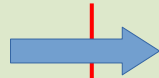# Simplex method
Additional example

Example

$$\min f(x) = -2x_1 + x_2 - 3x_3$$

Subject to :

$$x_1 + x_2 + x_3 \leq 10$$

$$2x_2 - 3x_3 \geq -12$$

$$x_1, x_2, x_3 \geq 0$$

Subject to :

$$x_1 + x_2 + x_3 + s_1 \qquad = 10$$

$$-2x_2 + 3x_3 \qquad + s_2 = 12$$

$$x_1, x_2, x_3, s_1, s_2 \geq 0$$

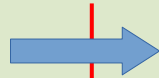$x_1 = 6$, $x_2 = 0$ and $x_3 = 4$.
min f(x) = -24

Example

max ?

$$\min f(x) = -2x_1 + x_2 - 3x_3$$

Subject to :

$$x_1 + x_2 + x_3 \le 10$$

$$2x_2 - 3x_3 \ge -12$$

$$x_1, x_2, x_3 \ge 0$$

Subject to :

$$x_1 + x_2 + x_3 + s_1 \qquad = 10$$

$$-2x_2 + 3x_3 \qquad + s_2 = 12$$

$$x_1, x_2, x_3, s_1, s_2 \ge 0$$

Method 1

$$x_1 + x_2 + x_3 + s_1 \qquad = 10$$

$$-2x_2 + 3x_3 \qquad + s_2 = 12$$

$$\min f(x) = -2x_1 + x_2 - 3x_3$$

$$
\begin{array}{ccccc}
X_1 & X_2 & X_3 & S_1 & S_2 \\
\end{array}
$$

$$
\left[
\begin{array}{ccccc|c}
1 & 1 & 1 & 1 & 0 & 10 \\
0 & -2 & 3 & 0 & 1 & 12 \\
\hline
-2 & 1 & -3 & 0 & 0 & 0 \\
\end{array}
\right]
$$

Basic variables

Method 1

$$x_1 + x_2 + x_3 + s_1 \qquad = 10$$

$$-2x_2 + 3x_3 \qquad + s_2 = 12$$

$$\min f(x) = -2x_1 + x_2 - 3x_3$$

max

$$
\begin{array}{ccccc}
X_1 & X_2 & X_3 & S_1 & S_2 \\
\end{array}
$$

$$
\left[
\begin{array}{ccccc|c}
1 & 1 & 1 & 1 & 0 & 10 \\
0 & -2 & 3 & 0 & 1 & 12 \\
\hline
-2 & 1 & -3 & 0 & 0 & 0 \\
\end{array}
\right]
$$

Basic variables

Most positive as entering variable

x2 enter, s1 quit

Method 1

$$x_1 + x_2 + x_3 + s_1 = 10$$

$$-2x_2 + 3x_3 + s_2 = 12$$

$$\min f(x) = -2x_1 + x_2 - 3x_3$$

max

$$
\begin{array}{ccccc}
x_1 & x_2 & x_3 & s_1 & s_2 \\
\end{array}
$$

$$
\left[
\begin{array}{ccccc|c}
1 & 1 & 1 & 1 & 0 & 10 \\
0 & -2 & 3 & 0 & 1 & 12 \\
\hline
-2 & 1 & -3 & 0 & 0 & 0 \\
\end{array}
\right]
$$

Basic variables

$$
\begin{array}{ccccc|c}
1 & 1 & 1 & 1 & 0 & 10 \\
2 & 0 & 5 & 2 & 1 & 32 \\
\hline
-3 & 0 & -4 & -1 & 0 & -10 \\
\end{array}
$$

R1
R2+2R1
R3-R1

x_1 = 0, x_2 = 10 and x_3 = 0.
max f(x) = -(-10) = 10

Method 2

$$x_1 + x_2 + x_3 + s_1 \qquad = 10$$

$$-2x_2 + 3x_3 \qquad + s_2 = 12$$

$$\min f(x) = -2x_1 + x_2 - 3x_3$$

max f(x) = − min −f(x)

$$
\begin{array}{ccccc}
\text{X}_1 & \text{X}_2 & \text{X}_3 & \text{S}_1 & \text{S}_2 \\
\end{array}
$$

$$
\left[
\begin{array}{ccccc|c}
1 & 1 & 1 & 1 & 0 & 10 \\
0 & -2 & 3 & 0 & 1 & 12 \\
\hline
-2 & 1 & -3 & 0 & 0 & 0 \\
\end{array}
\right]
$$

Basic variables

Method 2

$$x_1 + x_2 + x_3 + s_1 = 10$$

$$-2x_2 + 3x_3 + s_2 = 12$$

$$\min f(x) = -2x_1 + x_2 - 3x_3$$

max f(x) = – min –f(x)

= – min 2x₁ – x₂ + 3x₃

$$
\begin{array}{ccccc}
x_1 & x_2 & x_3 & s_1 & s_2 \\
\end{array}
$$

$$
\left[
\begin{array}{ccccc|c}
1 & 1 & 1 & 1 & 0 & 10 \\
0 & -2 & 3 & 0 & 1 & 12 \\
\hline
-2 & 1 & -3 & 0 & 0 & 0 \\
\end{array}
\right]
$$

2   -1   3

Basic variables

Method 2

$$x_1 + x_2 + x_3 + s_1 \qquad = 10$$

$$-2x_2 + 3x_3 \qquad + s_2 = 12$$

$$\min f(x) = -2x_1 + x_2 - 3x_3$$

max f(x) = – min –f(x)

= – min 2x₁ – x₂ + 3x₃

X_1	X_2	X_3	S_1	S_2	
1	1	1	1	0	10
0	-2	3	0	1	12
-2	1	-3	0	0	0
2	-1	3			

x₂ enter, s₁ exit

1	1	1	1	0	10	R1
2	0	5	2	1	32	R2+2R1
3	0	4	0	0	10	R3+R1

x₁ = 0, x₂ = 10 and x₃ = 0.
max f(x) = -(-10) = 10