WQD7011
Numerical Optimization

Mathematical Modelling

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- Model and Modelling
- Building Models
- Example

Model

- Models describe our beliefs about how the world functions – an abstraction of reality.
- In mathematical modelling, we translate those beliefs into the language of mathematics.
- However, systems in real world are far too complicated to model – models are not perfectly accurate!

Purpose of Modelling

- Understanding problems better
- Communicating with others
- Formulate ideas and identify underlying assumptions
- Perform computation with computers

Objectives of Modelling

- Develop scientific understanding
  - Knowns and unknowns are clearly discussed
- Test the effect of change in a system
- Aid decision making – include optimization!

Type of models

- Deterministic vs stochastic model
  - Deterministic models : ignore random variations
  - Stochastic model : uncertainty is present – deal with randomness
  - In the real world, uncertainty is a part of everyday life, so a stochastic model could literally represent anything.
  - However, in some cases, we may want to ignore the randomness to make the modelling simpler.

"A good theory" (or model) "should be as simple as possible, but not simpler."

– Albert Einstein

Type of models

- Mechanistic vs Empirical model
  - Mechanistic models : use a large amount of theoretical information is used to describe the relation between variables
  - Empirical model : Mainly based on data.

Type of models

- Static vs Dynamic models
  - Static models: systems in steady state
  - Dynamic models : systems that change over time. Usually have a time parameter in the models.

Stages of Modelling

1) Building
2) Studying
3) Testing
4) Use

May revert back to building stage if required

Building Model

- Make sure we are clear about the objective – this determine the direction of our project.
  - Want to optimize something?
  - Max or min?
  - To predict something?

Building Models – Systems Analysis

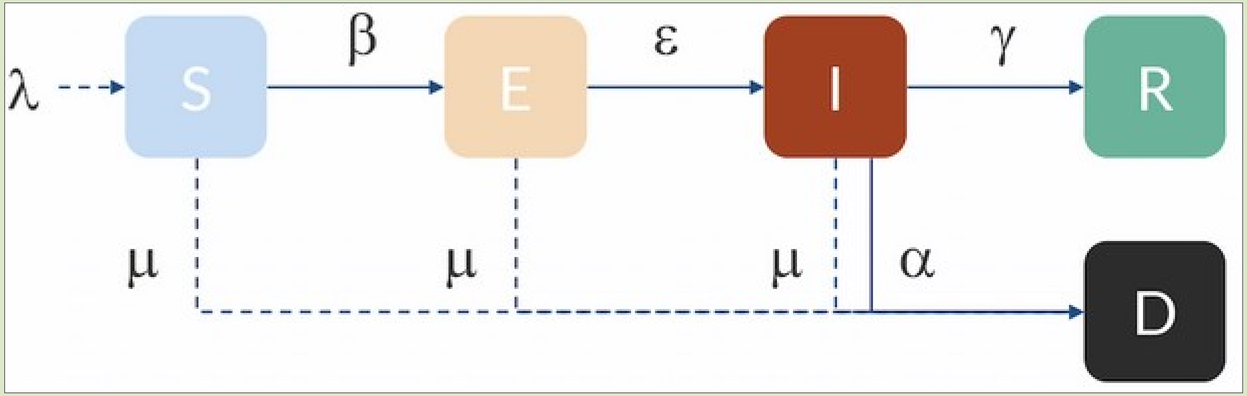
- In this stage, we build the basic framework of the model.
- This normally start with listing a set of assumptions (our believes on the way of the system work)
- Future works are based on the believes
- Newton assume mass is a constant – he develop classical mechanics

  Einstein assume mass can be depend on velocity – he developed theory of relativity

  (Newton may develop nothing if he assumed mass is a variable)

Building Models – Systems Analysis

- We may want to draw diagram to visualize how the system work, especially if the system is complex.
- Example : Modelling the Spread of COVID-19



If conservation law is obeyed:
$E\varepsilon = I$
$R + D = S$
… …

S = susceptible
E = exposed
I = infectious
R = recovered
D = Dead
μ - natural death rate

Building Models – Equations Formation

- Determine the equations that describe the system.
  - May come from literature
    - Someone may have develop a model that similar to yours.
    - However, due to different environment / data / …, these equation may not be used directly.
  - Analogies from physics
    - Used a physical system that well developed by physicists and similar to yours
  - Data Exploration
    - Explore data and fit your equations to it.

Building Models – Solving Equations

- Analytically – manipulation on the equations we obtain to find the solution. But this is not easy especially if the model is stochastic.
- Numerically.

Example:

- A chemical company has 2 factories F_1 and F_2 and a dozen retail outlets $R_1, R_2, \ldots, R_{12}$. Each factory F_i can produce a_i tons of a certain chemical product each week. a_i is called the capacity of the plant. Each retail outlet R_j has a known weekly demand of b_j tons of the product. The distance of factory F_i to retail outlet R_j is denoted as d_{ij}. Assume that x_{ij} is the amount of the chemical product received by outlet R_j from factory F_i, develop a model that represent the minimization of the shipping cost.

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Objective : minimize the total cost of shipping.

Assumptions:

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Assumptions:

- ✔ Uncertainty of travelling is small → deterministic model.
- ✔ The transporter only travel between factories and outlets, not between outlets and outlets
- ✔ The shipping cost per ton is linearly proportional to distance, d_{ij}

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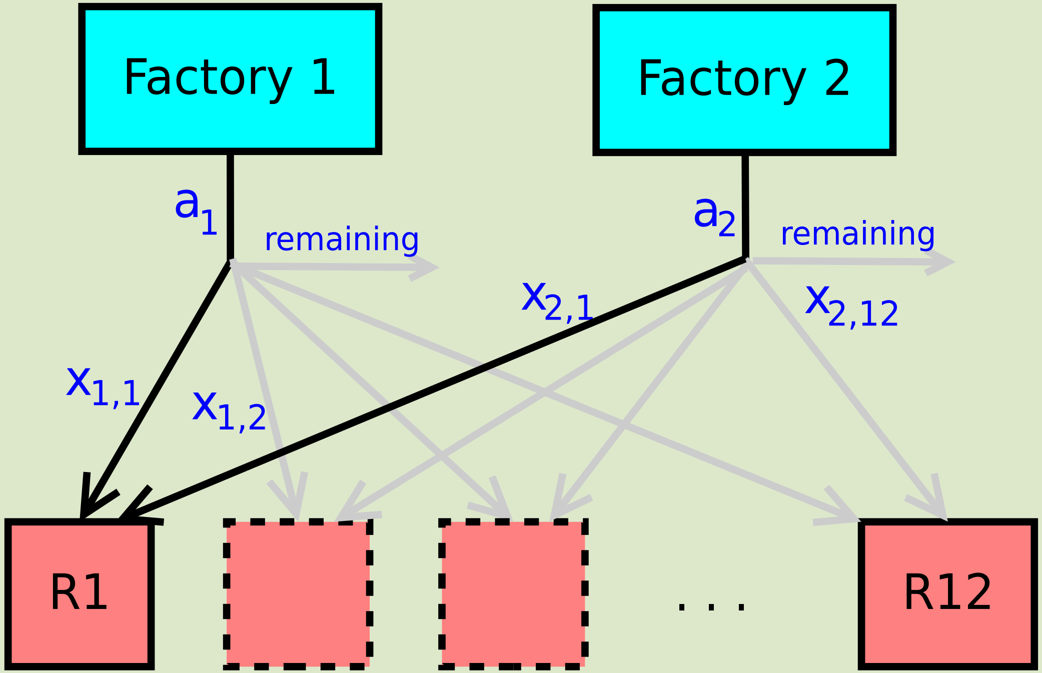

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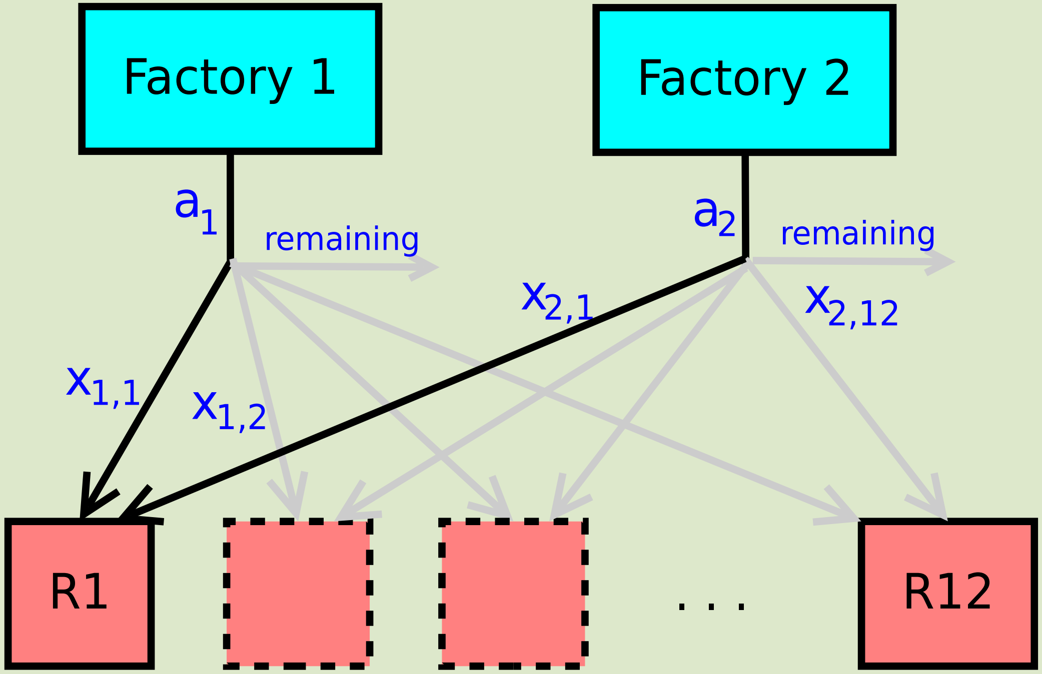

$$a_1 = x_{1,1} + x_{1,2} + \cdots + x_{1,12} + \text{remaining}$$

$$a_1 \geq \sum_{j=1}^{12} x_{1j}$$

$$b_1 \leq x_{1,1} + x_{2,1}$$

Shipping cost from F1 to R1

$\propto$ distance x shipping cost per ton

$\propto$ $d_{ij}\, x_{ij} = c_{ij}\, x_{ij}$

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Objective function : $\min \sum_{ij} c_{ij} x_{ij}$

$$\text{subject to} \sum_{j=1}^{12} x_{ij} \leq a_i, \quad i = 1, 2,$$

$$\sum_{i=1}^{2} x_{ij} \geq b_j, \quad j = 1, \ldots, 12,$$

$$x_{ij} \geq 0, \quad i = 1, 2, \quad j = 1, \ldots, 12.$$

Constraints