Fuzzy logic

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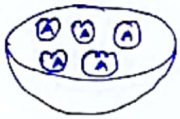
M.Sc. in CSE Program
Neural Network and Fuzzy logic
Sheet-6

Fuzzy logic:

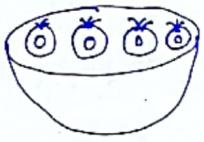
- A way to represent variation or imprecision in logic
- Fuzzy in the real-word sense "partly cloudy" - the distinctions that people use in decision-making all the time, but that computers and other advanced technology haven't been able to handle.

Crisp logic:



Is this a bowl of oranges?

Ans: NO

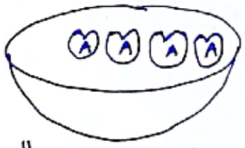


Is this a bowl of oranges?

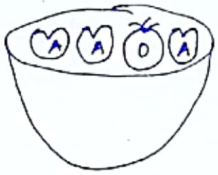
Ans: YES

Answer: {NO, YES}
: {0, 1}

Thinking Fuzzy logic:

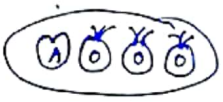


"Thinking fuzzy" about a bowl of oranges.



Fuzzy bowl of apples

Fuzzy bowl of apples (continued)



Fuzzy bowl of apples (continued)

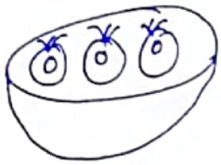
racteristics of fuzziness:

1. Word based, not number based. For instance, hot; not 85°
2. Analog (ambiguous), not digital (YES/NO)
3. Nonlinear changeable.

Crisp values:

Orangle ??

Ans: NO

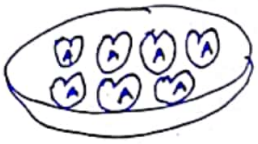


Orange ??

Ans: YES

Crisp set ≡ {NO, YES} ≡ {0, 1}

Fuzzy Values:

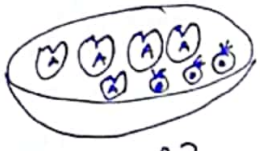


orange ??

Ans: NO

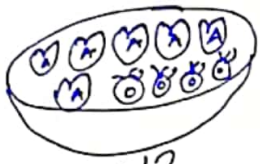
orange ??

slightly



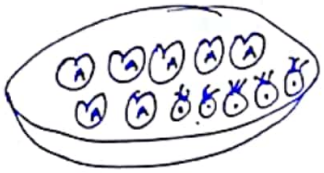
orange ??

somewhat



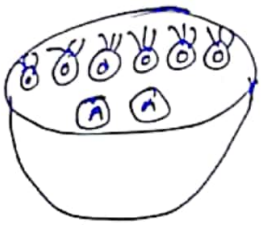
orange ??

sort of

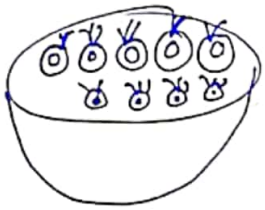


orange ??

few



Mostly



YES, Absolutely.

Fuzzy set ≡ {NO, slightly, somewhat, sort of, few, mostly, absolutely}

zzy words:

- Quantification:
  all, most, many, ~~tall~~ about half, few and no.
- Usuality:
  Always, frequently, often, occasionally, seldom and never
- Likely hood:
  certain, likely, uncertain, unlikely, and certainly not.

Fuzzy words are called linguistic variable.

Inventor of fuzzy logic:

Lotfi Zadeh in 1965.

Advantage of Fuzzy logic for System Control

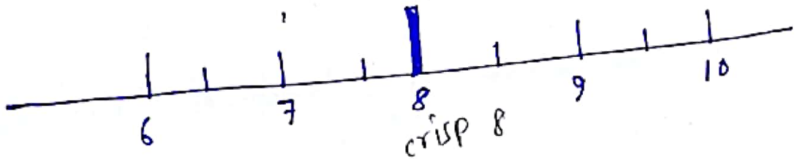
- Fewer rules, values and decisions.
- Linguistic variables are used.
- Relation of input and output.
- Simplicity.
- Rapid prototyping.
- Easier to design.
- Increased Robustness.
- Simple knowledge representation.
- Few rules for great complexity.

Drawbacks:

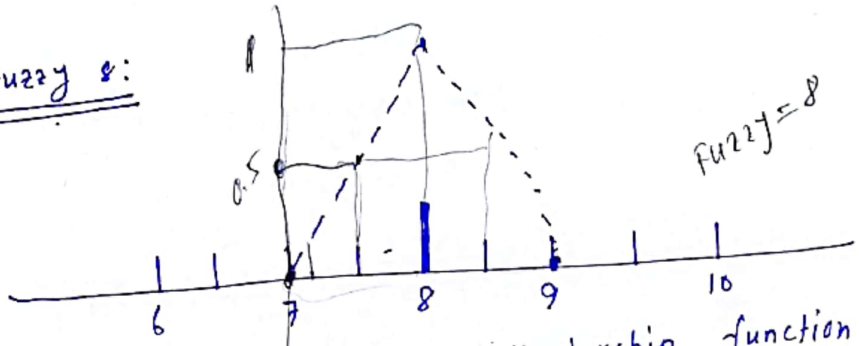
- Tunning of membership functions.
- May not scale well to large or complex problems.
- Deals with imprecision and vagueness, but not uncertainty.

zy Numbers:

A crisp 8:



A Fuzzy 8:



Eightness with a triangular Membership function:

Member	Degree of Membership
7	0
7.5	0.5
8	1
8.5	0.5
9	0

Traditional Representation of logic (Crisp logic):

```
boolean speed
get the speed
if speed = 0
    speed is slow
else
    speed is fast
```

y logic Representation:


```
float speed
get the speed
if   speed >= 0.0 and speed < 0.25
        speed is slowest
else if  speed >= 0.25 and speed < 0.5
        speed is slow
else if  speed >= 0.5 and speed < 0.75
        speed is fast

else
        speed is fastest
```

Crisp and Fuzzy arithmetic Operations:

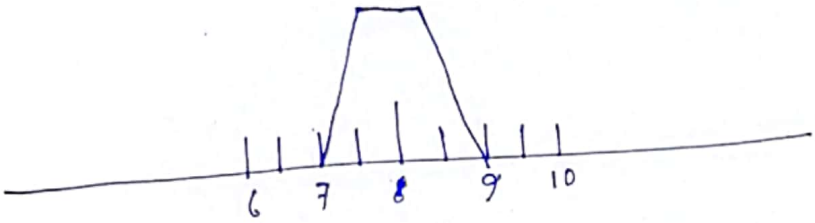
Crisp	Fuzzy
$a = 3 \quad b = 2$	$a = (-2, 3, 8)$ $b = (-1, 2, 7)$
Addition: $a+b$ $3+2=5$	$(-2,3,8) + (-1,2,7)$ $= (5-9, 5, 5+9)$ $= (-4, 5, 14)$
Subtraction: $a-b$ $3-2=1$	$(-2,3,8) - (-1,2,7)$ $= (1-9, 1, 1+9)$ $= (-8, 1, 10)$
Multiplication: $a \times b$ $3 \times 2 = 6$	$(-2,3,8) \times (-1,2,7)$ $= (6-9, 6, 6+9)$ $= (-3, 6, 15)$
Division: ~~a/b~~ a/b $3/2 = 1.5$	$(-2,3,8) / (-1,2,7)$ $= (1.5-9, 1.5, 1.5+9)$ $= (-7.5, 1.5, 10.5)$

10 → 8 → $\frac{10+8}{2} = 9$

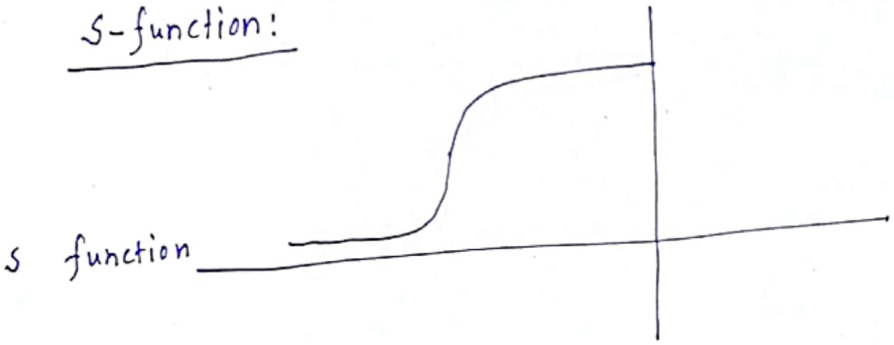
erent types of membership function:

Triangular:

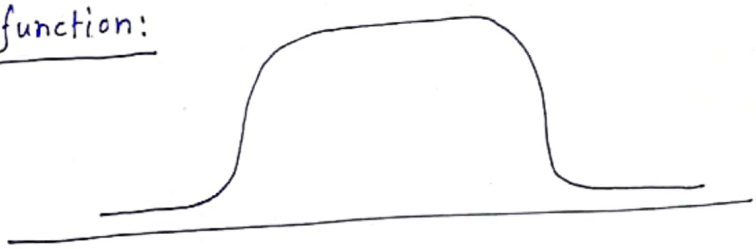
Trapezoid:



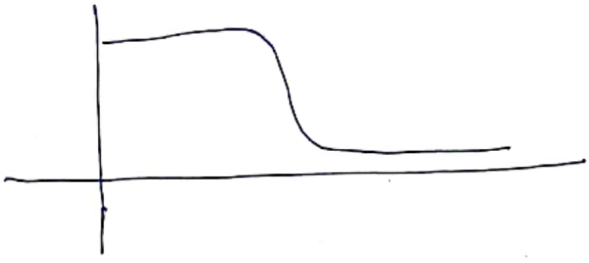
S-function:



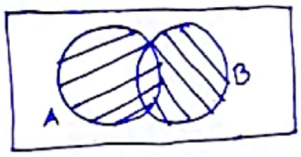
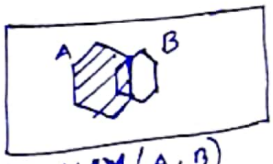
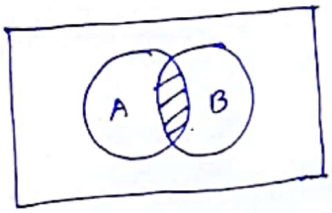
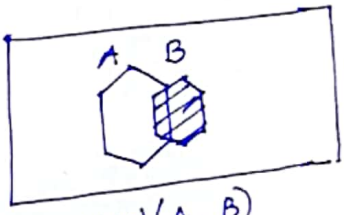
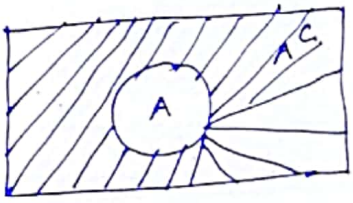
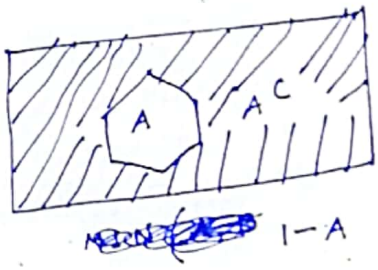
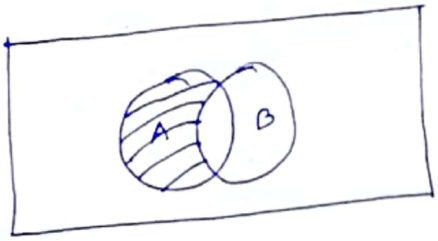
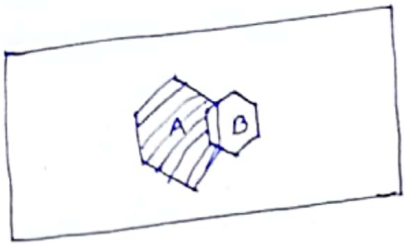
pi-function:



z-function:



theory

Operation	crisp	fuzzy
Disjunction or V or U		 MAX(A, B)
Conjunction or intersection		 MIN(A, B)
Complement		 1 − A
Difference	 A − B	 A − B
Implication	A implies B or, A → B	A → B

Fuzzy logic calculator:

$0.3 \wedge .8 = MIN(0.3, 0.8) = 0.3$

$0.2 \vee .0.6 = MAX(0.2, 0.6) = 0.6$

$\neg 0.9 = 1.0 - 0.9 = 0.1$

$\neg 0.6 = 1.0 - 0.6 = 0.4$

$0.8 \Longrightarrow 0.7$ $\quad 0.8 >= 0.7$ YES

$0.7 \Longrightarrow 0.8$ $\quad 0.7 >= 0.8$ NO

Fuzzy logic in Multielement sets :

$A \equiv (0.8 \quad 0.2 \quad 0.7)$

$B \equiv (1 \quad 0.3 \quad 0.4)$

Operation	Fuzzy logic
$A \cup B$	$(1.0 \quad 0.3 \quad 0.7)$
$A \cap B$	$(0.8 \quad 0.2 \quad 0.4)$
$B \subset A$	(NO NO YES)
	$(0 \quad 0 \quad 3)$
$A \setminus B$	$(0.2 \quad 0.8 \quad 0.3)$
A^C	

Crisp set:

$A \equiv (0.8 \quad 0.2 \quad 0.7)$

$B \equiv (1 \quad 0.3 \quad 0.4)$

fuzzy set

$\Rightarrow$

$A \equiv (1 \quad 0 \quad 1)$

$B \equiv (1 \quad 0 \quad 0)$

crisp set

and Fuzzy logic (Rules of inference)

Set theory is closely related to the truth-finding logical statements called the rules of inference.

⇒ Use rules of implication ($A \Rightarrow B$)

→ Modus ponens: (Affirmative mode)

If the apple is red
AND a red apple is a ripe apple
Then the apple is ripe

Modus tollens: (Denial mode)

If the apple is not ripe
AND a red apple is a ripe apple
→ Then the apple is not red

Fuzzy Rule 1: (Modus ponens)

As the apple is very red
And a red apple is a ripe apple
Then the apple is very ripe

Fuzzy Rule 2: (compositional rule of inference)

As Apple #1 is very ripe
And Apple #2 is not quite as ripe as Apple #1
Then Apple #2 is more or less ripe.

logical statements

crisp: $(\exists x)[\text{ripe}(\text{apple})]$ → Existential quantifier

There exists one example of a ripe apple.

Fuzzy:

- There exists
- Few

crisp: $(\forall x)[\text{apple}(x) \longrightarrow \text{ripeness}(x)]$ → Universal quantifier

All apples are ripe

Fuzzy

- For all
- Most

Traditional or Crisp Rule:

Ex: If the room temperature is less than 62 degrees
Then set the thermostat for 68 digress

* crisp rule gives precise terms.

Fuzzy logic

If-Then-style rules expressed by As-Then (general form) or As-Do (control frm).

Ex: As the room temperature is cool
Do turn on the heater to High

Fuzzy logic NOT

A	Not(A)
0	1
0.25	0.75
0.5	0.5
0.75	0.25
1	0

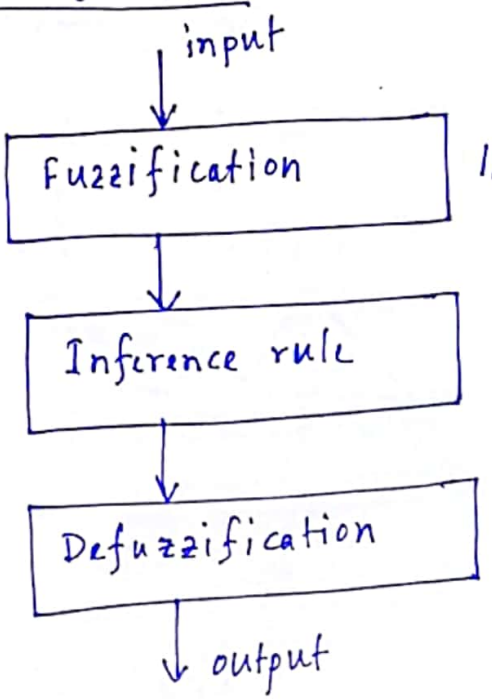
Fuzzy logic AND

A \ B	0	0.25	0.5	0.75	1.0
0	0	0	0	0	0
0.25	0	0.25	0.25	0.25	0.25
0.5	0	0.25	0.5	0.5	0.5
0.75	0	0.25	0.5	0.75	0.75
1.0	0	0.25	0.5	0.75	1.0

Fuzzy logic OR

A \ B	0	0.25	0.5	0.75	1.0
0	0	0.25	0.5	0.75	1.0
0.25	0.25	0.25	0.5	0.75	1.0
0.5	0.5	0.5	0.5	0.75	1.0
0.75	0.75	0.75	0.75	0.75	1.0
1.0	1.0	1.0	1.0	1.0	1.0

Fuzzy logic systems:



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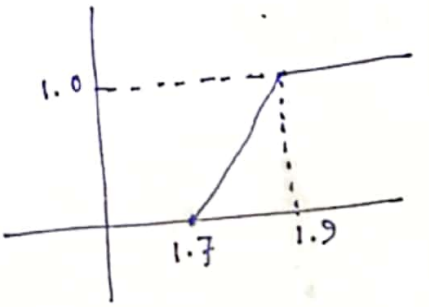
Membership Functions:

Better than listing membership values.

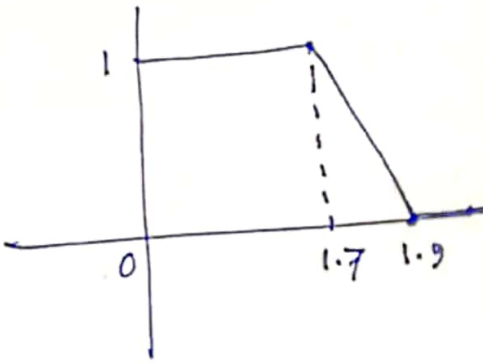
Example: Fuzzy tall

$$Tall(x) = \begin{cases} 1 & \text{if } x >= 1.9\,m \\ 0 & \text{if } x <= 1.7\,m \\ \frac{x-1.7}{0.2} & \text{otherwise} \end{cases}$$



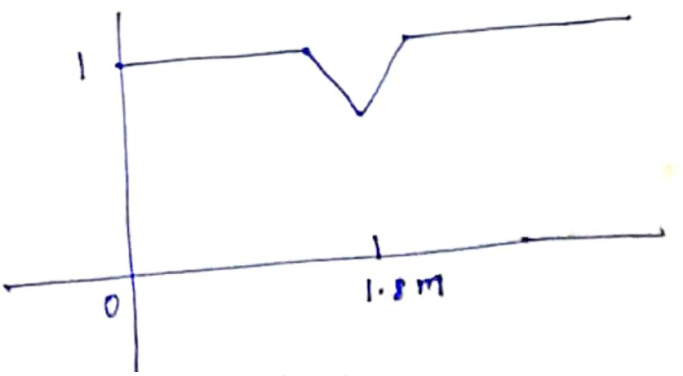
Example: Fuzzy short

$$short(x) = \begin{cases} 0 & \text{if } x >= 1.9\,m \\ 1 & \text{if } x <= 1.7\,m \\ \frac{1.9-x}{0.2} & \text{otherwise} \end{cases}$$



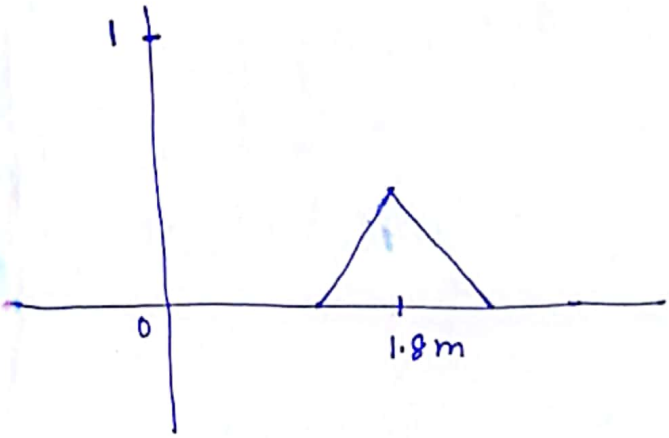
Union:

Tall (x) ∪ short (x):



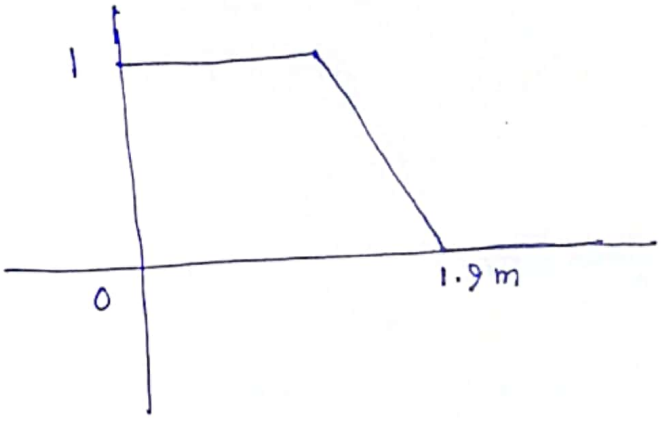
...iction:

Tall(x) and short(x)



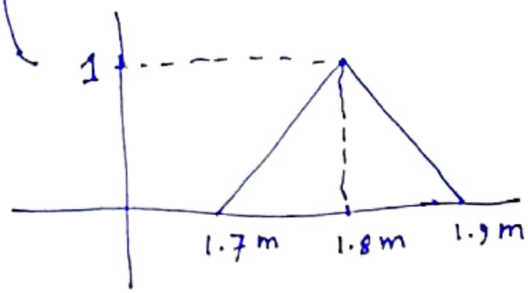
...mplement:

Not(Tall(x))



Example:

$$\text{Medium}(x) = \begin{cases} 0 & \text{if } x >= 1.90 \text{ or } x < 1.70 \\ \dfrac{1.9 - x}{0.1} & \text{if } x >= 1.80 \text{ and } x < 1.90 \\ \dfrac{x - 1.70}{0.1} & \text{if } x >= 1.70 \text{ and } x < 1.80 \end{cases}$$



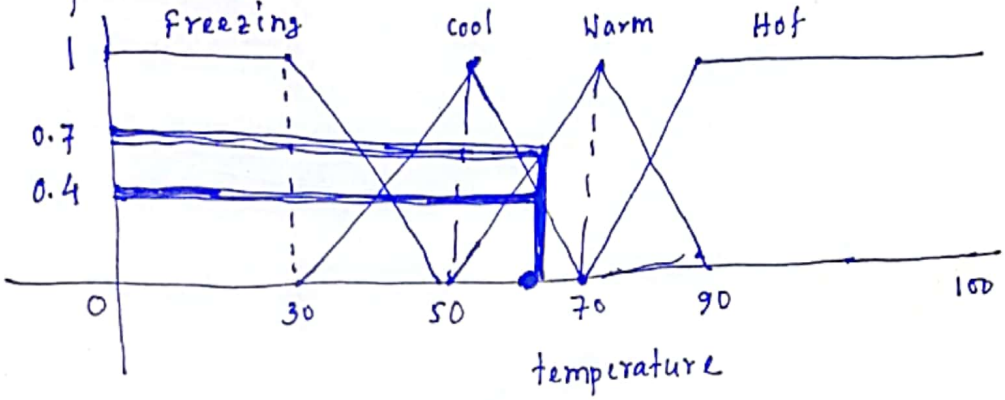
Input: Temperature { Freezing, Cool, Warm, Hot }
cloud Cover { Sunny, [illegible], Partly, Overcast }

Output: Speed { slow, Fast }

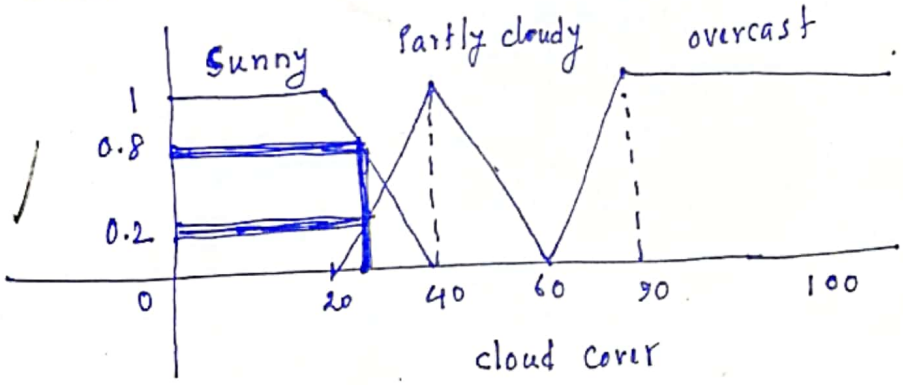
Fast will I go if it is 65 F° and 25% cloud cover?

Membership function / Fuzzification:

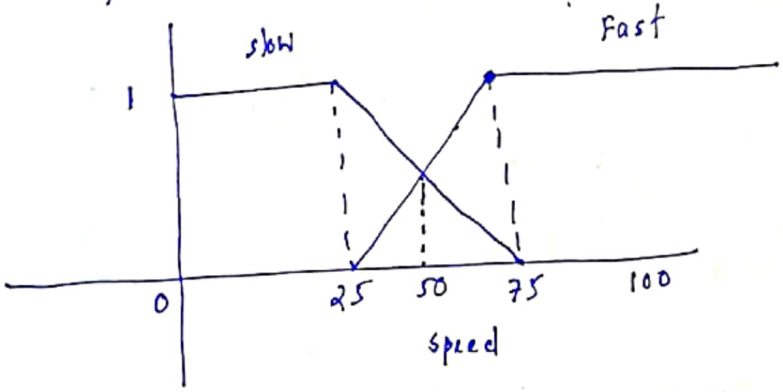
For Temperature:



For cloud Cover:



For output speed:



65°F ⇒ cool = 0.4, Warm = 0.7
25% Cover ⇒ Sunny = 0.8, cloudy = 0.2

s:

is sunny and warm, drive fast

it is Cloudy and cool, drive slow

sunny and warm = sunny ∧ warm

= 0.8 ∧ 0.7

= 0.7

By rule 1, Fast = 0.7 = 70%.

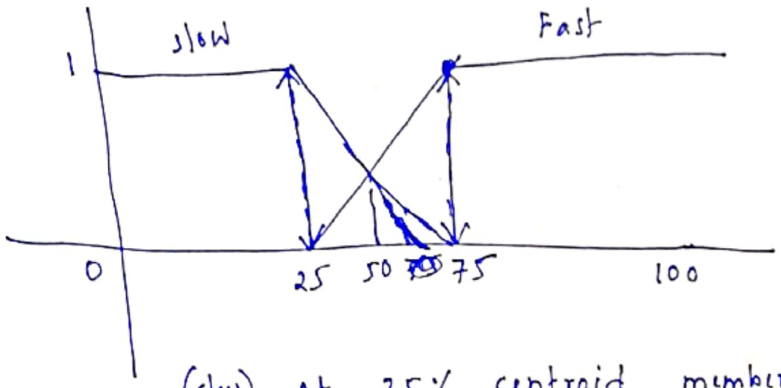
cloudy and cool = cloudy ∧ cool

= 0.2 ∧ 0.4

= 0.2

By rule 2, slow = 0.2 = 20%.

Defuzzification: (output speed)



(slow) At 25% centroid, membership is 100%.

(Fast) At 75% centroid, membership is 100%.

Generated speed = Weighted Mean

= $(2 \times 25 + 7 \times 75)/(2+7)$ = 63.8 mph Ans: