- What is the interference of light? Discuss the important conditions for the interference of light.

⇨ The redistribution of energy/light due to the superposition of two or more waves with the same or constant phase difference is called interference of light.

Conditions:

(i) **Monochromatic Light:**

In optics having a single frequency & wavelength are called monochromatic waves.

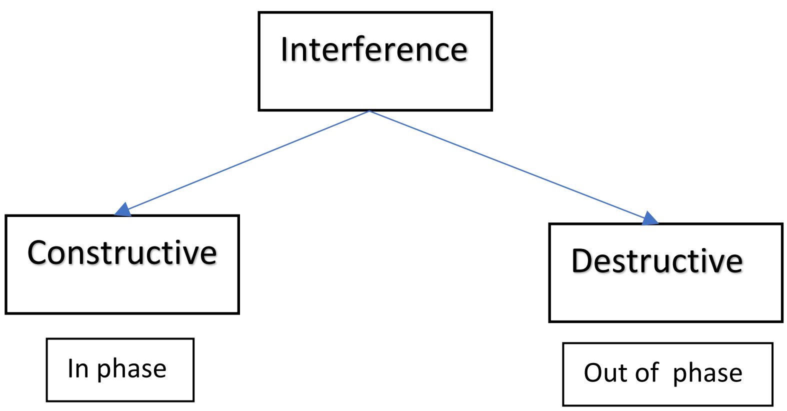
(ii) **Coherent Sources:** Same amplitude, frequency & constant phase difference.

Coherent source of light are those sources which emit a light wave having the same frequency, wavelength and in the same phase or they have a constant phase difference. A coherent source forms sustained interference patterns when superimposition of waves occurs and the positions of maxima and minima are fixed.

Two independent sources are never coherent or they cannot be considered coherent sources as all the above-mentioned factors cannot be present at the same time.

Coherent sources have the following characteristics:

- The waves generated have a constant phase difference(are in phase with each other)
- The waves are of a single frequency.
- The waves should have the same amplitude.



Producing Coherent Sources:

Path difference:

$\boldsymbol{\lambda}$ **=>** $\boldsymbol{2\pi}$

x => $\boldsymbol{\frac{2\pi}{\lambda}x}$

Huygens's Principle:

1. Every point on a primary wave front may be considered as a secondary source disturbance

2. Secondary wave same velocity to original

3. Envelops

- Discuss interference of light analytically and obtain conditions of maximum and minimum intensities.

(a) Analytical Method: Let us assume that the electric field components of the two waves arriving at point P vary with time as

$$E_A = E_1 \sin \omega t \tag{14.15}$$

and

$$E_B = E_2 \sin (\omega t + \delta) \tag{14.16}$$

where δ is the phase difference between them. According to Young's principle of superposition, the resultant electric field at the point P due to the simultaneous action of the two waves is given by

$$E_R = E_A + E_B \tag{14.17}$$

$$\begin{aligned} &= E_1 \sin \omega t + E_2 \sin (\omega t + \delta) \\ &= E_1 \sin \omega t + E_2 (\sin \omega t \cos \delta + \cos \omega t \sin \delta) \\ &= (E_1 + E_2 \cos \delta) \sin \omega t + E_2 \sin \delta \cos \omega t \end{aligned} \tag{14.18}$$

Equ. (14.18) shows that *the superposition of two sinusoidal waves having the same frequency but with a phase difference produces a sinusoidal wave with the same frequency but with a different amplitude E.*

Let $\quad E_1 + E_2 \cos \delta = E \cos \phi \quad (14.19)$

and $\quad E_2 \sin \delta = E \sin \phi \quad (14.20)$

where E is the amplitude of the resultant wave and ϕ is the new initial phase angle. In order to solve for E and ϕ, we square the equ.(14.19) and (14.20) and add them.

$$(E_1 + E_2 \cos \delta)^2 + E_2^2 \sin^2 \delta = E^2 (\cos^2 \phi + \sin^2 \phi)$$

or

$$E^2 = E_1^2 + E_2^2 \cos^2 \delta + 2E_1E_2 \cos \delta + E_2^2 \sin^2 \delta$$

or

$$E^2 = E_1^2 + E_2^2 + 2E_1E_2 \cos \delta \tag{14.21}$$

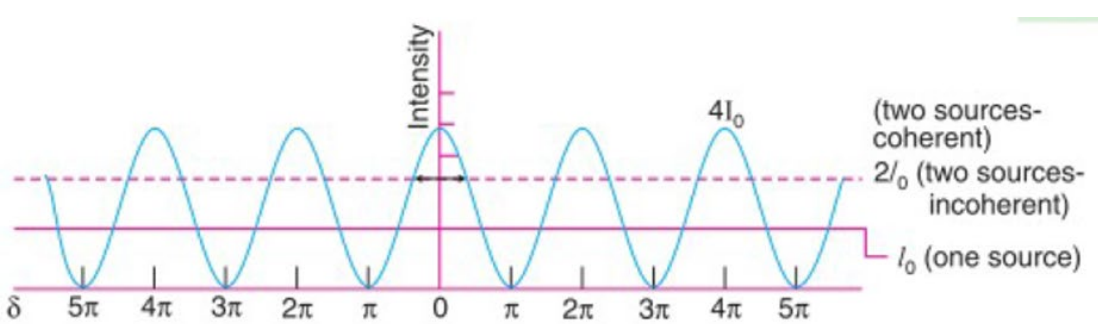
Thus, it is seen that the square of the amplitude of the resultant wave is not a simple sum of the squares of the amplitudes of the superposing waves, there is an additional term which is known as the *interference term*.

The intensity at a point is given by-

I = R^2

I = $4a^2\cos^2(\delta/2)$

For **$\delta/2$** = 0, I => max and for **$\delta/2$** = 90 or **δ** = 180, I => min



- Explain why two flashlights held close together do not produce an interference pattern on a distant screen.

- The light from the flashlights consists of many different wavelengths (that's why it's white) with random time difference between the light waves. There is no coherence between the two sources. The light from the two flashlights does not maintain a constant phase relationship over time. These three equivalent statements mean no possibility of an interference pattern.

What are coherent sources?

Coherent sources emit light waves with the same phase or with a constant phase difference.

Why should the source be coherent to obtain a sustained interference pattern?

The intensity of the bright fringe or the dark fringe does not fluctuate only when the source is coherent.

Why we cannot get an interference pattern with two different sources?

Two independent sources will not be coherent, so it cannot be used to produce an interference pattern.

Theory of interference:

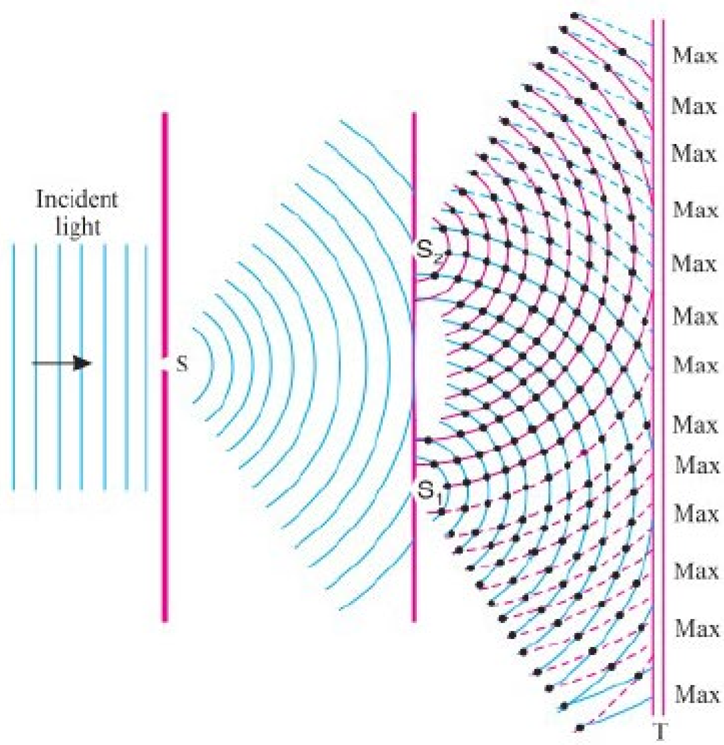


Fig. 14.10

14.5.1. OPTICAL PATH DIFFERENCE BETWEEN THE WAVES AT P:

Let P be an arbitrary point on screen T, which is at a distance D from the double slits. Let θ be the angle between MP and the horizontal line MO. Let S_1N be a normal on to the line S_2P. The distances

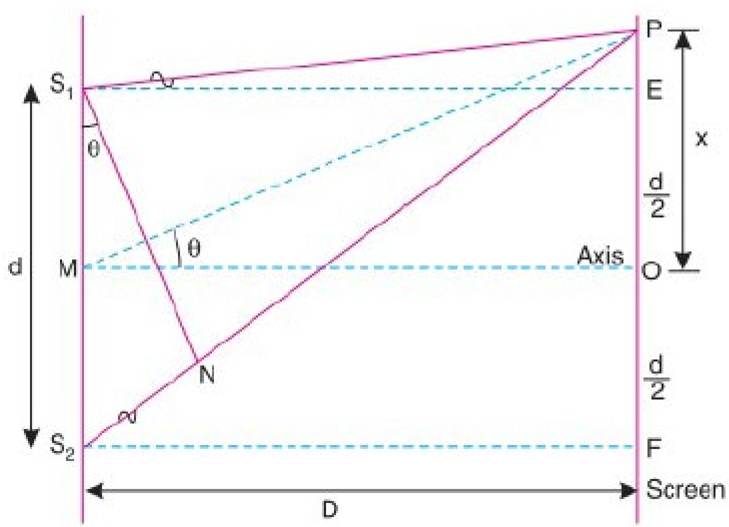


Fig. 14.11

PS_1 and PN are equal. The waves emitted at the slits, S_1 and S_2 are initially in phase with each other. The difference in the path lengths of these two waves is S_2N. We assume that the experiment is carried out in air. Therefore, the optical paths are identical with geometrical paths. The nature of the interference of the two waves at P depends simply on how many waves are contained in the length of the path difference S_2N. If S_2N contains an integral number of wavelengths, the two waves interfere constructively, producing a maximum in the intensity of light on the screen at P. If it contains an odd number of half-wavelengths, then the waves interfere destructively and produce a minimum intensity at P.

Let the point P be at a distance x from O (Fig.14.11). Then

PE $= x - d/2$ and PF $= x + d/2$.

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right]$$

$$(S_2P)^2 - (S_1P)^2 = 2xd$$

$$S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$$

We can approximate that $S_2P \cong S_1P \cong D$.

$$\therefore \text{ Path difference} = S_2P - S_1P = \frac{xd}{D} \qquad (14.28)$$

We now find out the conditions for observing bright and dark fringes on the screen.

If the optical path difference $\Delta = (\mu_2 r_2 - \mu_1 r_1)$ is equal to zero or an integral multiple of wavelength λ, then the waves arrive in phase at P and superpose with crest-to-crest correspondence. That is, if

$$\Delta = m\lambda \qquad (14.13)$$

where m is an integer and takes values, m $= 0,1,2,3,4,5,\ldots\ldots$, then the waves are in phase (see Fig.14.5a) and their overlapping at P produces constructive interference or brightness.

On the other hand, if the optical path difference $\Delta = (\mu_2 r_2 - \mu_1 r_1)$ is equal to an odd integral multiple of half-wavelength, $\lambda/2$, then the waves arrive out of phase at P and superpose with crest-to-trough correspondence. That is, if

$$\Delta = (2m+1)\frac{\lambda}{2} \qquad (14.14)$$

where m is an integer and takes values, m $= 0,1,2,3,4,5,\ldots\ldots$, then the waves are inverted with respect to each other (see Fig. 14.5b) and their overlapping at P produces destructive interference or darkness.

The regions of brightness and darkness are also known as regions of **maxima and minima**.

We now find out the conditions for observing bright and dark fringes on the screen.

14.5.2. BRIGHT FRINGES

Bright fringes occur wherever the waves from S_1 and S_2 interfere constructively. The first time this occurs is at O, the axial point. There, the waves from S_1 and S_2 travel the same optical path length to O and arrive in phase. The next bright fringe occurs when the wave from S_2 travels one complete wavelength further the wave from S_1. In general constructive interference occurs if S_1P and S_2P differ by a whole number of wavelengths.

The condition for finding a bright fringe at P is that

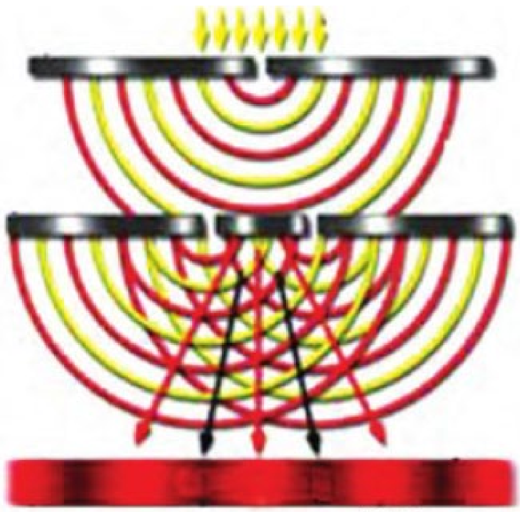
$$S_2P - S_1P = m\lambda$$

Using the equation (14.28), it means that

$$\frac{xd}{D} = m\lambda \qquad (14.29)$$

where m is called the **order of the fringe**.

The bright fringe B_o (at O), corresponding to m = 0, is called the *zero-order* fringe. It means the path difference between the two waves reaching at O is zero. Fringe at B_1 is the *first-order bright fringe* from the axis corresponding to m = 1; the path difference between the two waves reaching at B_1 is one λ. The *second order bright fringe* (m = 2) will be located where the path difference is 2λ and so on.



14.5.3. DARK FRINGES

The first dark fringe occurs when $(S_2P - S_1P)$ is equal to $\lambda / 2$. The waves are now in opposite phase at P. The second dark fringe occurs when $(S_2P - S_1P)$ equals $3\lambda / 2$. The m^{th} dark fringe occurs when

$$(S_2P - S_1P) = (2m + 1)\,\lambda / 2$$

The condition for finding a dark fringe is $\dfrac{xd}{D} = (2m+1)\dfrac{\lambda}{2}$ (14.30)

The *first-order dark fringe* D_1 (Fig. 14.12) from the axis corresponds to m = 0, where the path difference between the two waves is $\lambda/2$. The second order dark fringe (m = 1) will be produced where the path difference is $3\lambda/2$ and so on.

14.5.4. SEPARATION BETWEEN NEIGHBOURING BRIGHT FRINGES

The m^{th} order fringe occurs when $x_m = \dfrac{m\lambda\, D}{d}$

and the $(m+1)^{th}$ order fringe occurs when $x_{m+1} = \dfrac{(m+1)\lambda\, D}{d}$

The fringe separation, β is given by $\beta = x_{m+1} - x_m = \dfrac{\lambda D}{d}$ (14.31)

The same result will be obtained for dark fringes. Thus, the distance between any two consecutive bright or dark fringes is known as the ***fringe width*** and is the *same* everywhere on the screen. Further, the width of the bright fringe is equal to the width of the dark fringe. Therefore, the alternate bright and dark fringes are *parallel*. From the equ.(14.31), we find the following:

(*i*) The fringe width β is independent of the *order* of the fringe.It is directly proportional to the wavelength of light, i.e.$\beta \propto \lambda$. The fringes produced by red light are less closer compared to those produced by blue light.

(*ii*) The width of the fringe is *directly proportional* to the distance of the screen from the two slits, $\beta \propto D$. The farther the screen, the wider is the fringe separation.

(*iii*) The width of the fringe is *inversely proportional* to the distance between the two slits. The closer are the slits, the wider will be the fringes.

Text Books:

Book No.	Title	Author (s)	Edition
T-1	Physics for Engineers	Dr. Giasuddin Ahmad	1st
T-2	A Text Book of Optics	N. Subrahmanyam, Brijlal	22nd
T-3	Fundamentals of Optics	Francis A. Jenkins, White	4th

Interference of light:

If two beams of light cross each other at a certain point, in the region of cross over where both the beams are acting simultaneously, according to **superposition principle** a modification in their intensity is expected. The resultant intensity will be either great or less than that which would be given by one beam alone. This modification of intensity due to superposition of two or more beams of light is known as **interference of light**.

Superposition principle:

According to Thomas Young, when a medium is disturbed simultaneously by more than one wave, the instantaneous resultant displacement of medium at every point at every instant is the algebraic sum of the displacement of the medium that would be produced at the point by the individual wave trains if each were present alone. After the superposition at the region of crossover, the wave trains emerge unimpeded as if they have not met each other at all. Each wave train retains its individual characteristics. Each wave train behaves as if others are absent.

Suppose two trains cross each other at a certain point and let y_1 be the displacement of the point produced by the first wave in the absence of second wave. If y_2 be the displacement of the same point produced by the second wave in the absence of the first wave, then the resultant displacement y of the point due to the two waves acting together is expressed by

$y = y_1 + y_2$..(1)

If the two waves cross each other in phase then Eq. (1) can be written as-

$y = y_1 + y_2$ ....(2)

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If the two waves cross each other out of phase. Then Eq. (1) can be written as-

$y = y_1 \sim y_2$ ....(3)

From equation (2) we can say that, if the two individual displacements are in the same direction, the resultant displacement will be enhanced. So, two waves reinforce each other and are said to produce constructive interference. For example, in [Fig-1.1 (a)], two waves are of the same frequency but of different amplitudes, say a and b where a > b. when they reach a certain point in phase each other, then resultant displacement or amplitude is equal to the sum of the two amplitude i.e., (a + b).

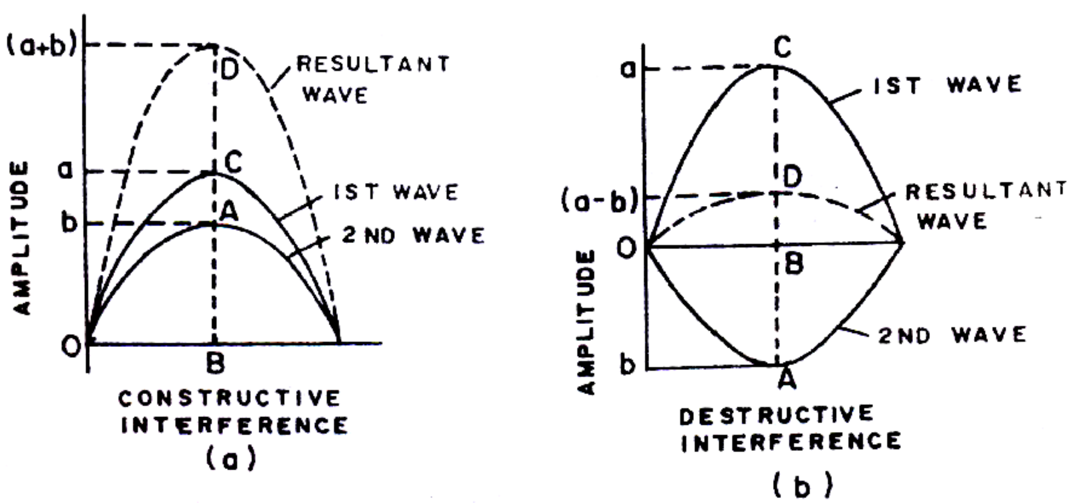

Fig.-1.1 (a) Constructive interference, (b) Destructive interference

From equation (3) we can say that, if the two individual displacements are in the opposite direction, the resultant displacement will be diminished. So, two waves neutralize each other and are said to produce **destructive interference**. For example, in Fig.-[1.1(b)], two waves are π radians or 180^0 out of phase with each other, the resultant amplitude is equal to the difference of the two amplitudes i.e., (a-b). If in addition, a = b, then the resultant amplitude is zero.

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Conditions of the interference:

a) The two beams of light which interfere must be coherent i.e., must originate from the same source of light.
b) The two interfering waves must have the some amplitude.
c) The original source must be monochromatic.
d) The fringe-width should reasonably be as large as possible and the separation between the two sources should be as small possible while the distance of the screen form the sources should be as large as possible.
e) The two interfering waves must be propagated in almost the same direction.

Uses of interference:

1. To determine the refractive index of liquids and gases.
2. To determine the width of a liquid and glass plate.
3. To determine the smoothness of a liquid and glass plate.
4. To determine the wavelength of light.
5. To measure the thickness of a very thin film.

Coherent Sources:

Two sources are said to be coherent if they emit light waves of the same frequency, nearly the same amplitude and are always in phase with each other. It means that the two sources must emit radiations of the same colour (wavelength). In actual practice it is not possible to have two independent sources which are coherent. But for experimental purposes, two virtual sources formed from a single source can act as coherent sources.

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Relation between path difference and phase difference:

If the path difference between the two waves is λ, the phase difference = 2π
Suppose for a path difference x, the phase difference is δ.
For a path difference λ, the phase difference is = 2π
According to unitary method, we can write
The path difference λ is equal to phase difference 2π

$\therefore$ 1 $\frac{2\pi}{\lambda}$

$\therefore$ x $\frac{2\pi}{\lambda}$ x

So, phase difference $\delta = \frac{2\pi}{\lambda}$ x

Or, phase difference $\delta = \frac{2\pi}{\lambda} \times$ path difference(x)

Or, $\frac{\delta}{2\pi} = \frac{x}{\lambda}$.

Interference of two light waves: analytical treatment

Consider a monochromatic source of light S emitting waves of wavelength λ and two slits A and B (Fig.-1.2). A and B are equivalent from S and act as two virtual sources. Let a be the amplitude of the waves. The phase difference between the two waves reaching the point P, at any instant, is δ.

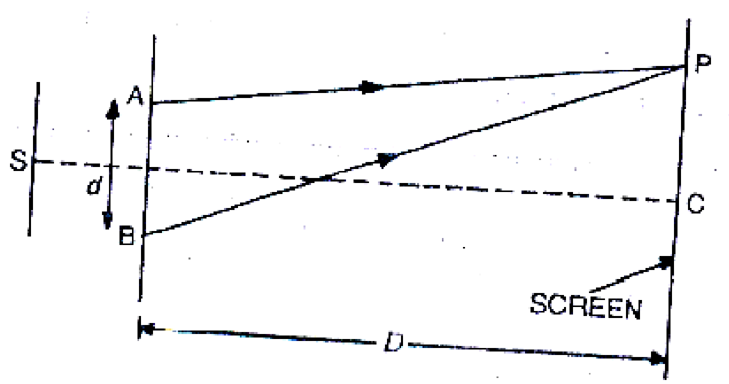

Fig.-1.2

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Let y_1 be the displacement of the particle P due to waves emanating from the source A alone (in the absence of waves coming from B) at any instant of time.

$y_1 = a \sin \omega t$ ....(1)

Let y_2 be the displacement of the particle P due to waves emanating from the source B alone (in the absence of waves coming from A) at any of time.

$y_2 = a \sin(\omega t + \delta)$ ....(2)

If y be the resultant displacement of the particle P at that instant of time then according to principle of superposition-

$$y_1 + y_2 = a \sin \omega t + a \sin(\omega t + \delta)$$
$$= a \sin \omega t + a \sin \omega t \cos\delta + a \cos \omega t \sin\delta$$
$$= a \sin \omega t (1 + \cos\delta) + a \cos \omega t \sin\delta \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Taking $a(1+\cos\delta) = R\cos\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$

And $a\sin\delta = R\sin\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$

$$\therefore y = R \sin \omega t \cos\theta + R \cos \omega t \sin\theta$$
$$= R\sin(\omega t + \theta)$$

which represents the equation of simple harmonic vibration of amplitude R. Squaring (4) and (5) and adding

$$R^2 \sin^2\theta + R^2 \cos^2\theta = a^2 \sin^2\delta + a^2 (1+\cos\delta)^2$$
$$R^2 = a^2 \sin^2\delta + a^2 + a^2 \cos^2\delta + 2a^2 \cos\delta$$
$$R^2 = 2a^2 + 2a^2 \cos^2\delta = 2a^2 (1+\cos\delta) = 2a^2.2.\cos^2\frac{\delta}{2}$$
$$R^2 = 4a^2\cos^2\frac{\delta}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(6)$$

We know that, the intensity at a point is given by the square of the amplitude

$$I = R^2$$
$$I = R^2 = 4a^2\cos^2\frac{\delta}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Special case:

(1) When the phase difference $\delta = 0, 2\pi, 2(2\pi), 3(2\pi), \ldots\ldots\ldots n(2\pi)$ or the path difference $x = 0, \lambda, 2\lambda, \ldots\ldots\ldots n\lambda$, The intensity is

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$I = 4a^2$

Intensity is maximum when the phase difference is a whole number multiple of 2π of the path difference is a whole number multiple of wavelength.

(2) When the phase difference $\delta = \pi, 3\pi, \ldots\ldots\ldots (2n+1)\pi$ or the path difference $x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \ldots\ldots\ldots (2n+1)\frac{\lambda}{2}$, The intensity is

$I = 0$

Intensity is minimum when the phase difference is an odd number multiple of π or the path difference is an odd number multiple of half wavelength.

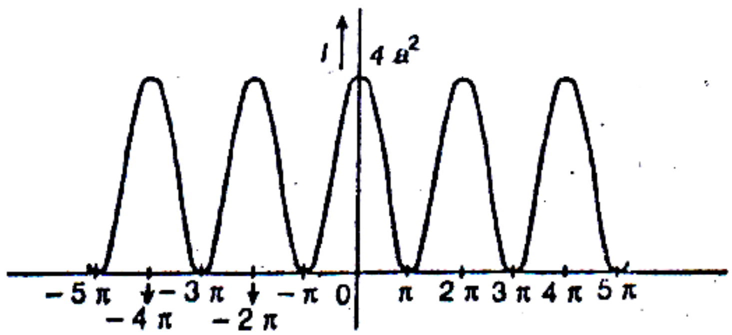

Fig.-1.3

From equation (7), it is found that the intensity at bright points is $4a^2$ and at dark points it is zero. According to the law of conservation of energy, the energy can not be destroyed. Here also the energy is not destroyed but only transferred form the points of minimum intensity to the points of maximum intensity.

Young's double-slit experiment

Historically, the phenomenon of interference of light was first demonstrated by Thomas Young in about 1801 by a simple experiment. Young allowed sunlight to pass through a pin hole S and then at some distance through two sufficiently close pin holes $S_1$and S_2 in an opaque screen. Finally the light was received on a screen on which he observed an uneven distribution of light intensity consisting of many alternate bright and dark spots. The corpuscular theory was found to be totally inadequate to explain this. On the other hand, Young was able to explain this due to superposition of two light waves. This

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experiment, described below, was regarded as crucial one at that time, since it definitely established the wave nature of light.

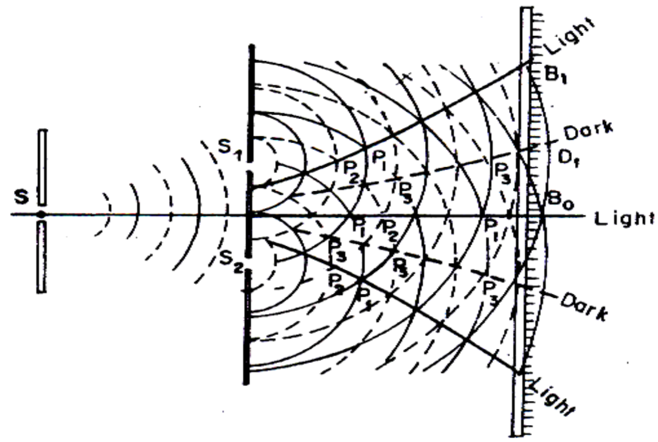

Fig.-1.3a

In accordance with the modern laboratory techniques Young's experiment is performed by illuminating a narrow parallel slit S with monochromatic light of wavelength λ. The light coming out of the slit S is then allowed to fall on two more narrow parallel equidistant slits S_1 and S_2 on a screen placed at a certain distance to the right of S (Fig.-1.3a). According to Huygen's principle, cylindrical waves spread out from slit S and reach the slits S_1 and S_2. As the slits S_1 and S_2 are equidistant from S the waves reach the slits at the same time i.e., S_1 and S_2 are on the same wavefront. A train of secondary wavelets, having the same amplitude, velocity, wavefront and precisely the same phase at the start, therefore, diverge to the right from both of these slits. Let the crest and the trough in each wave be represented by continuous and dotted circular arcs respectively. Furthermore, let the points where a crest of one wave is superposed on the crest of another wave or a trough of one wave is superposed on the trough of another wave be marked by P_1 and P_2 respectively and the points where the crest of one wave is superposed on the trough of another wave be marked by P_3. If a screen be placed at a certain distance from the slits S_1 and S_2, solid lines connecting the points marked P_1s and P_2s will intersect the screen at points B_o, B_1. Since the resultant intensity along these lines is always maximum (constructive interference), the points B_o, B_1 will appear as bright lines on the screen. Similarly D_1s, the points of intersection of lines, connecting the points marked P_3s with the screen will represent points of zero intensity (destructive interference) and consequently appear on the screen as dark lines. Thus the result of

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interference between waves coming from the slits S_1 and S_2 will appear on the screen as alternate bright and dark lines. As long as the experimental arrangement remains undisturbed, the alternate bright and dark lines on the screen remain stationary. This is known as interference pattern. That the observed pattern is truly due to interference of two waves of light can be demonstrated by covering one of the slits. Then the well defined dark and bright lines on the screen are replaced by a pattern much coarser due to diffraction of light by the uncovered single light. Thus a point on the screen, bright when only one slit is uncovered changes to dark when both the slits are uncovered. This cannot be explained on the basis of corpuscular theory of light, but can be readily explained on the basis of interference of two waves of light. The dark and bright lines are usually referred to as fringes.

Theory of interference fringes: expression for the width of a fringe.

Consider a narrow monochromatic source S and two slits A and B, equidistance form S. A and B act as two coherent sources separated by a distance d. Let a screen be placed at a distance D from the coherent sources. The point C on the screen is equidistance form A and B. Therefore, the path difference between the two waves is zero. Thus, the point C has maximum intensity.

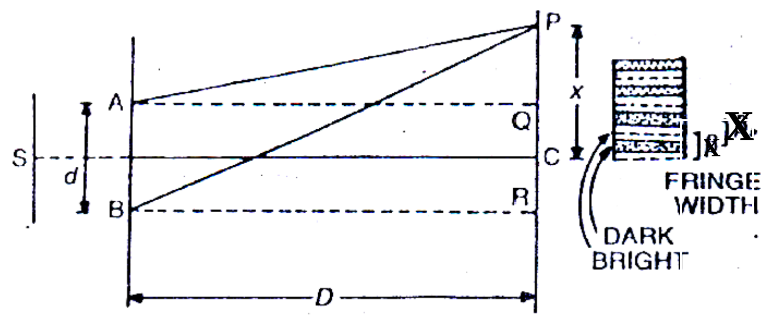

Fig.-1.4

Consider a point P at a distance x from C. The wavelength reaches at the point P from A and B.

Here, PQ = $x - \frac{d}{2}$, PR = $x + \frac{d}{2}$

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$(\mathrm{AP})^2 = \left[D^2 + \left(x - \frac{d}{2}\right)^2\right]$, $(\mathrm{BP})^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right]$

$$(\mathrm{BP})^2 - (\mathrm{AP})^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right]$$

(BP-AP) (BP +AP) = 2xd

$$\mathrm{BP} - \mathrm{AP} = \frac{2\mathrm{xd}}{\mathrm{BP} + \mathrm{AP}}$$

But BP = AP = D (approximately)

$$\text{Path difference} = \frac{\mathrm{xd}}{\mathrm{D}}$$

$$\text{Phase difference} = \frac{2\pi}{\lambda}\left(\frac{\mathrm{xd}}{\mathrm{D}}\right)$$

(1) Bright fringes:

If the path difference is a whole multiple of wavelength λ, the point P is bright.

$$\therefore \frac{xd}{D} = n\lambda$$

Where n = 0, 1, 2, 3, … … …

Or, $x_n = \frac{n\lambda D}{d}$

This equation gives the distance of the bright fringe form the point C. At C, the path difference is zero and a bright fringe is formed.

When n = 1, $x_1 = \frac{\lambda D}{d}$

n = 2, $x_2 = \frac{2\lambda D}{d}$

n = 3, $x_3 = \frac{3\lambda D}{d}$

Therefore the distance between any two consecutive bright fringes

$$x_2 - x_1 = \frac{2\lambda D}{d} - \frac{\lambda D}{d} = \frac{\lambda D}{d} \ldots\ldots\ldots\ldots\ldots(1)$$

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Thus the distance between any two consecutive bright fringes is same or all bright fringes are equally spaced.

(2) **Dark fringes:**

If the path difference is an odd number multiple of half wavelength λ, the point P is dark.

$$\therefore \frac{\text{xd}}{D} = (2n+1)\frac{\lambda}{2}$$

Where n = 0, 1, 2, 3, ...

$$\text{Or, } x_n = \frac{(2n+1)\lambda D}{2d}$$

This equation gives the distance of the dark fringe form the point C.

When n = 1, $x_1 = \frac{3\lambda D}{2d}$

n = 2, $x_2 = \frac{5\lambda D}{2d}$

n = 3, $x_3 = \frac{7\lambda D}{2d}$

Therefore the distance between any two consecutive bright fringes

$$x_2 - x_1 = \frac{5\lambda D}{2d} - \frac{3\lambda D}{2d} = \frac{\lambda D}{d} \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Thus the distance between any two consecutive dark fringes is same or all dark fringes are equally spaced.

Moreover, from equations (1) and (2), it is clear that the width of the bright fringe is equal to the width of the dark fringe. This distance between any two consecutive bright or dark fringes is called the **fringe-width (X).**

$$\text{X} = \frac{\lambda \text{D}}{\text{d}}$$

From this equation it can be seen that

$$\lambda = \frac{\text{Xd}}{\text{D}}$$

Thus if the fringe-width, the distance of separation of the sources and the distance of the screen from the sources are known, then the phenomenon of interference can be employed to determine the wavelength of unknown monochromatic light.

Interference in thin film:

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Everyone is familiar with the brilliant colours produced by a thin film of oil on the surface of water and a thin film of a soap bubble. The explanation of the origin of this colour phenomenon was given by Young, in 1802, in terms of the interference of light waves reflected from the upper and the lower surface of the thin film. It has been observed that interference in the case of thin film takes place due to both reflected as well as transmitted light.

Interference due to reflected light from a plane parallel (thin) film:

Consider transparent film of thickness t and refractive index μ, bounded by two parallel surfaces MN and PQ. A ray AB of monochromatic light is incident on the upper (Fig.-1.5) surface at the point B. A part of it is reflected along BC and a part is refracted along BD. The refracted beam is again partly reflected at the point D back into the medium along DE and the rest refracts into the surrounding medium along DV. The ray along DE suffers both reflection and refraction at point E on the upper surface MN. The refracted ray goes along EF. The difference in path between BC and EF can be calculated. Draw, El normal to BC and BR normal to DE. Also produce ED to meet BT produced at S.

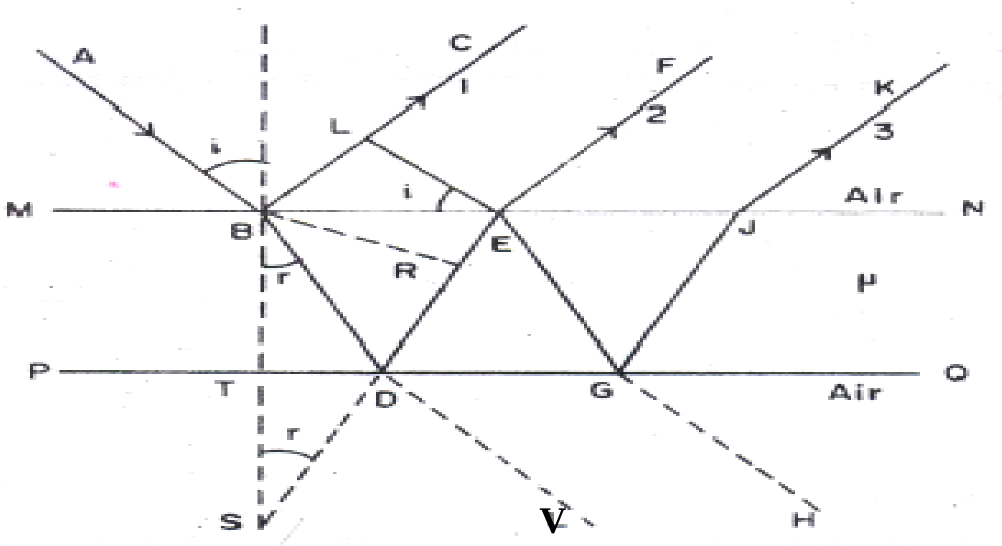

Fig.-1.5: Interference due to reflected light (thin) film

Let the angle of incidence and refraction be i and r respectively. The optical path difference

$x = \mu\,(BD + DE) - BL$

$= \mu\,(BD + DR + RE) - BL$ ....(1)

Also, BD=DE

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Here, μ = $\frac{\sin i}{\sin r} = \frac{BL/BE}{RE/BE} = \frac{BL}{RE}$

Or, BL=μRE ....(2)

$\therefore$ The path difference x = μ (RD + DS) = μRS

In the triangle BSR,

cos r = $\frac{RS}{BS}$

RS = BS cos r

= (BT +TS) cos r

=2t cos r ....(3)

Where BT = TS = t

$\therefore$ x = μ RS = 2t μ cos r ....(4)

Since an abrupt change of π (equivalent to a path difference of $\frac{\lambda}{2}$) is introduced whenever a ray is reflected from a surface backed by a denser medium, equation (4) does not represent the total path difference between BC and BDEF. Taking into account this additional path difference of $\frac{\lambda}{2}$ for reflection suffered at the point D the total path difference will be

x = 2t μcos r $\pm \frac{\lambda}{2}$

Therefore, for constructive interference or brightness

2t μcos r $\pm \frac{\lambda}{2}$ = nλ

2t μcos r = $(2n \pm 1)\frac{\lambda}{2}$ ………………………………Bright

In terms of phase difference, there will be constructive interference or brightness, when the total phase difference

$$\delta = \frac{2\pi}{\lambda}(2n \pm 1)\frac{\lambda}{2}$$

$= (2n \pm 1)\pi$ …………………………………Bright

And for destructive interference or darkness

2t μcos r $\pm \frac{\lambda}{2}$ = $(2n \pm 1)\frac{\lambda}{2}$

2t μcos r = nλ ……………………………..Dark

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In terms of phase difference, there will be destructive interference or darkness, when the total phase difference

$$\delta = \frac{2\pi}{\lambda}\, n\lambda \;=\; 2\pi\, n \text{ ……………………………..Dark}$$

Where n = 0, 1, 2, 3, … … …etc.

It should be remembered that the interference pattern will not be perfect because the amplitudes of the rays BC and EF are not same.

Interference due to reflected light from a plane of varying thickness (wedge shaped film):

Suppose the film is not parallel sided but is in the shape of thin wedge i.e., its surfaces make an angle θ with each other. Consider the film to be illuminated with monochromatic light. The incident light wave propagating along AB will give rise to two light waves- the directly reflected ray along BR and the internally reflected ray along B_1R_1. The rays BR and B_1R_1 are, therefore, coherent and capable of producing interference.

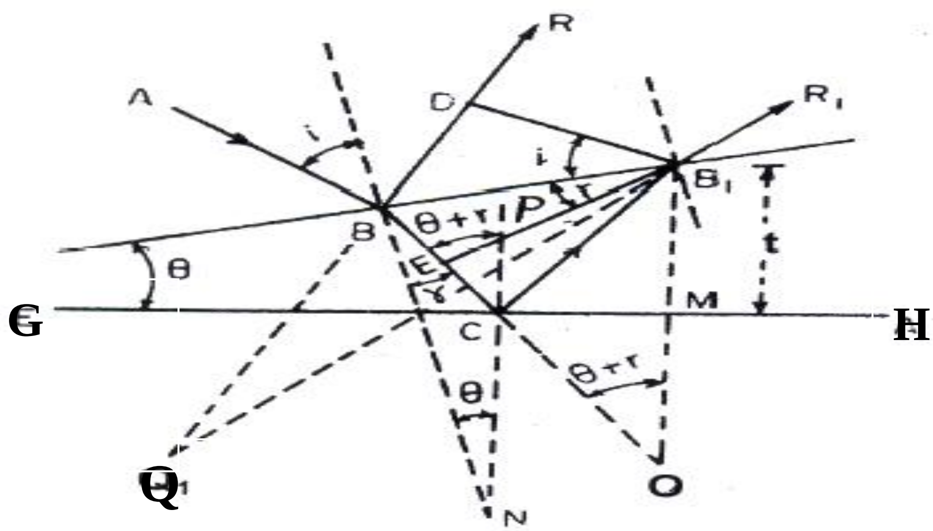

Fig.-1.6: Interference due to reflected light (thin film of varying thickness)

To derive an expression for the optical path difference between them, let us draw an arc with the optical path difference between them, let us draw an arc with the point Q as the center and QB_1 as the radius. The arc B_1D will be approximately straight and perpendicular to BR. From points B_1 and D onwards, the two waves or rays travel equal distances. Let us also draw a perpendicular B_1E from B_1 on BC. The optical path

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difference x, between the waves BR and B_1R_1 in reaching the arc B_1D from B, the point of their origination, is expressed by

$x = \mu\,(BC + CB_1) - BD$

$x = \mu\,(BE + EC + CB_1) - BD$....................(1)

Now the angle $\angle DB_1B = i$ = angle of incident and

Angle $\angle BB_1E = r$ = angle of refraction

$\sin i = \dfrac{BD}{BB_1}$ and $\sin r = \dfrac{BE}{BB_1}$

Hence $\dfrac{\sin i}{\sin r} = \mu = \dfrac{BD}{BE}$

Or, $BD = \mu\, BE$..............................(2)

Hence equation (1) becomes

$x = \mu\,(BE + EC + CB_1) - \mu BE$

$= \mu\,(EC + CB_1)$..........................(3)

Let BN and CN be the normal to the upper and lower surfaces of the film respectively. Hence we get

$\angle CNB = \theta$

$\angle PCB = \angle CBN + \angle CNB = \theta + r$ and $\angle PCB_1 = \angle PCB = \theta + r$

In the fig-1.6, B_1M is perpendicular from B_1 of the lower surface of the film and BC when produced further intersects it at O. Thus we have the relation

$\angle B_1OC = \angle PCB = \theta + r$.

Also $\angle CB_1O = \angle PCB_1 = \theta + r = \angle B_1OC$. Thus B_1OC is an isosceles triangle. Hence $CB_1 = CO$.

Equation (3), therefore reduces to

$x = \mu\,(EC+CO) = \mu\, EO = \mu B_1O \cos(\theta+r)$.............................(4)

If this thickness of the film at the point B_1 and since $B_1M = MO = t$,

$x = \mu\, 2t \cos(\theta+r) = 2t\,\mu \cos(\theta+r)$ ....(5)

The path difference x, therefore, varies both on account of changing thickness as well changing angle of incidence, provided the broad light source is at a finite distance from the film. Equation (5) does not, however, represent the total path difference between the rays. Now we have to consider a path difference of $\frac{\lambda}{2}$ introduced as a result of reflection

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at the point B which represents reflection at a surface backed by a denser medium. Thus the total path difference between the rays is

$$x = 2t\,\mu\cos(\theta + r) \pm \frac{\lambda}{2}$$

Therefore, for constructive interference or brightness

$$2t\mu\cos(\theta + r) \pm \frac{\lambda}{2} = n\lambda$$

$$2t\mu\cos(\theta + r) = (2n \pm 1)\frac{\lambda}{2} \text{ ....Bright}$$

In terms of phase difference, there will be constructive interference or brightness, when the total phase difference

$$\delta = \frac{2\pi}{\lambda}(2n \pm 1)\frac{\lambda}{2}$$

$$= (2n \pm 1)\pi \text{ ………………………………Bright}$$

And for destructive interference or darkness

$$2t\mu\cos(\theta + r) \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$$

$$2t\mu\cos(\theta + r) = n\lambda \text{ ……………………………..dark}$$

In terms of phase difference, there will be destructive interference or darkness, when the total phase difference

$$\delta = \frac{2\pi}{\lambda} n\lambda = 2\pi n \text{ ……………………………..dark}$$

Where n = 0, 1, 2, 3, … … …etc.

Consider the wedge shaped film to be illuminated by a parallel beam of monochromatic light of wavelength λ. Then the angle of incidence i, will be constant at every point of the film and so will be r, the angle of refraction. The total optical path difference will, therefore, be only due to variation of the thickness, t, from point to point of the film. At the edge of the wedge, since t = 0, the film appears perfectly dark and the two interfering waves are π out of phase. At distances from the edge where the total path difference

$$x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots\ldots\ldots \text{etc. the film is bright.}$$

While at distances where

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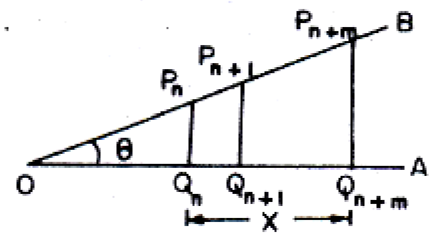
$x = \lambda, 2\lambda, 3\lambda, 5\lambda, \ldots\ldots\ldots$ etc. the film appears to be dark.

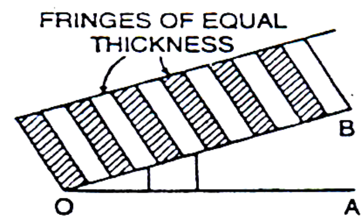
Thus as we go along the wedge in the direction of increasing thickness there will be alternate dark and bright bands parallel to the edge of the film.

To determine the fringe-width produced in a wedge shaped thin film

Consider two plane surfaces OA and OB inclined at an angle θ and enclosing a wedge shaped air film. The thickness of the air film increase from O to A (Fig-1.7). When the air film is viewed with reflected monochromatic light, a system of equidistance interference fringes is observed which are parallel to the line of intersection of the two surfaces. The effect is best observed when the angle of incidence is small.



a. Wedge shaped thin film



b. Fringes

Fig.-1.7: Fringe-width produced in a wedge shaped thin film

Suppose the rays are incident normally on the film which is in air, i.e. μ = 1. Then the angle of refraction is very small and since angle θ is also very small cos (θ+r) = 1. Under this condition the n^{th} bright fringe occurs at the point P_n where the thickness of the film, $t = P_nQ_n$. (Fig-1.7a).

Applying the relation for bright fringe (for reflected light),

$$2t\mu\cos(\theta + r) = (2n \pm 1)\frac{\lambda}{2}$$

$$2P_nQ_n = (2n \pm 1)\frac{\lambda}{2} \qquad [\mu = 1, \cos(\theta+r) = 1]$$

The next bright fringe (n+1) will occur at P_{n+1} such that

$$2P_{n+1}Q_{n+1} = [2(n+1) \pm 1]\frac{\lambda}{2}$$

Subtracting

$$P_{n+1}Q_{n+1} - P_nQ_n = \frac{\lambda}{2}$$

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Thus the next bright fringe will occur at the point where the air-film thickness has increase by $\frac{\lambda}{2}$. Suppose P_{n+m} represents the position of the $(n+m)^{th}$ bright fringe. Hence, there will be m bright fringes between P_n and P_{n+m}.

$$P_{n+m}Q_{n+m} - P_nQ_n = m\frac{\lambda}{2}$$

Let the distance $Q_nQ_{n+m} = x$

Then the angle of inclination θ, between OA and OB is

$$\theta = \frac{P_{n+m}Q_{n+m} - P_nQ_n}{Q_nQ_{n+m}} = \frac{m\lambda}{2x}$$

Or, $x = \dfrac{m\lambda}{2\theta}$

Since x is the distance corresponding to m fringes, the fringe width

$$X = \frac{x}{m} = \frac{\lambda}{2\theta}$$

Newton's rings:

When a plano-convex or bi-convex lens of large radius of curvature is placed on a glass plate p, a thin air film of progressively increasing thickness in all directions form the point of contact between the lens and the glass plate is very easily formed (Fig.-1.9) The air film thus possesses a radial symmetry about the point of contact. When it is illuminated normally with monochromatic light, an interference pattern consisting of a series of alternate dark and bright circular rings, concentric with the point of contact is observe (Fig-1.8). The fringes are the loci of points of equal optical film thickness and gradually become narrower as their radii increase until the eye or the magnifying instrument can no longer separate them. The rings are localized in the air film. Since the phenomenon was first examined in detail by Newton, the rings are termed as Newton's rings.

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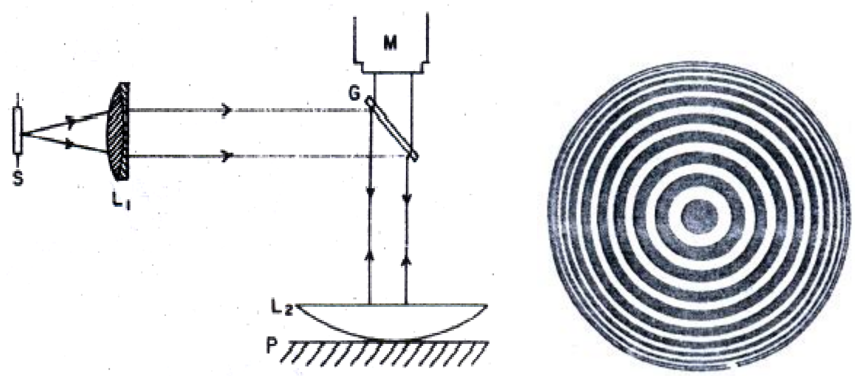

(a) **(b)**

Fig.-1.8: (a) Newton's rings experiment apparatus, (b) Newton's rings

Theory of Newton's rings:

Let us consider a ray of monochromatic light AB from an extended source to be incident at the point B on the upper surface of the film (Fig-1.9). One portion of the ray is reflected from point B on the glass-air boundary and goes upwards along BC. The other part refracts into the air film along BD. At point D, part of the light is again reflected along DEF but with an abrupt phase reversal of π (or a path difference of $\frac{\lambda}{2}$).

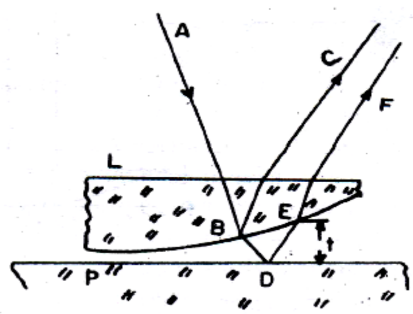

Fig.-1.9: Two virtual rays from one monochromatic ray

The two reflected rays BC and BDEF are derived from the same source and are coherent. They will produce constructive or destructive interference depending on their path difference. Let t be the thickness of the film at the point E and let the tangent to the convex surface at the point be inclined at an angle θ with the horizontal. Then the optical path difference between the two rays is given by

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$$2t\mu\cos(\theta + r) \pm \frac{\lambda}{2}$$

Where r is the angle of refraction at the point B and μ is the refractive index of the film with respect to air.

Thus the two rays will interfere constructively when

$$2t\mu\cos(\theta + r) \pm \frac{\lambda}{2} = n\lambda$$

$$2t\mu\cos(\theta + r) = (2n-1)\frac{\lambda}{2} \text{...Bright (1)}$$

The minus sing has been chosen purposely since n con not have a value of zero for bright fringes seen in reflected light.

The rays will interfere destructively when

$$2t\mu\cos(\theta + r) \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$$

$$2t\mu\cos(\theta + r) = n\lambda \text{..Dark (2)}$$

λ is the wavelength of light in air.

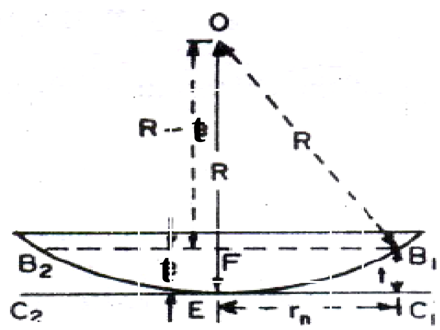

Fig.-1.10: Thickness of the air film

In practice, however, a thin lens of extremely small curvature is used in order to keep the film enclosed between the lens and the plane glass plate extremely thin. As a consequence, the angle θ becomes negligibly small as compared to r. Furthermore, the experimental arrangement is so designed (Fig-1.10) that the light is incident almost normally on the film and is viewed from nearly normal directions by reflected light, so that cos (θ+ r) = 1. Accordingly eqns. (1) and (2) reduce to

$$2t\mu = (2n-1)\frac{\lambda}{2} \text{...............................Bright (3)}$$

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And

$2t\mu = n\lambda$ ………..……………………….Dark (4)

Let us now compute the radius of any ring. Let R be the radius of curvature of the convex surface which rests on the plane glass surface (Fig.-1.10). From the right angled triangle OFB_1, we get the relation

$$R^2 = r_n^2 + (R - t)^2$$

$$r_n^2 = 2Rt - t^2$$

Where r_n is the radius of the circular ring corresponding to the constant film thickness t. As outlined above, the condition of the experiment makes t extremely small; so to a sufficient degree of accuracy, t^2 may be neglected compare to 2Rt. Then

$$t = \frac{r_n^2}{2R}$$

Substituting the value of t in the above expressions [(3) and (4)] for bright and dark rings, we have

$$r_n^2 = (2n-1)\frac{\lambda R}{2\mu} \text{ ……………………………Bright (5)}$$

And $r_n^2 = \frac{n\lambda R}{\mu}$ …………………………….…Dark (6)

The square of the diameters of the bright and dark rings are, therefore, given by the expressions

$$D_n^2 = 2(2n-1)\frac{\lambda R}{\mu} \text{ ……………………………Bright (7)}$$

$$D_n^2 = \frac{4n\lambda R}{\mu} \text{ ………………………………..…Dark (8)}$$

Thus the diameters of the different bright rings may be written as

1st ring, $D_1 = \sqrt{1}\sqrt{\frac{2\lambda R}{\mu}}$

2nd ring, $D_2 = \sqrt{3}\sqrt{\frac{2\lambda R}{\mu}}$

3rd ring, $D_3 = \sqrt{5}\sqrt{\frac{2\lambda R}{\mu}}$ and so on.

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Hence it can be seen that the diameters (also radii) of bright rings are proportional to the square root of the odd natural numbers.

Similarly, the diameters of the different dark rings can be written as

Central ring $D_0 = 0$

1^{st} ring, $$D_1 = 2\sqrt{1}\sqrt{\frac{\lambda R}{\mu}}$$

2^{nd} ring, $$D_2 = 2\sqrt{2}\sqrt{\frac{\lambda R}{\mu}}$$

3^{rd} ring, $$D_3 = 2\sqrt{3}\sqrt{\frac{\lambda R}{\mu}}$$ and so on.

It is obvious that the diameters (also radii) of the dark rings are proportional to the square root of the natural numbers.

At the point of contact of the lens and the glass plate, t = 0; therefore, the total phase difference between directly and internally reflected rays reduces to π. As a consequence, when Newton's rings are viewed in reflected light, the central spot appears to be dark. This central spot is surrounded alternately by a large number of bright and dark rings. This is very interesting result. How could you get bright at central spot?

If we consider the difference in diameters of the 5^{th} and 4^{th} dark rings, then

$$D_5 - D_4 = 2\left(\sqrt{5} - \sqrt{4}\right)\sqrt{\frac{\lambda R}{\mu}} = 0.46\sqrt{\frac{\lambda R}{\mu}}$$

And that between the 17^{th} and 16^{th} dark rings

$$D_{17} - D_{16} = 2\left(\sqrt{17} - \sqrt{16}\right)\sqrt{\frac{\lambda R}{\mu}} = 0.26\sqrt{\frac{\lambda R}{\mu}}$$

Thus it is clear that the alternate bright and dark rings surrounding the central dark spot in Newton's rings gradually become narrower as their radii increases.

This is also very interesting result. Do you know why it is happened?

Determination of wavelength:

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In the laboratory, the diameters of the Newton's rings can be measured with traveling microscope. Usually a little away from the centre, a bright (or dark) ring is chosen which is clearly visible and its diameter measured. Let it can be the nth order ring. For an air film $\mu = 1$. Then we have

$$D_n^2 = 2(2n-1)\lambda R \text{ ……………………………Bright (9)}$$

and

$$D_n^2 = 4n\lambda R \text{ ………………………………..…Dark (10)}$$

The wavelength of the monochromatic light employed to illuminate the film can be computed either of the above equations, provided R is known.

However, in actual practice, another ring, p rings from this ring onwards, is selected. The diameter of this (n+p)th ring is also measured. Then we have

$$D_{n+p}^2 = 2[2(n+p)-1]\lambda R$$

$$D_{n+p}^2 = 2(2n+2p-1)\lambda R \text{ ……………………………Bright (11)}$$

and

$$D_{n+p}^2 = 4(n+p)\lambda R \text{ ………………………………..….Dark (12)}$$

Subtracting either eqn. (9) from (11) or eqn. (10) from (12), we get for both the dark and bright rings the relation

$$D_{n+p}^2 - D_n^2 = 4p\lambda R$$

Or, $\lambda = \dfrac{D_{n+p}^2 - D_n^2}{4pR}$ …………………………..(13)

Thus, if the radius of curvature of the surface of the lens is known the eqn. (13) can be used to determine the wavelength of the light used.

Determination of radius of curvature of the lens:

Equation (13) can be rearranged as

$$R = \frac{D_{n+p}^2 - D_n^2}{4p\lambda} \text{ ……………………………(14)}$$

Thus, if the wavelength of the light used is known then eqn. (14) can be used to determine the radius of curvature of the surface of the lens in contact with the plane glass plate.

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Determination of refractive index of a liquid with Newton's rings:

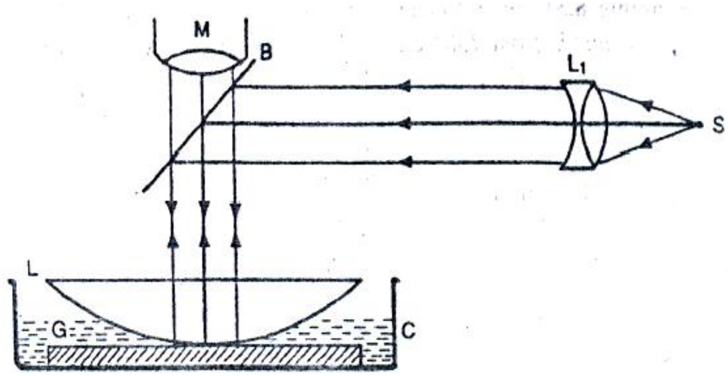

Fig.-1.11: Refractive index of a liquid with Newton's rings

It is possible to determine the refractive index of a liquid by Newton's rings method. The diameters of two particular rings, say the n^{th} and $(n + p)^{th}$, obtained in Newton's rings with an air film, are measured. The difference in diameters of the two rings is

$$(D^2_{n+p} - D^2_n)_{air} = 4\,p\lambda R \text{ ……………………..(15)}$$

Then a drop of liquid, whose refractive index is to be measured, is carefully introduced into the air film. This liquid is drawn in at the centre by the capillary action forming a liquid film between the lens and the plate. When the film is illuminated with the same monochromatic light, another set of Newton's rings is obtained. The diameters of the same two rings (n^{th} and $(n + p)^{th}$) are then measured. The difference in diameters of the two rings for the two films are

$$(D^2_{n+p} - D^2_n)_{liquid} = \frac{4\,p\lambda R}{\mu} \quad ; \qquad \mu = \frac{4\,p\lambda R}{(D^2_{n+p} - D^2_n)_{liquid}}$$

Where μ is the refractive index of the liquid.

Then,

$$\mu = \frac{(D^2_{n+p} - D^2_n)_{air}}{(D^2_{n+p} - D^2_n)_{liquid}} \text{ …………………………..(16)}$$

By using equation (16) we can get refractive index of a liquid.

Mathematical problem:

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1. Two coherent sources of monochromatic light of wavelength **6000 Å** produce an interference pattern on a screen kept at a distance of **1 m** from them. The distance between two consecutive bright fringes on the screen is **0.5 mm**. Find the distance between two coherent sources.

Solution:

Here $\lambda = 6000\,\text{Å} = 6000 \times 10^{-10} = 6 \times 10^{-7}$ m.

D = 1 m, X = 0.5 mm = 5×10^{-4} m and d = ?

$$X = \frac{\lambda D}{d}$$

$$\text{or, } d = \frac{\lambda D}{X} = \frac{6 \times 10^{-7} \times 1}{5 \times 10^{-4}} = 1.2 \times 10^{-3}\ \text{m} = 1.2\,\text{mm}.$$

2. Two straight and narrow parallel slits **1mm** apart are illuminated by monochromatic light. Fringes formed on the screen held at a distance of **100 cm** from the slits are **0.50 mm** apart. Calculate the wavelength of light.

Solution:

Here D = 100 cm = 1 m, X = 0.5 mm = 5×10^{-4} m and d = 1 mm = 1×10^{-3} m

$$X = \frac{\lambda D}{d}$$

$$\text{or, } \lambda = \frac{X\,d}{D} = \frac{5 \times 10^{-4} \times 1 \times 10^{-3}}{1} = 5 \times 10^{-7}\ \text{m} = 5000\,\text{Å}.$$

3. In Young's double slit experiment the separation of the slits is **1.9 mm** and the fringe spacing is **0.31 mm** at a distance of **1 m** from the slits. Calculate the wavelength of light.

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Solution:

Here D = 1 m, X = 0.31 mm = 3.1×10^{-4} m and d = 1.9 mm = 1.9×10^{-3} m

$$X = \frac{\lambda D}{d}$$

$$\text{or, } \lambda = \frac{X\,d}{D} = \frac{3.1\times10^{-4}\times1.9\times10^{-3}}{1} = 5.89\times10^{-7}\ \text{m} = 5890\,\text{Å}.$$

4. Newton's rings are observed by keeping a spherical surface of 100 cm radius on a plane glass plate. If the diameter of the 15th bright ring is 0.590 cm and the diameter of the 5th ring is 0.336 cm, what is the wavelength of light used?

Solution:

Here D_5 = 0.336 cm, D_{15} = 0.590 cm, R = 100 cm, p = 10 and $\lambda = ?$

$$R = \frac{D_{n+p}^2 - D_n^2}{4\,p\lambda}$$

$$\text{or, } \lambda = \frac{D_{n+p}^2 - D_n^2}{4\,pR} = \frac{D_{15}^2 - D_5^2}{4\,pR}$$

$$= \frac{(0.596)^2 - (0.336)^2}{4\times10\times100}$$

$$= 5880\times10^{-8}\,cm = 5880\,\text{Å}$$

<u>Physics for Engineers- Dr. Giasuddin Ahmad (1st Edition)</u>

Mathematical Problem:

Example: 27.1, 27.4, 27.16-27.27.

<u>A Text Book of Optics- N. Subrahmanyam, Brijlal (22nd Edition)</u>

Mathematical Problem:

Example: 8.1-8.4, 8.6, 8.7, 8.8, 8.47-8.56.

<u>Exercises</u>

1. What is mean by interference of light? Explain.
2. What do you mean by coherent sources? Derive a relation between path difference and

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Phase difference.

3. State the fundamental conditions for the production of interference fringes.

4. Discuss interference of light analytically and obtain the conditions of maximum and minimum intensities.

5. What do you mean by fringe width? Show that for bright and dark fringe, the fringe width is $X = \frac{\lambda D}{d}$, where the symbols have their usual meaning.

or,

5. Prove that the distance β between two successive bright fringes formed in Young's experiment is given by $\beta = \frac{\lambda D}{d}$ **,** where the symbols have their usual meaning.

6. For both constructive and destructive interference derive an expression of phase difference due to reflected light from a plane parallel (thin) film.

or,

6. Establish the expression of phase difference for both constructive and destructive interference due to reflected light from a plane parallel (thin) film.

7. Describe and explain the formation of Newton's rings in the reflected light. Show that **(i)** the diameters (also radii) of bright rings are proportional to the square root of the odd natural numbers and diameters (also radii) of the dark rings are proportional to the square root of the natural numbers. **(ii)** Account for the perfect blackness of the central spot in Newton's rings. **(iii)** What will happen if a little water is introduced between the lens and glass plate? (iv) Show that the alternate bright and dark rings surrounding the central dark spot in Newton's rings gradually become narrower as their radii increases.

or,

7. Explain the formation of Newton's rings. Describe with necessary theory the Newton's rings method of measuring wavelength of monochromatic light.

8. Describe the phenomena of interference due to reflected light from a plane of varying thickness (wedge shaped film).

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Text Books:

Book No.	Title	Author (s)	Edition
T-1	Physics for Engineers	Dr. Giasuddin Ahmad	1st
T-2	A Text Book of Optics	N. Subrahmanyam, Brijlal	22nd
T-3	Fundamentals of Optics	Francis A. Jenkins, White	4th

Diffraction of Light:

When light from a narrow linear slit is incident on the sharp edge of an obstacle, it will be found that there is illumination to some extent within the geometrical shadow of the obstacle. This show that light can be bend round an obstacle and this bending of light round an obstacle is called diffraction of light.

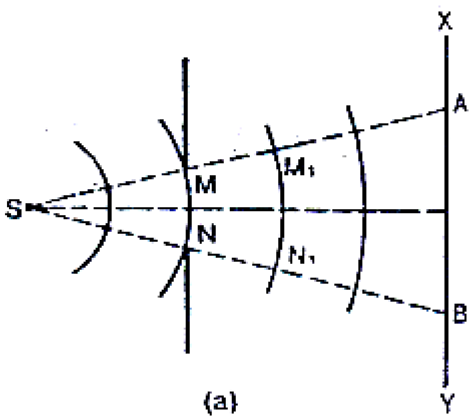

Fig.-1.1(a)

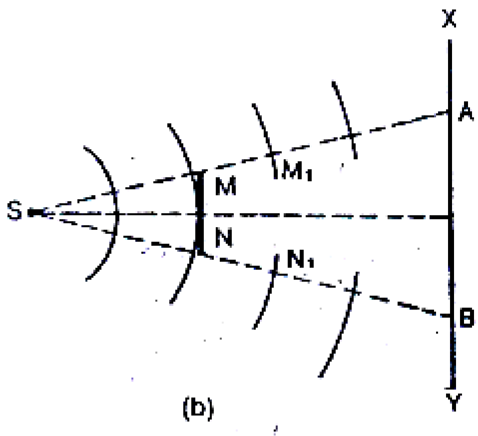

Fig.-1.1(b)

Explanation:

Let, S is a source of mono-chromatic light and MN is a small aperture, XY is the screen placed in the path of light, AB is the illuminated portion of the screen and above A and below B is the region of the geometrical shadow.

Considering MN as the primary wave-front, according to the Huygen's construction, if the secondary wavefronts are drawn the encroachment of light in the geometrical shadow is expected. This bending of light round the edges of an obstacle or the encroachment of

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light within the geometrical shadow is called diffraction (Fig-1.1-a). Similarly, if an opaque obstacle MN is placed in the path of light (Fig-1.1-b), there should be illuminated in the geometrical shadow region AB also.

Classification of the diffraction:

Diffraction phenomena can be divided into two groups. These are following:

1. Fresnel diffraction phenomena

2. Fraunhofer diffraction phenomena

1. Fresnel diffraction phenomena:

In the Fresnel class of diffraction, the source or the screen or both are at finite distances from the aperture or obstacle causing diffraction.

2. Fraunhofer diffraction phenomena:

In the Fraunhofer class of diffraction phenomena, the source or the screen on which the pattern is observed are at infinite distances from the aperture or an obstacle causing diffraction.

Distinction between Fresnel and Fraunhofer diffraction.

The diffraction between Fresnel and Fraunhofer diffraction is given below:

Fresnel diffraction	Fraunhofer diffraction
1. The source or the screen or both are at finite distances from the aperture.	1. The source and the screen are at infinite distance from the aperture.
2. Observation of Fresnel diffraction phenomena dose not require any lenses.	2. The incoming light is rendered parallel with a lens and the diffracted beam focused on the screen with another lens.
3. Theoretical treatment is complex.	3. Theoretical treatment is simpler.
4. The wave-fronts are spherical and hence divergent.	4. The wave-fronts are planer
5. It is used to construct a zone plate.	5. It is used to construct plane transmission grating or concave reflection grating.
6. Spectral lines that are seen are less	6. Spectral Lines that are seen are more

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brighter.	brighter.
7. Centre of the screen can be both bright and dark. It depends upon the width of the slit.	7. Centre of the screen is always bright.

An expression for the intensity pattern due to single slit diffraction The maxima and minima of the pattern.

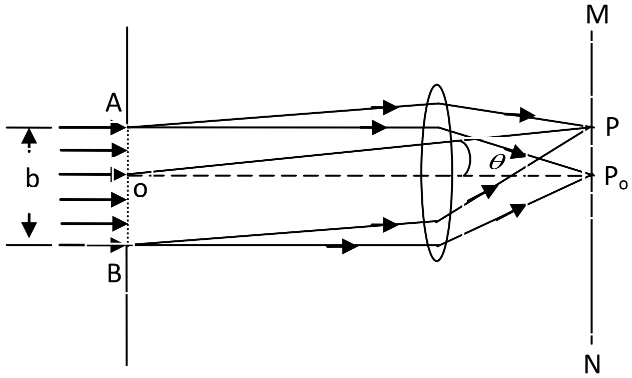


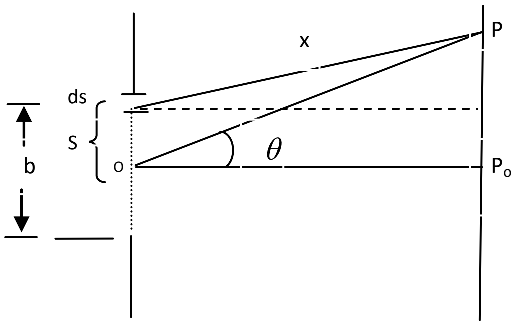
Fig.-1.2: Fraunhofer diffraction by a single slit

Let a parallel beam of mono-chromatic light of wavelength λ be incident normally on a

narrow slit AB of width b through witch parallel rays are passing. Let ds be the small element at a distance s from the origin O.

The part of each secondary wave which travels normal to the plane of the slit will be focused at Po, while those which travel at any angle θ will reach P. Considering first the wave-length emitted by the element ds situated at the origin. The amplitude will be proportional to the length ds and inversely proportional to the distance, so

$$dy_0 = \frac{\text{ads}}{\text{x}}\sin\left(\text{wt - kx}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$



Where, a = amplitude of wave

$$k = \frac{2\pi}{\lambda}$$

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As the position ds varies the displacement it produces will vary in phase because of the different path length to P. When ds is situated at a distance S from the origin O, then,

$$dy_s = \frac{\mathrm{ads}}{\mathrm{x}} \sin\left[\mathrm{wt} - \mathrm{k}(\mathrm{x} + \Delta\mathrm{x})\right]$$

$$= \frac{\mathrm{ads}}{\mathrm{x}} \sin\left[\mathrm{wt} - \mathrm{k}(\mathrm{x} + \mathrm{S}\sin\theta)\right]$$

$$= \frac{\mathrm{ads}}{\mathrm{x}} \sin\left[\mathrm{wt} - \mathrm{kx} - \mathrm{kS}\sin\theta\right] \text{……………………… (2)}$$

$$\Delta\mathrm{x} = \mathrm{S}\sin\theta$$
$$= \text{Phase difference}$$

Similarly,

$$\mathrm{dy}_{-\mathrm{s}} = \frac{\mathrm{ads}}{\mathrm{x}} \sin\left[\mathrm{wt} - \mathrm{kx} + \mathrm{kS}\sin\theta\right] \text{…………………… (3)}$$

We know,

$$dy = \mathrm{dy}_{\mathrm{s}} + \mathrm{dy}_{-\mathrm{s}}$$

$$= \frac{\mathrm{ads}}{\mathrm{x}} \sin\left[\mathrm{wt} - \mathrm{kx} - \mathrm{kS}\sin\theta\right] + \frac{\mathrm{ads}}{\mathrm{x}} \sin\left[\mathrm{wt} - \mathrm{kx} + \mathrm{kS}\sin\theta\right]$$

$$= \frac{\mathrm{ads}}{\mathrm{x}} \left[\sin\left(\mathrm{wt} - \mathrm{kx} - \mathrm{kS}\sin\theta\right) + \sin\left(\mathrm{wt} - \mathrm{kx} + \mathrm{kS}\sin\theta\right)\right]$$

$$= \frac{\mathrm{ads}}{\mathrm{x}} \left[2\cos(\mathrm{kS}\sin\theta)\sin\left(\mathrm{wt} - \mathrm{kx}\right)\right] \text{……………..… (4)}$$

Now, integrating equation (4) between the limits 0 to b/2 and x is constant, we get

$$y = \frac{2\mathrm{a}}{\mathrm{x}} \sin\left(\mathrm{wt} - \mathrm{kx}\right) \int_0^{b/2} \cos(\mathrm{kS}\sin\theta)\, ds$$

$$y = \frac{2\mathrm{a}}{\mathrm{x}} \sin\left(\mathrm{wt} - \mathrm{kx}\right) \left[\frac{\sin(kS\sin\theta)}{k\sin\theta}\right]_0^{b/2}$$

$$= \frac{2\mathrm{a}}{\mathrm{x}} \left[\frac{\sin\left(\frac{1}{2}kb\sin\theta\right)}{k\sin\theta}\right] \sin\left(\mathrm{wt} - \mathrm{kx}\right)$$

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$$y = \frac{ab}{x} \frac{\sin\left(\frac{1}{2}kb\sin\theta\right)}{\frac{1}{2}kb\sin\theta} \sin(wt - kx) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The resultant vibration will therefore be a simple harmonic one

We can write equation (5) as,

$$y = A\sin(wt - kx)$$

Where, amplitude $A = \frac{ab}{x} \frac{\sin\left(\frac{1}{2}kb\sin\theta\right)}{\frac{1}{2}kb\sin\theta}$

$$= \frac{ab}{x} \frac{\sin\beta}{\beta} = A_0 \frac{\sin\beta}{\beta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

We know, the intensity is square of the amplitude, so intensity,

$$I = A^2$$

$$= A_0^2 \frac{\sin^2\beta}{\beta^2} \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Which is the required expression.

From (7)

$$I = A_0^2 \frac{\sin^2\beta}{\beta^2} = I_0 \frac{\sin^2\beta}{\beta^2}$$

Differentiating,

$$dI = I_0 \left[\frac{\beta^2\, 2\sin\beta\cos\beta - (\sin^2\beta)2\beta}{\beta^4}\right] d\beta$$

For I to be maximum, $\frac{dI}{d\beta} = 0$

$$\beta^2 (2\sin\beta\cos\beta) - (\sin^2\beta)2\beta = 0$$

or, $\beta^2 (2\sin\beta\cos\beta) = (\sin^2\beta)2\beta$

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$\therefore\ \tan\beta = \beta$; transcendental equation.

The maxima and minima of the pattern

When $\beta = 0$, then $\lim_{\beta\to 0} \frac{\sin\beta}{\beta} = 1$

$$I = A_0^2$$

Which corresponds to maximum intensity. The intensity is zero when $\beta = m\pi$ and $m \neq 0$.

Substituting the value of β, we get

$$\beta = \frac{1}{2}\,kb\sin\theta$$

$$m\pi = \frac{1}{2}\,kb\sin\theta$$

$$m\pi = \frac{1}{2}\frac{2\pi}{\lambda}\,b\sin\theta \qquad \left[\because k = \frac{2\pi}{\lambda}\right]$$

$b\sin\theta = m\lambda$ …………….. (8)

Here m = 1, 2, 3, …………, etc and we will get minimum intensity distribution.

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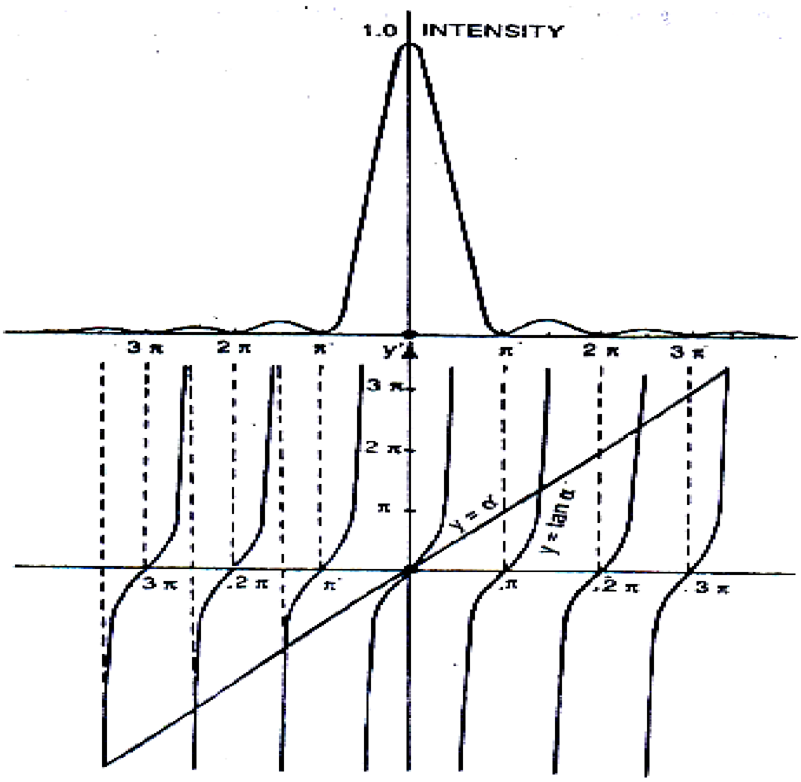

The position of secondary maxima are obtained by putting $\beta = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, ...etc.$

From equation (8) we see that when the breadth b is wide $\sin\theta$ will be small. Then the minimum and maximum intensity will approach to the central maximum intensity at P_o. The value of θ will be large when the slit breadth is small. In this case the maximum and maximum diffraction will be observed on both sides of the principle maximum.

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Fraunhofer diffraction by a double slit

Our goal is now to study the Fraunhofer diffraction pattern by two parallel slits, The arrangement is similar to that of Young's double slit experiment which helped to demonstrate the wave nature of light. In fig-1.3, AB and CD are two rectangular slits parallel to one another and perpendicular to the plane of the paper.

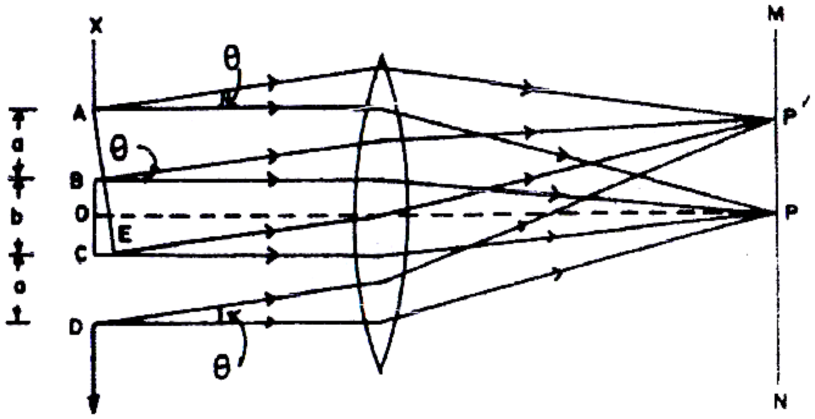

Fig-1.3: Fraunhofer diffraction by a double slit

Let, a be the width of each slit and b be width of the opaque portion separating them. P is a point on the serene MN such that OP is perpendicular to the screen. When a plane wave front incident on the surface of the slits, the transmitted rays are brought to focus at different points on the screen by convex lens L. All secondary waves traveling in a direction parallel to OP will be brought to focus at P, which, therefore, corresponds to the position of the central bright maximum. In order to derive an equation for the intensity in a double alit diffraction it is merely necessary to change the limits of integration for single slit to include the two portions of the wave length transmitted by double slit. Thus, if the origin is chosen as the centre of one of the slits, them the integration is to extend from $s = -\frac{a}{2}$ to $s = +\frac{a}{2}$ and from $\left(d - \frac{a}{2}\right)$ to $\left(d + \frac{a}{2}\right)$, Where, $d = a + b$. We therefore, have

$$y = \int_{-a/2}^{+a/2} \sin(\phi - \psi)\, ds + \int_{d-a/2}^{d+a/2} \sin(\phi - \psi)\, ds$$

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Where $\phi = 2\pi\left(\frac{\mathrm{t}}{\mathrm{T}} - \frac{\mathrm{x}}{\lambda}\right)$ and $\psi = 2\pi\,\mathrm{s}\,\frac{\sin\theta}{\lambda}$

$$y = \int_{-\frac{a}{2}}^{+\frac{a}{2}} (\sin\phi\cos\psi - \cos\psi\sin\phi)\,ds + \int_{d-\frac{a}{2}}^{d+\frac{a}{2}} \sin(\sin\phi\cos\psi - \cos\phi\sin\psi)\,ds$$

$$= \left[\frac{\mathrm{s}\sin\psi}{\psi}\sin\phi + \frac{\mathrm{s}\cos\psi}{\psi}\cos\phi\right]_{-\frac{a}{2}}^{+\frac{a}{2}} \quad + \quad \left[\frac{\mathrm{s}\sin\psi}{\psi}\sin\phi + \frac{\mathrm{s}\cos\psi}{\psi}\cos\phi\right]_{d-\frac{a}{2}}^{d+\frac{a}{2}}$$

Substituting in the limits and combining the terms, we obtain

$$\mathrm{y} = 2\mathrm{a}\frac{\sin\frac{(\pi\,\mathrm{a}\sin\theta)}{\lambda}}{\frac{(\pi\,\mathrm{a}\sin\theta)}{\lambda}} \quad \cos\frac{(\pi\,\mathrm{d}\sin\theta)}{\lambda} \times \sin 2\pi\left(\frac{\mathrm{t}}{\mathrm{T}} - \frac{x}{\lambda} - \frac{d\sin\theta}{2\lambda}\right)$$

$$= 2\mathrm{a}\,\frac{\sin\beta}{\beta}\,\cos\gamma\,\sin 2\pi\left(\frac{\mathrm{t}}{\mathrm{T}} - \frac{x}{\lambda} - \frac{d\sin\theta}{2\lambda}\right)$$

Where, as before,

$\beta = \frac{\pi\,\mathrm{a}\sin\theta}{\lambda}$ And where $\gamma = \frac{\pi\,\mathrm{d}\sin\theta}{\lambda}$

Replacing a by R_0 as before,

$$\mathrm{y} = 2\mathrm{R}_0\,\frac{\sin\beta}{\beta}\,\cos\gamma\,\sin 2\pi\left(\frac{\mathrm{t}}{\mathrm{T}} - \frac{x}{\lambda} - \frac{d\sin\theta}{2\lambda}\right)$$

The intensity is proportional to the square of the amplitude in the above equation. Hence,

$$\mathrm{I} = \mathrm{R}^2 = 4\mathrm{R}_0^2\,\frac{\sin^2\beta}{\beta^2}\,\cos^2\gamma$$

As can be seen, the intensity is the product of two terms: the first term $\frac{\sin^2\beta}{\beta^2}$ represents the diffraction pattern produced by a single slit of width a and the second term $\cos^2\gamma$ represents the interference pattern produced by two point sources of equal intensity and phase difference $\gamma = \frac{\delta}{2}$ and separated by a distance d. The central inference maximum of

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this pattern has intensity four times greater than that of the central maximum in the single slit diffraction and is bordered by other interference maxima of gradually decreasing intensity. Indeed, if the width of the slits is very small so that there is almost no variation of the $\frac{\sin^2\beta}{\beta^2}$ term with θ, then one simply obtains the interference pattern produced in Young's double slit experiment. Thus each slit produces a diffracted beam in which the intensity distribution depends upon the slit-width; these diffracted beams then interfere with each other to produce the final diffraction pattern.

Position of maxima and minima

Equation (2) tells us that the resultant intensity will be zero when either of the two terms is zero. For the first term this will occur when

$$\beta = \pi, 2\pi, 3\pi, \ldots\ \ldots\ \ldots$$

And for the second term, when

$$\gamma = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\ \ldots\ \ldots$$

Since by definition $\beta = \frac{\pi\, \mathrm{a} \sin\theta}{\lambda}$ and $\gamma = \frac{\pi\, \mathrm{d} \sin\theta}{\lambda}$

The corresponding angles of diffraction are given by the following relations

$$a \sin\theta = \lambda, 2\lambda, 3\lambda, \ldots\ \ldots\ = \mathrm{m}\lambda \qquad \text{minima}$$

Where $m = 1, 2, 3, \ldots\ \ldots\ \ldots$

And $$d \sin\theta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots\ \ldots\ \ldots\ = \left(\mathrm{n} + \frac{1}{2}\right)\lambda \qquad \text{minima}$$

Where $n = 0, 1, 2, 3,$ etc. This equation simply expresses the condition that for minima, the path between the parallel rays diffracted from any pair of corresponding point in the two slits should be odd multiple of $\frac{\lambda}{2}$ on reaching the focal plane of the focusing lens.

The positions of the maxima will then be determined solely by the $\cos^2\gamma$ factor. The interference maxima then occur

When $\gamma = 0, \pi, 2\pi, \ldots\ \ldots\ \ldots$

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Or, when $d\sin\theta = 0, \lambda, 2\lambda, 3\lambda, \ldots\ \ldots\ \ldots, n\lambda$ maxima

The whole number n represents physically the number of wavelengths in the path difference from corresponding points in the two slits and represents the order of interference. Points separated by a distance $(a+b)$ in the two slits are known as corresponding points.

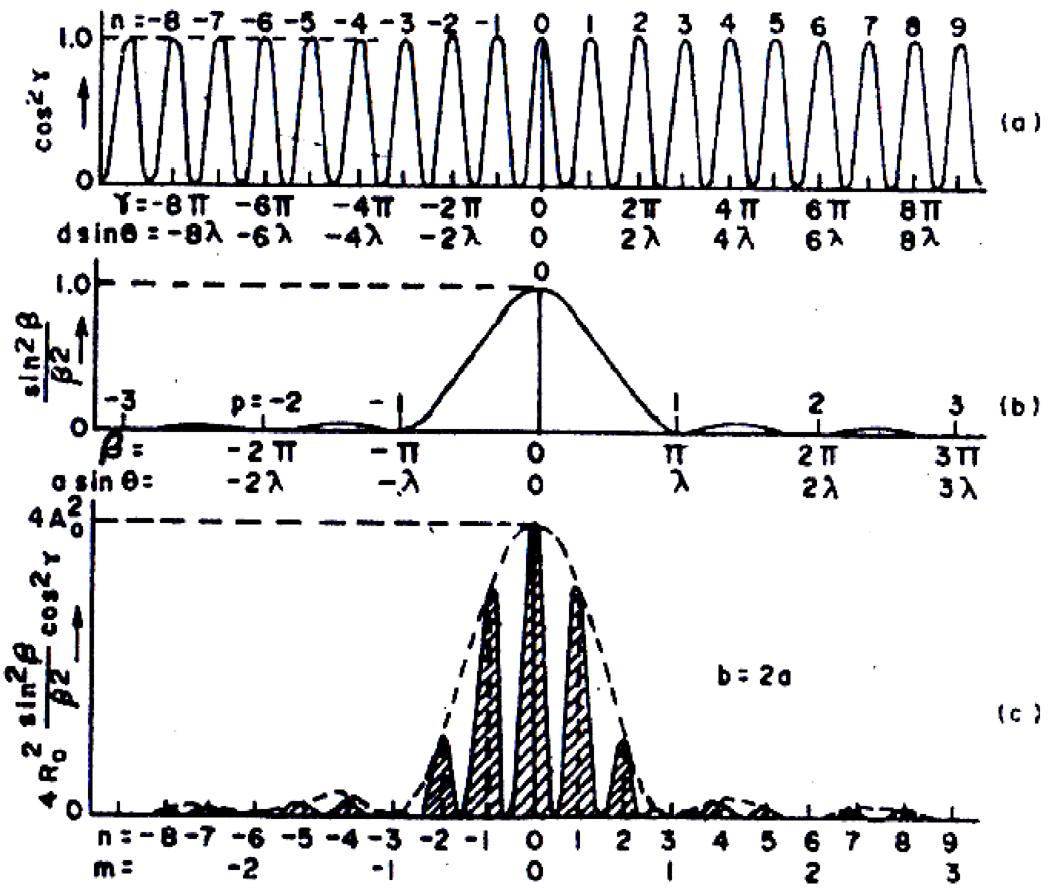

Fig. - 1.4: Intensity curves for a double slit where *d = 3a*.

Fig. - 1.4 shows the intensity curve for the double slit diffraction pattern. The upper curve is a plot of $\cos^2\gamma$ against γ and the values of the order and path difference are indicated for the various maxima. All these maxima are of equal intensity and equidistant. The single slit diffraction pattern is given by the curve in the middle. The complete double slit pattern as given by the equation (2) is the product of these two factors. This is obtained by multiplying the ordinates of the upper curve by those the middle curve and the constant $4R_o^2$. The resultant pattern is shown in the lowest curve of the figure.

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Missing orders in double slit diffraction pattern

If the separation between the slits b is varied, keeping the width of the individual slits a constant, the scale of the interference pattern varies, but that of the diffraction pattern remains the same. A study of the resultant diffraction pattern reveals that certain order of interference maxima are missing.

Let us suppose that for a certain values of θ, the following relations simultaneously hold

$a\sin\theta = \pm \mathrm{m}\lambda$ ………………………..Dark (diffraction)

$(a+b)\sin\theta = \mathrm{d}\sin\theta = \pm \mathrm{n}\lambda$ ……………Bright (interference)

But the first relation expression the condition of zero intensity in the diffraction pattern while the second equation expresses the condition of maximum intensity in the interference pattern. Thus for certain values of θ, the position of certain interference maxima correspond to the diffraction minima at the same position on the screen and consequently will be missing (or at least reduced to two maxima of very low intensity).

From the above two equations we have

$$\frac{d}{a} = \frac{\mathrm{n}}{\mathrm{m}}$$

Since m and n are both integers, $\frac{d}{a}$ must be in the ratio of two integers in order to have missing order. This ratio determines the orders which are missing.

For example,

(1) Let $\frac{d}{a} = 2 = \frac{\mathrm{n}}{\mathrm{m}}$. But $d = a + b$. This means $b = a$. Hence $n = 2m$. Giving m integral values, we get the corresponding values of missing orders n. If m = 1,2,3,4, etc. n=2, 4, 6, 8, etc. Thus the orders 2, 4, 6, of the interference maxima will be missing in the diffraction pattern and there will be three interference maxima (the zero order and the two first maxima) within the central diffraction maximum.

(2) Let $\frac{d}{a} = 3 = \frac{\mathrm{n}}{\mathrm{m}}$. That is $b = 2a$. Then $n = 3m$.

Therefore, when m = 1, 2, 3, 4, etc.

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The missing order are n=3, 6, 9, 12, etc.

Thus there will now be five interference maxima (zero order, two first order and two second order maxima) within the central diffraction maximum.

In this way as the ratio $\frac{d}{a}$ increases, the number of interference maximum within the central diffraction maximum also increases. When $\frac{d}{a} = 1$, the two slits exactly join and all orders should be missing. However, the diffraction patterns then observed on the screen is similar to that due to a single slit of width equal to 2a.

Comparison the diffraction pattern of a single slit with that of a double slit.

The comparison/distinction between single and double slit diffraction pattern is given below:

1. The Single slit diffraction pattern consists of a central bright maximum with secondary maxima and minima of gradually decreasing intensity.

The double slit diffraction pattern consists of equally spaced inference maxima and minima within the central maximum.

2. The intensity of the central maximum in the diffraction pattern due to a double slit is four times that of the central maximum due to diffraction at a single slit.

3. The spacing of the diffraction maxima and minima depends on the slit width and the spacing of the interference maxima and minima depends of the slit width and the opaque spacing between the two slits.

4. Since, the resultant intensity ($\mathrm{I} = \mathrm{A}_0^2 \frac{\sin^2 \beta}{\beta^2}$) is proportional to the square of sine, so the maxima is seen to be clear.

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Since, the resultant intensity ($I = 4A_0^2 \dfrac{\sin^2\beta}{\beta^2} \cos^2\gamma$) is proportional to the square of sine and cosine, so the maxima is seen to be hazy.

5. Intensity distribution curves for both cases.

Distinctions between interference and diffraction

Difference between inference and diffraction are given below:

Interference	Diffraction
1. Interference is the result of interaction of light waves coming from the different wave fronts originating from same source.	1. Diffraction pattern is the result of interaction of light waves coming from the different parts of the same wave-fronts.
2. In the interference pattern, the intensity of all bright fringes is equal and symmetrical.	2. In the diffraction pattern,the intensity of all bright fringes is not equal.
3. The interference fringes are of equal width.	3. The diffraction fringes are not of equal width.
4. Points of minimum light intensity are perfectly dark.	4. Points of minimum light intensity are not perfectly dark.
5. All the interference fringes are clear.	5. The diffraction fringes are not clear without the central bright fringe.
6. Interference is of two types: i). Constructive ii). Destructive.	6. Diffraction is of two types: i). Fresnel diffraction ii). Fraunhofer diffraction

Diffraction grating

Any arrangement which is equivalent in its action a number of parallel of equidistant slits of the same width is called a diffraction grating. It is an extremely useful device. There are a large number of narrow slits side by side. The slits are separated by opaque space.

There are three kinds of diffraction grating:

1. Plane diffraction grating
2. Concave reflecting grating and
3. Echelon grating.

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Plane diffraction grating:

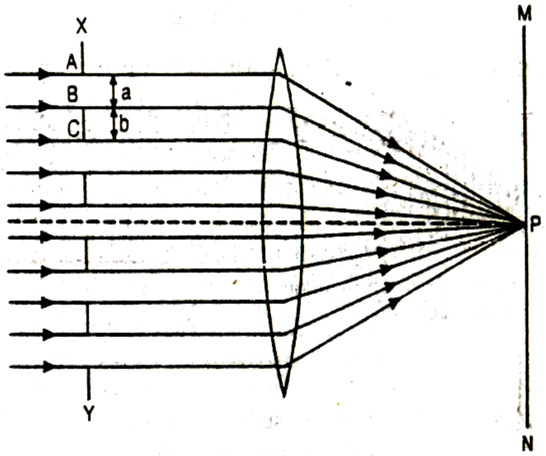
A plane diffraction grating is an arrangement which is equivalent in action to a large number of parallel and equidistant slits of the same width.

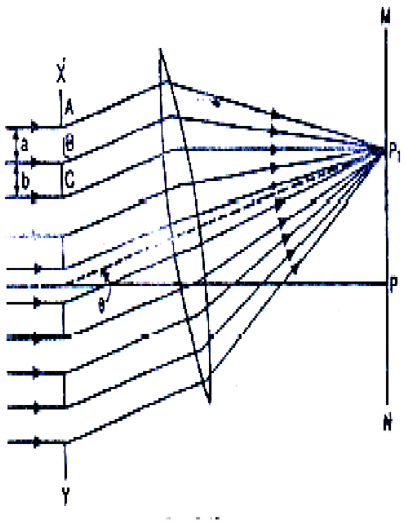
Construction:

It is constructed by ruling equidistant lines on a transparent material, such as glass plate by means of a fine diamond point worked with a ruling engine. The ruled lines act like opaque wires and thus do not allow the incident light to pass through the spaces in between the lines. These are called transparencies. Such surfaces act as transmission gratings. The number of lines on a plane diffraction grating is of the order of 15000 lines/inch.

Working Details or the theory of plane transmission grating.

Let XY is the grating surface and MN is the screen, both perpendiculars to the plane of the paper. The slit are all parallel to one another and perpendicular to the plane of the paper. Here AB is a slit and BC is an opaque portion. The width of each slit is *a* and the opaque spacing between any two consecutive slits is b. This distance $(a+b)$ is called the grating element or grating space.





Let a plane wavefront be incident on the grating surface. Consider the secondary waves traveling in a direction inclined at an angle θ with the direction of the incident light. The collecting lens also is suitably rotated such that the axis of the lens is parallel to the direction of the secondary waves. These secondary waves come to focus at the point P_1 on the screen. The intensity at P_1 will depend on the path difference between the secondary

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waves originating from the corresponding points A and C of two neighbouring slits. The path difference between the secondary waves starting from A and C is equal to AC sin θ .

But, AC = AB + BC = a + b.

$$\therefore \text{ Path difference} = \text{AC} \sin\theta$$
$$= (a + b)\sin\theta$$

The point P_1 will be of maximum intensity if this path difference is equal to integral multiples of λ, where λ is the wavelength of light. In this case, all the secondary waves originating from the corresponding points of the neighbouring slits reinforce one another and the angle θ , gives the direction of maximum intensity. In general,

$$(a + b)\sin\theta_n = n\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (1)$$

Where θ_n is the direction of the n^{th} principle maximum. Putting n = 1, 2, 3, etc, the angles $\theta_1, \theta_2, \theta_3, \ldots \ldots$, etc, corresponding to the directions of the principle maximum can be obtained.

If the incident light consists of more than one wavelength, the beam gets dispersed and angle of diffraction is different for different wavelengths. If for two nearly equal wavelengths λ and $(\lambda + d\lambda)$, the corresponding angles of diffractions are θ and $(\theta + d\theta)$, then in first order,

$$(a + b)\sin\theta = \lambda$$

$$(a + b)\sin(\theta + d\theta) = \lambda + d\lambda$$

Thus, for white light, the diffraction pattern on the screen will consist of central bright maximum, surrounded on both sides by spectrum corresponding to different wavelengths of the constituents of white light.

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Mathematical Problem:

1. Deduce the missing orders for a double slit Fraunhofer diffraction pattern if the slits widths are **0.088 mm (0.12 mm / 0.16 mm / 8.8×10^{-3} cm)** and they are **0.44 mm (0.6 mm / 0.8 mm / 4.4×10^{-2} cm)** apart.

Solution:

The direction of interference maxima are given by the equation,

$$(a+b)\sin\theta = \pm\,\mathrm{n}\lambda \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

The direction of diffraction minima are given by the equation,

$$a\sin\theta = \pm\,\mathrm{m}\lambda \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

From the above two equations we have

$$\frac{(a+b)}{a} = \frac{\mathrm{n}}{\mathrm{m}}$$

Here a = 0.088 mm and b = 0.44 mm

$$\therefore \frac{(a+b)}{a} = \frac{0.088+0.44}{0.088} = \frac{0.528}{0.088} = 6 = \frac{\mathrm{n}}{\mathrm{m}}$$

$$or,\ \mathrm{n} = 6\mathrm{m}.$$

For values of m = 1, 2, 3, etc.

n = 6, 12, 18, etc.

Thus the orders 6, 12, 18, etc. of the interference maxima will be missing in the diffraction pattern.

2. How many orders will be visible if the wavelength of incident radiation be **5000 Å** and the number of lines on the grating be **14000 an inch**?

Solution:

Grating constant, $\mathrm{N} = 14000\,\mathrm{lines/inch}$

$$= \frac{14000\,\mathrm{lines}}{1\,\mathrm{inch}} = \frac{14000\,\mathrm{lines}}{2.54\,\mathrm{cm}}$$

$$\mathrm{N} = \frac{1}{\mathrm{a}+\mathrm{b}} = \frac{14000\,\mathrm{lines}}{2.54\,\mathrm{cm}}$$

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$$\therefore\ a + b = \frac{2.54}{14000}\, cm$$

Here wavelength, $\lambda = 5000 \text{Å} = 5000 \times 10^{-8}$ cm.

The maximum observable angle of diffraction is 90^o.

Hence, from $(a+b)\sin\theta = m\lambda$, we have

$$m = \frac{(a+b)\sin\theta}{\lambda} = \frac{2.54 \times \sin 90^0}{14000 \times 5000 \times 10^{-8}}$$

$$= \frac{2.54}{70 \times 10^{-2}} = 3.63$$

Therefore, Three (3) orders will be visible in the grating spectra.

Physics for Engineers- Dr. Giasuddin Ahmad (1st Edition)

Mathematical Problem:

Example: 28.6, 28.9, 28.10, 28.11, 28.12, 28.13, 28.14, 28.16.

A Text Book of Optics- N. Subrahmanyam, Brijlal (22nd Edition)

Mathematical Problem:

Example: 9.17, 9.22, 9.23, 9.29, 9.30, 9.34, 9.36, 9.37.

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Exercises

1. Explain diffraction of light.

2. Derive the intensity expression for Fraunhofer single slit diffraction and discuss the condition for maxima and minima pattern with draw the intensity distribution for the diffraction pattern.

or,

2. Discuss the Fraunhofer diffraction at a single slit. Draw the intensity distribution for the diffraction pattern.

3. Derive the intensity expression for Fraunhofer double slit diffraction pattern and discuss the condition for maxima and minima pattern with draw the intensity distribution for the diffraction pattern. .

4. How can you get missing orders in double slit diffraction pattern?

5. Draw the resultant curve for Fraunhofer diffraction due to double slit when **b = a, b = 2a**.

6. Distinguish between interference and diffraction.

7. Discuss the construction and working principle of plane transmission grating and show how you would use it to find the wavelength of light.

8. Distinguish between the Fresnel and Fraunhofer diffraction phenomena.

9. Distinguish between the Fraunhofer diffraction at a single slit and double slit.

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