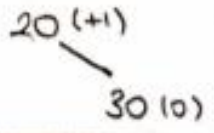
Question 1:

o AVL Tree :

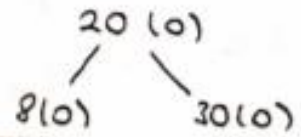
- Insert 20

20(0)

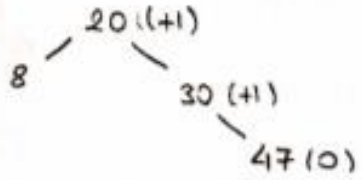
- Insert 30



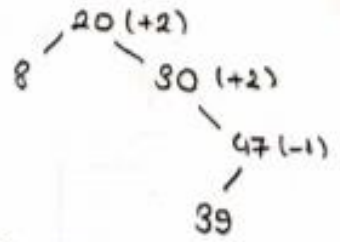
- Insert 8



- Insert 47

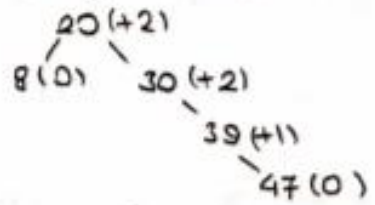


- Insert 39

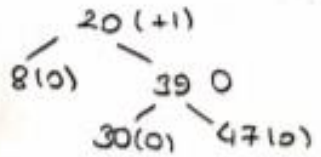


Right-left on 30
- Rotate right around 47
- Rotate left around 30

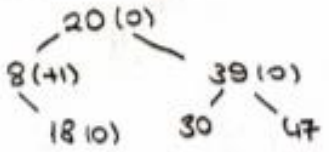
- Rotate right around 47



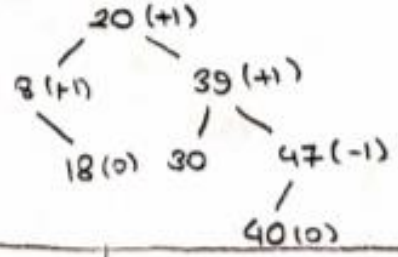
- Rotate left around 30



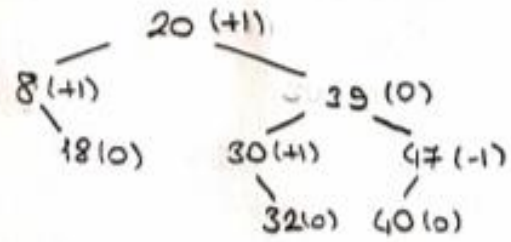
- Insert 18



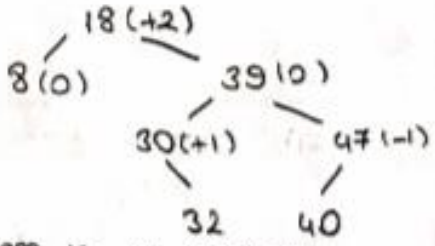
- Insert 40



- Insert 32

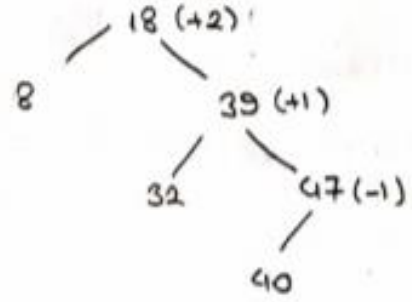


- Remove 20
  - Assign 20 as 20's left most right child (18)



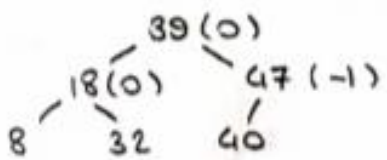
- There is no critically unbalanced trees

- Remove 30
  - Assign 30 as 30's left most right child (32)

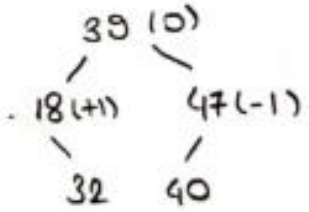


- Right-right on 18

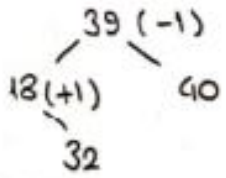
- Rotate left around 18



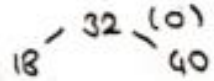
- Remove 8



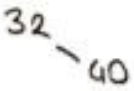
- Remove 47
  - Assign 47 as its left most right child (40



- Remove 39
  - Assign 39 as its left most right child (32)



- Remove 18



- Remove 40

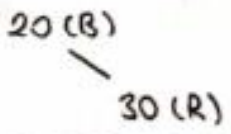
- Remove 32

o Red - Black Tree:

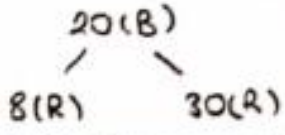
- Insert 20

20 (B)

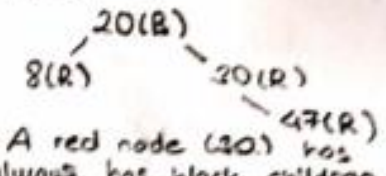
- Insert 30



- Insert 8

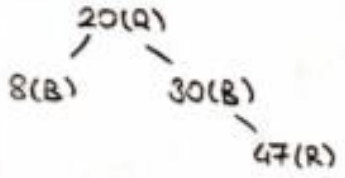


- Insert 47



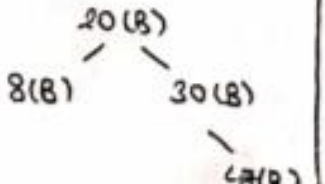
o A red node (30) has always has black children

- If parent is red, and its sibling is also red, they can both be changed to black, and the grandparent to red

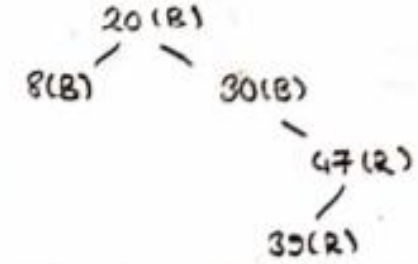


o The root always black

- The root can be changed to black

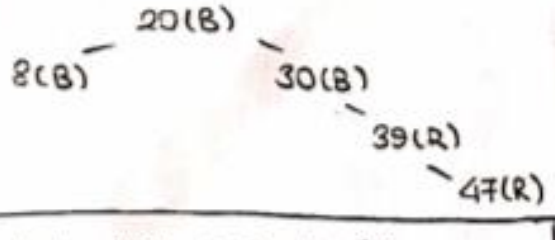


- Insert 39

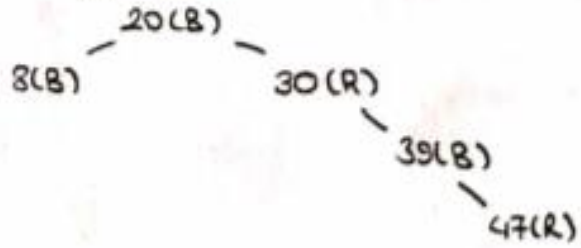


o A red node (47) always has black children

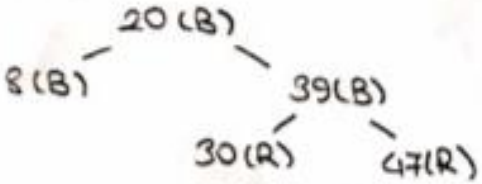
- Rotate right about the parent (47) so that the red child is on the same side of the parent as the parent is to the grandparent.



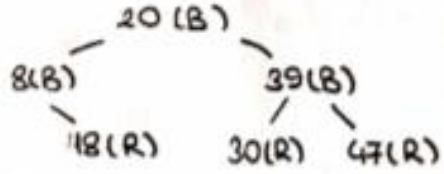
- Change colors



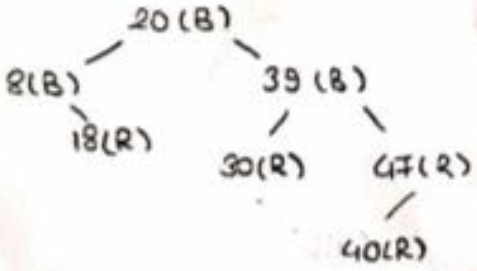
- Rotate left around 30



- Insert 18

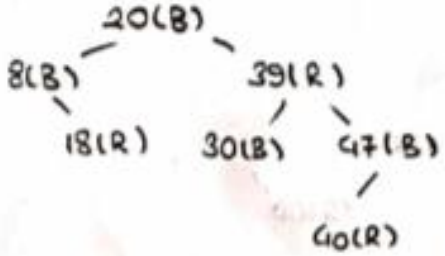


- Insert 40

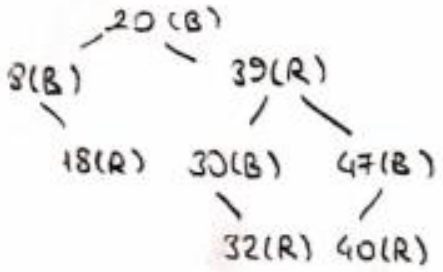


o 30 and 47 both are red then change colors

- Change colors



- Insert 32



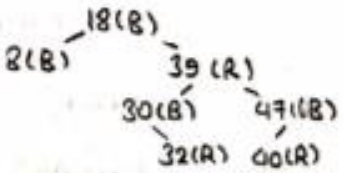
- Balanced

- Remove 20

- Find predecessor of 20 (18)

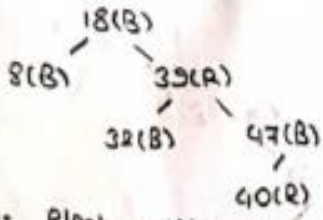
18 is red with no child

Tree is still balanced



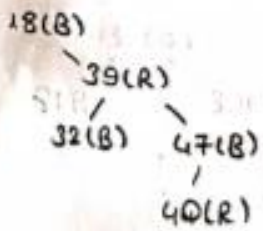
- Remove 30

- Find predecessor of 30 (32)



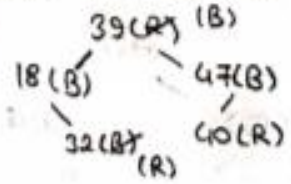
30 is Black with one red child Tree is still balanced.

- Remove 8



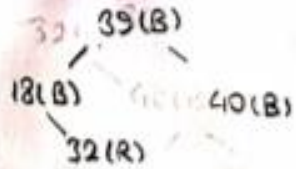
Left fixed up is required

- Rotate left around 39



- Balanced

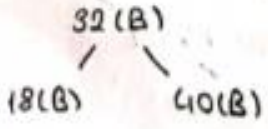
- Remove 47 . predecessor (40)



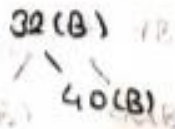
- 47 is black with one red child. Replace and change color.

- Remove 39

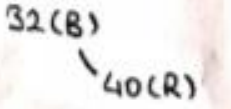
Predecessor 42



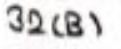
- Remove 18



- Unbalanced
- Change 40's color to red



- Remove 40



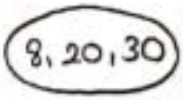
- Remove 32

2-3 Tree

- Insert 20

- Insert 30

- Insert 8



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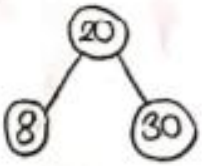
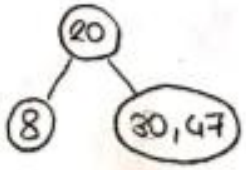
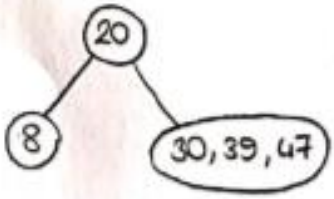


- Insert 47



- Insert 39



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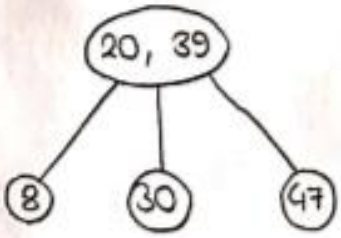
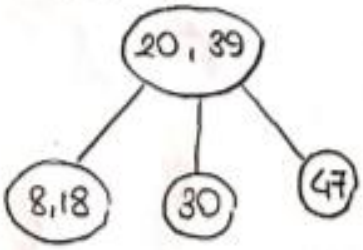
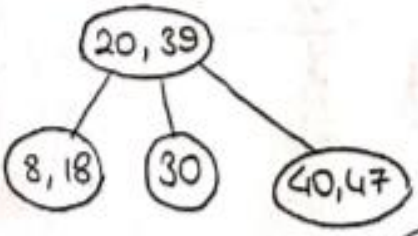


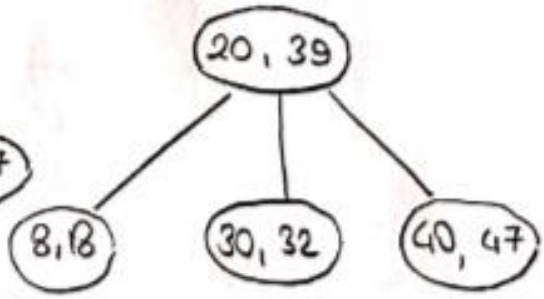
- Insert 18



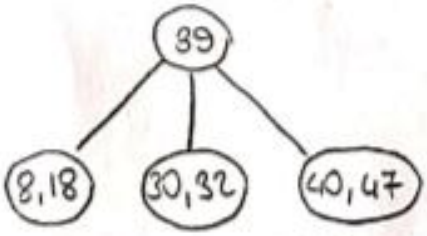
- Insert 40



- Insert 32

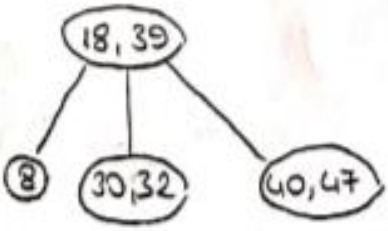


- Remove 20

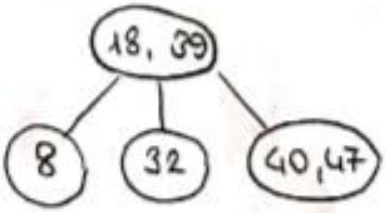


but there is a problem

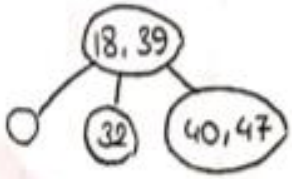
- Merge 18 into 39



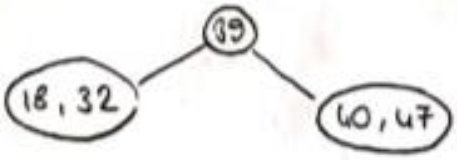
- Remove 30



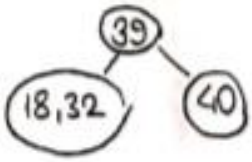
Remove 8



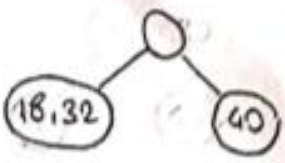
- Merge 18 and 32



- Remove 47



- Remove 39



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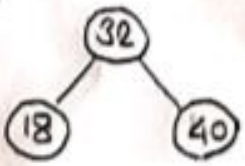


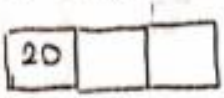
- Remove 18

- Remove 40

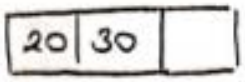
- Remove 32

o B- Tree with order 4

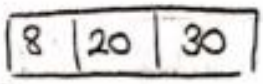
- Insert 20



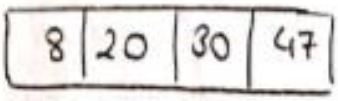
- Insert 30



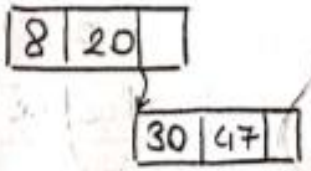
- Insert 8



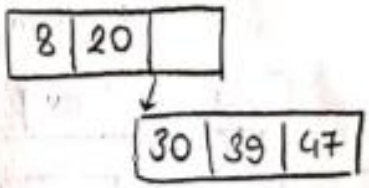
- Insert 47



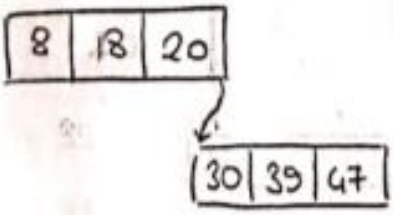
oSplit



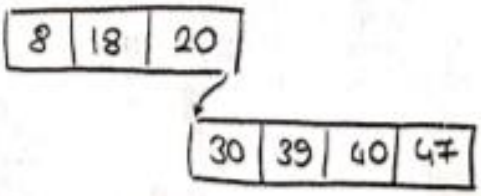
- Insert 39



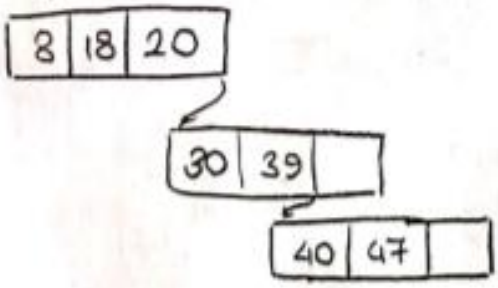
- Insert 18



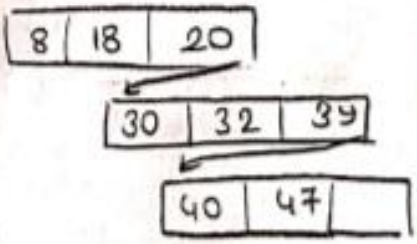
- Insert 40



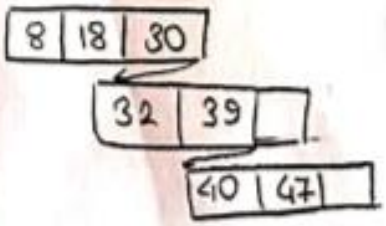
o Split



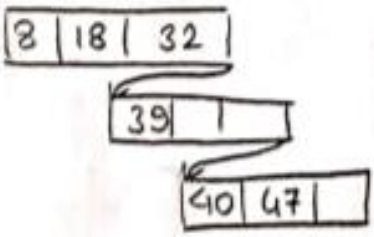
- Insert 32



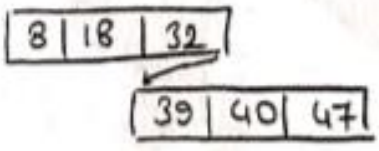
- Remove 20



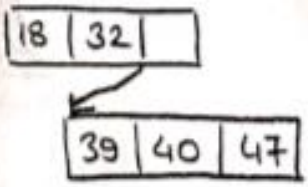
- Remove 30



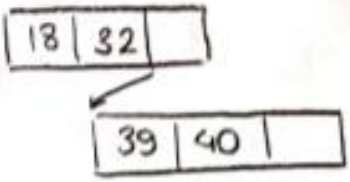
o Merge



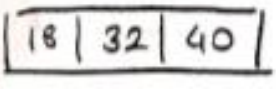
- Remove 8



- Remove 47



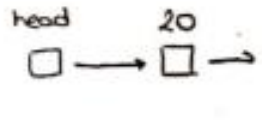
- Remove 39



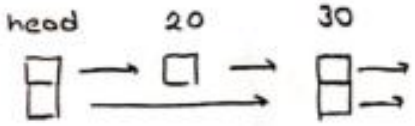
- Remove rest

o SKip-List

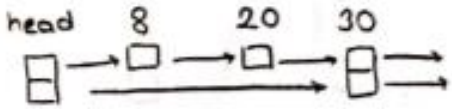
-Insert 20



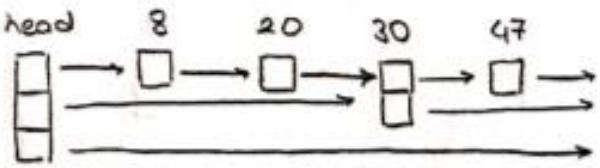
-Insert 30



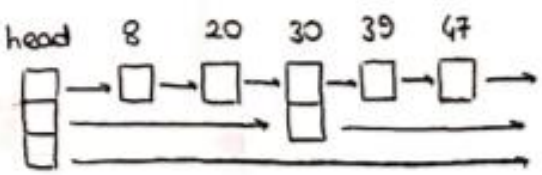
-Insert 8



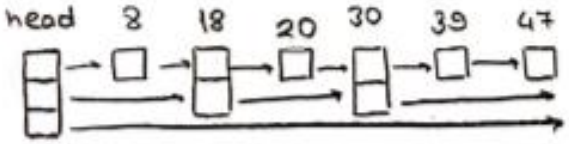
-Insert 47



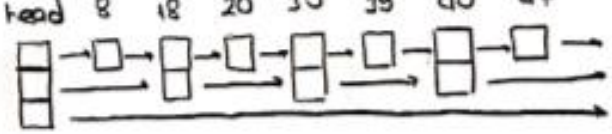
-Insert 39



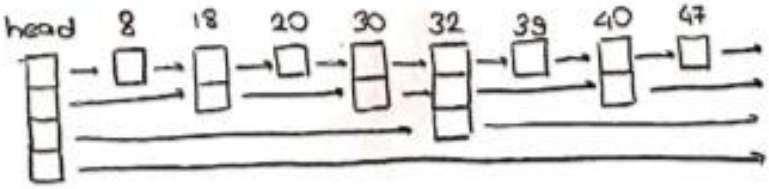
-Insert 18



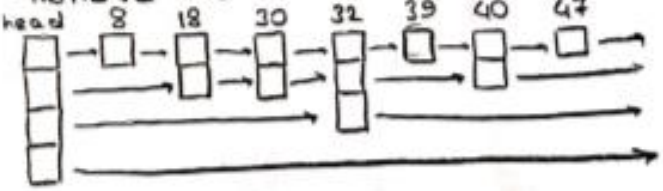
-Insert 40



-Insert 32



- Remove 20



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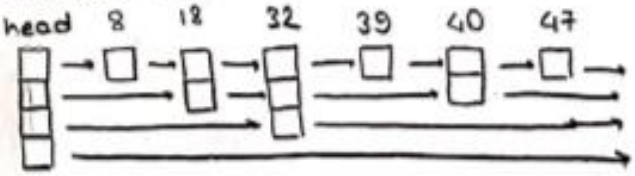
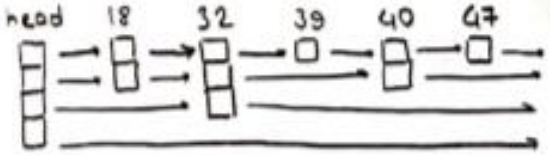
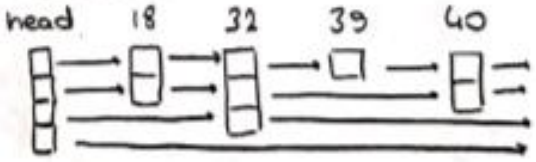


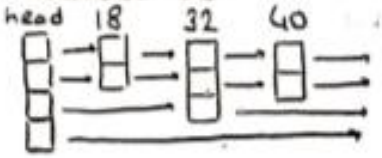
-Remove 8



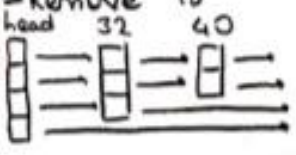
-Remove 47



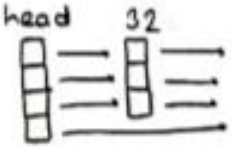
-Remove 39



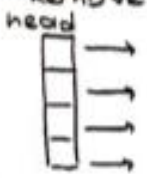
-Remove 18



-Remove 40



-Remove 32



Question 2:

Problem Solution:

- I need implement Skip-List data structure from my book first.
- It's remove method is missing so I searched online and found remove method for it.
- Then, our task is design a data structure so each node can keep several elements instead of one element.
- To do this, I changed SLNode<E> data field to an ArrayList (I could use a default java array but I guess it needs to be shifted after add and remove methods so I thought ArrayList is a better solution)
- In constructor of MySkipList an integer is given as parameter. This integer decides how many elements will be in a single node at maximum. And any node cannot hold less than this integer's half size except first node of the list.
- Then, I need to change add and remove methods.
- In normal skip list in the book, it first finds the predecessors of the node that it will be added and makes assignments so it can be added easily to anywhere in the list. Firstly, I found predecessors of that element and the element to node of predecessors first link. If it passes its capacity, I need to divide this node. (Example: Order is 5 -> If a node reaches capacity, It will hold 3 and 2 elements in two nodes.)
- Remove method pretty much the same. Just checks any node can not hold lesser than order/2, it will be removed and elements will be added to node's predecessors.

Test Cases:

Test Subject	Test Number	Pass/Fail
Create a test object with order 5	T1	PASS
Add first element	T2	PASS
Add more elements	T3	PASS
Divide first node	T4	PASS
Add more elements	T5	PASS
Remove nodes	T6	PASS
Remove elements and node	T7	PASS
Test it again with order 4	T8	PASS

Running and Results:

Test T1:
Test Data:

```
MySkipList<Integer> test = new MySkipList<>(5);
```

Excepted: test: “Empty”
Result: Passed

Test T2:
Test Data:

```
test.add(-5);
```

Excepted:
Head 1->-5 --> {-5}1: |->null|
Result:Passed

Test T3:
Test Data:

```
test.add(5);
test.add(10);
```

Excepted:
Head 1->-5 --> {-5, 0, 5, 10}1: |->null|
Result: Passed

Test T4:
Test Data:

```
test.add(3);
```

Excepted:
Head 2->-5->null --> {-5, 0, 3}1: |->5| --> {5, 10}1: |->null|
Result: Passed

Test T5:
Test Data:

```
test.add(20);
test.add(-10);
test.add(9);
test.add(7);
test.add(17);
test.add(25);
```

Excepted:
Head 2->-10->15 --> {-10, -5, 0, 3}1: |->5| --> {5, 7, 9, 10}1: |->15| --> {15, 17, 20, 25}2: |->null->null|
Result: Passed

Test T6:
Test Data:

```
test.remove(5);
test.remove(10);
```

Excepted:
Head 2->-10->15 --> {-10, -5, 0, 3}1: |->7| --> {7, 9}1: |->15| --> {15, 17, 20, 25}2: |->null->null|
Result: Passed

Test T7:
Test Data:

```
test.remove(7);
```

Excepted:
Head 2->-10->3 --> {-10, -5, 0}1: |->3| --> {3, 9}1: |->15| --> {15, 17, 20, 25}2: |->null->null|
Result: Passed

Test T8:
Test Data:

```
MySkipList<Integer> test = new MySkipList<>(4);
test.add(-5);
test.add(0);
test.add(5);
test.add(10);
test.add(3);
test.add(15);
test.add(20);
test.add(-10);
test.add(9);
test.add(7);
test.add(17);
test.add(25);
test.remove(5);
test.remove(10);
test.remove(7);
```

Excepted:
Head 3->-10->15->null --> {-10, -5}1: |->0| --> {0, 3, 9}1: |->15| --> {15, 17}2: |->20->null| --> {20, 25}1: |->null|
Result: Passed

Question 3:

1- Insert a collection of randomly generated numbers. Perform this operation 10 times for 10.000, 20.000, 40.000 and 80.000 random numbers (10 times for each). So, you will have 10 instances of each data structure for each 4 different sizes. There should be 240 data structure in total.

```
//Random arrays
Random rand = new Random();
Integer[][] k1 = new Integer[10][10000];
Integer[][] k2 = new Integer[10][20000];
Integer[][] k4 = new Integer[10][40000];
Integer[][] k8 = new Integer[10][80000];
for (int i = 0; i < 10; i++) {
    for (int j = 0; j < 10000; j++)
        k1[i][j] = rand.nextInt();
    for (int j = 0; j < 20000; j++)
        k2[i][j] = rand.nextInt();
    for (int j = 0; j < 40000; j++)
        k4[i][j] = rand.nextInt();
    for (int j = 0; j < 80000; j++)
        k8[i][j] = rand.nextInt();
}
//Data structures
BinarySearchTree<Integer>[][] bst = new BinarySearchTree[4][10];
RedBlackTree<Integer>[][] rbt = new RedBlackTree[4][10];
TreeSet<Integer>[][] ts = new TreeSet[4][10];
ConcurrentSkipListSet<Integer>[][] skip = new ConcurrentSkipListSet[4][10];
SkipList<Integer>[][] book_skip = new SkipList[4][10];
MySkipList<Integer>[][] my_skip = new MySkipList[4][10];
```

```
//Initialize all structures
for (int i = 0; i < 4; i++) {
    for (int j = 0; j < 10; j++) {
        bst[i][j] = new BinarySearchTree<>();
        rbt[i][j] = new RedBlackTree<>();
        ts[i][j] = new TreeSet<>();
        skip[i][j] = new ConcurrentSkipListSet<>();
        book_skip[i][j] = new SkipList<>();
        my_skip[i][j] = new MySkipList<>( order: 5);
    }
}
```

```
long start = System.currentTimeMillis();
//Add all arrays to the data structures
for (int j = 0; j < 10; j++) {
    for (int k = 0; k < 10000; k++) {
        bst[0][j].add(k1[j][k]);
        rbt[0][j].add(k1[j][k]);
        ts[0][j].add(k1[j][k]);
        skip[0][j].add(k1[j][k]);
        book_skip[0][j].add(k1[j][k]);
        my_skip[0][j].add(k1[j][k]);
    }
    for (int k = 0; k < 20000; k++) {
        bst[1][j].add(k2[j][k]);
        rbt[1][j].add(k2[j][k]);
        ts[1][j].add(k2[j][k]);
        skip[1][j].add(k2[j][k]);
        book_skip[1][j].add(k2[j][k]);
        my_skip[1][j].add(k2[j][k]);
    }
    for (int k = 0; k < 40000; k++) {
        bst[2][j].add(k4[j][k]);
        rbt[2][j].add(k4[j][k]);
        ts[2][j].add(k4[j][k]);
        skip[2][j].add(k4[j][k]);
        book_skip[2][j].add(k4[j][k]);
        my_skip[2][j].add(k4[j][k]);
    }
```

```
    for (int k = 0; k < 80000; k++) {
        bst[3][j].add(k8[j][k]);
        rbt[3][j].add(k8[j][k]);
        ts[3][j].add(k8[j][k]);
        skip[3][j].add(k8[j][k]);
        book_skip[3][j].add(k8[j][k]);
        my_skip[3][j].add(k8[j][k]);
    }
}
long end = System.currentTimeMillis();
System.out.println(end - start);
```

Total Time: 7693ms

2- Verify that data structures are built correctly (for example, for BST, perform an inorder traversal).

```
//Check all are correctly builded.
try {
    FileWriter myWriter = new FileWriter( fileName: "output.txt");
    myWriter.append(bst[2][2].toString());
    myWriter.append(rbt[2][2].toString());
    myWriter.append(ts[2][2].toString());
    myWriter.append(skip[2][2].toString());
    myWriter.append(book_skip[2][2].toString());
    myWriter.append(my_skip[2][2].toString());
    myWriter.close();
} catch (IOException e) {
    e.printStackTrace();
}
```

Output.txt attached to zip file.

3- Compare the run-time performance of the insertion operation for the data structures. Insert 10 extra random numbers into the structures you built. Measure the running time and calculate the average running time for each data structure and four different problem size. Compare the running times and their increase.

BST Insertion Test(nanoseconds)

	N = 10000	N = 20000	N = 40000	N = 80000
1-	19500	18600	20200	23400
2-	22700	17900	21700	35500
3-	17700	17800	16300	21400
4-	15200	17300	19700	19800
5-	15900	14700	19800	20200
6-	16100	19300	23500	25500
7-	15600	18600	21300	22500
8-	17700	17400	17400	21100
9-	12800	16600	22600	22400
10-	12500	19000	22800	30900
Average	16570	17720	20530	24.270

Red-Black Tree from book Insertion Test(nanoseconds)

	N = 10000	N = 20000	N = 40000	N = 80000
1 -	35200	23600	22000	39800
2 -	16900	17200	23700	28400
3 -	15800	15500	21000	27700
4 -	13400	13600	20900	25100
5 -	14100	16000	20100	30100
6 -	13400	13500	19000	29000
7 -	14000	14500	18900	27700
8 -	13500	20800	20400	27800
9 -	13800	15100	20900	27800
10 -	13000	13900	22000	2594500
Average	16310	16370	20890	285790

Red-Black Tree Insertion in Java(nanosecond)

	N = 10000	N = 20000	N = 40000	N = 80000
1 -	17700	20000	28500	28600
2 -	18400	20100	21900	25500
3 -	17000	33700	18800	22600
4 -	15000	18400	23100	19300
5 -	17000	19000	17900	22500
6 -	15700	21000	20000	23100
7 -	23300	16900	20400	19300
8 -	16300	18700	21200	27700
9 -	14500	17900	17400	26500
10 -	13000	18200	21800	37200
Average	16790	20390	21100	25230

Skip-List in Java Insertion(nanosecond)

	N = 10000	N = 20000	N = 40000	N = 80000
1 -	27000	36500	35600	42600
2 -	26600	41300	43400	41700
3 -	25900	34200	35700	38300
4 -	25800	44500	28100	30500
5 -	25500	34400	31000	36800
6 -	24100	33700	33700	34300
7 -	22900	31600	30100	34500
8 -	21800	24700	38900	37700
9 -	22100	28300	32600	90300
10 -	21300	26800	60500	32200
Average	24300	33600	36960	41890

Skip-List from my book Insertion(nanosecond)

	N = 10000	N = 20000	N = 40000	N = 80000
1 -	24800	52600	34200	44500
2 -	27300	32100	49700	53000
3 -	24000	40200	28200	39800
4 -	25900	33200	27000	33800
5 -	26000	49300	27200	35200
6 -	23300	31800	36300	63200
7 -	19900	29200	24200	36800
8 -	20100	25300	33200	32800
9 -	23800	36100	25800	30600
10 -	25300	26600	34800	52800
Average	24040	35640	32060	42250

Skip-List from Question 2(nanosecond)

	N = 10000	N = 20000	N = 40000	N = 80000
1 -	36000	36000	48100	69100
2 -	28600	32800	51500	68300
3 -	31300	37800	39600	56200
4 -	30200	41400	38500	44400
5 -	33200	38100	42100	52500
6 -	25800	38000	35200	50000
7 -	38100	35200	36700	39800
8 -	27800	32800	42000	46200
9 -	30900	33800	38400	37800
10 -	24300	31600	42100	55200
Average	30620	35750	41420	51950

4-Compare the run-time performance of the deletion operation for the data structures. Perform 10 successful deletion operations from the structures you built. Measure the running time and calculate the average running time for each data structure and four different problem size. Compare the running times and their increase.

BST Remove Test(nanoseconds)

	N = 10000	N = 20000	N = 40000	N = 80000
1 -	51700	51400	60100	62800
2 -	44000	47500	44200	66500
3 -	58000	44400	42100	45600
4 -	43800	43100	44500	51900
5 -	43800	56400	48400	54400
6 -	38800	47600	43800	58300
7 -	42500	46100	46100	71700
8 -	37300	46700	40300	57900
9 -	41300	47200	48500	60900
10 -	41000	42100	42000	54100
Average	44220	47250	46000	58410

Red-Black Tree from my book Remove Test(nanoseconds)

	N = 10000	N = 20000	N = 40000	N = 80000
1 -	66600	56400	67700	61800
2 -	44500	54500	55700	55800
3 -	43900	46700	49600	51600
4 -	50000	50100	50000	52400
5 -	42500	46900	48800	53600
6 -	40600	49300	50300	59100
7 -	55900	47700	51100	56800
8 -	42400	47500	47100	60300
9 -	44100	48400	50300	54900
10 -	55600	49700	46600	56200
Average	48610	49720	51720	56250

Red-Black Tree in Java Remove Test(nanoseconds)

	N = 10000	N = 20000	N = 40000	N = 80000
1 -	69000	46500	47300	61500
2 -	37300	46500	49200	59200
3 -	37200	46400	45900	52500
4 -	36300	41800	45500	42400
5 -	38700	38600	46900	42700
6 -	36100	39800	53300	51500
7 -	34100	38500	59700	51000
8 -	37000	36400	85900	54700
9 -	34600	37400	51700	58800
10 -	33700	55400	75200	62500
Average	39400	42730	56060	53680

Skip-List in Java Remove Test(nanoseconds)

	N = 10000	N = 20000	N = 40000	N = 80000
1 -	292800	122800	152000	138800
2 -	111700	133600	122400	145100
3 -	112600	138000	119800	144100
4 -	102600	123400	111400	110300
5 -	94400	112400	115600	163000
6 -	105700	119200	115400	168900
7 -	116700	130900	107800	167200
8 -	105200	119400	103600	166400
9 -	99000	122000	116700	126000
10 -	97900	120100	110000	117300
Average	123860	124180	117470	144710

Skip-List from my Book Remove Test(nanoseconds)

	N = 10000	N = 20000	N = 40000	N = 80000
1 -	45100	49800	40900	49500
2 -	35300	36600	37700	40000
3 -	37800	31000	30500	43200
4 -	46200	28500	30600	32700
5 -	35800	26200	55900	40300
6 -	30400	31300	34600	37700
7 -	30800	32000	28100	32600
8 -	33500	27600	34200	31400
9 -	36200	29200	29200	41900
10 -	35000	27800	26900	29900
Average	36610	32000	34860	37920

Skip-List in Q2 Remove Test(nanoseconds)

	N = 10000	N = 20000	N = 40000	N = 80000
1 -	139600	118400	146100	125100
2 -	115000	170000	131800	129300
3 -	92800	141300	141700	118600
4 -	84600	128600	149000	113700
5 -	93200	133500	140300	99800
6 -	96700	145000	3647300	126200
7 -	100900	170000	126800	112900
8 -	94900	109000	112700	116400
9 -	90300	117600	116100	128700
10 -	86600	117300	108000	116800
Average	99460	135070	481980	118750

Question 4:

Problem Solution:

- I need to implement a menu-driven program for managing a software store.
- System will have two types of user:
- Administrators and users
- Administrators enters the system with a password, to be able to add, delete, update information.
- Users who browse software.
- To able to store, I should use a search tree interface for the table.
- Add, get, set methods use table's add, get, set methods
- Two types of users will be nested class in the system.
- There is method to instantiate admin and user classes.
- All methods will be in these two classes.
- There is a toString method override to print all table

Test Cases:

Test Subject	Test Number	Pass/Fail
Create Admin	T1	Pass
Adds software	T2	Pass
Prints store	T3	Pass
Create User	T4	Pass
Search By Name	T5	Pass
Buy	T6	Pass
Admin delete	T7	Pass
Admin update	T8	Pass

Running and Results:

Test T1:
Test Data:

```
SoftwareStore st = new SoftwareStore(new RedBlackTree<>());
SoftwareStore.Admin admin = st.createAdmin("password");
```

Excepted: No Error
Result: Passed

Test T2:
Test Data:

```
admin.add("Adobe Flash 4.0","FREE");
admin.add("Windows 10", "10$");
admin.add("Adobe Photoshop 6.0", "120$");
```

Excepted: No errors
Result: Passed

Test T3:
Test Data:

```
System.out.println(st.toString());
```

Excepted:
Black: Software: Adobe Photoshop 6.2, Price: 40$, Quantity: 1
Black: Software: Adobe Flash 4.0, Price: FREE, Quantity: 2
Red: Software: Adobe Flash 3.3, Price: FREE, Quantity: 1

null
null
Red: Software: Adobe Photoshop 6.0, Price: 120$, Quantity: 2
null
null
Black: Software: Norton 5.5, Price: 20₺, Quantity: 1
Red: Software: Norton 4.5, Price: 30$, Quantity: 1
null
null
Red: Software: Windows 10, Price: 10$, Quantity: 1
null
null
Result: Passed

Test T4:
Test Data:

```
SoftwareStore.User user = st.createUser();
```

Excepted: No errors
Result: Passed

Test T5:
Test Data:

```
user.searchByName("Windows 10");
```

Excepted:
Software: Windows 10, Price: 10$, Quantity: 1
Result: Passed

Test T6:
Test Data:

```
user.buy("Windows 10", "10$");
user.buy("Norton 5.5", "20₺");
```

Excepted: No errors
Result: Passed

Test T7:
Test Data:

```
admin.delete("Adobe Photoshop 6.0", "120$");
```

Excepted: No errors
Result: Passed

Test T8:
Test Data:

```
admin.update("Adobe Flash 4.0", "FREE" ,"1$");
```

Excepted: No errors
Result: Passed