



Assignment-01

Course Code: CSE 6131

Course Title: Computational Intelligence

Section: M

Submitted by

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Ans To The Q. No - 1 (a)

Fuzzy Logic: Fuzzy logic is an approach to variable processing that allows for multiple possible truth values to be processed through the same variable. Fuzzy logic attempts to solve problems with an open, imprecise spectrum of data and heuristics that makes it possible to obtain an array of accurate conclusions.

Distinguishment between Crisp and Fuzzy Logic

Crisp Logic	Fuzzy Logic
i. The degree of truth is 0 or 1.	i. The degree of truth is between 0 and 1.
ii. Elements is either the member of a set or not.	ii. Elements are allowed to be partially included in set.
iii. It can't deal with representation of human intelligence.	iii. It can deal with representation of human intelligence.
iv. Elements is either the member of a set or not.	iv. Elements are allowed to be partially included in set.
v. Used in Digital Design	v. Used in fuzzy controllers.
vi. Binary Logic.	vi. Continuous valued logic.

Ans To The Q.No → 1(b)

fuzzy logic based system is largely used in modern Control systems such as expert systems. Fuzzy logic is used with Neural Networks, because it can mimic how a person would make decisions only much faster. The importance of fuzzy based logic system in real life described below

- i. Simplicity
 - ii. Rapid Prototyping
 - iii. Easier to design
 - iv. Increased Robustness
 - v. Simple knowledge representation
 - vi. Few rules for great complexity
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Ans To The Q.No - 1(c)

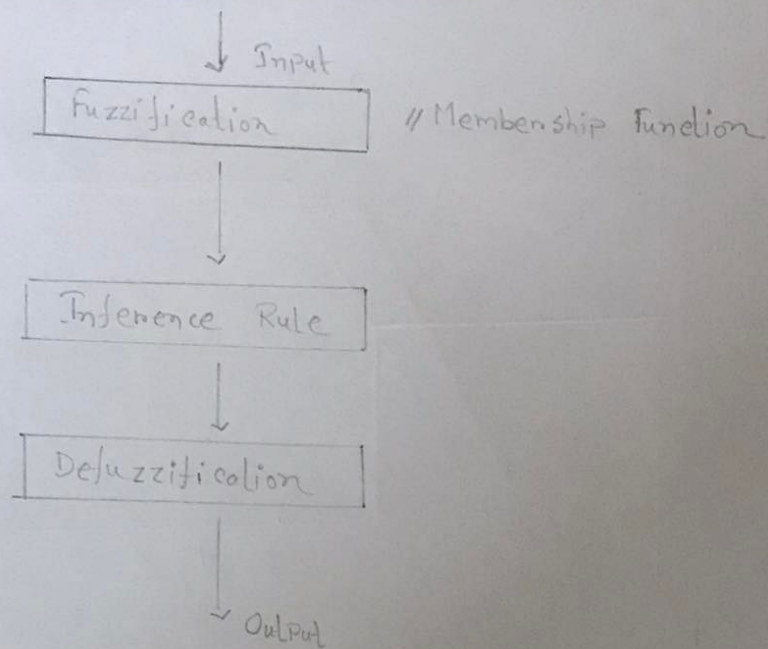
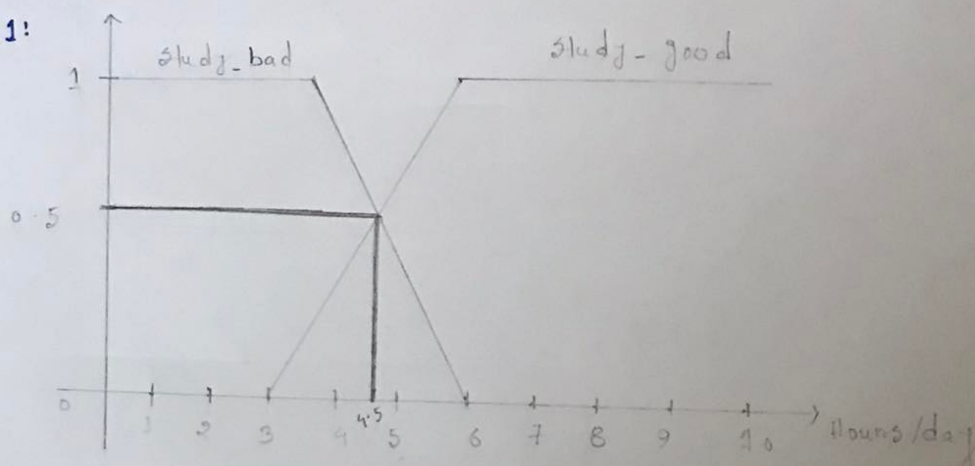


Fig: Diagram of a Fuzzy Logic System

The fuzzy logic system can be divided into three parts. In Fuzzification, the Component transforms raw inputs into fuzzy sets. In rule of inference, set theory is closely related to the truth finding logical statements. In de-fuzzification fuzzy sets are transformed into explicit output. Membership function is a graphical representation of a fuzzy set. The whole thing is explained with an example. Let's say a Problem statement given to find obtained mark of a student who studied 4.5 hours & slept 7.5 hours in a day. Regarding this three sets are given with graphs STUDY-HOUR {study-bad, study-good}, SLEEPING-HOUR {under-sleep, well-sleep, over-sleep} & STUDENT {GOOD, BAD}.

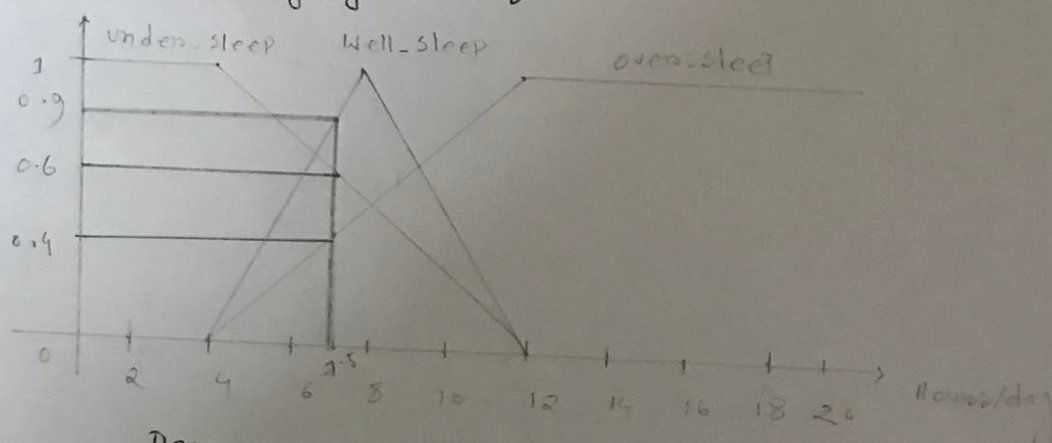
At first from graphs we will find out Degree of Membership. Which means Fuzzification Technique will be applied.

Step 1:



Degree:

$$\begin{aligned}\text{Study-bad} &= 0.5 \\ \text{Study-good} &= 0.5\end{aligned}$$



Degree:

$$\begin{aligned}\text{Well-sleep} &= 0.9 \\ \text{Under-sleep} &= 0.6 \\ \text{Over-sleep} &= 0.4\end{aligned}$$

Step 2:

For Inference Rule, Lets assume 2 rules been given
 Rule 1: If a student studies & sleeps well, he will be a good student.

According to Rule 1:

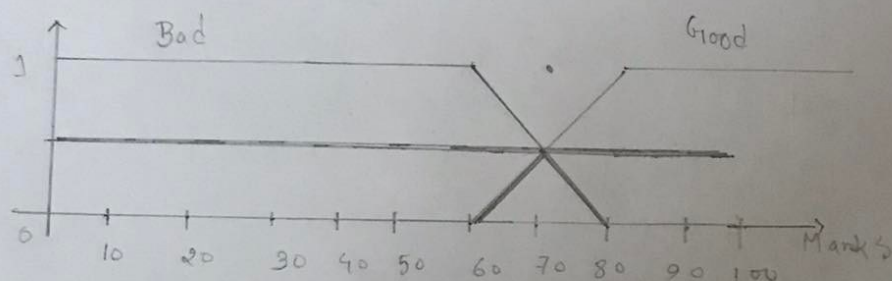
$$\begin{aligned}\text{Study-good} \wedge \text{Well-sleep} &= 0.5 \wedge 0.9 \\ &= \min(0.5, 0.9) \\ &= 0.5 \text{ (good)}\end{aligned}$$

Rule 2: If a student studies bad, and sleeps bad on over, he will be bad student.

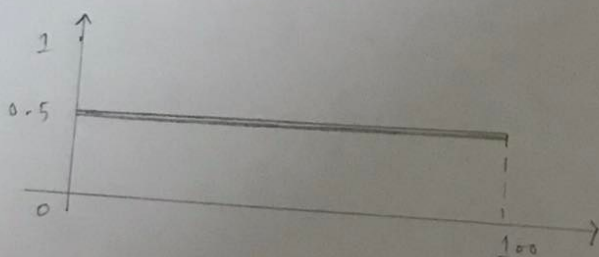
According to Rule 2:

$$\begin{aligned} \text{Study_bad} \wedge (\text{under_sleep} \vee \text{over_sleep}) &= 0.5 \wedge (0.6 \vee 0.9) \\ &= 0.5 \wedge 0.6 \\ &= \min(0.5, 0.6) \\ &= 0.5 (\text{bad}) \end{aligned}$$

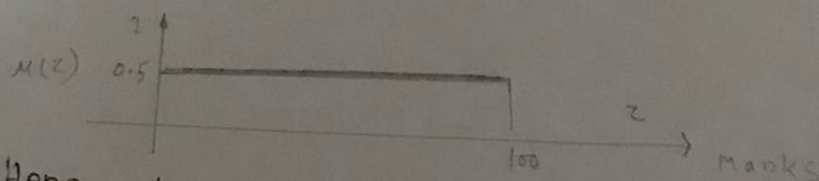
From Membership Function of a student, in the graph now we will draw envelop.



After ORing operation

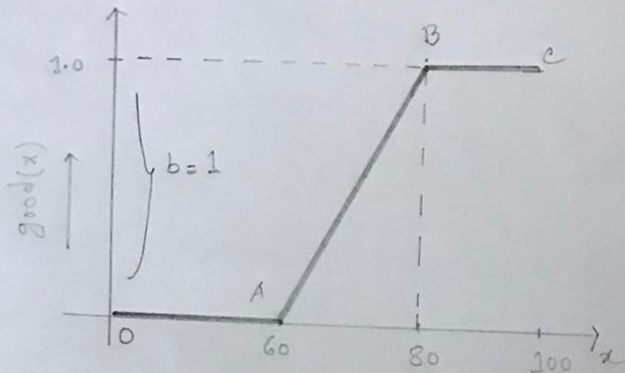


Step 3: Applying Centroid defuzzification



Hence the graph represents a Rectangular area

$$\begin{aligned} \text{So, obtained Marks} &= \mu(z) * z \\ &= 0.5 * 100 \\ &= 50 \end{aligned}$$

Ans To The Q.No - 2(a)Fuzzification of good(x):

$$\text{good}(x) = \begin{cases} OA, & \text{where } 0 \leq x \leq 60 \\ AB, & \text{where } 60 < x < 80 \\ BC, & \text{where } 80 \leq x \leq 100 \end{cases}$$

For OA

OA is on x-axis

So, the equation of OA:

$$y = 0$$

$$\Rightarrow \text{good}(x) = 0$$

For AB

$$A \equiv (60, 0)$$

$$B \equiv (80, 1)$$

Equation of AB:

$$\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$

$$\Rightarrow \frac{y - 0}{0 - 1} = \frac{x - 60}{60 - 80}$$

$$\Rightarrow \frac{y}{-1} = \frac{x - 60}{-20}$$

$$\Rightarrow \frac{y}{1} = \frac{x - 60}{20}$$

$$\Rightarrow 20y = x - 60$$

$$\Rightarrow y = \frac{x - 60}{20}$$

$$\Rightarrow \text{good}(x) = \frac{x - 60}{20}$$

For Be

Be is Parallel to x axis

Equation of Be:

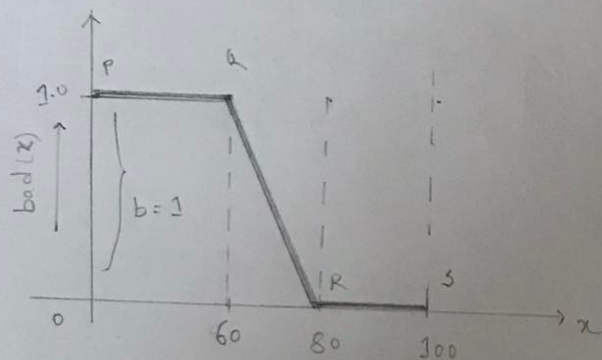
$$\gamma = b$$

$$\Rightarrow \gamma = 1$$

$$\Rightarrow \text{good}(x) = 1$$

$$\text{So, } \text{good}(x) = \begin{cases} 0 & 0 \leq x \leq 60 \\ \frac{x-60}{20} & 60 < x < 80 \\ 1 & 80 \leq x \leq 100 \end{cases}$$

Fuzzification of bad(x):



$$\text{bad}(x) = \begin{cases} \text{PQ} & , \text{ where } 0 \leq x \leq 60 \\ \text{QR} & , \text{ where } 60 < x < 80 \\ \text{RS} & , \text{ where } 80 \leq x \leq 100 \end{cases}$$

For PQ

PQ is Parallel to x-axis
So, the equation of PQ:

$$\gamma = b$$

$$\Rightarrow \text{bad}(x) = 1$$

For QR

$$Q = (60, 1)$$

$$R = (80, 0)$$

So, the equation of QR:

$$\frac{\gamma - \gamma_1}{\gamma_1 - \gamma_2} = \frac{x - x_1}{x_1 - x_2}$$

$$\Rightarrow \frac{\gamma - 1}{1 - 0} = \frac{x - 60}{60 - 80}$$

$$\begin{aligned}
 \Rightarrow \frac{y-1}{1} &= \frac{x-60}{-20} \\
 \Rightarrow -20y + 20 &= x - 60 \\
 \Rightarrow 20y &= -x + 60 + 20 \\
 \Rightarrow 20y &= 80 - x \\
 \Rightarrow y &= \frac{80-x}{20} \\
 \Rightarrow \text{bad}(x) &= \frac{80-x}{20}
 \end{aligned}$$

For RS

RS is on x-axis
So, the equation of RS:

$$y=0$$

$$\Rightarrow \text{bad}(x) = 0$$

$$\text{So, } \text{bad}(x) = \begin{cases} 1 & 0 \leq x \leq 60 \\ \frac{80-x}{20} & 60 < x < 80 \\ 0 & 80 \leq x \leq 100 \end{cases}$$

good(73)

where $x = 73$

which means $60 < x < 80$ is applicable

$$\begin{aligned}
 \text{good}(x) &= \frac{x-60}{20} \\
 &= \frac{73-60}{20} \\
 &= \frac{13}{20} \\
 &= 0.65
 \end{aligned}$$

bad(52)

where $x = 52$

which means $0 \leq x \leq 60$ is applicable

$$\text{bad}(x) = 1$$

So, NOT (good(73) AND bad(52))

$$= \text{NOT} (0.65 \text{ AND } 1)$$

$$= \text{NOT} (0.65)$$

$$= 0.35$$

$$[\text{NOT} (0.65) = 1 - 0.65 = 0.35]$$

Ans To The Q.No - 2(b)

Maximum defuzzification technique is also called the height method. It's only accurate for peaked output. Mean of Max is closely related to the Max Membership Principle.

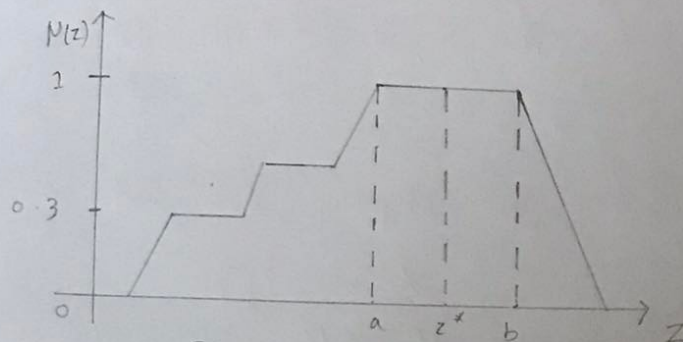


Fig: Max & Mean of Max Defuzzification

For Maximum Defuzzification

$$\mu(z^*) \geq \mu(z) \text{ for all } z \in Z$$

z^* is the defuzzified value

For Mean of Maximum Defuzzification

$$z^* = \frac{a+b}{2}$$

z^* is the defuzzified value.

Ans To The Q.No - 2(c)

Given,

The mixture of apples & oranges in the ratios 0:6, 1:5, 2:4, 3:3, 4:2, 5:1 and 6:0

For the given Problem,

Crisp set $\equiv \{ \text{No, No, No, No, No, No, YES} \}$

$\equiv \{ 0, 0, 0, 0, 0, 0, 1 \}$

Fuzzy set $\equiv \{ \text{No, slightly, somewhat, sort of, Few, Mostly,}$

$\text{Absolutely} \}$

$\equiv \{ 0, 1/6, 2/6, 3/6, 4/6, 5/6, 6/6 \}$

Ans To The Q.No - 3

Step 1: Fuzzification or Membership Function Construction

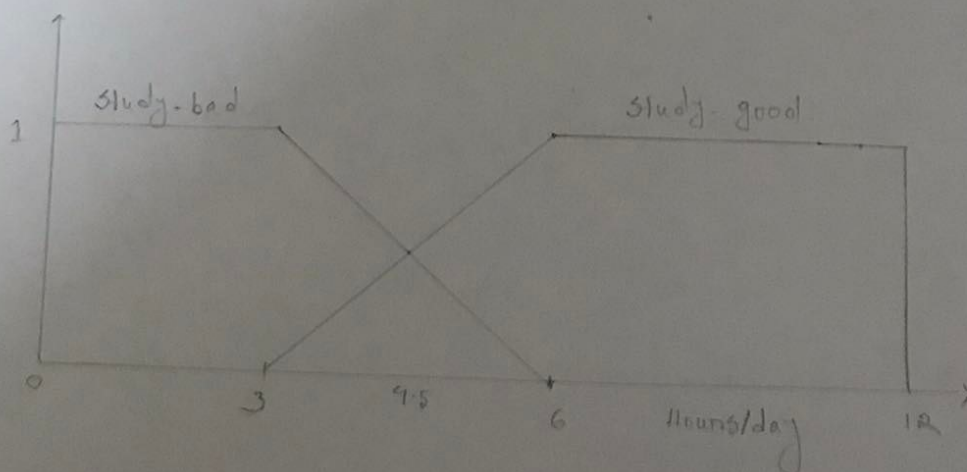


Fig. Membership function for STUDY HOUR

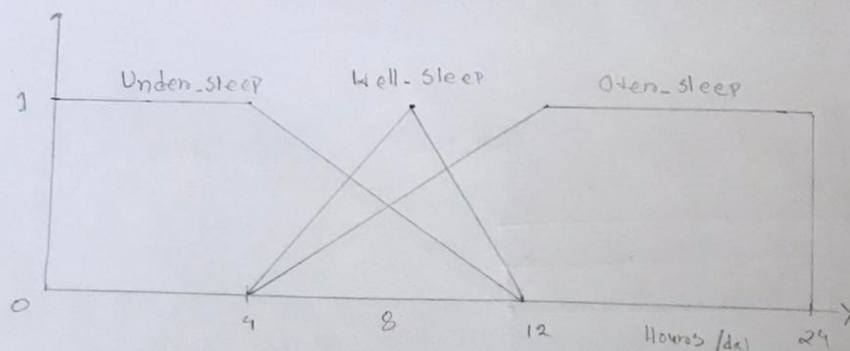


Fig: Membership function for SLEEPING HOUR

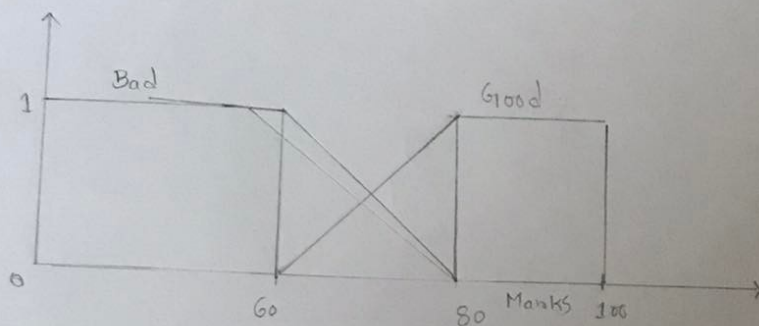
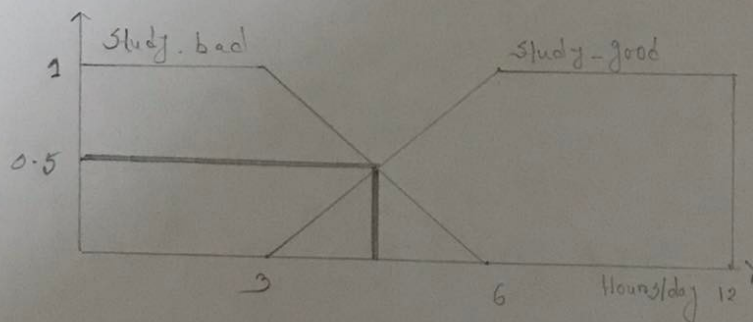


Fig: Membership function for a STUDENT

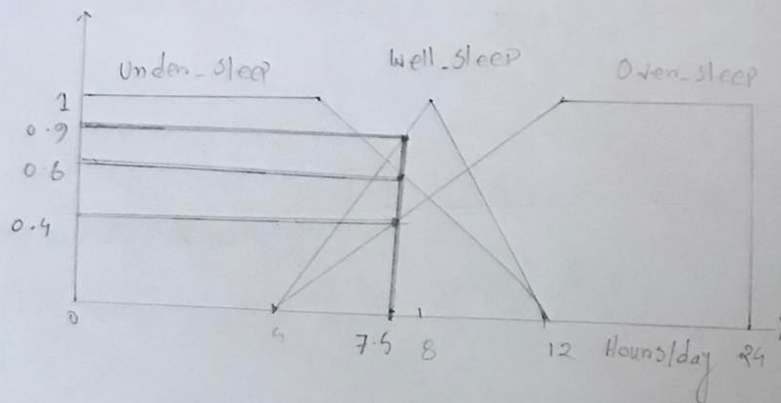
STUDY HOUR {Study-bad, Study-good}



Degree :

$$\begin{aligned} \text{Study-bad} &= 0.5 \\ \text{Study-good} &= 0.5 \end{aligned}$$

SLEEPING HOUR {Under-Sleep, Well-Sleep, Over-Sleep}



Degree:

$$\begin{aligned}\text{Under-Sleep} &= 0.6 \\ \text{Well-Sleep} &= 0.9 \\ \text{Over-Sleep} &= 0.4\end{aligned}$$

Step 2: Applying Fuzzy Rule

Rule 1: If a student studies and sleeps well, he will be good student.

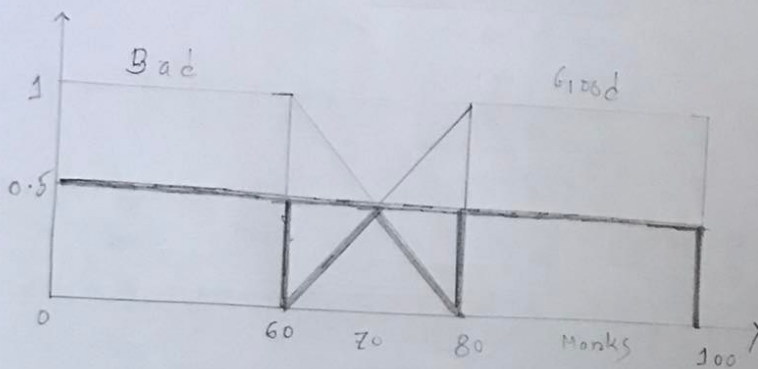
Rule 2: If a student studies bad, and sleeps bad or over, he will be bad student.

According to Rule 1:

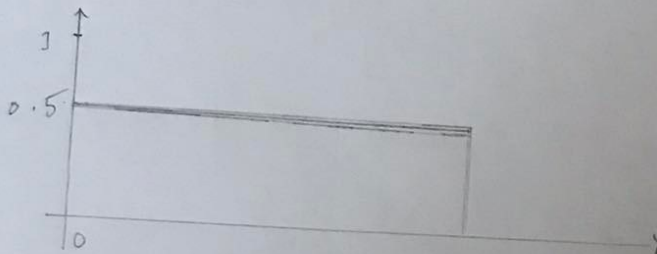
$$\begin{aligned}\text{Study-good} \wedge \text{Well-Sleep} &= 0.5 \wedge 0.9 \\ &= \min(0.5, 0.9) \\ &= 0.5 \text{ (good)}\end{aligned}$$

According to Rule 2:

$$\begin{aligned}\text{Study-good} \\ \text{Study-bad} \wedge (\text{Under-sleep} \vee \text{Over-sleep}) &= 0.5 \wedge (0.6 \vee 0.4) \\ &= 0.5 \wedge 0.6 \\ &= \min(0.5, 0.6) \\ &= 0.5 \text{ (bad)}\end{aligned}$$

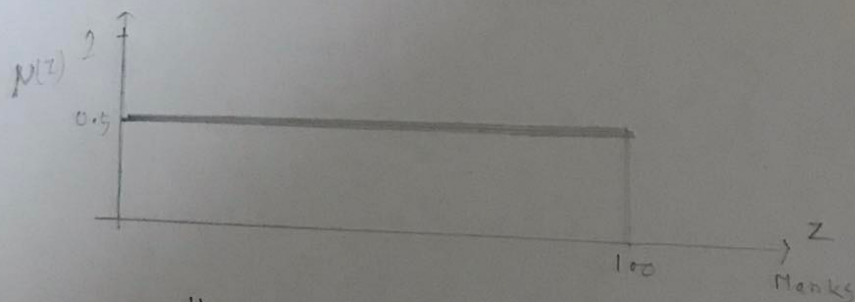


After ORing Operation



Step 3: Defuzzification Technique

We are applying Centroid Defuzzification Technique.



Here, The bold horizontal line is Parallel to z axis.

$$\text{So, } y = b$$

$$\Rightarrow \mu(z) = 0.5$$

$$\begin{aligned}\text{Obtained Marks} &= \frac{\int_0^{100} z \cdot u(z) dz}{\int_0^{100} u(z) dz} \\ &= \frac{\int_0^{100} z \cdot 0.5 dz}{\int_0^{100} 0.5 dz} \\ &= \frac{N}{D}\end{aligned}$$

$$\begin{aligned}N = \text{Numerator} &= \int_0^{100} z \cdot 0.5 dz \\ &= 0.5 \int_0^{100} z dz \\ &= 0.5 \left[\frac{z^2}{2} \right]_0^{100} \\ &= 0.5 \left[\frac{(100)^2}{2} - \frac{(0)^2}{2} \right] \\ &= 0.5 \times \frac{10000}{2} \\ &= 0.5 \times 5000 \\ &= 2500\end{aligned}$$

$$\begin{aligned}D = \text{Denominator} &= \int_0^{100} 0.5 dz \\ &= 0.5 \int_0^{100} dz \\ &= 0.5 [z]_0^{100} \\ &= 0.5 [100 - 0] \\ &= 0.5 \times 100 \\ &= 50\end{aligned}$$

$$\begin{aligned}\text{So Obtained Marks} &= \frac{N}{D} = \frac{2500}{50} \\ &= 50\end{aligned}$$

Ans To The Q. No - 4(a)

Page-15

Given, $A = \{0.8, 0.9, 0.1\}$ and $B = \{0.9, 0.5, 0.8\}$

Fuzzy Logic in Multi element Sets:

$$A \equiv (0.8 \quad 0.9 \quad 0.1)$$

$$B \equiv (0.9 \quad 0.5 \quad 0.8)$$

Operation	Fuzzy logic
$A \cup B$	$(0.9 \quad 0.9 \quad 0.8)$
$A \cap B$	$(0.8 \quad 0.5 \quad 0.1)$
$B \subset A$	$(\text{NO} \quad \text{YES} \quad \text{NO})$
$A \setminus B$	$(0 \quad 0.4 \quad 0)$
A^c	$(0.2 \quad 0.1 \quad 0.9)$
B^c	$(0.1 \quad 0.5 \quad 0.2)$

Crisp Logic in Multi element Sets:

From $A \cup B$ we got $(0.9 \quad 0.9 \quad 0.8)$ [Above Table]

In Crisp set it would be $(1 \quad 1 \quad 1)$

From $A \cap B$ we got $(0.8 \quad 0.5 \quad 0.1)$ [Above Table]

In Crisp set it would be $(1 \quad 1 \quad 0)$

From $B \subset A$ we got $(\text{NO} \quad \text{YES} \quad \text{NO})$ [Above Table]

In Crisp set it would be $(0 \quad 1 \quad 0)$

From $A \setminus B$ we got $(0 \quad 0.4 \quad 0)$ [Above Table]

In Crisp set it would be $(0 \quad 1 \quad 0)$

From A^c we got $(0.2 \quad 0.1 \quad 0.9)$ [Above Table]

In Crisp set it would be $(0 \quad 0 \quad 1)$

From B^c we got $(0.1 \quad 0.5 \quad 0.2)$ [Above Table]

In Crisp set it would be $(0 \quad 1 \quad 0)$

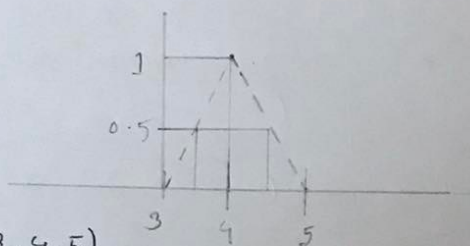
Ans To The Q.No-4(b)

Given $((a^3 + b^3) / (a - b))$

also

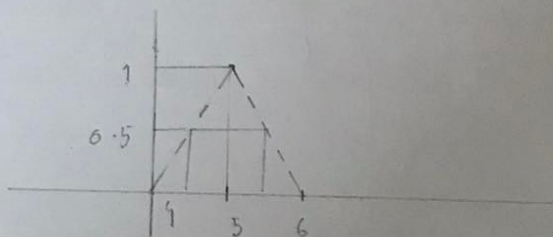
$a = \text{fuzzy } 4$ and $b = \text{fuzzy } 5$

A Fuzzy 4 with a triangular Membership Function



So, $a = (3, 4, 5)$

A Fuzzy 5 with a triangular Membership Function



So, $b = (4, 5, 6)$

Hence,

$$((a^3 + b^3) / (a - b)) = \frac{a^3 + b^3}{a - b}$$

a^3 means, $a \times a \times a$

b^3 means, $b \times b \times b$

$$\text{So, } a \times a = (16 - 2, 16, 16 + 2) = (14, 16, 18)$$

Now

$$(a \times a) \times a = \text{~~14, 16~~} (64 - 3, 64, 64 + 3) = (61, 64, 67)$$

Again

$$b \times b = (25 - 2, 25, 25 + 2) = (23, 25, 27)$$

Now

$$(b \times b) \times b = (125 - 3, 125, 125 + 3) = (122, 125, 128)$$

$$\text{So, } a^3 = (61, 64, 67) \text{ and } b^3 = (122, 125, 128)$$

$$a^3 + b^3 = (189 - 6, 189, 189 + 6) \\ = (183, 189, 195)$$

$$a - b = (-1 - 2, -1, -1 + 2) \\ = (-3, -1, 1)$$

Now

$$\frac{a^3 + b^3}{a - b} = (-189 - 8, -189, -189 + 8) \\ = (-197, -189, -181)$$

(Showed)

Ans To The Q.No - 4(c)

Given

Fuzzy 4 as (3, 4, 5)

Fuzzy 5 as (4, 5, 6)

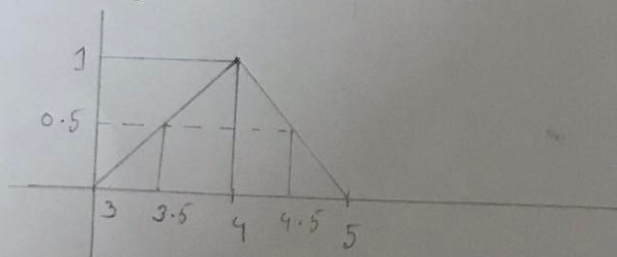


Fig: Founness with a -triangular Membership Function

Members	Degree Me. of Membership (μ)
3	0
3.5	0.5
4	1
4.5	0.5
5	0

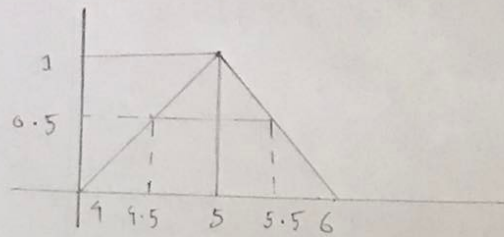


Fig: Fiveness with a Triangular Membership Function

Member	Degree of Membership (μ)
4	0
4.5	0.5
5	1
5.5	0.5
6	0

$$\text{So, } \mu(3.5) + \mu(5.5) = 0.5 + 0.5 = 1$$

[Value of $\mu(3.5)$ got from first table, Value of $\mu(5.5)$ got from second table]

$$\text{So, } \mu(3.5) + \mu(5.5) = 1.$$

Ans To The Q.No-4(d)

Fuzzy Inference Rule: It is a method that interprets the values in the input vector and based on some sets of rules, assigns values to the output vector.

Importance of fuzzy inference rule

- i. Computationally efficient
- ii. Functions well with linear technique

Ans To The Q.No-4(e)

Diagrams of different fuzzification techniques on Membership function.

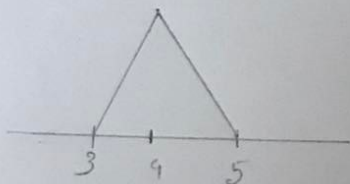


Fig: Triangular

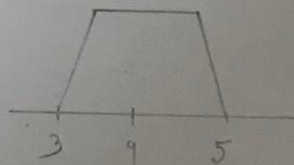


Fig: Trapezoid

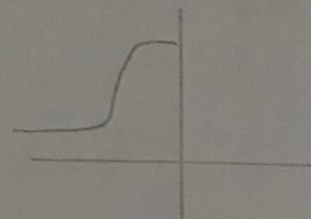


Fig: S Function

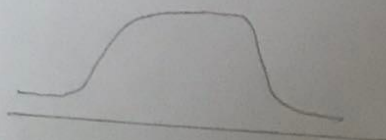


Fig: Pi Function

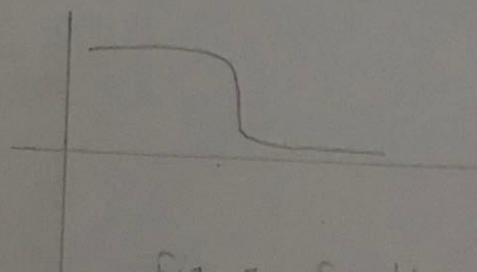


Fig: Z Function