Sorbonne University

M2 Internship Synopsis

Modified gravity under scrutiny – testing MOND with pairs of SDSS MaNGA galaxies

Özgen Tunç TÜRKER

Observatoire astronomique de Strasbourg | ObAS

supervised by
Oliver MÜLLER & Benoît FAMAEY

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1 Introduction

In this synopsis, we will give the details of the M2 internship “Modified gravity under scrutiny – testing MOND with pairs of SDSS MaNGA galaxies”. It starts with an introduction consisting of a brief description of the project, scientific goal, and the team’s presentation in which the project is realized. In the next section, a literature review of the subject will be presented, an overview and some problems of the current standard model of the cosmology, the modified gravity theory (Modified Newtonian Dynamics, MOND), and spiral galaxies will be introduced. In the following section, the planning of the project will be discussed. And finally, the last section will be dedicated to the conclusion.

1.1 Project Description

The current standard model of cosmology, Lambda + Cold Dark Matter model (ΛCDM), which describes well the observations on large scales, is constituted of 70% of dark energy, 25% of dark matter and 5% baryonic matter at z=0. However, the non-detection of dark sector contents, and some disagreements in small scales (kpc), motivates to look for some alternatives. Modified Newtonian Dynamics (MOND), providing an alternative to dark matter, was introduced at the same time as ΛCDM. Instead of requiring an additional unseen component, MOND modifies gravitational laws in low acceleration regimes, which is the typical regime for galaxy outskirts. By applying MOND, different consequences and predictions than the ΛCDM model can be found. One crucial consequence is having a constant external gravitational field affecting the system’s internal dynamics, which is not the case for general relativity. When the moon is orbiting around the earth, in the general theory of relativity, the system is not affected by the Milky Way’s external gravitational field, but in the MOND theory with the External Field Effect, it is affected.

This External Field Effect (EFE) will be tested in this project. The project is supervised by Dr. Oliver Müller, who is an expert on observations of galaxies, and Dr. Benoît Famaey, who is one of the world’s experts on MOND.

1.2 Scientific Goals

In this project, we will use data from spiral galaxies observed with the Sloan Digital Sky Survey (more specifically with the SDSS MaNGA survey) to test the EFE. The expected decrease of the internal gravitational field will be analytically modeled with synthetic galaxy models as a function of distance, and observational data [1] will be used to confront these expectations.

1.3 GALHECOS Team

The "Galaxies, High Energy, Cosmology, Compact Objects & Stars" team (GALHECOS) studies the formation and evolution of galaxies and our Galaxy in a cosmological context, through their stellar populations, their star and dark matter dynamics, and their backreaction effects linked to their central black hole. The team is interested in galactic and extragalactic

X-ray emitting sources, compact objects (neutron stars, white dwarfs, etc.), and active nuclei of galaxies. It is involved in the SSC-XMM, an international consortium of laboratories selected by ESA and labeled by INSU as an Observation Service, which is in charge of providing complete catalogs of X sources observed by the XMM-Newton satellite to the international community. The team participates in other National Observation Services: SVOM, Large Spectroscopic Surveys with WEAVE, Gaia, and the Besançon Galaxy model. A large part of the GALHECOS team, working on galaxies dynamics, is closely working in this context on testing various dark matter models and gravity theories by confronting them to galaxy scale data. Multiple ANR and ERC fundings are dedicated to this topic within the GALHECOS team.

2 "State of the Art" of The Subject

There is a known discrepancy between the predictions of general relativity with the observed matter-energy and the galactic and extra-galactic observations. Two different deductions can be made out of this disagreement; the existence of additional matter-energy components which haven't been observed yet or the breakdown of the general relativity [2].

In each scenario, there are ongoing researches for years with no general agreement so far. In this project, we will be working on an alternative gravitational theory introduced by Mordehai Milgrom, namely *Modified Newtonian Dynamics (MOND)* [3]. It is worth mentioning that MOND is a survivor alternative gravity among all the other attempts thanks to its behavior of being close becoming a universal force law in a complete set of systems, namely galaxies, as they are considered to contain only the visible matter [2]. The MOND phenomenology has also survived, with observations that appeared long after it had been hypothesized. Some of the most challenging observations for MOND and its relativistic extensions have been gravitational waves or the angular power spectrum of the Cosmic Microwave background, but it has recently been shown that, while many of such extensions are ruled out indeed [4, 5], some can survive and explain these observations [6, 7].

The problem that MOND is intended to solve is to explain the appearance of *the universal critical acceleration constant* in different observations on galaxy dynamics (which we will explain in detail in section 2.3), rather than getting rid of the unexplained exotic matter content [2].

2.1 Mass Discrepancies

Mass discrepancies manifest themselves in a various range of data. In the 1930s, Jan Oort noticed a discrepancy in the Milky Way between the luminous matter mass and the mass that provides the force to keep the stars moving. Then Fritz Zwicky, by studying velocities of the galaxies in the Coma Cluster, noticed that velocities are higher than what is needed them to be bounded [2, 8, 9]. At the epoch, these studies were not considered seriously. These disagreements have more audiences in the 1970s. In order to deal with the "bar-like" instabilities in the "cold" galaxies which have stars with small velocity dispersion, i.e., Milky Way, in 1973, Ostriker and Peebles proposed for the first time "an unseen disk component" corresponding to dark matter halo [10]. Following the studies, most out-breaking work is done on the rotation curves of spiral galaxies. Bosma and Rubin showed that the rotation curves of galaxies are not obeying Newton's gravity. Instead of decreasing with $V \propto r^{-1/2}$, they tend to be constant with increasing radius [2].

There are also mass discrepancies in other type of galaxies than the spirals, for instance, dwarf spheroidal galaxies that are orbiting the Milky Way and Andromeda systems. These galaxies are good examples to measure the inconsistency as they are close enough to measure individual star line-of-sight speeds. Moreover, this measurement gives the mass of the system roughly $M \sim r\sigma^2/G$, a higher disagreement with the luminous mass. This is interpreted as dimmer galaxies have more dark matter. However, the relation is not exactly between circular speed and the mass but the circular speed and the surface brightness [2].

In addition to all these mass-related problems, there are also other motivations for proposing an unseen matter content. The small ratio of the baryonic mass density to the critical density from Big Bang Nucleosynthesis with the result inferred from the large scale structure growth in the Hubble time by a factor of $\sim 10^5$ it is understood that there is much more matter than the observable ($\Omega_m > \Omega_b$). Since only baryonic content is taken into account, this growth factor should be around 10^2 [2].

2.2 ΛCDM Model

Facing the problems mentioned above motivates the current standard cosmological model, ΛCDM, with non-baryonic, dissipationless and collisionless particles (Cold Dark Matter, CDM) with non-zero cosmological constant (Λ). It was proposed by Ostriker and Steinhardt in 1995 [11] in the framework Friedmann–Lemaître–Robertson–Walker metric as being the exact solution of Einstein's equation in the general theory of relativity. The model is supported by the observational data on large scales and has successful predictions; accelerated expansion rate, the baryon acoustic oscillations, Cosmic Microwave Background (CMB) power spectrum, and the abundance of light elements from Big bang Nucleosynthesis [2, 12].

There is a price to pay for having these extraordinary achievements of ΛCDM, a new physics in the dark sector [2].

2.2.1 Dark Energy

A flat universe with density parameter 1 is favored by the inflation theory. Knowing the fact that Ω_m falls short of fulfilling this requirement of inflation and the disagreement of measured Hubble parameter and age of the universe, ΛCDM was proposed with a non-zero cosmological constant, which represents vacuum energy or *Dark Energy*. This constant is the same as the one Einstein proposed in 1917 and then banished. Later, the discovery of late-time acceleration of the expansion of the universe, as the universe in the dark energy dominated era, is strong support to accept the model with Einstein's cosmological constant being dark energy with equation of state $p/\rho = -1$.

The Density of the matter content in the ΛCDM universe is known to be $\Omega_m = 0.27$, adding up to 1 as it is expected in the dark energy dominated flat universe by the model itself with the dark energy density of $\Omega_\Lambda = 0.73$.

2.2.2 Dark Matter and The Candidate Particles

In the total density of matter 0.27, the dark matter to the baryonic matter ratio is 5:1, from the Big Bang Nucleosynthesis [2]. The conclusion is to be done that the unseen matter is not just a dark matter, but it should be non-baryonic dark matter. Additionally, dark matter is believed to be cold, which refers to the fact that when it decouples from the photon-baryon plasma, it was already slow, non-relativistic [2]. It condensed and started forming structures that eventually let the ordinary matter, starting to be formed after the recombination, fall in. Today we call these structures dark matter halos, where the luminous structures are inhabiting.

Dark Matter Candidates

Other than the evolution of the cold dark matter (CDM) more crucial question may be about its nature. First things to mention that dark matter candidate should be out of the Standard Model of Particle Physics, a new particle that has not been observed yet. It is known that dark matter particles are interacting with ordinary matter gravitationally.

Candidate particles are lying on an extensive range of mass, as it is seen in figure 1.

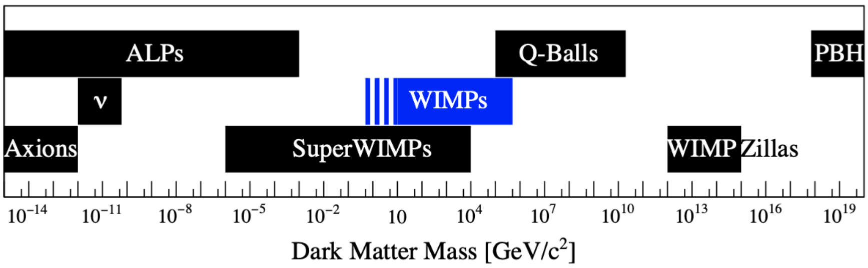

Figure 1: Different dark matter candidates on a many order of magnitude mass range.[13]

Dark matter is known that it is not interacting with ordinary matter via electromagnetic interaction. However, it may also possess a tiny electrical charge [14]. It is also assumed that dark matter particle is collisionless, but self-interacting candidates also exist [14].

Among all the candidate particles, there is one preferred which "best-developed researches" focus on, Weakly Interacting Massive Particles (WIMPs) [14]. They are electromagnetically neutral and they were non-relativistic when they decoupled from plasma, which makes them "cold" dark matter particles. Their mass range is 1 to 10^5 GeV, and their abundance naturally corresponds to the dark matter density parameter [13].

2.2.3 Challenges

In this section, it will be discussed tests and problems waiting to be solved for ΛCDM Model. In the first place, the detection of the hypothesized dark sector component is a very important test. Especially for the WIMPs, there will be no room to hide by the ongoing researches for decades [15]. By the improvements of the detectors it will be reached to the "neutrino floor" where it will be an "irreducible background" for the WIMP signal [13]. As they are elaborated in [2], there are lists of challenges waiting for satisfying explanations. Even there are some claims showing that some of the challenges, i.e., missing baryon challenge, are solved [16], there still need to have general agreement on the challenges.

The First thing to mention as the biggest unexplained problem of ΛCDM, so-called "vacuum catastrophe" [17], is the disagreement on the value of the vacuum energy density between the observation and the prediction of the quantum field theory. Additionally, there is also a cosmic coincidence problem which arises from the fact that dark energy and matter density are of the same order, 0.7 and 0.3, respectively. Because the matter density decreases as the universe expands, two densities come to coincide today, after around 15 billion years [18], raises a "why now?" question [19].

Apart from these unexplained coincidences, there are also questions waiting for an explanation. As the review [2] categorized, there are two types of challenges; one group consists of the disagreements between the model and the observations; the other one has the empirical results waiting for theoretical description by the model.

Observations versus Predictions

ΛCDM Model is predicting rather slow structure formation than observed data. This conflict manifests itself in the high-z clusters challenge and the local void challenges [2]. Another disagreement is about the prediction of the population of dark subhalos. According to the model, there must be around 100 dark subhalos that have sufficient mass to form observable galaxies, orbiting around Milky Way size dark halos [20]. Despite ongoing searches, there is still inconsistency with the observed number of satellite galaxies, and they are also not isotropically distributed as predicted [2]. There are some other unexpected observations as the high number of bulgeless disk, bar, and spiral formations in the low surface brightness disk galaxies, and the dark matter distribution disagreement which is constant in the center instead of the expected steep decrease [2].

Unexplained Results

Baryonic Tully-Fisher Relation (BTFR), which is purely empirical, tells the relation between the baryonic mass of the galaxy and its rotation velocity. Which is described by a power law,

$$\log M_b = \alpha \log V_f - \log \beta \tag{1}$$

where M_b is detected baryonic mass, and V_f is the flat portion of the rotation curve. Here the slope α is equal to 4, which is consistent with *the universal critical acceleration constant* $a = V_f^4/(GM_b)$. This acceleration scale ($a_0 \simeq 10^{-10}$ ms$^{-2} \simeq \Lambda^{0.5}$) will play a crucial role in the MOND formalism manifesting itself in the data [2]. In ΛCDM context, the explanation of the BTFR requires fine-tunings rather than having a natural explanation. The problem arises from its small intrinsic scatter [21].

There also exists a limit that defines surface brightness limit, which shows a stability limit for disks, the so-called *Freeman Limit*. In the dark matter context, there is no reason to have a relation between baryonic surface density and the acceleration scale [2].

2.3 Milgrom's law and MOND

So far, all the discussion was about the observations and the ΛCDM model. Nevertheless, it is also reasonable to consider other approaches than putting the dark matter in the game, to solve the mass discrepancies, such as Mordehai Milgrom did in 1983. He thought there might be a breakdown of Newtonian gravitational acceleration g_N in galactic scales. This scale, rather than being a length scale, is an acceleration scale which is typical for a star in a galaxy, $\sim 10^{-10}$ ms^{-2}.

New natural constant a_0 can be seen as the same way of the speed of light or Planck constant. When the acceleration is much higher than a_0 ($F/m = g >> a_0$), everything can be explained purely Newtonian, $g = g_N$, as it is $a_0 \rightarrow 0$ (like having pure classical mechanics when h goes to 0), however when the acceleration is less than the constant ($F/m = g << a_0$) then

$$g = \sqrt{g_N a_0} \tag{2}$$

where g is real gravitational acceleration and g_N is Newtonian.

Similarly, centripetal acceleration of a point particle around the mass M, in weak acceleration turns out to be:

$$\frac{V_c^2}{r} = \sqrt{\frac{GMa_0}{r^2}} \tag{3}$$

So it is seen that circular velocity no longer depends on the radius.

$$V_c^2 = GMa_0 \tag{4}$$

In a more general way, equation 2 can be written as following, (Milgrom's law)

$$g\,\mu(\frac{g}{a_0}) = g_N \tag{5}$$

where $\mu(x)$ is interpolating function helps to have a smooth transition between the weak and strong acceleration regimes.

Rather than applying equation 5 directly, it is derived from the action. Thus, the action of the particles which have the mass m_i in a gravitational field defined by the Newtonian potential Φ_N and created by the mass density $\rho = \sum_i m_i \delta(x - x_i)$, is:

$$S_N = S_{kin} + S_{in} + S_{grav} = \int \frac{\rho v^2}{2} d^3x dt - \int \rho \Phi_N d^3x dt - \int \frac{|\nabla \Phi_N|^2}{8\pi G} d^3x dt \tag{6}$$

Modifying the kinetic part of equation 6 will give the MOND effect with "modified inertia" and modifying the S_{grav} will give it with "modified gravity".

2.3.1 Modified Inertia

By modifying the kinetic term, instead of having $d^2x/dt^2 = -\nabla \Phi_N$, the left hand side of the equation of motion will have acceleration constant dependence. A being a functional of the trajectory $x(t)$

$$A\{\{x(t)\}, a_0\} = -\nabla \Phi_N \tag{7}$$

Then for bound circular trajectories in axisymmetric potentials, equation 7 is written as follows [2, 22]

$$\frac{V_c^2}{R}\,\mu\Big(\frac{V_c^2}{Ra_0}\Big) = -\frac{\partial \Phi_N}{\partial R} \tag{8}$$

where R is radius and V_c is orbital speed.

2.3.2 Modified Gravities

There are 2 different modified gravity versions of MOND. One is about changing the matter action (Bekenstein–Milgrom MOND) the other one is about adding an auxiliary field (QUMOND) [2].

Bekenstein–Milgrom MOND

By changing gravitational action, the Poisson equation is modified. Gravitational action of Bekenstein–Milgrom MOND is written as follows

$$S_{grav\,BM} = -\int \frac{a_0^2\, F\big(|\nabla\Phi_N|^2/a_0^2\big)}{8\pi G} d^3x dt \tag{9}$$

where F is any dimensionless function. New defined action not being explicitly quadratic in $|\nabla\Phi_N|^2$ lets this theory be named Aqudaratic Lagrangian theory (AQUAL).

Then modified Poisson equation becomes:

$$\nabla\,.\,\Big[\mu\Big(\frac{|\nabla\Phi|}{a_0}\Big)\nabla\Phi\Big] = 4\pi G\rho \tag{10}$$

where μ is the function which has the behavior as the interpolation function.

Following the modified Poisson equation, relation between true ($g = -\nabla\Phi$) and Newtonian accelerations ($g_N = -\nabla\Phi_N$) is

$$g\,\mu\Big(\frac{g}{a_0}\Big) = g_N + S \tag{11}$$

where S is a solenoidal vector field that has no net flow in closed surface [2]. So equation 5 is retrieved up to a field correction.

Far from a mass M, in a vacuum, isopotentials diminish and as well as the curl field in "deep-MOND" regime where acceleration is way below a_0, the potential is defined as follows:

$$\Phi(r) \sim \sqrt{GMa_0}\,\ln(r) \tag{12}$$

QUMOND

In QUMOND, the first action is written with the auxiliary acceleration field ($g_N = -\nabla\Phi_N$) without modifying Newtonian gravity:

$$S_{grav\,N} = -\frac{1}{8\pi G}\int (2\nabla\Phi.g_N - g_N^2) d^3x\, dt \tag{13}$$

Now MONDian modification is done by replacing g_N^2 with a non-linear function as it is done in AQUAL:

$$S_{grav\,\mathrm{QUMOND}} = -\frac{1}{8\pi G}\int \Big[2\nabla\Phi.\nabla\Phi - a_0^2\, Q\Big(|\nabla\Phi_N|^2/a_0^2\Big)\Big]\, d^3x\, dt \tag{14}$$

and varying the action with respect to MOND field gives $\nabla^2\Phi_N = 4\pi G\rho$ and respect to Newtonian (auxiliary) potential gives

$$\nabla^2\Phi = \nabla\,.\Big[\nu\Big(\frac{|\nabla\Phi_N|}{a_0}\Big)\nabla\Phi_N\Big] \tag{15}$$

ν function is corresponding μ function in the inverted version of equation 5, $g = v(g_N/a_0)\, g_N$.

If we take equation 15 as it is a standard Poisson equation, then right hand side would be the source that creates MOND potential $\hat{\rho}$. The difference between sources create MOND potential and the Newtonian potential is called *dark matter density*.

$$\rho_{ph} = \hat{\rho} - \rho \tag{16}$$

In the MOND context, this ρ_{ph} referred to *phantom dark matter*, which is the wrong interpretation of gravitational law [23].

As it is derived in AQUAL, from equations 10 to 11, Milgrom's law is also retrieved in QUMOND but with a different curl field. This shows that AQUAL formalism and QUMOND have differences far from the high symmetries [2].

2.3.3 External Field Effect

External Field Effect (EFE) is a very powerful consequence of the MOND theories. Because MOND is a theory which acceleration-based, what matters is the total acceleration [2]. With this feature, it is breaking the fundamental principle of general relativity, which is Strong Equivalence Principle.

So, the MOND effects are not only considering the internal acceleration of the system but also external gravitational acceleration. To have the standard MOND effect, both internal gravity (g) and external gravity (g_e) should go below a_0 [2].

- $g_e < g < a_0$, MOND effect
- $g < a_0 < g_e$, pure Newtonian
- $g < g_e < a_0$, Newtonian with a renormalized gravitational constant.

2.3.4 Relativistic Extension

MOND theory needs to have a relativistic extension to be tested in the cosmological situation such as CMB since it requires the fluctuations to be treated relativistically. In [7], a new relativistic extension of MOND is produced. This new relativistic MOND (rMOND) reproduces lensing and galactic phenomenology and succeeds in reproducing the fundamental cosmological observables; the CMB and matter power spectra (MPS). rMOND is meeting the phenomenological facts such as returning to Newtonian gravity when $\nabla\Phi >> a_0$ and reproducing MOND effect when $\nabla\Phi << a_0$. It should also be in accord with CMB and MPS results, gravitational lensing and propagation of the tensor modes, gravitational waves [7].

2.3.5 Spiral Galaxies

Before passing the planning, here is the right place to talk about the spiral galaxies which will use to confront the analytical results.

Spiral galaxies consist of stars, gas, and dark matter. Their rotation curves are getting flattened in increasing radius. However, without the existence of dark matter, it is expected to reduce with radius. In the figure

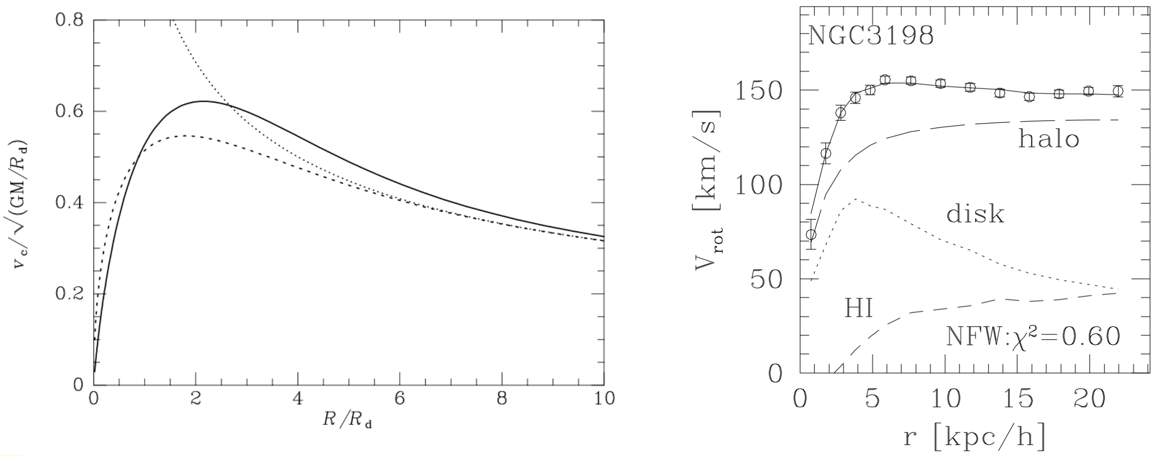

Figure 2: **Left:** Rotation curves calculated by exponential disk (full line), point mass (dotted), spherical body (dashed) [24], **Right:** Rotation curves with gas (HI), disk and dark matter halo contents [25]

Dark matter halo, which contains most of the galaxy's mass, comes to help at this stage by having the Navarro–Frenk–White (NFW) profile that extends far beyond the virial radius of the galaxy [25, 26].

NFW profile is dominant at the higher radius compared to the disc mass profile.

The surface brightness in spiral galaxy disks, which traces the radial distribution of stars, obeys the exponential law.

3 Planning of The Project

In the planning of the project first weeks are dedicated to reading and understanding the state of the art of the subject as it is briefly explained in the previous section. To do so, review article "Modified Newtonian Dynamics (MOND): Observational Phenomenology and Relativistic Extensions" of Benôit Famaey and Stacy S. McGaugh is taken as a guideline.

Then, the ultimate goal of the project, testing the External Field Effect of the MOND, will be done by following the path, retrieving the pure Newtonian rotation curves, transforming them to the MONDian rotation curves, and finally adding the external field effect.

To start with, as it is the common surface brightness profile, we will consider an exponential disk, approximated to be infinitesimally thin. Then, the surface density profile,

$$\Sigma(R) = \Sigma(0)e^{-R/R_d} \tag{17}$$

where $\Sigma(0)$ is the central density $\Sigma(0) = M_b/(2\pi R_d^2)$ with M_b the total baryonic mass of the disk and R_d the disk scale length.

With the enclosed mass as a function of radius R

$$M(R) = M_b\big[1 - e^{-R/Rd}(1 + \frac{R}{Rd})\big] \tag{18}$$

Then the circular speed of the disk comes from solving the Poisson equation, given in equation 2.165 in [24]:

$$V_c^2(R) = GM_b\frac{R^2}{2R_d^3}\big[I_0(y)K_0(y) - I_1(y)K_1(y)\big] \tag{19}$$

where the I_n and K_n are the modified Bessel functions.

Then, by taking baryonic masses (M_b) corresponding to the range of masses in the observational articles, and a large range of R_d, we will create a library of models. When we have the library, we will transform the Newtonian rotation curves to the MONDian ones and add External Field Effect. EFE will be applied by a new formula developed by the GALHECOS Team.

By adding the External Field Effect, finally, we aim to explain on one of the results Douglass and Demina have in [1]. That is, the galaxies located in the region with denser local environments have a smaller ratio of total mass to the visible mass (see Figure 3), which is hard to have a satisfying explanation with the ΛCDM model. With the external field effect, we plan to create pairwise galaxies, control the separation between them and calculate EFE as a function of radius. With EFE, we expect to find a negative phantom dark matter density to confirm the MOND theory. In the Netwonian gravity context, this negative phantom matter corresponds to a decrease in the galaxy mass as it is expected in the absence of a dark matter halo.

4 Conclusion

By the existence of External Field Effect (EFE), the successful general theory of relativity is in danger. Since, by construction, EFE contradicts the fundamental principle of GR, which is the Strong Equivalence Principle (SEP), MOND has to break down SEP. Thus, testing EFE may bring out much discussion about the viability of GR, as it is done in [27] by using The Spitzer Photometry and Accurate Rotation Curves database.

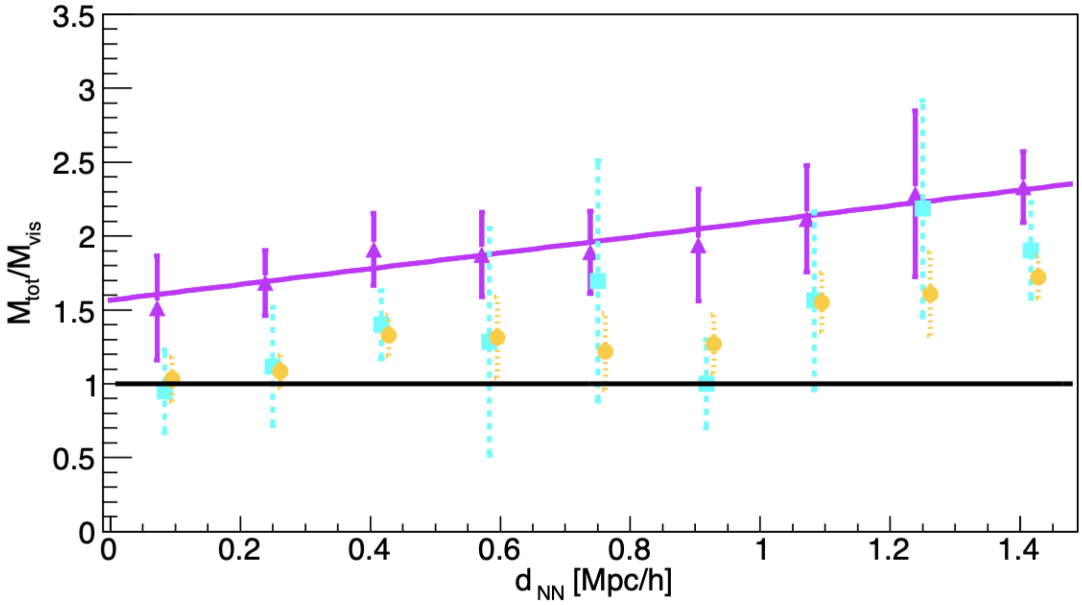

Figure 3: The relation between the ratio of masses and the nearest neighbor distance. Magenta points are M_{tot}/M_* (which is linear fitted), cyan is $M_{tot}/(M_* + M_{Hi})$, and orange is $M_{tot}/(M_* + M_{Hi,est})$. Where M_{tot} is the total mass of the galaxy, M_* is stellar mass, M_{Hi} neutral hydrogen mass, and $M_{Hi,est}$ neutral hydrogen mass estimated with some parameters in the article. Spectroscopic observations from the SDSS MaNGA DR15 are used. [1]

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