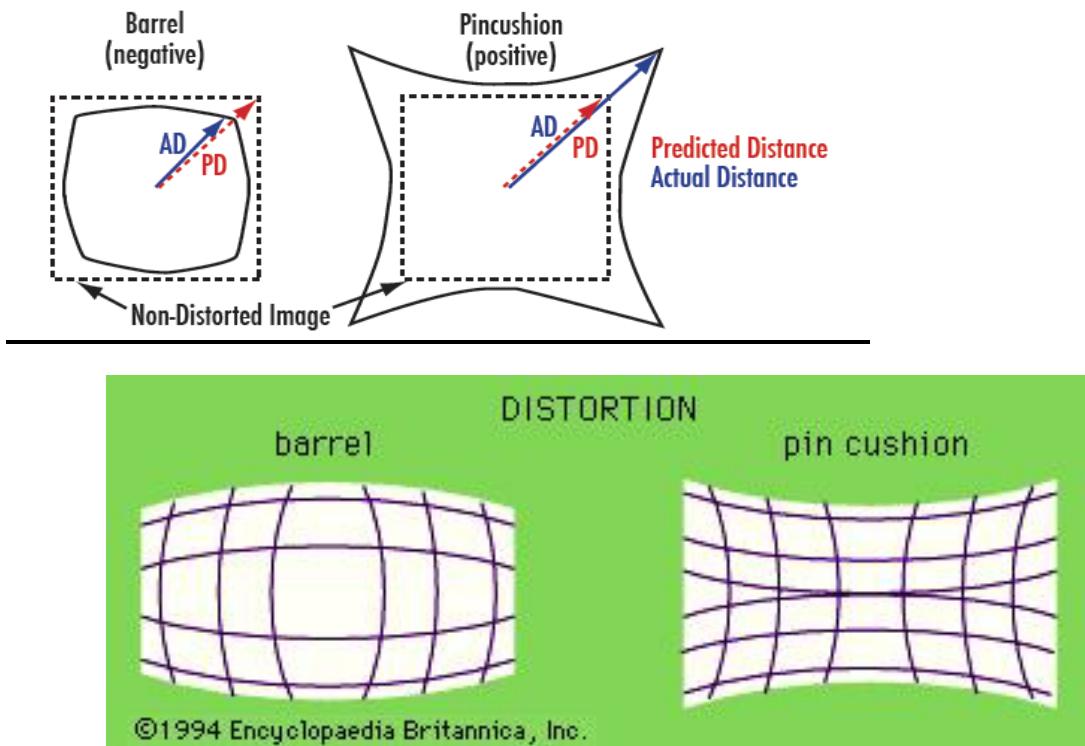
In barrel distortion, image magnification decreases with distance from the optical axis. The apparent effect is that of an image which has been mapped around a sphere (or barrel). Fisheye lenses, which take hemispherical views, utilize this type of distortion as a way to map an infinitely wide object plane into a finite image area. In a zoom lens barrel distortion appears in the middle of the lens's focal length range and is worst at the wide-angle end of the range.

Pincushion distortion

In pincushion distortion, image magnification increases with the distance from the optical axis. The visible effect is that lines that do not go through the centre of the image are bowed inwards, towards the centre of the image, like a pincushion

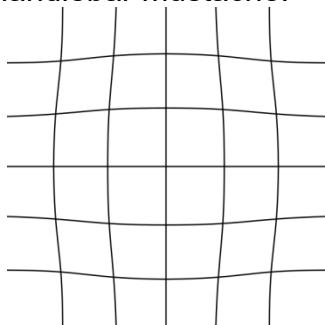


RESTRICTED

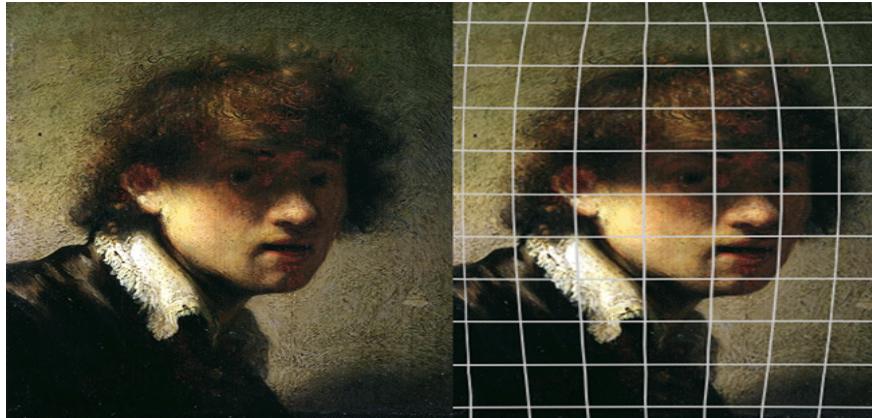


Mustache distortion

A mixture of both types, sometimes referred to as mustache distortion (moustache distortion) and complex distortion, is less common but not rare. It starts out as barrel distortion close to the image center and gradually turns into pincushion distortion towards the image periphery, making horizontal lines in the top half of the frame look like a handlebar mustache.



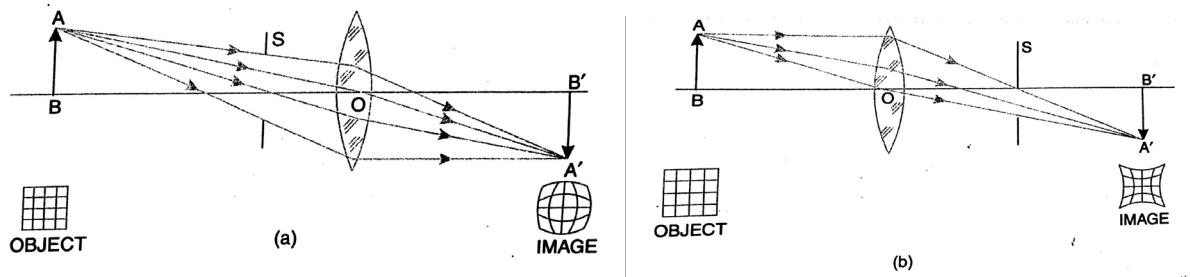
Mathematically, barrel and pincushion distortion are quadratic, meaning they increase as the square of distance from the center.



Barrel distortion is most prominent with wide angles.

Reduction of Distortion:

In the case of optical instruments intended mainly for visual observation, a little amount of distortion may be present. Distortion can be minimized by introducing suitable stops on the lens axis. If stop is used before the lens the distortion is barrel shaped and if a stop is place after the lens, the distortion is pin-cushioned type.



To eliminate such distortions, a stop is placed in between two symmetrical lenses, so that the pin cushion distortion produced by the first lens is compensated and barrel shaped distortion produced by the second lens. Camera and projection lenses are constructed in this manner.

Find the condition of achromatism when two lenses are in contact .

Or

The ratio of the dispersive powers of the materials of the lenses is equal to the ratio of the focal lengths of two lenses when they places in contact.

Or

RESTRICTED

Show that, $\frac{w}{f} + \frac{w^1}{f^1} = 0$; or, $\frac{w_1}{f_1} + \frac{w_2}{f_2} = 0$

[Prof Gias udin, page 1070-25.16]

Chromatic aberration is ordinarily corrected by suitably combining two lenses such that the combination is free from axial chromatic aberration for the two given colors (say, blue and red), the combination being itself considered as a thin lens.

If two lenses of focal length f_1 and f_2 are placed in contact then the equivalent focal length f is given by

$$f = \frac{1}{f_1} + \frac{1}{f_2} \quad \dots \dots \dots \text{(i)}$$

Now $\frac{1}{f_1} = (\mu_1 - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$ and

$$= k_1(\mu_1 - 1) \quad \dots \dots \dots \text{(ii)}$$

Now $\frac{1}{f_2} = (\mu_2 - 1) \left(\frac{1}{R_1^1} - \frac{1}{R_2^1} \right)$ and

$$= k_2(\mu_2 - 1) \quad \dots \dots \dots \text{(iii)}$$

[When f_1 and f_2 refer to the respective focal lengths for the mean ray (Yellow) of the colors considered]

Where μ_1 and μ_2 are the refractive indices of the mean ray for the materials of the first and second lenses respectively.

Then, equ (i) reduces to

$$\frac{1}{f} = k_1(\mu_1 - 1) + k_2(\mu_2 - 1) \quad \dots \dots \dots \text{(iv)}$$

Equ (iv) shows that equivalent focal length f is the function of wavelength (λ), since both μ_1 and μ_2 are also function of wavelength.

Differentiating equ (4) w.r.t λ we get,

$$\frac{d}{d\lambda} \left(\frac{1}{f} \right) = k_1 \frac{d\mu_1}{d\lambda} + k_2 \left(\frac{d\mu_2}{d\lambda} \right) \quad \dots \dots \dots \text{(v)}$$

RESTRICTED

Achromatism of the combination is attended when the focal length f or $\left(\frac{1}{f}\right)$ does not change with color, i.e. wavelength. Hence, for achromatism $\frac{d}{d\lambda}\left(\frac{1}{f}\right)$ should be zero.

Thus the condition of achromatism is given by

$$k_1 \frac{d\mu_1}{d\lambda} + k_2 \left(\frac{d\mu_2}{d\lambda} \right) = 0 \dots \dots \dots \text{(vi)}$$

From equation (ii) and (iii) we have

$$k_1 = \frac{1}{f_1(\mu_1 - 1)} \quad \text{and} \quad k_2 = \frac{1}{f_2(\mu_2 - 1)}$$

Equ μ (vi), therefore, becomes

$$\frac{1}{f_1(\mu_1 - 1)} \frac{d\mu_1}{d\lambda} + \frac{1}{f_2(\mu_2 - 1)} \frac{d\mu_2}{d\lambda} = 0 \dots \dots \dots \text{(vii)}$$

If we deal with a finite change in refractive indices ($d\mu$) for a finite change in wavelength ($d\lambda$) of light between the two given colors (blue and red), then we may write,

$$f_1 \frac{d\mu_1}{(\mu_1 - 1)} + \frac{1}{f_2} \frac{d\mu_2}{(\mu_2 - 1)} = 0 \times d\lambda = 0 \dots \dots \dots \text{(viii)}$$

$$\text{But, } \frac{d\mu_1}{(\mu_1 - 1)} = w_1 \quad \text{and} \quad \frac{d\mu_2}{(\mu_2 - 1)} = w_2$$

Where w_1 and w_2 are the dispersive power of the materials of the two lenses for the blue and red rays.

Then from equ (viii) we get,

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \dots \dots \dots \dots \dots \text{(ix)}$$

Equ (ix) is the required condition of achromatism for two lenses in contact.

From equ (ix) it follows that,

- (1) One of the lens is convergent and the other is divergent.

RESTRICTED

Since $\mu_b > \mu r$, d_μ is positive and since $\mu > 1$ the dispersive power w_1 and w_2 are always positive. Hence, in order that the equ (ix) satisfied f_1 and f_2 should be of opposite signs.

(2) The lenses should be made of different Materials:

If we take two lenses of same material, then $w_1 = w_2 = w$ (say). Then condition (equ ix) reduces to,

$$w \left(\frac{1}{f_1} + \frac{1}{f_2} \right) = 0 \quad \text{or, } w \cdot \frac{1}{f} = 0$$

But, Since w cannot be Zero, $\frac{1}{f} = 0$ or, $f = \infty$, The combination will behave like a lens. This, achromatism cannot be achieved by taking two lenses of the same material in contact.

(3) The choice of the focal length and dispersive power of the lens is governed by (equ ix)