Answer To The Question No – 3(a) (i)

The steepest Descent Direct would be

$$P_k = -\nabla f_k$$

Given $x_0 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$\nabla f(x) = \begin{bmatrix} 6x_1^2 x_2 - 3x_2^2 + \sin(x_2) \\ 2x_1^3 - 6x_1 x_2 + x_1 \cos(x_2) \end{bmatrix}$$

$$\cancel{\nabla f(x) =} \; \nabla f\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 6(1)^2 \times 2 - 3(2)^2 + \sin(2) \\ 2(1)^3 - 6 \times 1 \times 2 + 1\cos(2) \end{bmatrix}$$

$$= \begin{bmatrix} 0.035 \\ -9.0 \end{bmatrix}$$

So the direction will be $P_k = \begin{bmatrix} -0.035 \\ 9.0 \end{bmatrix}$

Answer To The Question No – 3(a) (ii)

Using Steepest Descent

$$x_{k+1} = x_1 = x_k + \alpha P_k$$

$$= x_k + \alpha(-\nabla k)$$ [We got P_k from 3(a)(i)]

$$= \begin{bmatrix}1\\2\end{bmatrix} + 1 \times \begin{bmatrix}-0.035\\9.0\end{bmatrix}$$

$$= \begin{bmatrix}1\\2\end{bmatrix} + \begin{bmatrix}-0.035\\9.0\end{bmatrix}$$

$$= \begin{bmatrix}0.965\\11\end{bmatrix}$$

The cost function value at $x_0 = \begin{bmatrix}1\\2\end{bmatrix}$ would be

$$f(x) = 2x_1^3 x_2 - 3x_1 x_2^2 + x_1 \sin(x_2)$$

$$= 2 \times (1)^3 \times 2 - 3 \times 1 \times (2)^2 + 1 \sin(2)$$

$$= -7.965$$

The cost function value at $x_1 = \begin{bmatrix}0.965\\11\end{bmatrix}$ would be

$$f(x) = 2x_1^3 x_2 - 3x_1 x_2^2 + x_1 \sin(x_2)$$

$$= 2 \times (0.965)^3 \times 11 - 3 \times 0.965 \times 11 + 0.965 \sin(11)$$

$$= 19.77 - 31.85 + 0.184$$

$$= -11.896$$

So for x_1 the cost function value decreased

Answer To The Question No – 3(b)

I can not agree with Caine's suggestion. Because if we use Newton's Method, then in every step to calculate the direction, we have to use the Hessian as well. Which will cost our calculation time.

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