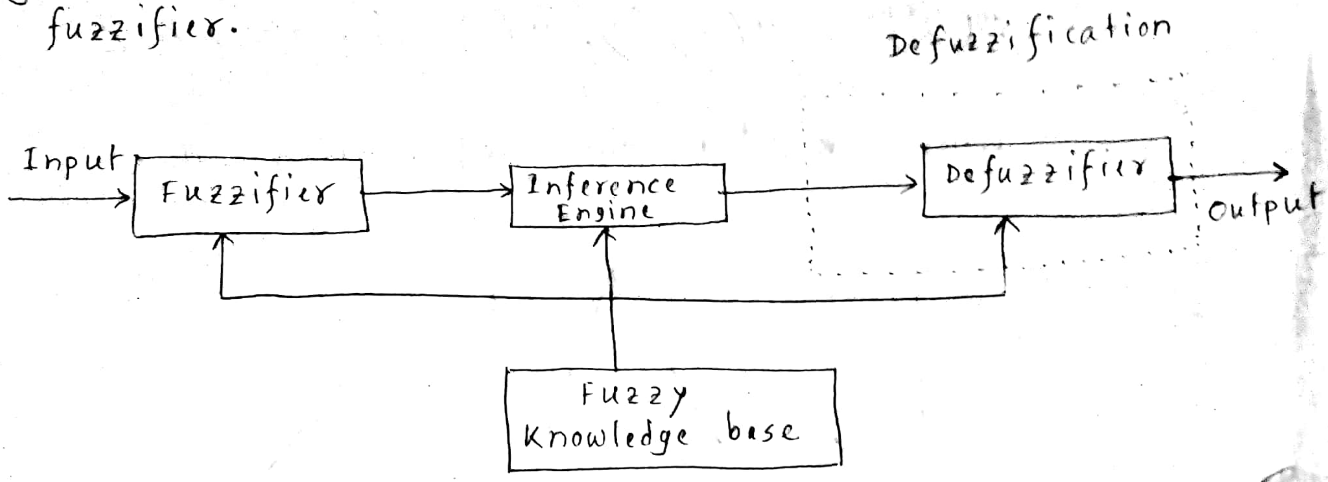
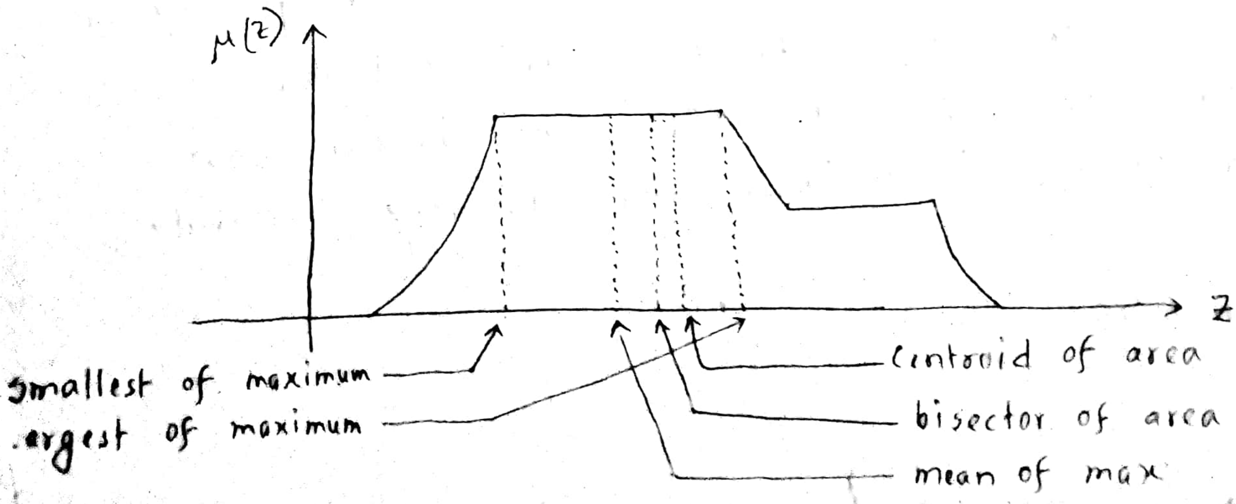
Defuzzification

- Converts the fuzzy output of the inference engine to crisp using membership functions analogous to the ones used by the fuzzifier.



Defuzzification Techniques

- Maximum Defuzzification Technique ~~(Largest of max~~
- Bisector of area
- Mean of maximum
- Smallest of maximum
- ~~Largest of maximum~~ - Largest of maximum
- Centroid Defuzzification Technique
- Weighted Average Defuzzification Technique
- Center of Sums method



Maximum Defuzzification Technique:

- Gives the output with the highest membership
- This defuzzification technique is very fast but is only accurate for peaked output

$$\mu(z^*) \geq \mu(z) \quad \text{for all } z \in Z$$

where z^* is the defuzzified value

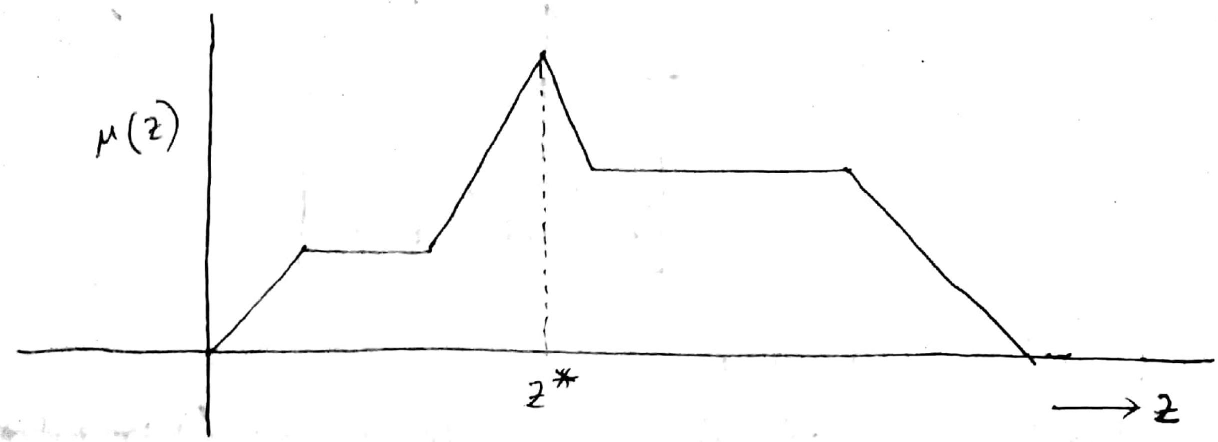


Figure: Max-membership defuzzification method.

Centroid Defuzzification Technique

$$z^* = \frac{\int \mu(z)\, z\, dz}{\int \mu(z)\, dz}$$

- Where z^* is the defuzzified value

This method is also known as center of gravity or center of area defuzzification. This technique was developed by Sugeno in 1985. This is the most commonly used Technique and is very accurate.

Example:

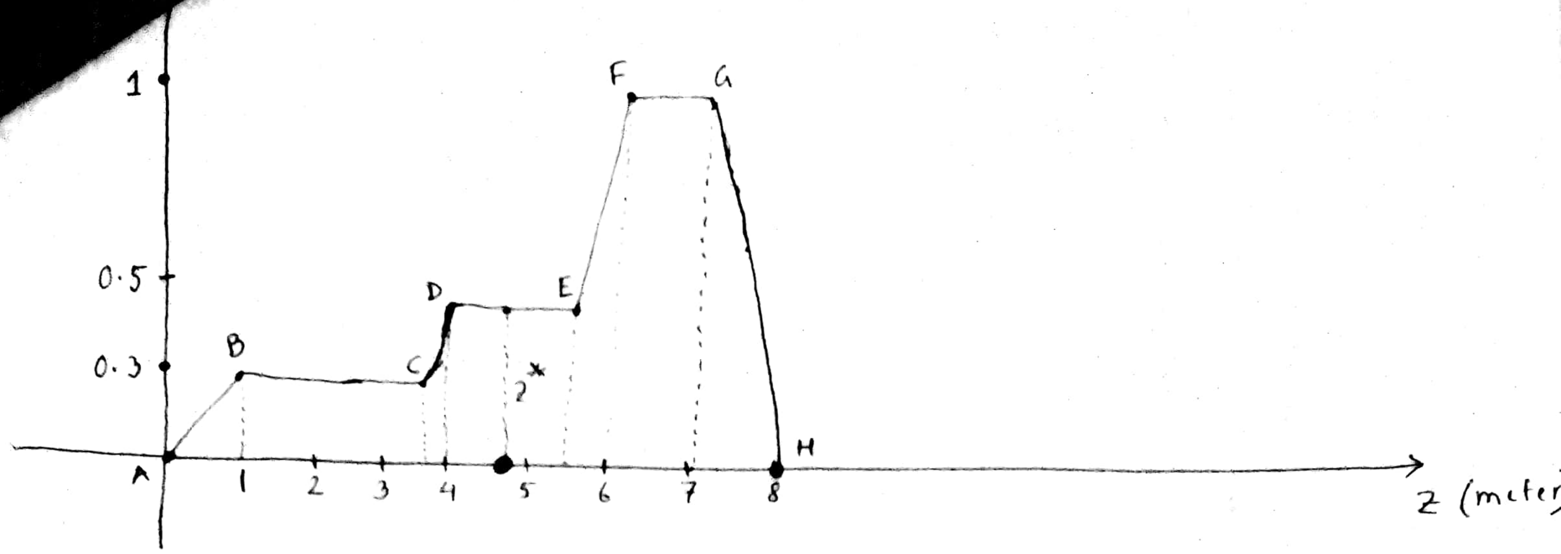


Figure : The centroid method for finding z^*

$A \equiv (0,0)$	$C \equiv (3.6, 0.3)$	$E \equiv (5.5, 0.5)$	$G \equiv (7,1)$
$B \equiv (1, 0.3)$	$D \equiv (4, 0.5)$	$F \equiv (6, 1)$	$H \equiv (8, 0)$

Equation of AB $\equiv$

$$y = mx$$

$$\Rightarrow \mu(z) = \frac{\text{লম্ব}}{\text{ভূমি}} * z$$

$$= \frac{0.3}{1} * z$$

$$= 0.3z$$

Equation of BC $\equiv$

$$\mu(z) = 0.3$$

Equation of CD $\equiv$

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$$

$$\Rightarrow \frac{z - 3.6}{3.6 - 4} = \frac{\mu(z) - 0.3}{0.3 - 0.5}$$

$$\Rightarrow \frac{z - 3.6}{-0.4} = \frac{\mu(z) - 0.3}{-0.2}$$

$$\Rightarrow \frac{z - 3.6}{2} = \mu(z) - 0.3$$

$$\Rightarrow \mu(z) = \frac{z - 3.6}{2} + 0.3 = \frac{z - 3.6 + 0.}{2}$$

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Equation of DE $\Rightarrow$

$$\mu(z) = 0.5$$

Equation of EF $\Rightarrow$

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$$

$$\Rightarrow \frac{z - 5.5}{5.5 - 6} = \frac{\mu(z) - 0.5}{0.5 - 1}$$

$$\Rightarrow \frac{z - 5.5}{-0.5} = \frac{\mu(z) - 0.5}{-0.5}$$

$$\Rightarrow z - 5.5 = \mu(z) - 0.5$$

$$\Rightarrow \mu(z) = z - 5.5 + 0.5$$

$$= z - 5$$

Equation of FG:

$$\mu(z) = 1$$

Equation of GH:

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$$

$$\Rightarrow \frac{z - 7}{7 - 8} = \frac{\mu(z) - 1}{1 - 0}$$

$$\Rightarrow \frac{z - 7}{-1} = \mu(z) - 1$$

$$\Rightarrow -z + 7 = \mu(z) - 1$$

$$\Rightarrow \mu(z) = 1 - z + 7$$

$$\Rightarrow \mu(z) = 8 - z$$

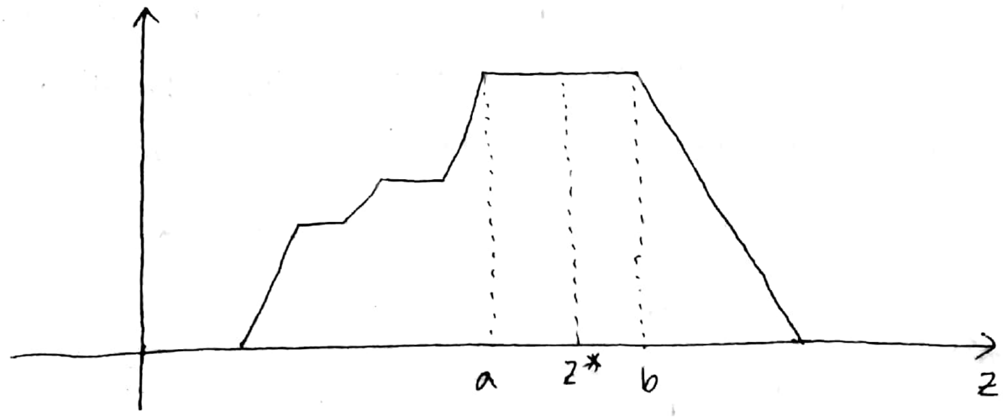
$$\frac{\int \mu(z)\, z\, dz}{\int \mu(z)\, dz}$$

$$= \frac{\int AB \cdot z\, dz + \int BC\ z\, dz + \int CD \cdot z\, dz + \int DE \cdot z\, dz + \int EF \cdot z\, dz + \int FG \cdot z\, dz + \int GH \cdot z\, dz}{\int AB \cdot dz + \int BC\ dz + \int CD \cdot dz + \int DE \cdot dz + \int EF \cdot dz + \int FG \cdot dz + \int_{GH} \cdot dz}$$

$$= \frac{\int_0^1 0.3z \cdot z\, dz + \int_1^{3.6} 0.3\, z\, dz + \int_{3.6}^{4} \frac{z-3}{2} \cdot z \cdot dz + \int_4^{5.5} 0.5\, z\, dz + \int_{5.5}^{6} (z-5) \cdot z\, dz + \int_6^7 1 \cdot z\, dz + \int_7^8 (8-z)\, z\, dz}{\int_0^1 0.3z\, dz + \int_1^{3.6} 0.3\, dz + \int_{3.6}^{4} \frac{z-3}{2}\, dz + \int_4^{5.5} 0.5\, dz + \int_{5.5}^{6} (z-5)\, dz + \int_6^7 1 \cdot dz + \int_7^8 (8-z)\, dz}$$

$= 4.9$ meter

Mean of Maximum Method :



$$z^* = \frac{a+b}{2}$$

Where z^* is the defuzzied value

Example :

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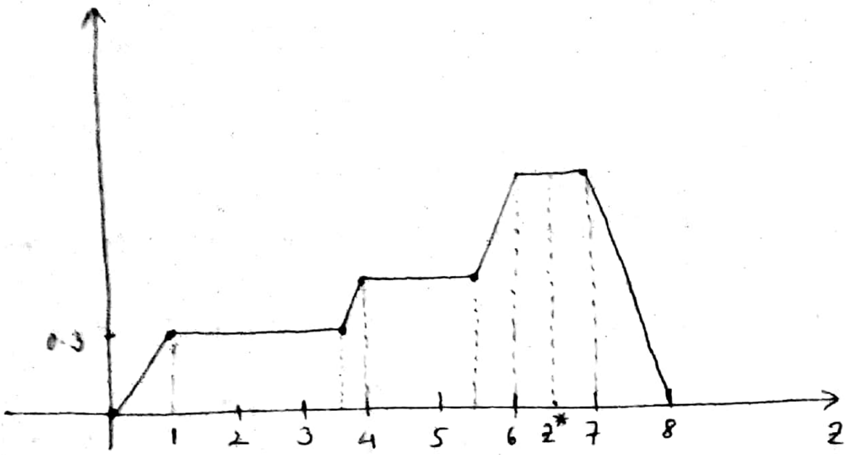


$a = 6$

$b = 7$

$$z^* = \frac{6+7}{2}$$

$= 6.5$ meter

Weighted Average Defuzzification Technique:

In this method the output is obtained by the weighted average of each output of the set of rules stored in the knowledge base of the system. The weighted average defuzzification technique can be expressed as

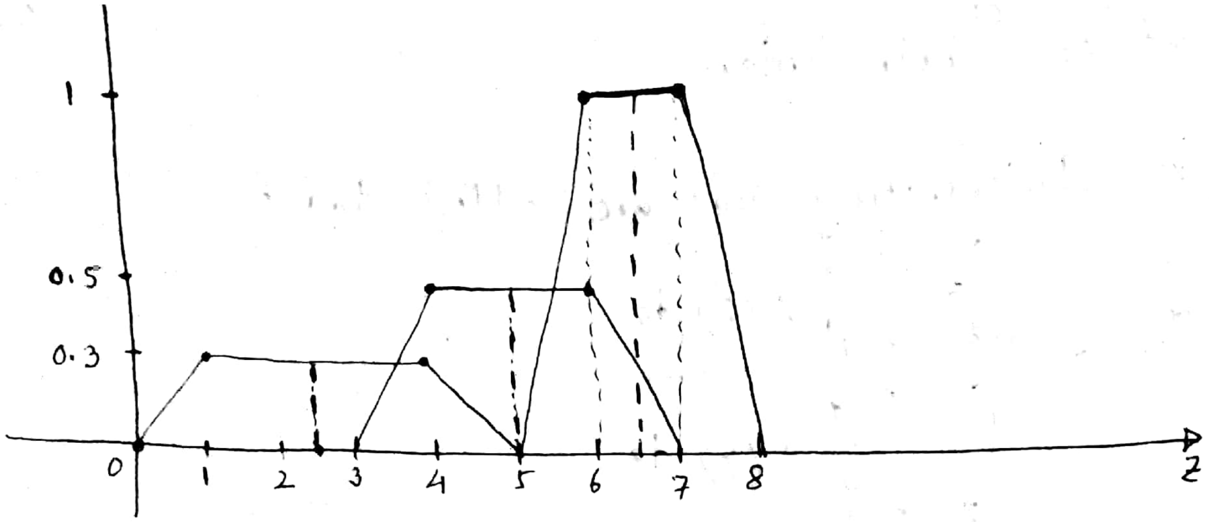
$$z^* = \frac{\sum_{i=1}^{n} m^i W_i}{\sum_{i=1}^{n} m^i}$$

Where z^* is the defuzzified output

m^i is the membership of the output of each rule

W_i is the weight associated with each rule

This method is computationally faster and easier and gives fairly accurate result.



$m^1 = 0.3 \quad W_1 = 2.5$

$m^2 = 0.5 \quad W_2 = 5$

$m^3 = 1 \quad W_3 = 6.5$

$$z^* = \frac{\sum_{i=1}^{n} m^i W_i}{\sum_{i=1}^{n} m^i}$$

$$= \frac{\sum_{i=1}^{3} m^i W_i}{\sum_{i=1}^{3} m^i}$$

$$= \frac{m^1 W_1 + m^2 W_2 + m^3 W_3}{m^1 + m^2 + m^3}$$

$$= \frac{0.3 * 2.5 + 0.5 * 5 + 1 * 6.5}{0.3 + 0.5 + 1} = 5.4 \text{ meters}$$

Center of Sums Method

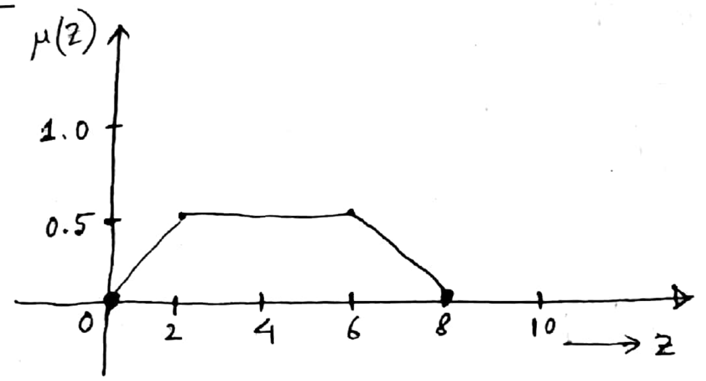
- Faster than any defuzzification method.
- Involves algebric sum of individual output fuzzy sets instead of their union.

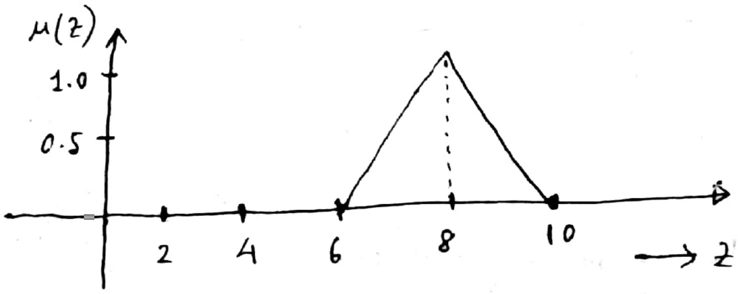
Drawback: Intersecting areas are added twice.

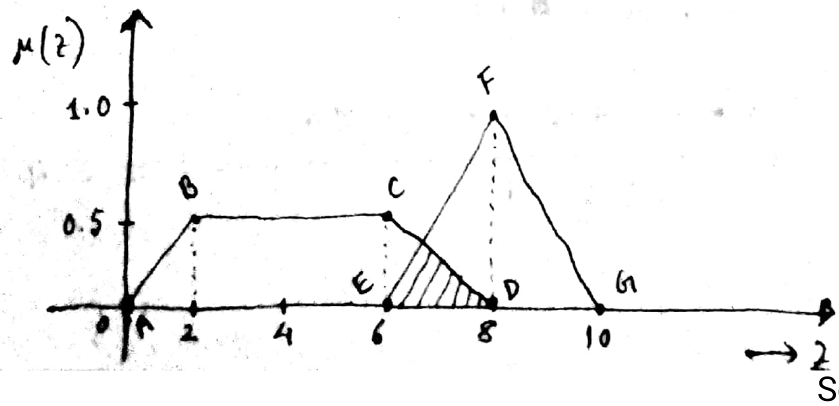
$$z^* = \frac{\int_z z \sum_{k=1}^{n} \mu(z)\, dz}{\int_z \sum_{k=1}^{n} \mu(z)\, dz}$$

It is similar to the weighted average method, but the weights are the areas, instead of individual membership values.

Example:







$$s_1 = \frac{1}{2}\times 2\times 0.5 + 4 * 0.5 + \frac{1}{2}\times 0.5\times 2 = 0.5 + 2 + 0.5 = 3$$

2 = 8 $\quad s_2 = \frac{1}{2}\times(10-6)\times 1 = \frac{1}{2}\times 4\times 1 = 2$

Similar to Weighted Average method

$$z^* = \frac{\sum_{i=1}^{n} z_i s_i}{\sum_{i=1}^{s} s_i}$$

$$= \frac{z_1 s_1 + z_2 s_2}{s_1 + s_2}$$

$$= \frac{4 * 3 + 8 * 2}{3 + 2}$$

$$= \frac{28}{5} = 5.6$$

~~Equation of AB~~ = $\Rightarrow y = mx$

$\Rightarrow \mu(z) = \frac{\text{লম্ব}}{\text{ভূমি}} \times z$

~~A = (0, 0)~~

~~B = (2, 0.5)~~ $\quad = \frac{0.5}{2} * z$

Using Formula

~~Equation of AB~~

$A \equiv (0,0)$	$C \equiv (6, 0.5)$	$E \equiv (6, 0)$	$G \equiv (10, 0)$
$B \equiv (2, 0.5)$	$D \equiv (8, 0)$	$F \equiv (8, 1)$	

Equation of AB, $y = mx$

$\Rightarrow \mu(z) = \frac{\text{লম্ব}}{\text{ভূমি}} z$

$\Rightarrow \mu(z) = \frac{0.5}{2} z$

$= 0.25 z$

Equation of BC, $\mu(z) = 0.5$

Equation of CD, $\dfrac{x - x_1}{x_1 - x_2} = \dfrac{y - y_1}{y_1 - y_2}$

$\Rightarrow \dfrac{z - 6}{6 - 8} = \dfrac{\mu(z) - 0.5}{5 - 0}$

$\Rightarrow \dfrac{z - 6}{-2} = \dfrac{\mu(z) - 0.5}{0.5}$

$\Rightarrow \mu(z) - 0.5 = 0.5 \left(\dfrac{z - 6}{-2}\right)$

$\Rightarrow \mu(z) = 0.5 - 0.25(z - 6)$

$= 0.5 - 0.25z + 1.5$

$= 2 - 0.25z$

$= -0.25z + 2$

Equation of EF, $\dfrac{x - x_1}{x_1 - x_2} = \dfrac{y - y_1}{y_1 - y_2}$

$\Rightarrow \dfrac{z - 6}{6 - 8} = \dfrac{\mu(z) - 0}{0 - 1}$

$\Rightarrow \dfrac{z - 6}{-2} = \dfrac{\mu(z)}{-1}$

$\Rightarrow \mu(z) = \dfrac{z - 6}{2} = 0.5z - 3$

Equation of FG, $\dfrac{x - x_1}{x_1 - x_2} = \dfrac{y - y_1}{y_1 - y_2}$

$\Rightarrow \dfrac{z - 8}{8 - 10} = \dfrac{\mu(z) - 1}{1 - 0}$

$\Rightarrow \dfrac{z - 8}{-2} = \mu(z) - 1$

$\Rightarrow \mu(z) = 1 - \dfrac{z - 8}{2}$

$= \dfrac{2 - z + 8}{2} = \dfrac{10 - z}{2} = -0.5z + 5$

$$\frac{\int_z z \sum_{k=1} \mu(z)\, dz}{\int \sum_{k=1}^{n} \mu(z)\, dz} = \frac{\text{Neumerator}}{\text{Denominator}} = \frac{N}{D}$$

$$N = \int_z z \sum_{k=1}^{n} \mu(z)\, dz$$

$$= \int_0^2 AB.\, z\, dz + \int_2^6 BC.\, z\, dz + \int_6^8 CD.\, z\, dz$$

$$+ \int_6^8 EF.\, z\, dz + \int_8^{10} FG.\, z\, dz$$

$$= \int_0^2 0.25z.\, z\, dz + \int_2^6 0.5\, z\, dz + \int_6^8 (-0.25z+2).\, z\, dz$$

$$+ \int_6^8 (0.5z-3)\, z\, dz + \int_8^{10} (-0.5z+5)\, z\, dz$$

$$= 28$$

$$D = \int_0^2 AB.\, dz + \int_2^6 BC.\, dz + \int_6^8 CD.\, dz + \int_6^8 EF.\, dz + \int_8^{10} FG.\, z\, dz$$

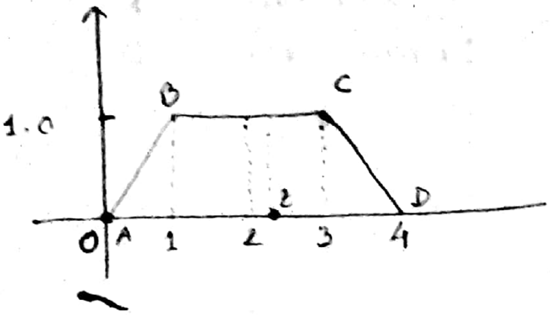
$$= \int_0^2 0.25z.\, dz + \int_2^6 0.5\, dz + \int_6^8 (-0.25z+2)\, dz$$

$$+ \int_6^8 (0.5z-3)\, dz + \int_8^{10} (-0.5z+5)\, dz$$

$$= 5$$

$$z^* = \frac{N}{D} = \frac{28}{5} = 5.6$$

Bisector of Area :



$$\int_{\alpha}^{z_{BOA}} \mu(z)\cdot dz = \int_{z_{BOA}}^{\beta} \mu(z)\cdot dz$$

$\alpha = 0$

$\beta = 4$

AB Equation, $y = mx$

$$\Rightarrow \mu(z) = \tan\theta \cdot z$$
$$= \frac{1}{1}\cdot z = z$$

BC Equation, $\mu(z) = 1$

CD Equation, $C \equiv (\overset{x_1}{3}, \overset{y_1}{1}),\ D \equiv (\overset{x_2}{4}, \overset{y_2}{0})$

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$$

$$\Rightarrow \frac{z-3}{3-4} = \frac{\mu(z) - 1}{1 - 0}$$

$$\Rightarrow \frac{z-3}{-1} = \mu(z) - 1$$

$$\Rightarrow \mu(z) = 1 - (z-3)$$
$$= 4 - z$$

$$\int_0^{z} \mu(z)\cdot dz = \int_{z}^{4} \mu(z)\cdot dz$$

$$\Rightarrow \int_0^1 z\cdot dz + \int_1^z 1\, dz = \int_z^3 1\, dz + \int_3^4 (4-z)\cdot dz$$

$$\Rightarrow \left[\frac{z^2}{2}\right]_0^1 + \left[z\right]_1^z = \left[z\right]_z^3 + \left[4z - \frac{z^2}{2}\right]_3^4$$

$$\Rightarrow \frac{1}{2} + (z-1) = 3 - z + \left[\left(16 - \frac{16}{2}\right) - \left(12 - \frac{9}{2}\right)\right]$$

$$\Rightarrow \frac{1}{2} + (z-1) = 3 - z + 8 - 12 + \frac{9}{2} \Rightarrow 2z = 1 - \frac{1}{2} + 3 + \ldots$$

$$\Rightarrow 2z = 4 \Rightarrow z = 2 \quad \textbf{Ans:}$$