1. These questions are given as a revision questions, not tips!
2. Since this is a revision, triggering studetns to learn and think is for more important than the solution itself. So, please don't ask for the answers of the questions.

Four searches for the minima of a function *f*, with different initial points or parameters have been performed. Figure 1 shows the contour plots of the searches using default colourmap.
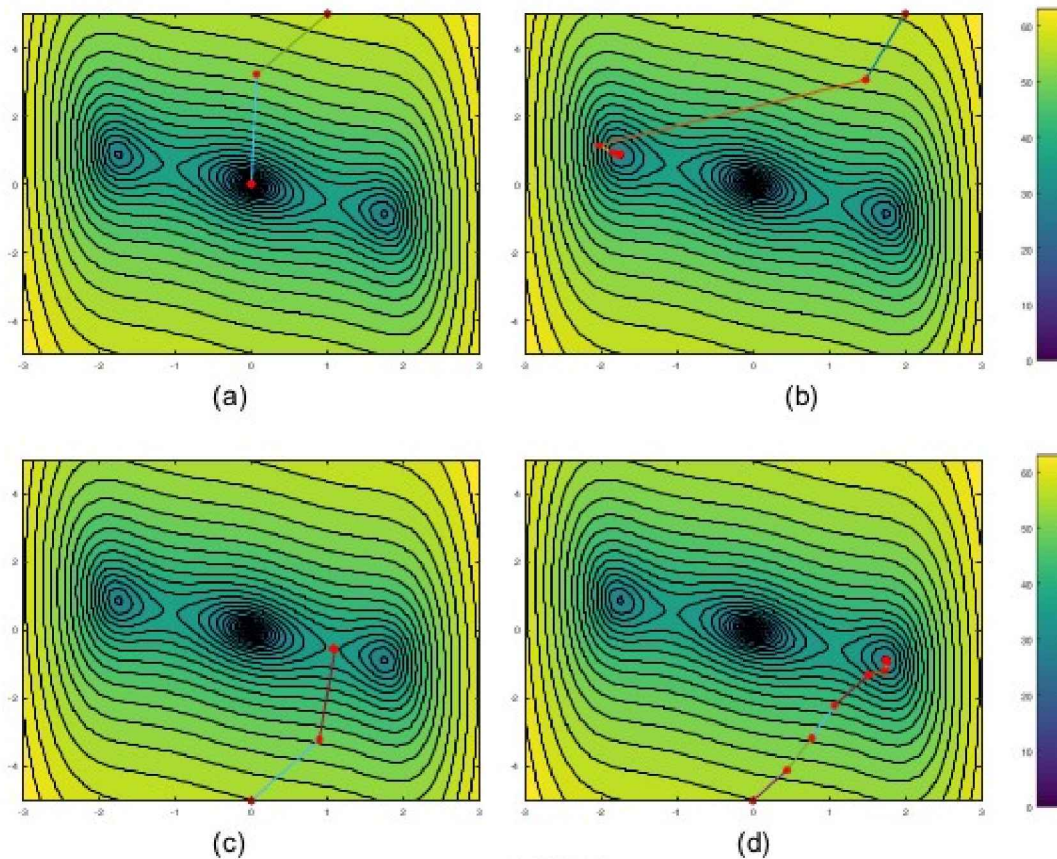

(a)

(b)

(c)

(d)

Figure 1

i. Write your observation about the function *f*.

ii. Write your comment about each of these searches.

A function is defined as follow:

$$f(x,y) = \sin(x+y) + (x-y)^2 - 1.5x + 2.5y + 1$$

a) Find the gradient and Hessian of this function.

b) Compute the gradient and Hessian of the function at (-1,-2). What is the the search direction at this point if Steepest Descent method is used? Show your work.

c) Given that the initial point is (-1,-2). Write a program to perform a sequence of 20 searches for a minimizer using Newton method. Assume that the Hessian is always positive definite along the path and a fixed step length 0.1 will help you to reach the minimizer at the end of the searches. Print the coordinate of the point the searches stop at the end of your program.

Refer to the following program written with Octave / Matlab:

```
C = [4; 6; -3; 5];
A = [2 6 2 0 ; 0 -3 5 4];
b = [20; 72];
lb = [0;0;0;0];
ub=[];
cType = "SU";
varType = "CCCC";
sense = 1;
[x, f, status, extra] = glpk (C,A,b,lb,ub,cType,varType,sense);
```

a) The program is to find the maximizer or minimizer of a function? Give your reason.

b) Besides requiring all the variables involved must be non-negative, how many constraints you can find in this optimization problem? Explain your answer.

c) By using x_1, x_2 and etc. as the notations of the variables, write the function this program wants to optimize.

d) By using x_1, x_2 and etc. as the notations of the variables, write the first constraint in this optimization problem.

e) After the execution of the program, a command `fprintf("%f\t",x)` is executed, and the following is the output of this command:

0.000000	0.000000	10.000000	0.000000

Comment on the output of the command.

f) If the following command is executed, what will be the output?
```
disp(f);
```

Consider the following function $f(x):\mathbb{R}^3 \rightarrow \mathbb{R}$ in a linear programming problem:

$$\max f(x) = 3x_1 + 2x_2 - 5x_3$$

subject to :
$$x_1 + 2x_2 - 3x_3 = 0$$
$$-2x_1 + x_2 \geq 5$$
$$2x_1 + 3x_3 \leq 18$$
$$x_1, x_2, x_3 \geq 0$$

Find the optimal solution of the function using simplex method as follow:

a) Add slack and artificial variables to convert this problem to a standard form. Identify an augmented matrix that representing the problem.

b) From the tableau formed in part (a), what is the value of $f(x)$ if the initial basic feasible solution is used?

c) Start the pivot process to identify the optimal solution of the problem. State the values of x and $f(x)$.