UNIVERSITI MALAYA

ALTERNATIVE ASSESSMENT FOR THE DEGREE OF MASTER OF DATA SCIENCE

ACADEMIC SESSION 2023/2024 : SEMESTER II

WQD7011 : Numerical Optimization

June 2024 Time : 2 hours

INSTRUCTIONS TO CANDIDATES :

Answer **ALL** questions (40 marks).

(This question paper consists of 4 questions on 3 printed pages)

Question 1

a) Plot the contour of the following function in the interval of [-2.8,2.8] for both *x* and *y* using Octave/Matlab. Set the number of levels of the contour to 30 and show the `colorbar` next to the contour. Save the plotted contour to a file named "`myplot.jpg`" and the program as "`myprogram.m`".

Submit both the program and `myplot.jpg`.

$$f(x) = 3\sin(x+y)\cos(x-y) + 6\sin(2x)\cos(2y)$$ (5 marks)

b) From the plot of the function, identify the number of local minimizer you can observe.

(1 mark)

c) Discuss what you can find around the point (0.8, 0.1). (2 marks)

Question 2

Given a two dimensional function *f* as follow:

$$f(x) = x_1^2 + x_2^2 + \left(\frac{1}{2}x_1 + x_2\right)^2 + \left(\frac{1}{2}x_1 + x_2\right)^4$$

The gradient and Hessian of the function is also given as follow:

$$\nabla f(x) = \begin{bmatrix} 2.5x_1 + x_2 + 2(0.5x_1 + x_2)^3 \\ x_1 + 4x_2 + 4(0.5x_1 + x_2)^3 \end{bmatrix}$$

$$\nabla^2 f(x) = \begin{bmatrix} 2.5 + 3(0.5x_1 + x_2)^2 & 1 + 6(0.5x_1 + x_2)^2 \\ 1 + 6(0.5x_1 + x_2)^2 & 4 + 12(0.5x_1 + x_2)^2 \end{bmatrix}$$

A search starts at the initial point $x_0 = (3, 5)$.

a) Determine the direction of a search using steepest descent method starting at x_0 .

(3 marks)

b) John assumes that α = 0.01 is a good choice of step length. Find the point this search ends and determine whether John is correct. (4 marks)

c) Caine does not agree to John. He suggests to use backtracking line search starts from α = 0.01 for a better result. Assume that contraction factor ρ = 0.9, do you think that the result after 2 iterations fulfills Armijo condition? (3 marks)

d) Determine the direction of a search using Newton method starting at x_0 .

(2 marks)

Question 3

Function $f(x)$:$R^3 \rightarrow R$ is defined as follow:

$$f(x) = 3x_1 - 5x_2 + 4x_3$$

Linear programming is to used to find max f(x) subject to the following constraints:

$$\begin{aligned} 2x_1 + 3x_2 &\leq 8 \\ -2x_2 - 5x_3 &\geq -10 \\ 8x_1 + 10x_2 + 15x_3 &\leq 15 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

a) Add slack or surplus variables to convert this problem to a standard form.
(2 marks)

b) Form an augmented matrix that representing the problem.
(2 marks)

c) Start the pivot process to identify the optimal solution of the problem. State the values of x and $f(x)$ when the solution is optimal.
(4 marks)

d) Instead of finding max $f(x)$, find min $f(x)$ by writing an Octave program.
(5 marks)

Question 4

a) Explain the concept of trust region methods.
(3 marks)

b) Discuss how the performance of the last iteration in trust region methods affect the next iteration.
(2 marks)

c) In every step of the trust region method, ρ, the ratio between actual reduction and predicted reduction is always computed. Explain why ρ with a negative value is not acceptable.
(2 marks)

END