CSE 231 – Introduction to Algorithms

Homework-1

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1) For each following statement, specify whether it is true or not. Explaining your reasoning for each of them

a) $\log_2 n^2 + 1 \in O(n)$:

$O(\log n) \in O(n)$ ✓

$O(\log_2 n^2 + 1) = O(2\log n + 1) = O(\log n)$

base does not matters

b) $\sqrt{n(n+1)} \in \Omega(n)$:

$\sqrt{n^2} \leq \sqrt{n(n+1)}$

$n \leq \sqrt{n(n+1)}$

$n \in \Omega(n)$

$\sqrt{n(n+1)} \in \Omega(n)$

c) $n^{n-1} \in \Theta(n^n)$:

$\lim_{n \to \infty} \frac{f(x)}{g(x)} = \lim_{n\to\infty} \frac{(n^{n-1})}{(n^n)} = 1$ so statement is true

d) $O(2^n + n^3) \subset O(4^n)$:

$O(4^n) = O(2^{2n}) = O(2^n)$

$O(2^n + n^3) = O(2^n)$

because 2^n has higher order

Statement is false

$O(2^n) \subseteq O(2^n)$

not $\subset$

e) $O(2\log_3 \sqrt[3]{n}) \subset O(3\log_2 n^2)$:

$O(2\log_3 n^{(1/3)}) = O(\frac{2}{3}\log_3 n) = O(\log n)$

so $O(\log n) \subseteq O(\log n)$

not $\subset$

Statement is false

$O(3\log_2 n^2) = O(6\log_2 n) = O(\log n)$

f) $\log_2 \sqrt{n}$ and $(\log_2 n)^2$ are of the same asymptotical order:

$$\lim_{n\to\infty} \frac{\log_2 \sqrt{n}}{(\log_2 n)^2} = \lim_{n\to\infty} \frac{1}{2\log_2 n} = \frac{1}{2} \lim_{n\to\infty} \frac{1}{\log_2 n} = \frac{1}{2} \cdot \frac{1}{\infty} = 0$$

so they are not the same asymptotical order

2) Order the following functions by growth rate and explain your reasoning for each of them.

$n^2 \in O(n^2)$, $n^3 \in O(n^3)$, $n^2 \log n \in O(n^2 \log n)$, $\sqrt{n} \in O(\sqrt{n})$

$\log n \in O(\log n)$, $10^n \in O(10^n)$, $2^n \in O(2^n)$, $8^{\log n} \in O(2^{\log n})$

$\lim_{n\to\infty} \frac{\log n}{\sqrt{n}} = 0$ $O(\log n) \subset O(\sqrt{n})$ $\lim_{n\to\infty} \frac{n^3}{2^{\log n}} = 0$ $O(n^3) \subset O(2^{\log n})$

$\lim_{n\to\infty} \frac{\sqrt{n}}{n^2} = 0$ $O(\sqrt{n}) \subset O(n^2)$ $\lim_{n\to\infty} \frac{2^{\log n}}{2^n} = 0$ $O(2^{\log n}) \subset O(2^n)$

$\lim_{n\to\infty} \frac{n^2}{n^2 \log n} = 0$ $O(n^2) \subset O(n^2 \log n)$ $\lim_{n\to\infty} \frac{2^n}{10^n} = 0$ $O(2^n) \subset O(10^n)$

$\lim_{n\to\infty} \frac{n^2 \log n}{n^3} = 0$ $O(n^2 \log n) \subset O(n^3)$

$O(\log n) \subset O(\sqrt{n}) \subset O(n^2) \subset O(n^2 \log n) \subset O(n^3) \subset O(8^{\log n}) \subset O(2^n) \subset O(10^n)$

3) What is the time complexity of the following programs? Explain by giving details.

a)

```
void f(int my-array[]){
    for(int i=0; i<sizeofArray; i++){        → n times
        if (my_array[i] < first element){
            second_element = first_element;      } c1.3 times
            first_element = my_array[i];
        }
        else if (my_array[i] < second_element){
            if (my_array[i] != first_element){    } c2.3 times
                second_element = my_array[i];
            }
        }
    }
}
```

c_1 and c_2 are non-negative integer numbers

So $f(n) = (3c_1 + 3c_2)n$

$O(f(n)) = O(n)$

b)

```
void f(int n){
   int count =0;
   for (int i=2;i<=n;i++){
     if(i%2 ==0){
       count++;          } 1
     }
     else{
       i=(i-1)*i;        } logn
     }
   }
}
```

In this example, integer i multiplies itself with i-1. So for loop works $O(\log n)$ times

$f(n) = O(\log n)$

4) Find the complexity classes of the following functions using the integration method.

a) $\sum_{i=1}^{n} i^2 \log i \rightarrow g(x) = i^2 \log i$ is a non decreasing function

$$\int_0^n g(x)\,dx \leq \sum_{i=1}^{n} i^2 \log i \leq \int_1^{n+1} g(x)\,dx$$

$$\int_0^n x^2 \log x = \left[\frac{1}{3}x^3 \ln(x) - \frac{x^3}{9}\right]_0^n = \frac{1}{3}n^3 \log n - \frac{n^3}{9}$$

$$\int_1^{n+1} g(x)\,dx = \left[\frac{1}{3}x^3 \ln(x) - \frac{x^3}{9}\right]_1^{n+1} = \frac{1}{3} \log n\,(n+1)(n+1)^3 - \frac{(n+1)^3}{9} + \frac{1}{9}$$

$$\frac{1}{3}n^3 \log n - \frac{n^3}{9} \leq \sum_{i=1}^{n} i^2 \log i \leq \frac{1}{3} \log(n+1)(n+1)^3 - \frac{(n+1)^3}{9} + \frac{1}{9}$$

$$\sum_{i=1}^{n} i^2 \log i \in O(n^3 \log n)$$
$$\in \Omega(n^3 \log n)$$
$$\in \Theta(n^3 \log n)$$

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b) $\sum_{i=1}^{n} i^3$ is a non decreasing function

$$\int_0^n g(x)\,dx \le f(n) \le \int_1^{n+1} g(x)\,dx$$

$$\frac{x^4}{4}\Big|_0^n \le f(n) \le \frac{x^4}{4}\Big|_1^{n+1}$$

$f(n) \in O(n^4)$
$\in \Omega(n^4)$
$\in \Theta(n^4)$

c) $\sum_{i=1}^{n} \frac{1}{2\sqrt{i}}$ is a non increasing function

$$\int_1^{n+1} g(x)\,dx \le \sum_{i=1}^{n} \frac{1}{2\sqrt{i}} \le \int_0^n g(x)\,dx$$

$$\int \frac{1}{2\sqrt{x}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{x}}\,dx = \frac{1}{2}\left(\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right) = \sqrt{x}$$

$$\sqrt{x}\,\Big|_1^{n+1} \le \sum_{i=1}^{n} \frac{1}{2\sqrt{i}} \le \sqrt{x}\,\Big|_0^n$$

$$\sqrt{n+1} - 1 \le \sum_{i=1}^{n} \frac{1}{2\sqrt{i}} \le \sqrt{n}$$

$\sum_{i=1}^{n} \frac{1}{2\sqrt{i}} \in O(\sqrt{n})$
$\in \Omega(\sqrt{n})$
$\in \Theta(\sqrt{n})$

d) $\sum_{i=1}^{n} 1/i$ is a non decreasing function

$\int \frac{1}{x}\,dx = \ln|x|$

$$\int_1^{n+1} g(x)\,dx \le \sum_{i=1}^{n} 1/i \le \int_0^n g(x)\,dx$$

$$\ln|x|\,\Big|_1^{n+1} \le \sum_{i=1}^{n} 1/i \le \ln|x|\,\Big|_0^n$$

$$\ln|n+1| - \ln 1 \le \sum_{i=1}^{n} 1/i \le \ln|n| - \ln|0|$$

So, this integration method works for finding a lower bound but not for an upper bound.

$$\sum_{i=1}^{n} 1/i = 1 + \sum_{i=2}^{n} 1/i \le 1 + \int_1^n g(x)\,dx \le \ln|n| + 1$$

$$\ln|n+1| \le \sum_{i=1}^{n} 1/i \le \ln|n| + 1$$

$\sum_{i=1}^{n} 1/i \in O(\log n)$
$\in \Omega(\log n)$
$\in \Theta(\log n)$

5) Find the best case and worst case complexities of linear search with repeated elements, that is, the elements in the list need not be distinct. Show your analysis.

If all elements in the list are the same, the element that is searching for is either on the list or it is not in the list.

<u>Best-Case:</u> If the first element $x = L[1]$, then best case occurs.

$B(n) = 1 \in \Theta(1)$

<u>Worst-Case</u>; If the first element $x \neq L[1]$, then worst case occurs because it will traverse the entire list.

$W(n) = n \in \Theta(n)$