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CSE 321

HW 3

1)

a) $T(n) = 27\,T(n/3) + n^2$

- $T(n/3) = 27T(n/9) + n^2/9$

$T(n) = 27(27\,T(n/9) + n^2/9) + n^2$

$= 27^2\,T(n/9) + 3n^2 + n^2$

- $T(n/9) = 27T(n/27) + n^2/81$

$T(n) = 27^2\,T(n/9) + 4n^2$

$= 27^2(27T(n/27) + n^2/81) + 4n^2$

$= 27^3\,T(n/27) + 9n^2 + 3n^2 + n^2$

$= 27^3\,T(n/27) + 13n^2$

General Form:

$T(n) = 27^k\,T(n/3^k) + (3^{k-1} + 3^{k-2} + \dots + 3^0)\,n^2$

$= 27^k\,T(n/3^k) + \left(\sum_{i=0}^{k-1} 3^i\right)n^2$

$= 27^k\,T(n/3^k) + \left(\frac{1-3^k}{1-3}\right)n^2 = 27^k\,T(n/3^k)$ +

NOTE: $\sum_{i=0}^{k-1} 3^i$ geometric sequence

$\frac{1-3^k}{1-3}$

$n = 3^k$

$\log_3 n = k$

$T(n) = 27^{\log_3 n}\,T(n/3^{\log_3 n}) + \left(\frac{1-3^{\log_3 n}}{-2}\right)n^2$

$= 3n\,T(n/n) + \frac{1-n}{-2}\,n^2$

$= 3n\,T(1) + \left(\frac{n-1}{2}\right)n^2$

$= 3n \cdot 0 + \frac{n^3-n^2}{2} = \underline{\frac{n^3-n^2}{2}} \in \underline{\Theta(n^3)}$

b)

$T(n) = 9T(n/4) + n$

$\circ\ T(n/4) = 9T(n/16) + n/4$

$T(n) = 9(9T(n/16) + n/4) + n$

$= 9^2 T(n/16) + \frac{9n}{4} + n$

$\circ\ T(n/16) = 9T(n/64) + n/16$

$T(n) = 9^2(9T(n/64) + n/16) + \frac{9n}{4} + n$

$= 9^3 T(n/64) + \frac{9^2 n}{16} + \frac{9n}{4} + n$

General Form :

$T(n) = 9^k\ T(n/4^k) + \left(\frac{9^{k-1}}{4^{k-1}} + \frac{9^{k-2}}{4^{k-2}} + \dots + \frac{9^0}{4^0}\right)n$

$= 9^k\ T(n/4^k) + \left(\sum_{i=0}^{k-1} \frac{9^i}{4^i}\right)n$ $\qquad \sum_{i=0}^{k-1} \frac{9^i}{4^i} = \frac{1-\left(\frac{9}{4}\right)^k}{1-\frac{9}{4}} = \frac{1-\frac{9^k}{4^k}}{+\frac{5}{4}}$

$n = 4^k$

$\log_4 n = k$

$T(n) = 9^{\log_4 n}\ T(1) - \frac{4}{5}(n - 9^{\log_4 n})$ $\qquad \frac{4}{5}($ -

$= 9^{\log_4 n}\ \underbrace{T(1)}_{0} - \frac{4n}{5} + \frac{9^{\log_4 n} \cdot 4}{5}$

$= \frac{9^{\log_4 n} \cdot 4}{5} - \frac{4n}{5} \longrightarrow \theta(n^{\log_2 3})$

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c) $T(n) = 2T(n/4) + \sqrt{n}$

$\circ\ T(n/4) = 2T(n/16) + \sqrt{\frac{n}{4}}$

$$T(n) = 2\left(2T(n/16) + \sqrt{\frac{n}{4}}\right) + \sqrt{n} = 4T(n/16) + 2\sqrt{\frac{n}{4}} + \sqrt{n}$$

$\circ\ T(n/16) = 2T(n/64) + \sqrt{\frac{n}{16}}$

$$T(n) = 4\left(2T(n/64) + \sqrt{\frac{n}{16}}\right) + 2\sqrt{\frac{n}{4}} + \sqrt{n}$$

$$= 8T(n/64) + 4\sqrt{\frac{n}{16}} + 2\sqrt{\frac{n}{4}} + \sqrt{n}$$

General form :

$$T(n) = 2^k T(n/4^k) + \left(2^{k-1} \cdot \frac{1}{2^{k-1}} + \text{---} + 2^0 \cdot \frac{1}{2^0}\right)\sqrt{n}$$

$$+ \sum_{i=0}^{k-1} 1 = k-1$$

$$= 2^k T(n/4^k) + (k-1)\sqrt{n}$$

$n = 4^k$

$\log_4 n = k$

$$= 2^{\log_4 n} \underbrace{T(1)}_{0} + (\log_4 n - 1)\sqrt{n}$$

$$= \frac{\sqrt{n}\log_2 n}{2} - \sqrt{n} \rightarrow \theta\left(\frac{\sqrt{n}\log_2 n}{2}\right)$$

d) $T(n) = 2T(\sqrt{n}) + 1$

Assume $n = 2^k$

$T(2^k) = 2T(2^{k/2}) + 1$ $\qquad$ $T(2^k) = S(k)$

$S(k) = 2S(k/2) + 1$ $\qquad$ Apply Moster's Theorem $a=2$ $b=2$

$S(k) = k^{\log_2 2}$

$S(k) = \theta(k)$

$S(k) = T(2^k)$

$T(2^k) = \theta(k)$ $\qquad$ $n = 2^k \rightarrow k = \log_2 n$

$T(n) = \theta(\log_2 n)$

e) $T(n) = 2T(n-2)$, $T(0)=1$, $T(1)=1$

$T(0) = 1$
$T(1) = 1$
$T(2) = 2T(0) = 2$
$T(3) = 2T(1) = 2$
$T(4) = 2T(2) = 4$
$T(5) = 2T(3) = 4$

$T(n) = 2T(n-2)$

$T(n) - 2T(n-2) = 0$

$\alpha^2 - 2 = 0$

$\alpha = \mp\sqrt{2}$

so $T(n) = c_1(\sqrt{2})^n + c_2(-\sqrt{2})^n$

$T(0)=1 \qquad c_1 + c_2 = 1$

$T(1)=1 \qquad \sqrt{2}c_1 - \sqrt{2}c_2 = 1 \qquad c_1 - c_2 = \frac{1}{\sqrt{2}}$

$$\begin{array}{r} c_1 + c_2 = 1 \\ + \quad c_1 - c_2 = \frac{1}{\sqrt{2}} \\ \hline 2c_1 = 1 + \frac{1}{\sqrt{2}} = \frac{\sqrt{2}+1}{\sqrt{2}} \end{array}$$

$c_1 = \frac{1+\sqrt{2}}{2\sqrt{2}} \qquad c_2 = \frac{\sqrt{2}-1}{2\sqrt{2}}$

$$\underline{T(n) = \frac{(\sqrt{2}+1)}{2\sqrt{2}}(\sqrt{2})^n + \frac{\sqrt{2}-1}{2\sqrt{2}}(-\sqrt{2})^n}$$

so $\underline{T(n) \in \Theta((\sqrt{2})^n)}$

f) $T(n) = 4T(n/2) + n, \quad T(1) = 1$

$\circ\ T(n/2) = 4T(n/4) + n/2$

$T(n) = 4(4T(n/4) + n/2) + n = 16T(n/4) + 2n + n$

$\circ\ T(n/4) = 4T(n/8) + n/4$

$T(n) = 16(4T(n/8) + n/4) + 2n + n$

$= 64T(n/8) + 4n + 2n + n$

General Form :

$$T(n) = 4^k T(n/2^k) + \underbrace{(2^{k-1} + 2^{k-2} + \dots + 2^0)}_{\sum_{i=0}^{k-1} 2^i = \frac{1-2^k}{1-2} = 2^k - 1} n$$

$T(n) = 4^k T(n/2^k) + 2^k - 1$

$n = 2^k$

$\log_2 n = k$

$T(n) = 4^{\log_2 n} T(n/n) + 2^{\log_2 n} - 1$

$= 2n \cdot T(1) + n - 1$

$= 2n + n - 1$

$= 3n - 1 \in \theta(n)$

g) $T(n) = 2T(\sqrt[3]{n}) + 1, \quad T(3) = 1$

Assume $n = 3^k$

$T(3^k) = 2T(3^{k/3}) + 1 \qquad T(3^k) = S(k)$

$S(k) = 2S(k/3) + 1 \qquad a=2 \quad b=3 \quad$ Apply Master's Theorem

$S(k) = k^{\log_3 2}$

$S(k) \in \theta(k)$

$S(k) = T(3^k)$

$T(3^k) \in \theta(k) \qquad n = 3^k \quad \log_3 n = k$

$T(n) \in \theta(\log_3 n)$

2)

```
function f(n)
    if n<=1:
        print_line("**") — 1 times
    else:
        for i=1 to n — n times
            f(n/2)
        end for
```

$f(1) = 2$

$f(n) = f(n/2) \cdot n$

Assume n is a power of 2

Backward Substitution :

$f(n) = f(n/2)\,n$

$\circ\, f(n/2) = f(n/4)\,n/2$

$f(n) = f(n/4) \cdot n/2 \cdot n$

$\circ\, f(n/4) = f(n/8) \cdot n/4$

$f(n) = f(n/8) \cdot n/4 \cdot n/2 \cdot n$

Generalized Form :

$f(n) = f(n/2^k) \cdot (n/2^{k-1}) \cdot (n/2^{k-2}) \ldots n$

$f(n) = f(n/2^k) \cdot n^k / 2^{\frac{k(k-1)}{2}}$

Let $\frac{n}{2^k} = 1$

$n = 2^k$

$k = \log_2 n$

$f(n) = f(1) \cdot \frac{(2^k)^k}{2^{\frac{k(k-1)}{2}}} = \frac{2 \cdot 2^{k^2}}{2^{\frac{k^2-k}{2}}} = 2^{\frac{k^2+k+2}{2}}$

$f(n) = 2^{(\log_2 n)^2 + \log_2 n + 2}$

$= 2^{\log_2 n} \cdot 2^{(\log_2 n)^2 + 2}$

$= n \cdot 2^{(\log_2 n)^2 + 2}$

$= n\, 2^{c^2+2}$

if we assume n is a power of 2

$\log_2 n = c$ (some constant)

$n \cdot 2^{c^2+2}$ * will be printed

3)

```
function_f (A[0:n])
    if n=2 and A[0] > A[1]
        swap (A[0], A[1])

    if n>2
        function_f(A[0: 2n/3])
        function_f(A[n/3 : n])
        function_f(A[0: 2n/3])
```

$T(1) = 1$

$T(n) = 3T(2n/3) + 1$

3 recursive calls

size of 2n/3

Backward Substitution :

$T(n) = 3T(2n/3) + 1$

$\circ\ T(2n/3) = 3T(4n/9) + 1$

$T(n) = 3(3T(4n/9)+1) + 1 = 9T(4n/9) + 3 + 1$

$\circ\ T(4n/9) = 3T(8n/27) + 1$

$T(n) = 9(3T(8n/27)+1) + 3 + 1$

$= 27T(8n/27) + 9 + 3 + 1$

Generalized Form!

$3^k T(2^k n/3^k) + 3^{k-1} + 3^{k-2} + \dots + 1 = T(n)$

$\sum_{i=0}^{k-1} 3^i = \frac{1-3^k}{1-3} = \frac{3^k-1}{2}$

$\underbrace{T(n) = 3^k(2^k n/3^k) + \frac{3^k-1}{2}}$

Let $n = \frac{3^k}{2^k}$ $\quad T(n) = 3^k T(1) + \frac{3^k-1}{2}$

$n = \left(\frac{3}{2}\right)^k$

$\log_{\frac{3}{2}} n = k$

$\underbrace{T(n) = 3^{\log_{\frac{3}{2}} n} \cdot 1 + \frac{3^{\log_{\frac{3}{2}} n} - 1}{2}}$

$T(n) \in \Theta(3^{\log_{3/2} n})$

4)

```
procedure InsertionSort(L[1:n])
  for i=2 to n do
    current = L[i]
    position = i-1
    while (position >=1 and current < L[position]) do
        L[position +1] = L[position]
        position = position - 1
        count ++
    end while
    L[position+ 1] = current
  end for
end
```

```
procedure QuickSort ( L[low:high])
  if high > low :
    call Rearrange(L[low:high], position)
    call QuickSort(L[low:position -1])
    call QuickSort(L[position +1 : high])
  end if
end
```

```
procedure Rearrange(L[low:high], position)
  right = low
  left = high +1
  x = L[low]
  while right < left do
      repeat right ++ until L[right] >= x
      repeat left ++ until L[left] <= x
  if right < left
      interchange(L[left], L[right])
      count++
  end if
  end while
  position = left
  L[low] = L[position]
  L[position] = x
  count ++
end
```

Lets our array = [15, 12, 13, 14, 6, 10] and test it for insertion and quick sort in implementations which are given above.

There is 5 swaps for quicksort but 16 swaps for insertion sort

The complexity of insertion sort $\theta(n^2)$ and quick sort is $\theta(n\log n)$

We can say that quick sort is faster than Insertion sort

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5)

a) $T(n) = 5T(n/3) + O(n^2)$

b) $T(n) = 2T(n/2) + O(n^2)$

c) $T(n) = T(n-1) + O(n)$

Solve a : Master's Theorem

$T(n) = aT(n/b) + f(n)$

$T(n) = 5T(n/3) + O(n^2)$

$T(n) \in \Theta(n^k)$ if $a < b^k$

$T(n) \in \Theta(n^2)$

Solve b : Master's Theorem

$T(n) = aT(n/b) + f(n)$

$T(n) = 2T(n/2) + O(n^2)$

$T(n) \in \Theta(n^k)$ if $a < b^k$

$T(n) \in \Theta(n^2)$

Solve c : Substitution Method

$T(n) = T(n-1) + n$

$T(n) = T(n-1) + n-1 + n$

$T(n) = T(n-2) + n-2 + n-1 + n$

$T(n) = \frac{n.(n-1)}{2} \in \Theta(n^2)$

So three situations are giving same worst case time complexity.

You can pick any of them.