Optimizing Dietary Planning Using the Simplex Method

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WQD - 7011 : Numerical Optimization

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Introduction

Maintaining a balanced, healthy diet is crucial for general well-being in today's fast-paced society. However, it can be difficult to provide optimal nutrition while taking into account different dietary restrictions. Here, mathematical optimization approaches can be used to solve difficult food planning issues in a methodical manner.

A popular linear programming approach called the Simplex Method offers a potent tool for enhancing decision-making and resource allocation. The Simplex Method can be used to formulate dietary planning as a linear programming problem in order to discover the best solution that satisfies nutritional needs while taking into account restrictions like price, and daily nutrient requirements.

Goals

- Understanding the basic concepts of the simplex method
- Building the objective function and identifying the constraints
- Implement the simplex method using octave programming.

Problem Statement

Given a dataset containing information about various food items, including their nutritional content and prices, our goal is to determine the optimal combination of food items that minimizes the cost while meeting specific nutritional requirements.

We aim to minimize the cost of the diet plan, which is determined by the quantities of different food items included. The objective function is defined as the sum of the costs of each food item multiplied by its respective quantity.

Objectives

Minimize Cost: The primary objective is to minimize the cost of the diet plan by identifying the optimal combination of food items that meets the nutritional requirements. This involves determining the quantities of each food item that minimize the total cost while satisfying the given constraints.

Meet Nutritional Requirements: Ensuring that the diet plan meets specific nutritional requirements is a key objective. The constraints imposed on energy, protein, calcium, magnesium, vitamin C, and iron should be satisfied by the quantities of food items chosen in the diet plan. The optimized solution should provide a well-balanced diet that fulfills these nutritional needs.

Optimal Food Selection: Another objective is to recommend the most appropriate combination of food items that provide the necessary nutrients at the lowest cost.

Data

The "**Diet_KSA_2020**" dataset that we are using for our project is taken from Kaggle. This dataset offers thorough details on the most popular food items consumed in Saudi Arabia, including their nutrient profiles and price tags.

index	name	max_serv	size	energy	protein	calcium	magnesiu	vitaminc	iron	price
1	Oatmeal	4	28	110	4	2	26	0	6	11
2	Chicken	3	100	205	32	12	1	2	1	16
3	Eggs	2	2	160	13	54	10	0	1	2
4	Whole Mil	8	237	160	8	285	0	0	11	8
5	Cherry pie	2	170	420	4	22	9	1	2	25
6	Pork with	2	206	260	14	80	31	1	1	4
7	Yogurt	3	3	58	10	110	11	0	0	10
8	Rice	5	30	500	10	28	2501	9	9	10
9	Meat	6	143	143	26	6	29	0	1	40
10	Potato	5	54	323	2	12	28	19	1	3

Daily energy, protein, calcium, magnesium, vitamin C, and iron requirements based on Health.gov's nutritional goals

Table E3.1.A4. Nutritional goals for each age/sex group used in assessing adequacy of USDA Food Patterns at various calorie levels

	Source of goal*	child 1 - 3	female 4 - 8	male 4 - 8	female 9--13	male 9--13	female 14-18	male 14-18	female 19-30	male 19-30	female 31-50	male 31-50	female 51+	male 51+
Calorie level(s) assessed		1000	1200	1400, 1600	1600	1800	1800	2200, 2800, 3200	2000	2400, 2600, 3000	1800	2200	1600	2000
Macronutrients														
Protein, g	RDA	13	19	19	34	34	46	52	46	56	46	56	46	56
Protein, % kcal	AMDR	5--20	10--30	10--30	10--30	10--30	10--30	10--30	10--35	10--35	10--35	10--35	10--35	10--35
Carbohydrate, g	RDA	130	130	130	130	130	130	130	130	130	130	130	130	130
Carbohydrate, %kcal	AMDR	45--65	45--65	45--65	45--65	45--65	45--65	45--65	45--65	45--65	45--65	45--65	45--65	45--65
Dietary Fiber, g	14g/1000kcal	14	16.8	19.6	22.4	25.2	25.2	30.8	28	33.6	25.2	30.8	22.4	28
Total fat, %kcal	AMDR	30-40	25-35	25-35	25-35	25-35	25-35	25-35	20-35	20-35	20-35	20-35	20-35	20-35
Saturated fat, %kcal	DG	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%
Linoleic acid, g	AI	7	10	10	10	12	11	16	12	17	12	17	11	14
Linolenic acid, g	AI	0.7	0.9	0.9	1	1.2	1.1	1.6	1.1	1.6	1.1	1.6	1.1	1.6
Minerals														
Calcium, mg	RDA	700	1000	1000	1300	1300	1300	1300	1000	1000	1000	1000	1200	1200
Iron, mg	RDA	7	10	10	8	8	15	11	18	8	18	8	8	8
Magnesium, mg	RDA	80	130	130	240	240	360	410	310	400	320	420	320	420
Phosphorus, mg	RDA	460	500	500	1250	1250	1250	1250	700	700	700	700	700	700
Potassium, mg	AI	3000	3800	3800	4500	4500	4700	4700	4700	4700	4700	4700	4700	4700

Objective Function

Decision variables play a crucial role in formulating and solving optimization problems using the simplex method. They represent the unknown quantities or values that need to be determined in order to optimize the objective function while satisfying the given constraints. In a nutshell; Minimize cost, meet nutritional requirements through optimal food quantities.

In our dataset, we have 10 food items. So we can say that we have 10 decision variables.

Coefficients indicate the cost per unit or serving of each food item in the diet plan.

Min 11*x1 + 16*x2 + 2*x3 + 8*x4 + 25*x5 + 4*x6 + 10*x7 + 10*x8 + 40*x9 + 3*x10

x1 corresponds to the quantity of "**Oatmeal**", **x2** corresponds to the quantity of "**Chicken**", **x3** corresponds to the quantity of "**Eggs**", **x4** corresponds to the quantity of "**Whole Milk**", **x5** corresponds to the quantity of "**Cherry pie**", **x6** corresponds to the quantity of "**Pork with beans**", **x7** corresponds to the quantity of "**Yogurt**", **x8** corresponds to the quantity of "**Rice**", **x9** corresponds to the quantity of "**Meat**", **x10** corresponds to the quantity of "**Potato**"

Constraints

Linear programming constraints define requirements, limitations, and resource availability, guiding decision variables toward feasible solutions within given restrictions.

The dataset's max_serv and size columns, which describe the maximum serving size and the size of each food item, respectively, are not taken into account while formulating the constraint. Since the constraints are more concerned with the overall nutritional content than with the precise serving sizes or sizes of the food items, these numbers are not explicitly employed in the formulation of the constraints. The goal is to satisfy dietary needs while making the best possible choice of food items based on nutritional content and price.

Constraints

$110*x1 + 205*x2 + 160*x3 + 160*x4 + 420*x5 + 260*x6 + 58*x7 + 500*x8 + 143*x9 + 323*x10 \geq$ **Daily Energy Requirement**

$4*x1 + 32*x2 + 13*x3 + 8*x4 + 4*x5 + 14*x6 + 10*x7 + 10*x8 + 26*x9 + 2*x10 \geq$ **Daily Protein Requirement**

$2*x1 + 12*x2 + 54*x3 + 285*x4 + 22*x5 + 80*x6 + 110*x7 + 28*x8 + 6*x9 + 12*x10 \geq$ **Daily Calcium Requirement**

$26*x1 + 1*x2 + 10*x3 + 0*x4 + 9*x5 + 31*x6 + 11*x7 + 2501*x8 + 29*x9 + 28*x10 \geq$ **Daily Magnesium Requirement**

$0*x1 + 2*x2 + 0*x3 + 0*x4 + 1*x5 + 1*x6 + 0*x7 + 9*x8 + 0*x9 + 19*x10 \geq$ **Daily VitaminC Requirement**

$6*x1 + 1*x2 + 1*x3 + 11*x4 + 2*x5 + 1*x6 + 0*x7 + 9*x8 + 1*x9 + 1*x10 \geq$ **Daily Iron Requirement**

Here $x1, x2, x3, x4, x5, x6, x7, x8, x9, x10 \geq 0$

$110*x1 + 205*x2 + 160*x3 + 160*x4 + 420*x5 + 260*x6 + 58*x7 + 500*x8 + 143*x9 + 323*x10 \geq 2000$

$4*x1 + 32*x2 + 13*x3 + 8*x4 + 4*x5 + 14*x6 + 10*x7 + 10*x8 + 26*x9 + 2*x10 \geq 50$

$2*x1 + 12*x2 + 54*x3 + 285*x4 + 22*x5 + 80*x6 + 110*x7 + 28*x8 + 6*x9 + 12*x10 \geq 1000$

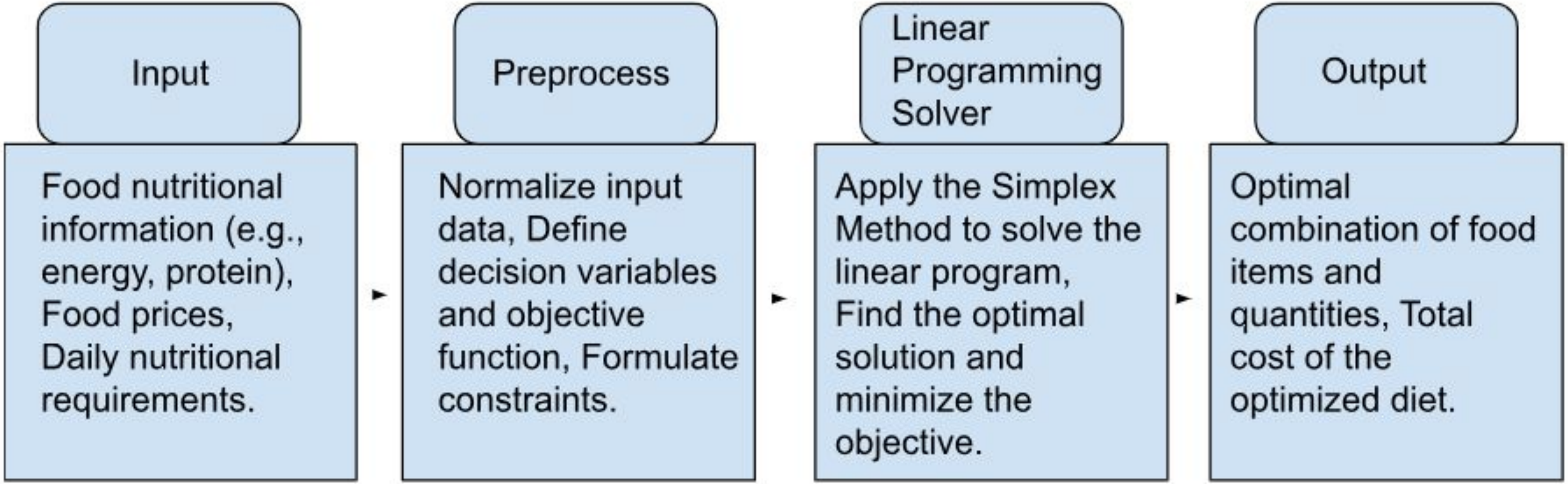
$26*x1 + 1*x2 + 10*x3 + 0*x4 + 9*x5 + 31*x6 + 11*x7 + 2501*x8 + 29*x9 + 28*x10 \geq 400$

$0*x1 + 2*x2 + 0*x3 + 0*x4 + 1*x5 + 1*x6 + 0*x7 + 9*x8 + 0*x9 + 19*x10 \geq 90$

$6*x1 + 1*x2 + 1*x3 + 11*x4 + 2*x5 + 1*x6 + 0*x7 + 9*x8 + 1*x9 + 1*x10 \geq 18$

6 Constraints and **10** Decision Variables. This means the number of **decision variables> constraints**, which satisfies the condition of the simplex method.

System Diagram



Modelling

For the identified constraints, slack variables convert them into equality constraints. The slack variables represent the surplus or slack in meeting the constraints.

Original constraint	Modified constraint	Slack variable
110x1 + 205x2 + 160x3 + 160x4 + 420x5 + 260x6 + 58x7 + 500x8 + 143x9 + 323x10 ≥ 2000	110x1 + 205x2 + 160x3 + 160x4 + 420x5 + 260x6 + 58x7 + 500x8 + 143x9 + 323x10 + slack1 = 2000	slack1 ≥ 0
4x1 + 32x2 + 13x3 + 8x4 + 4x5 + 14x6 + 10x7 + 10x8 + 26x9 + 2x10 ≥ 50	4x1 + 32x2 + 13x3 + 8x4 + 4x5 + 14x6 + 10x7 + 10x8 + 26x9 + 2x10 + slack2 = 50	slack2 ≥ 0
2x1 + 12x2 + 54x3 + 285x4 + 22x5 + 80x6 + 110x7 + 28x8 + 6x9 + 12x10 ≥ 1000	2x1 + 12x2 + 54x3 + 285x4 + 22x5 + 80x6 + 110x7 + 28x8 + 6x9 + 12x10 + slack3 = 1000	slack3 ≥ 0
26x1 + 1x2 + 10x3 + 0x4 + 9x5 + 31x6 + 11x7 + 2501x8 + 29x9 + 28x10 ≥ 400	26x1 + 1x2 + 10x3 + 0x4 + 9x5 + 31x6 + 11x7 + 2501x8 + 29x9 + 28x10 + slack4 = 400	slack4 ≥ 0
0x1 + 2x2 + 0x3 + 0x4 + 1x5 + 1x6 + 0x7 + 9x8 + 0x9 + 19x10 ≥ 90	0x1 + 2x2 + 0x3 + 0x4 + 1x5 + 1x6 + 0x7 + 9x8 + 0x9 + 19x10 + slack5 = 90	slack5 ≥ 0
6x1 + 1x2 + 1x3 + 11x4 + 2x5 + 1x6 + 0x7 + 9x8 + 1x9 + 1x10 ≥ 18	6x1 + 1x2 + 1x3 + 11x4 + 2x5 + 1x6 + 0x7 + 9x8 + 1x9 + 1x10 + slack6 = 18	slack6 ≥ 0

Simplex Method

x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	slack 1	slack 2	slack 3	slack 4	slack 5	slack 6	
110	205	160	160	420	260	58	500	143	323	1	0	0	0	0	0	2000
4	32	13	8	4	14	10	10	26	2	0	1	0	0	0	0	50
2	12	54	285	22	80	110	28	6	12	0	0	1	0	0	0	1000
26	1	10	0	9	31	11	2501	29	28	0	0	0	1	0	0	400
0	2	0	0	1	1	0	9	0	19	0	0	0	0	1	0	90
6	1	1	11	2	1	0	9	1	1	0	0	0	0	0	1	18
11	16	2	8	25	4	10	10	40	3	0	0	0	0	0	0	0

- No negative coefficients in the objective row.
- The glpk library or any other simplex algorithm implementation typically uses additional techniques, such as the "minimum ratio test" or "Bland's rule," to select the entering and exiting variables.
- The minimum ratio test involves calculating the ratios of the right-hand side values (RHS) divided by the corresponding coefficient of the entering variable in each row. The row with the smallest positive ratio would determine the exiting variable.
- Bland's rule is another technique used to break ties and ensure progress in degenerate cases. It suggests selecting the variable with the smallest index among those with the same minimum ratio.

Simplex Method

Objective Function and Constraints are input to **GNU Octave**. Using this "**glpk**" function solves the programming, using linear programming.

```
C = [11,16,2,8,25,4,10,10,40,3]; #Co-efficincets of the Objective Function
```

```
A = [
    110, 205, 160, 160, 420, 260, 58, 500, 143, 323;
    4, 32, 13, 8, 4, 14, 10, 10, 26, 2;
    2, 12, 54, 285, 22, 80, 110, 28, 6, 12;
    26, 1, 10, 0, 9, 31, 11, 2501, 29, 28;
    0, 2, 0, 0, 1, 1, 0, 9, 0, 19;
    6, 1, 1, 11, 2, 1, 0, 9, 1, 1]; #Co-efficinets of the Constraints
```

Simplex Method

```
b = [2000,50,1000,400,90,18]; #Column Array of the Constraints
lb = [0;0;0;0;0;0;0;0;0;0];
ub = [];
```

```
cType = "LLLLLL"; #Constraints Type
varType = "CCCCCCCCCC"; #Variable Type
sense = 1; # 1 means minimization problem
```

```
#Execute Function
[xmin,fmin,status,extra] = glpk(C,A,b,lb,ub,cType,varType,sense);
```



```
#Execute Function
[xmin,fmin,status,extra] = glpk(C,A,b,lb,ub,cType,varType,sense);
```

Simplex Method

Variable	Optimal Value
x1	0
x2	0
x3	1.1481
x4	3.0837
x5	0
x6	0
x7	0
x8	0.1029
x9	0
x10	4.6881

Objective Function Value (fmin): 42.059

Conclusion

Variables not included in the optimal solution: Both **x1 and x2** have an optimal value of 0, indicating that they are not included in the optimal diet plan. This suggests that increasing their quantities does not contribute to minimizing the objective function.

x3 and x4 are crucial for meeting nutritional requirements and minimizing cost in the optimal diet plan.

The variables **x5, x6, x7, and x9** have optimal values of 0, indicating their non-contribution to minimizing the objective function, making them dispensable in meeting nutritional requirements.

Variable **x8** has an optimal value of approximately 0.1029, making a moderate contribution to minimizing the objective function. Its inclusion in the optimal solution aids in cost reduction, albeit to a lesser extent than **x3 and x4.**

Key variable in the optimal solution: **x10** has a substantial optimal value of approximately 4.6881. Including x10 in the diet plan significantly contributes to minimizing the objective function while meeting the nutritional requirements.