AERODYNAMICS course.

$$\rho = \frac{m}{V}$$

- $P.V = nRT \Rightarrow P = \frac{nRT}{V} = \frac{mRT}{M.V} = \rho.\frac{R}{M}.T = \boxed{\rho.R.T}$ où $R = \frac{R}{M}$

gaz law; specific for air.

- $F = mg \Rightarrow \boxed{dp = d\rho R.T = -\rho g dh}$ ← hydrostatic law

- $T = T_0 + a(h - h_0)$; a = lapse rate.

- $\frac{P_1}{P_0} = \left(\frac{T_1}{T_0}\right)^{-\frac{g}{aR}} \Rightarrow P_1 = \left(\frac{T_0 + ah}{T_0}\right)^{-\frac{g}{aR}} . P_0$

$$\rho_1 = \left(\frac{T_0+ah}{T_0}\right)^{-\frac{g}{aR}-1} . \rho_0$$

- → With gliders you have to run to generate enough lift, often down hill to make it easier. is the wind direction of any influence?

↳ What matters is the Airspeed, not the ground speed the higher the Airspeed the higher the lift.

→ tailwind is a wind that blows in the direction of travel of an object

→ A headwind is a wind that blows against the dir of travel.

→ tailwind increases the objects speed and reduces the time required to reach destination, head wind has the opposite effect

→ a head wind is favorable in takeoffs and landings because an airfoil moving into a headwind is capable of greater lift generation that a tail wind of tranquil air.

→ for a straigh, horizontal, steady flight, V and alt. (altitude) are constant due to the following force equilibrium:

- $L = W$
- $D = T$

Where W: Weigh | Drag: D
L: Lift | Thrust: T

↳ these are the 4 forces acting on a plane in flight.

→ Lift : mainly generated by the wing (small contributions of tail surfaces)

→ Weight : composed of 3 main components : aircraft empty weight, fuel, payload (passengers + luggage, freight)
→ [Aircraft body (main)]

struct systems ...

→ Drag : caused by fuselage, wings, tail surfaces, ...

→ Thrust : provided by the engines.

• → One of the main rules in aerospace design is to create lightweight aircraft, Why ?

└→ Answer : If we reduce the empty weight (MTOW = cte),
we can | transport more payload (higher revenue)
| take more fuel on board (fly longer distances)
in general : Improve the performance of the aircraft.

—

• Lift

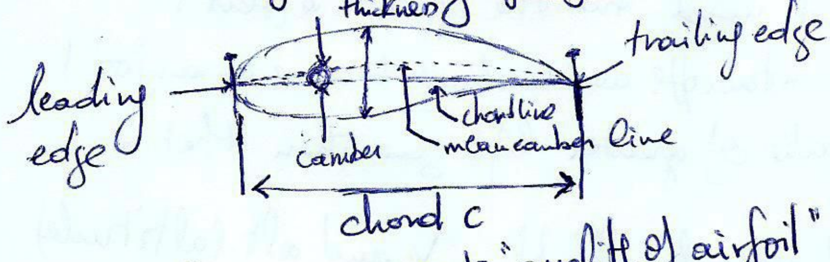
→ the formula for the Lift is : $\boxed{L = C_L \times \left(\frac{1}{2} \rho V^2\right) \times S}$

→ Lift parameters :

| C_L : Lift coefficient
| ρ : Density of the air [Kg/m^3]
| V : Air speed [m/s]
| S : Wing area [m^2]

• C_L (Lift coef) depends on :

↳ Airfoil or wing profile



• Rq : C_L represents "quality of airfoil" (ability to generate Lift)

↳ Angle of attack (α)

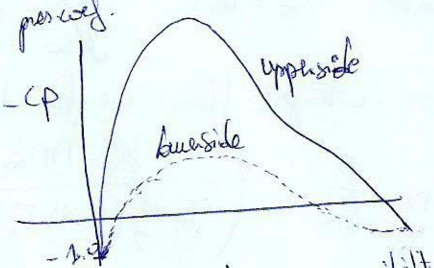
• ρ (Air density) depends on :
↳ Altitude & temperature (atmosphere)

• V (air speed) & S (wing area) are design parameters.

+ Airfoil theory +

→ Finite wing.

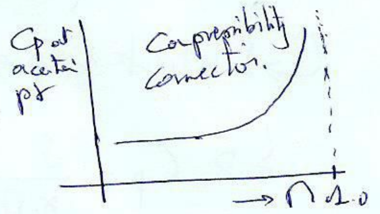
→ Aspect ratio: $A = \frac{b^2}{S}$ (b: span, S: Wing area)



→ Pressure coef:

→ Def: $C_p = \frac{p - p_\infty}{q_\infty} = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2}$

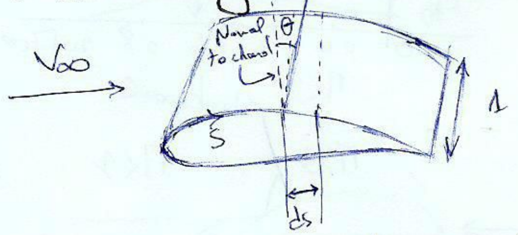
→ Compressibility correction of the pressure coefficient.

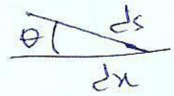


→ Approximate theoretical correction (valid for $0 < M < 0{,}7$):

$$C_p = \frac{C_{p0}}{\sqrt{1 - M_\infty^2}}$$ Prandtl - Glauert rule.

→ Obtaining normal force from pressure distribution





→ Normal force per meter span: $N = \int_{LE}^{TE} p_\ell \cos\theta \, ds - \int_{LE}^{TE} p_u \cos\theta \, ds = \int_0^c p_\ell \, dx - \int_0^c p_u \, dx \quad (ds\cos\theta = dx)$

↳ Write dimensionless force coefficient: $C_n = \frac{N}{\frac{1}{2}\rho V_\infty^2 c}$

↳ $C_n = \int_0^1 \frac{p_\ell - p_\infty}{q_\infty} d\left(\frac{x}{c}\right) - \int_0^1 \frac{p_u - p_\infty}{q_\infty} d\left(\frac{x}{c}\right) \Rightarrow \boxed{C_n = \int_0^1 (C_{p\ell} - C_{pu}) \, d\left(\frac{x}{c}\right)}$.

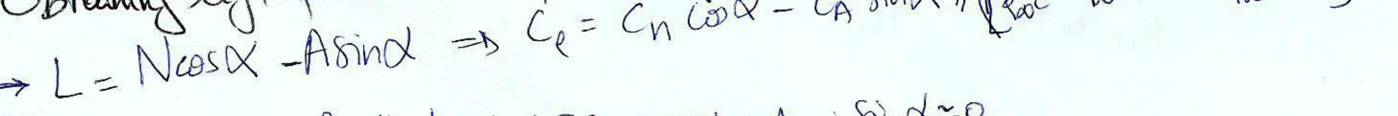
→ Obtaining lift from normal force coefficient:

→ $L = N\cos\alpha - A\sin\alpha \Rightarrow C_\ell = C_n \cos\alpha - C_A \sin\alpha \quad // \left[\frac{L}{q_\infty c} = \frac{N}{q_\infty c}\cos\alpha - \frac{A}{q_\infty c}\sin\alpha\right]$

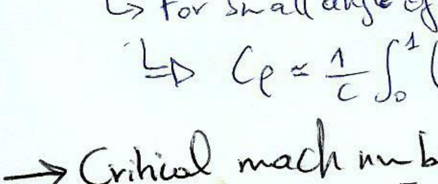
↳ For small angle of attack $\alpha \leq 5°$: $\cos\alpha \simeq 1$; $\sin\alpha \simeq 0$.

↳ $C_\ell \simeq \frac{1}{c}\int_0^1 (C_{p\ell} - C_{pu}) \, d(x)$

→ Critical mach number and critical pressure coefficient:



→ Critical pressure coefficient:



$C_{p_{cr}} = f(M_\infty)$

→ Thicker airfoil reaches critical pressure coefficient at a lower value of M_∞.

→ Def of pres coef.: $C_p = \dfrac{p - p_\infty}{q_\infty} = \dfrac{p_\infty}{q_\infty}\left(\dfrac{p}{p_\infty} - 1\right)$

→ Dynamic pres.: $q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}\dfrac{\rho_\infty}{\gamma p_\infty}\gamma p_\infty V_\infty^2 = \frac{1}{2}\dfrac{V_\infty^2}{\gamma p_\infty / \rho_\infty}\gamma p_\infty$

↳ or: $a_\infty^2 = \gamma R T = \dfrac{\gamma p_\infty}{\rho_\infty} \Rightarrow q_\infty = \frac{1}{2}\dfrac{V_\infty^2}{a_\infty^2}\gamma p_\infty = \boxed{\dfrac{\gamma}{2} p_\infty M_\infty^2}$

→ For isentropic flow we found: $\dfrac{p_0}{p} = \left(1 + \dfrac{\gamma - 1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$ also $\dfrac{p_0}{p_\infty} = \left(1 + \dfrac{\gamma-1}{2}M_\infty^2\right)^{\frac{\gamma}{\gamma-1}}$

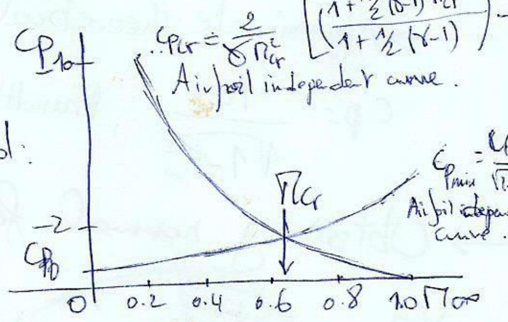
↳ $\dfrac{p}{p_\infty} = \left(\dfrac{1 + \frac{1}{2}(\gamma-1)M_\infty^2}{1 + \frac{1}{2}(\gamma-1)M^2}\right)^{\frac{\gamma}{\gamma-1}}$

↳ Substitute in Cp-eq.: $C_p = \dfrac{p_\infty}{q_\infty}\left(\dfrac{p}{p_\infty} - 1\right) = \dfrac{p_\infty}{\frac{1}{2}\gamma p_\infty M_\infty^2}\left[\left(\dfrac{1+\frac{1}{2}(\gamma-1)M_\infty^2}{1+\frac{1}{2}(\gamma-1)M^2}\right)^{\frac{\gamma}{\gamma-1}} - 1\right]$

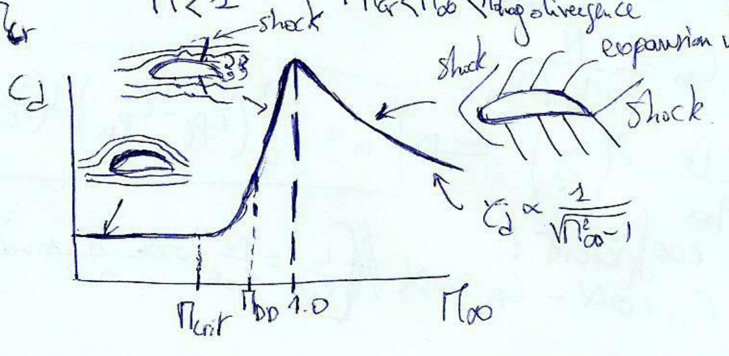
↳ $C_p = \dfrac{2}{\gamma M_\infty^2}\left[\left(\dfrac{1+\frac{1}{2}(\gamma-1)M_\infty^2}{1+\frac{1}{2}(\gamma-1)M^2}\right)^{\frac{\gamma}{\gamma-1}} - 1\right]$

→ The critical pres coef is found when $M = 1$ is reached:

↳ $C_{p_{cr}} = \dfrac{2}{\gamma M_\infty^2}\left[\left(\dfrac{2 + (\gamma-1)M_\infty^2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} - 1\right]$



→ Drag divergence:



+ Compressibility +

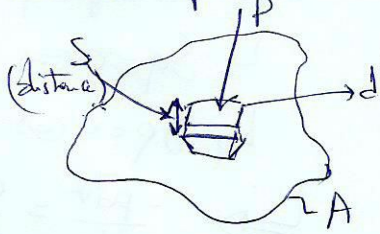
→ First law of thermodynamics:

→ Internal energy per unit mass [e] can only change when:
+ Heat is added or taken away from the system (δq is heat per unit mass)
+ Work is done on, or by, the system (δw is work done per unit mass)

↳ $de = \delta q + \delta w$.

→ Different forms of first law:

→ For the work at surface: Work is Force × Area × distance $= p dA s$



↳ $F = p dA$; $W = F.d = p dA . S$

↳ $\delta w = \int_A p dA S = p \underbrace{\int_A S dA}_{\text{change in volume / unit mass} \rightarrow \frac{dm^3}{kg}} = \boxed{-p dv}$ specific vol

↳ Substitute in 1st law: $\delta q = de - \delta w = \boxed{de + p dv}$

↳ Define enthalpy as: $h = e + pv \rightarrow dh = \underbrace{de + p dv}_{\delta q} + v dp \Rightarrow \boxed{\delta q = dh - v dp}$

→ 2 types of processes:

→ Constant volume process:

— rigid boundary = constant volume | → Heat is added, while volume keeps constant.

→ Constant Pressure proc:

| → Heat is added, piston moves such to keep the pressure constant.

→ Specific Heat:

→ def of specific Heat, (depends on type of process, thus, multivalued): $\boxed{c \equiv \frac{\delta q}{dT}}$

→ The resulting value of dT when adding δq depends on the type of proc:

$c_v = \left(\frac{\delta q}{dT}\right)_{\text{constant volume}}$ | $c_p = \left(\frac{\delta q}{dT}\right)_{\text{constant pressure}}$

→ constant vol proc $\Leftrightarrow dv = 0 \Leftrightarrow \delta q = de + \underbrace{p dv}_{=0} \Leftrightarrow \delta q = de = c_v dT \Rightarrow \boxed{e = c_v T}$

→ constant pres proc $\Leftrightarrow dp = 0 \Leftrightarrow \delta q = dh - \underbrace{v dp}_{=0} \Leftrightarrow \delta q = dh = c_p . dT \Rightarrow \boxed{h = c_p T}$

↳ Above relations hold as long as the gaz is a perfect gaz. [for any proc with $T < 600K$]

↳ For air: $c_v = 720 J/(kg.K)$ | $c_p = 1008 J/(kg.K)$

→ Isentropic flow:

→ We introduce one more concept to bridge thermodynamics and compressible aerodynamics: isentropic flow.

→ Def: + Adiabatic proc: $\delta q = 0$

+ Reversible proc: no frictional or dissipative effects.

+ Isentropic proc: both adiabatic & reversible.

↳ Though the flow can be isentropic, the T° might change from point to point.

→ Ratio of specific heats:

• For isentropic flow (or a perfect gas) ⇒ several important relations btw T, p, ρ:

↳ $\delta q = 0 \Leftrightarrow de + p\,dv = 0 \Leftrightarrow -p\,dv = de = c_v dT$

↳ $\delta q = 0 \Leftrightarrow dh - v\,dp = 0 \Leftrightarrow v\,dp = dh = c_p dT$

$$\Rightarrow \frac{-p\,dv}{v\,dp} = \frac{c_v}{c_p} \Leftrightarrow \frac{dp}{p} = -\left(\frac{c_p}{c_v}\right)\frac{dv}{v} \Leftrightarrow \boxed{\frac{dp}{p} = -\gamma\frac{dv}{v}}$$

• Ratio of specific heats $\boxed{\frac{c_p}{c_v} = \gamma = 1.4 \leftarrow \text{for air}}$

→ We integrate btw pt 1 & pt 2 on a streamline:

$$\int_1^2 \frac{dp}{p} = \int_1^2 -\gamma\frac{dv}{v} \Leftrightarrow \ln p_2 - \ln p_1 = -\gamma\left(\ln v_2 - \ln v_1\right) \Leftrightarrow \ln\frac{p_2}{p_1} = \ln\left(\frac{v_2}{v_1}\right)^{-\gamma} \Leftrightarrow \boxed{\frac{p_2}{p_1} = \left(\frac{v_2}{v_1}\right)^{-\gamma}}$$

↳ since the specific vol is $v_1 = \frac{1}{\rho_1}$ & $v_2 = \frac{1}{\rho_2}$ we find:

$$\frac{p_2}{p_1} = \left(\frac{\rho_1}{\rho_2}\right)^{-\gamma} \Leftrightarrow \boxed{\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}}$$

↳ Now with the eq of state $pv = RT$ we may write:

$$\frac{p_2}{p_1} = \left(\frac{p_2}{RT_2}\cdot\frac{RT_1}{p_1}\right)^{\gamma} = \left(\frac{p_2}{p_1}\cdot\frac{T_1}{T_2}\right)^{\gamma} \Rightarrow \left(\frac{p_2}{p_1}\right)^{1-\gamma} = \left(\frac{T_1}{T_2}\right)^{\gamma} \Leftrightarrow \left(\frac{p_2}{p_1}\right)^{1-\gamma} = \left(\frac{T_2}{T_1}\right)^{-\gamma}$$

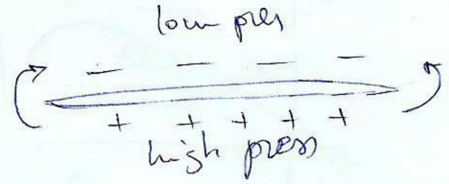
$$\Rightarrow \boxed{\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}}$$ for isentropic flow (only relevant to compressible flow).

$$\Rightarrow \frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

+ Finite & swept wings +

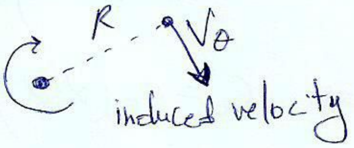
→ Characteristics of finite wings:

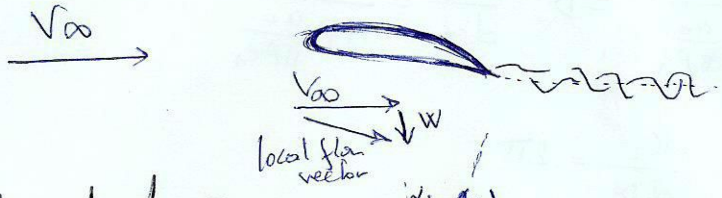
→ Flow around the wing tip generates tip vortices:



→ Tip vortices:

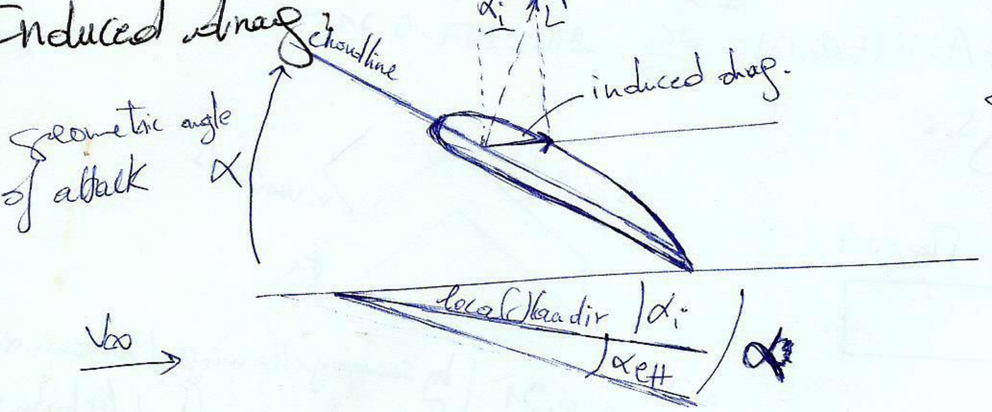
→ Cross section of tip vortex:





The trailing tip vortex causes downwash, w.

→ Induced drag

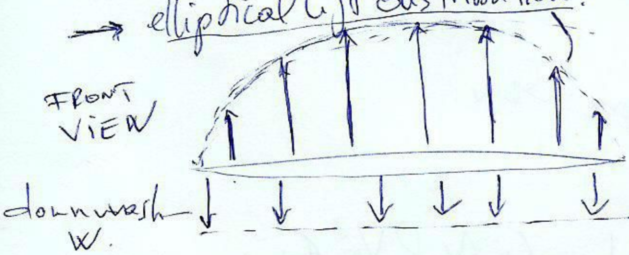


$D_i = L\sin\alpha_i$

Since α small: $\sin\alpha_i \approx \alpha_i$

↳ $\boxed{D_i = L\alpha_i}$

→ elliptical lift distribution:



Elliptical lift distribution results in constant downwash and therefore constant induced angle of attack.

↳ From incompressible flow theory: $\alpha_i = \dfrac{C_L}{\pi A}$ where $A = \dfrac{b^2}{S}$ (Aspect ratio) Thus $C_{D_i} = \dfrac{C_L^2}{\pi A}$

→ Span efficiency factor:

→ $C_{D_i} = \dfrac{C_L^2}{\pi A e}$ ← span efficiency factor (Oswald factor)

↳ Elliptical loading : $e = 1$ ⟹ minimum induced drag.

↳ Non-elliptical loading: $e < 1$ ⟹ higher induced drag.

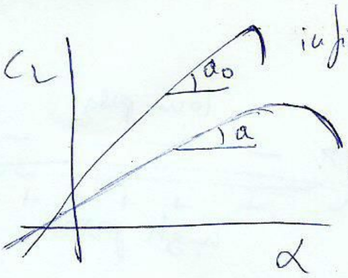
→ The total drag of the wing can now be written as: $C_D = \underbrace{C_{D_P}}_{\text{profile drag}} + \underbrace{\dfrac{C_L^2}{\pi A e}}_{\text{induced drag}}$

→ Lift curve slope:

→ The induced angle of attack reduces the local effective angle of attack: $\alpha_{eff} = \alpha - \alpha_i$

→ For a wing with a general plan form we may write:

$\alpha_i = \dfrac{C_L}{\pi A e_1}$: Angle in radians (span eff factor $\neq e$) | $\alpha_i = \dfrac{57.3\,C_L}{\pi A e_1}$ for angles in degrees.



infinite wing (effective angle of attack) $a_0 = \frac{dC_\ell}{d\alpha}$

finite wing (geometric angle of attack) $a = \frac{dC_L}{d\alpha}$

→ From thin airfoil theory : $\boxed{a_0 = 2\pi}$

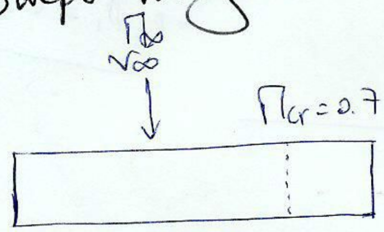
$$\rightarrow \frac{dC_L}{d\alpha_{eff}} = a_0 \Rightarrow C_L = a_0(\alpha - \alpha_i) + const = a_0\left(\alpha - \frac{C_L}{\pi A e_1}\right) + const$$

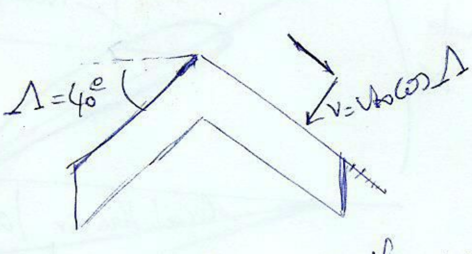
$$\hookrightarrow C_L = \frac{a_0 \alpha}{1 + \frac{a_0}{\pi A e_1}} + \frac{const}{1 + \frac{a_0}{\pi A e_1}} \Rightarrow \frac{dC_L}{d\alpha} = \frac{a_0}{1 + \frac{a_0}{\pi A e_1}}$$

→ Example :

- Infinite wing : $A = \infty$ then $\frac{dC_L}{d\alpha} = 2\pi$
- Finite wing : $A = 12$ (Fokker 50) : $\frac{dC_L}{d\alpha} = 2\pi \times 0.857 = 1.71\pi$

→ Swept wings :





→ M_{cr} for swept wings $= \frac{0.7}{\cos \Lambda} = 0.91$ | by sweeping the wings of subsonic aircraft, the drag divergence is delayed to higher mach numbers.

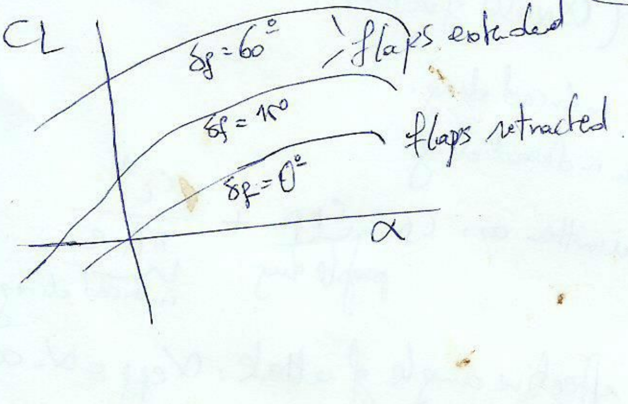
→ Total drag :

→ $C_D = C_{D profile} + \frac{C_L^2}{\pi A e}$ | $C_{D profile} = C_{Df} + C_{D pres} + C_{D w}$

→ Flaps :

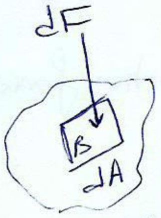
$W = L = C_L \frac{1}{2} \rho V_\infty^2 S$

$\hookrightarrow V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}}$ and $\boxed{V_{stall} = \sqrt{\frac{2W}{\rho_\infty S C_{Lmax}}}}$ | The landing speed is decreased when the maximum lift coefficient is increased



+ Fundamental quantities +

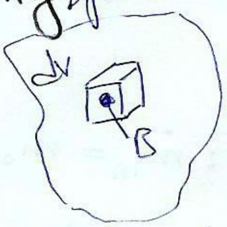
→ **Pressure**: is the normal force per unit area on a surface



dA: element surface around B.
dF: force on 1 side of dA

The pressure in point B is:
$$p = \lim_{dA \to 0} \frac{dF}{dA} \quad [N/m^2]$$

→ **Density** of a substance is it's mass per volume:



dv = element volume around B
dm = mass of gaz in dv

The density in pt B is:
$$\rho = \lim_{dv \to 0} \frac{dm}{dV} \quad [Kg/m^3]$$

→ **Temperature** is a measure of the average kinetic energy of the particles in the gaz.

→ $K_E = \frac{3}{2} kT$; k = Boltzmann constant (1.38×10^{-23} J/K) ; [T] = K | °C ; K = 273.15 + C

→ γ p, ρ, T : equation of state:

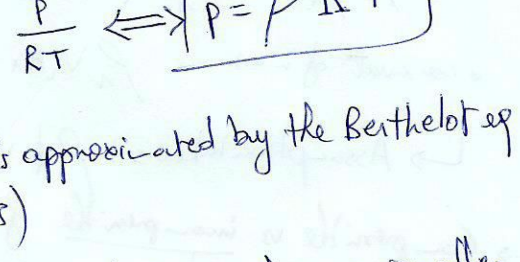
→ A perfect gaz is a gaz in which intermolecular forces are negligible:

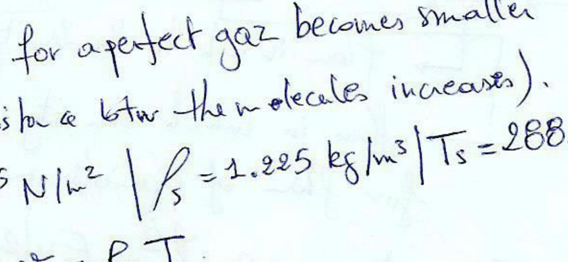
- The equation of state for a perfect gaz is: $\rho = \frac{p}{RT} \iff \boxed{p = \rho R T}$
  with gaz constant $R = 287.05$ J/kg.K
- For an actual gaz (non-perfect) the eq of state is approximated by the Berthelot eq:
  $$\frac{p}{\rho R T} = 1 + \frac{ap}{T} + \frac{bp}{T^3} \quad (a, b \text{ constants})$$
  ↳ The difference with the equation of state for a perfect gaz becomes smaller as p decreases or T increases (The dist. btw the molecules increases).
- Under standard condition: $p_s = 1.01325 \times 10^5$ N/m² | $\rho_s = 1.225$ kg/m³ | $T_s = 288.15$ K
- Definition of specific volume: $v = \frac{1}{\rho}$ and $pv = RT$
- Kg/m² = 1.01325×10^5 x atm.

→ **Velocity & streamlines**:

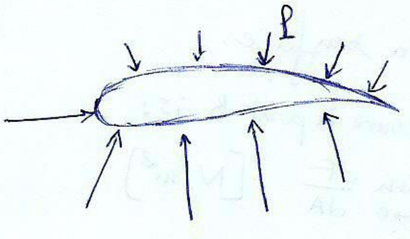
→ The velocity in pt B is the velocity of an infinitesimally small fluid element as it sweeps through B

$$\vec{V} = \frac{ds}{dt} \quad [m/s]$$





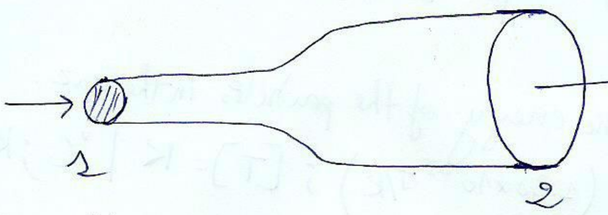
→ Aerodynamic forces:



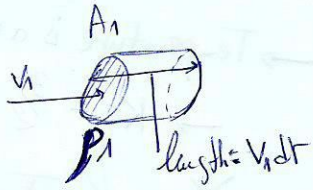
+ Pressure distribution.

+ Shear stress or friction force.

→ Continuity equat° : Time rate of change of the mass of a material region is zero. (mass is conserved)



| For 1-directional incompressible flow : $\dot{m}_{in} = \dot{m}_{out}$



• $\dot{m}$ = flow mass = $V_1 A_1 \rho_1 = \frac{m}{s} \cdot m^2 \cdot \frac{Kg}{m^3} = \frac{Kg}{s}$

• $m_1 = \rho_1 \cdot V = \rho_1 \cdot (A_1 \cdot V_1 dt) = \rho A V dt \Rightarrow \frac{dm}{dt} = \rho A V$

• conservat° of mass : $\rho_1 V_1 A_1 \cancel{dt} = \rho_2 A_2 V_2 \cancel{dt} \Rightarrow \boxed{\rho A V \text{ constant} = \dot{m}}$

↳ Assumption taken: ρ & V uniformly distributed over A, or ρ & V are mean values of.

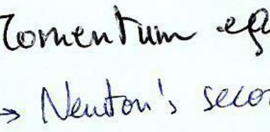
→ Compressible vs incompressible flow:

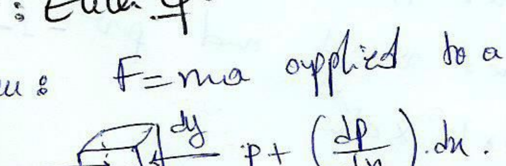
↳ flow in which the density of a fluid element can change from pt to pt.

↳ flow in which density changes of fluid elements are negligible. Assumption valid for flow of liquids and low speed air flow.

→ Momentum equat° : Euler eq.

→ Newton's second law: F = ma applied to a flowing gas:





→ In reality 3 forces act on this element: pressure | Friction | gravity.

↳ In the derivation we neglect 2 forces: gravity (small) | viscosity (no friction forces).

→ The force in the x-dir: $F = p\,dy\,dz - \left(p + \frac{dp}{dx} dx\right) dy\,dz = \boxed{-\frac{dp}{dx}\,dx\,dy\,dz}$ Force on fluid elmt due to pressure

→ The mass of the fluid element is: $m = \rho \cdot Vol = \rho\,dx\,dy\,dz$

→ Acceleration a of the fluid element is: $a = \frac{dV}{dt} = \frac{dV}{dx} \cdot \frac{dx}{dt} = \frac{dV}{dx} \cdot V$

↳ $F = ma \Leftrightarrow -\frac{dp}{\cancel{dx}} \cancel{dx\,dy\,dz} = \rho \cdot \cancel{dx\,dy\,dz} \cdot V \cdot \frac{dV}{\cancel{dx}} \Rightarrow \boxed{dp = -\rho V dV}$

Euler Equation: differential eq.

↳ Assumptions: Gravity neglected | Viscosity neglected | steady flow | Flow may be compressible |

→ Energy eq. for frictionless adiabatic flow, derived from 1st law of thermodynamics:

→ physical principal: Energy can neither be created or destroyed

→ $\delta q = dh - v\,dp \iff \underset{\text{Adiabatic flow}}{\delta q = 0} = dh - v\,dp \iff dh = v\,dp \iff \overset{\text{Euler}}{dh = -v\rho V dV}$

↳ since $v = \frac{1}{\rho}$ we find: $dh + VdV = 0$

↳ we integrate btw 2 pts in a stream line: $\int_1^2 dh + \int_1^2 VdV = 0$

↳ $h_2 - h_1 + \frac{1}{2}(V_2^2 - V_1^2) = 0 \iff h_2 + \frac{1}{2}V_2^2 = h_1 + \frac{1}{2}V_1^2 \Rightarrow \boxed{h + \frac{1}{2}V^2 = cte}$

↳ since $h = C_pT \Rightarrow \boxed{C_pT + \frac{1}{2}V^2 = cte}$

→ Summary of equations:

→ For steady, frictionless, incompressible flow:

+ $A_1V_1 = A_2V_2$ Continuity eq.

+ $p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$ bernoulli's eq.

→ For steady, isentropic (adiabatic & frictionless flow), compressible flow:

+ $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$ Continuity eq ($= \dot{m}$ (mass flow))

+ $\frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2}\right)^{\gamma} = \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$ Isentropic eqs. (relation)

+ $c_pT_1 + \frac{1}{2}V_1^2 = c_pT_2 + \frac{1}{2}V_2^2$ Energy eq.

+ $p = \rho RT$ Equation of state.

→ Speed of Sound:

$$\left.\begin{matrix} p \\ \rho \\ T \end{matrix}\right\} \begin{matrix} p + dp \\ \rho + d\rho \\ T + dT \end{matrix} \quad \equiv \quad \begin{matrix} p \\ \rho \\ T \\ \Rightarrow a \end{matrix} \left| \begin{matrix} p + dp \\ \rho + d\rho \\ T + dT \\ a + da \end{matrix} \right.$$

velocity of sound → a ⟸

moving sound wave with speed a

static observer

motionless sound wave

observer moving with sound wave.

→ Apply Continuity eq: $\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \iff \rho_1 A_1 a = (\rho + d\rho) A_2 (a + da)$

. 1-dir flow $\iff A_1 = A_2 \Rightarrow \rho a = (\rho + d\rho)(a + da) \Rightarrow \cancel{\rho a} = \cancel{\rho a} + a\,d\rho + \rho\,da + \cancel{d\rho\,da}$ small, ignore

↳ $\boxed{a = -\rho \frac{da}{d\rho}}$ (eq for speed of sound).

→ Now apply momentum eq (Euler eq) $\Rightarrow dp = -\rho V dV \Rightarrow dp = -\rho a\, da \Rightarrow da = -\frac{1}{\rho a} dp$

sub into $a = -\rho \frac{da}{d\rho} \Rightarrow a = \rho \frac{1}{\rho a} \cdot \frac{dp}{d\rho} \Rightarrow a^2 = \frac{dp}{d\rho} \Rightarrow \boxed{a = \sqrt{\left(\frac{dp}{d\rho}\right)_{isentropic}}}$

↳ Now apply the isentropic relations:

We want to rewrite $\frac{dp}{d\rho}$; with isentropic relation eq for γ: $a = \sqrt{\gamma \frac{p}{\rho}}$

with the eq. of state $p = \rho R T \Leftrightarrow \frac{p}{\rho} = RT \Rightarrow \boxed{a = \sqrt{\gamma R T}} \Rightarrow$ speed of sound in a perfect gas depends only on T.

→ Isentropic flow relations, second form.

→ Energy eq: $c_p T_1 + \frac{1}{2} V_1^2 = cte = c_p T_0 + \frac{1}{2} V_0^2$; Assuming index 0 = stagnation pt $\Rightarrow V_0 = 0$

↳ $c_p T_1 + \frac{1}{2} V_1^2 = c_p T_0 \Leftrightarrow \frac{T_0}{T_1} = 1 + \frac{V_1^2}{2 c_p T_1}$

→ $R = c_p - c_v$; $h = e + p v \Leftrightarrow c_p \cancel{T} = c_v \cancel{T} + R \cancel{T} \Leftrightarrow R = c_p - c_v$ $\left[p v = \frac{p}{\rho} = RT \right]$

↳ $\frac{R}{c_p} = 1 - \frac{c_v}{c_p} = 1 - \frac{1}{\gamma} = \frac{\gamma - 1}{\gamma} \Rightarrow c_p = \frac{\gamma R}{\gamma - 1} \xrightarrow{\text{subs.}} \frac{T_0}{T_1} = 1 + \frac{\gamma - 1}{2} \frac{V_1^2}{\gamma R T_1}$

↳ with $a_1^2 = \gamma R T \Rightarrow \frac{T_0}{T_1} = 1 + \frac{\gamma - 1}{2} \cdot \frac{V_1^2}{a_1^2} = \boxed{1 + \frac{\gamma - 1}{2} M_1^2}$

↳ Bring flow isentropically to rest → isentropic relation can be used:

$$\frac{p_0}{p_1} = \left(\frac{\rho_0}{\rho_1}\right)^{\gamma} = \left(\frac{T_0}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma - 1}} \quad \& \quad \boxed{\frac{\rho_0}{\rho_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{1}{\gamma - 1}}}$$

→ Summary of second form - Isentropic flow eqs:

→ $\frac{T_0}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2$

→ $\frac{p_0}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma - 1}}$

→ $\frac{\rho_0}{\rho_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\left(\frac{1}{\gamma - 1}\right)}$

For $M < 0.3$ the change in density is $< 5\%$

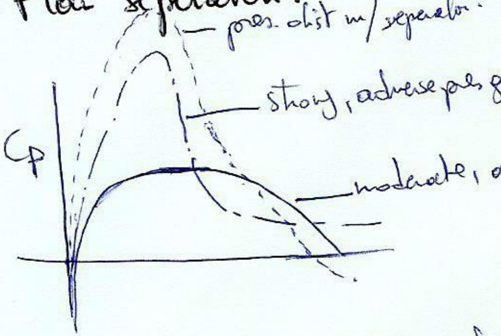
↳ Thus the flow can be treated as incompressible.

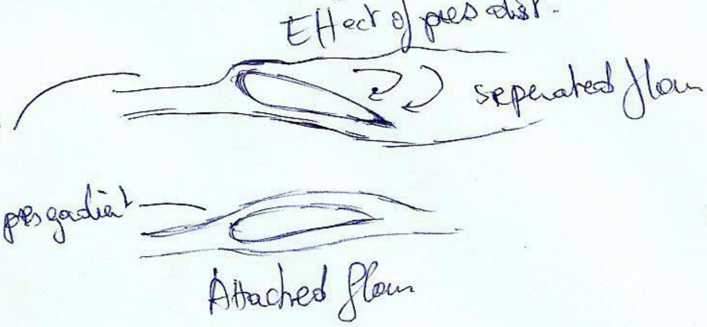
→ Laminar flow favorable:

→ We have seen that: $\tau_{w\,laminar} < \tau_{w\,turbulent}$

→ Vast majority of flows is turbulent ⇒ we may adapt the geometry of the airfoil such that it favors laminar flow. We then have laminar flow airfoils.

→ Flow separation:





→ Why is it important:
- Loss of lift (airfoil)
- Increase in pres drag.
- Generation of unsteady loads.

→ Effect of turbulence: Turbulent boundary layer has more flow kinetic energy near the surface. Thus flow separation may be postponed.

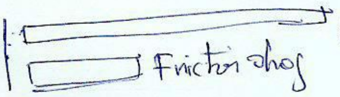
→ Viscous drag:

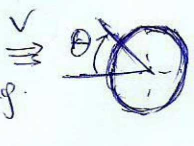
→ Drag due to viscous effects = friction drag + pressure drag = profile drag

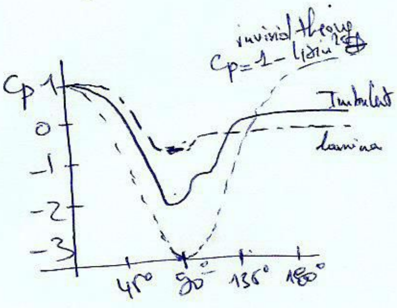
↳ $C_{D_p} = C_{D\,pressure} + C_{D\,friction}$

→ pressure drag:

→ Drag dist on cylinder:







→ A smooth sphere is laminar and a rough sphere is turbulent and it gives a lot less pressure drag.

+ Aerodynamics : from subsonic to hypersonic +

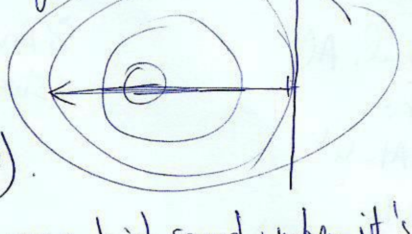
- Regimes of aerodynamic flow :
  - subsonic : $M < 1$
  - sonic : $M = 1$
  - supersonic : $M > 1$

or

subsonic	$M < 0{,}8$
transsonic	$0{,}8 < M < 1.2$
supersonic	$1.2 < M < 5$
hypersonic	$M > 5$

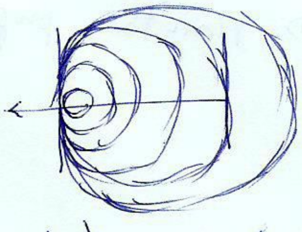
↳ M (the mach number) $= V/a \longrightarrow$ the speed of sound.

→ Subsonic :



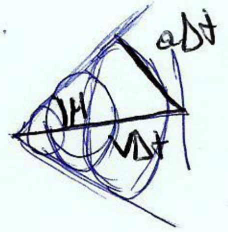
- Sound moves faster than object : $a > V$
- No coalescence ~~effect~~ of waves (sound of pressure).
- Doppler effect : when something comes to you, you hear a high sound, when it's passed you you hear a low sound.

→ Sonic :



- $a = V$
- Coalescence of waves : shock wave (pressure step) "barrier"

→ Supersonic : $V > a$ | Coalescence of waves : shock wave (pressure step).

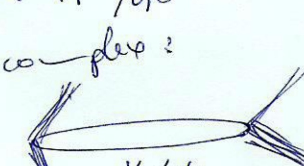


↳ A shockwave is a pressure difference (that bang!)

↳ μ (mach angle) $\Rightarrow \sin\mu = a/V = 1/M$

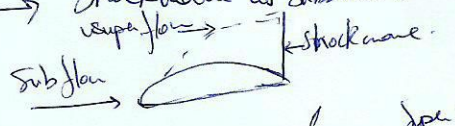
→ Air compressibility :
- Incompressible (at very low speeds) until $M = 0{,}3$
- Compressible : $M > 0{,}3$
- Supersonic complex :

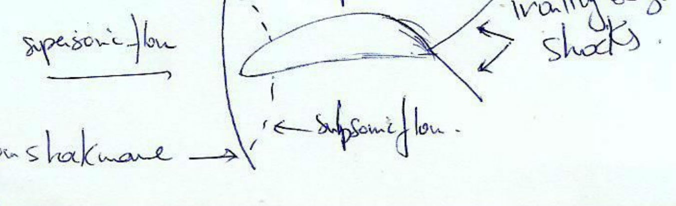
V_1
M_1
P_1
T_1



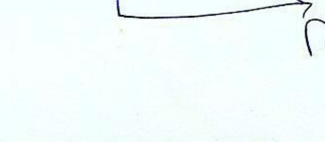
$V_2 < V_1$
$M_2 < M_1$
$P_2 > P_1$
$T_2 > T_1$

→ Shockwave at subsonic :



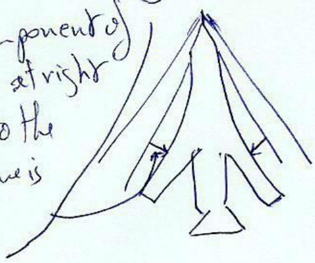


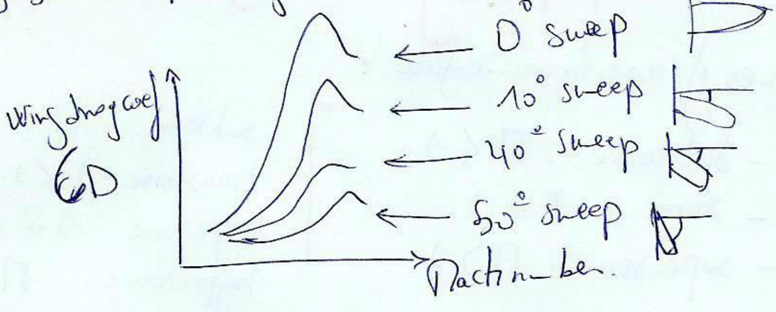
Shockwave induces : - reduction in lift
- increase in drag (wave drag)



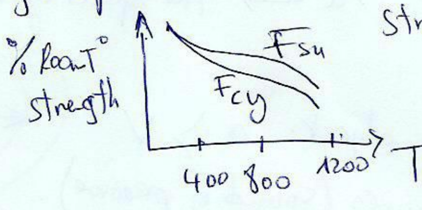
→ Reducing drag in supersonic flight : swept wings.

The component of the flow at right angles to the shockwave is subsonic





→ Special materials for high speeds.



Strength of T° exposure up to 1/2 hr.

· Conventional: AC.
· HT° mat:
+ Ti-6Al-4V
+ RVS 316
+ Inconel (Alloy).

↳ Maintain properties at high T°.

+ Helicopters & other vehicules +

→ **Principles of flight :**

- Three ways to fly :
  - + Floating by being lighter than air (Ballons, Airships)
  - + Push air downwards (Airplanes, Heli, VTOL, STOVL)
  - + Push something else downwards (Rockets, Jetpack)

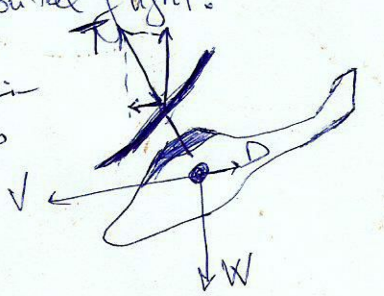
→ **Helicopters:**

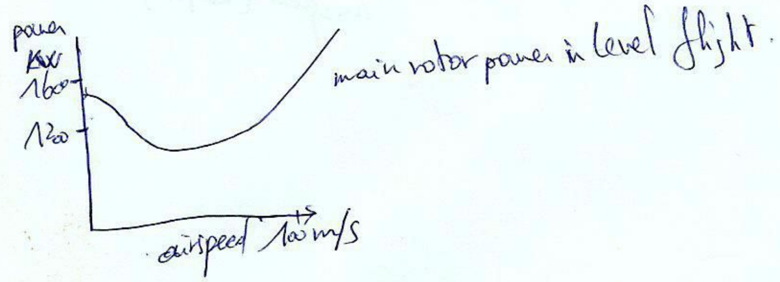
- A helicopter is nothing more than a large collection of parts flying in close formation held together with grease and copper wire.
- Helicopters are : – Creating more noise (internal & external)" – less fuel efficient – less environmently friendly – More expensive to buy & operate – Harder to fly. – less confortable for passengers..

But can : + Land & take off anywhere + Hover.

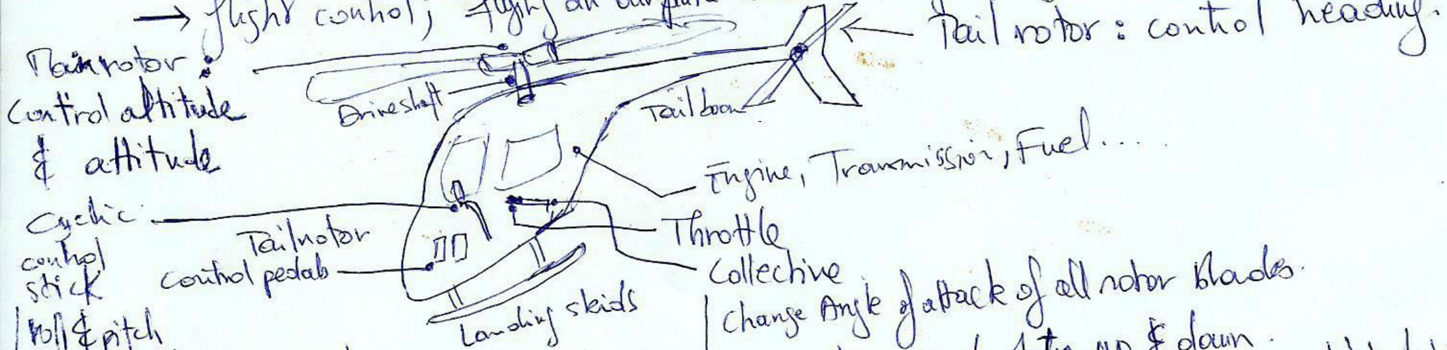
→ Horizontal flight :

- equilibrium of forces

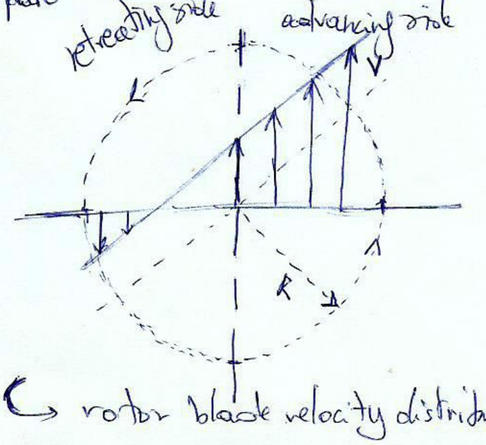




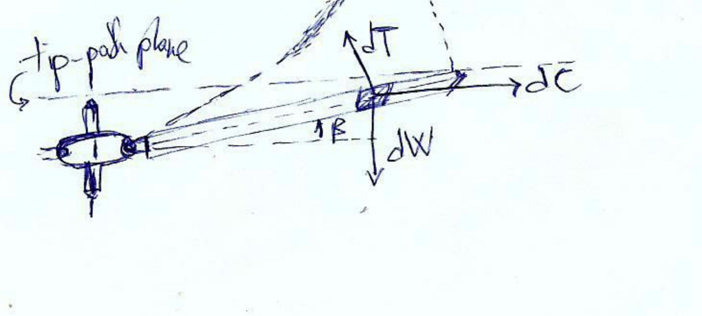
→ flight control; flying an airplane is like riding a bicycle, hovering a heli is like riding a unicycle.



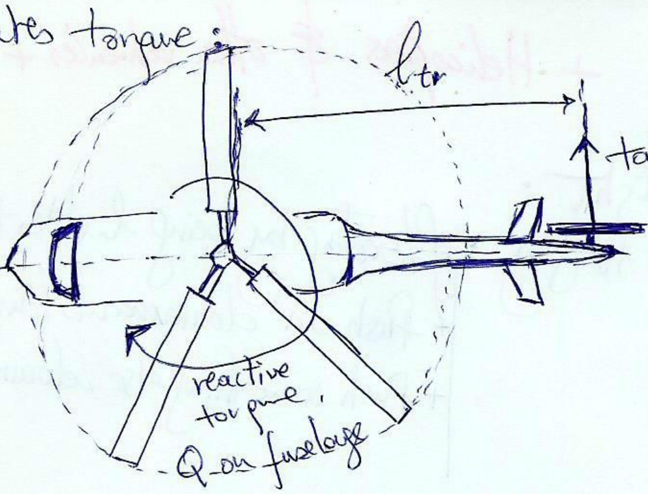
Change Angle of attack of all rotor blades.
move entire swash plate up & down.



↳ rotor blade velocity distribution.

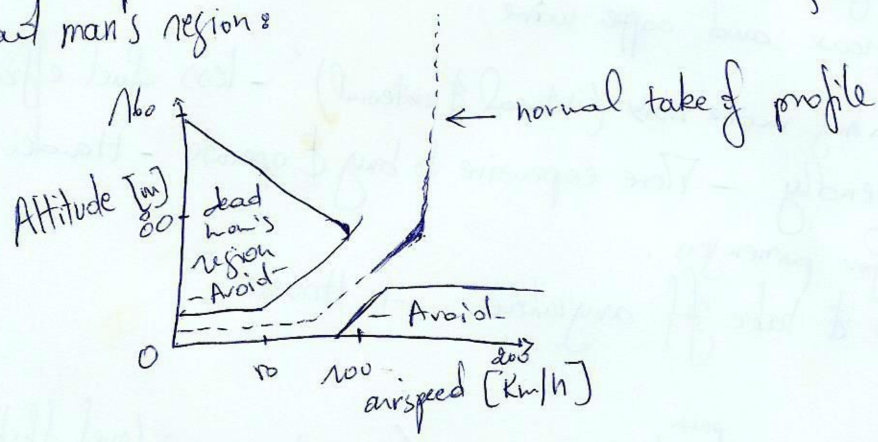


→ Tail rotor compensates torque:



→ Alternatives for tail rotor: + Jet + two rotor in opposite direction (to counter the torque effect) (Kamov principle).

→ Dead man's region:



+ Laminar & turbulent flow +

→ Supersonic wind tunel : area velocity relation.

→ Area velocity relation : $\frac{dA}{A} = (M^2 - 1)\frac{dV}{V}$

↳ Conc:
- subsonic flow if $dV > 0$ then $dA < 0$ and vice versa
- supersonic flow if $dV > 0$ then $dA > 0$ and vice versa.
- if the flow is sonic ($M = 1$) then:

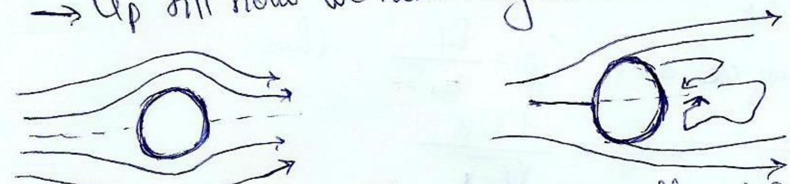
↳ $\frac{dV}{V} = \left(\frac{1}{M^2-1}\right)\frac{dA}{A} = \frac{1}{0}\frac{dA}{A}$! $\Rightarrow \frac{dV}{V} = \infty$; not possible at physical basis.

then we must have (for finite $\frac{dV}{V}$): $\frac{dA}{A} = 0 \Rightarrow \frac{dV}{V} = \frac{0}{0}$ = finite !

↳ if $\frac{dA}{A} = 0 \Rightarrow$ in the throat $M = 1$.

→ Viscous flow :

→ Up till now we have only dealt with frictionless flow, what's the effect of frict°?

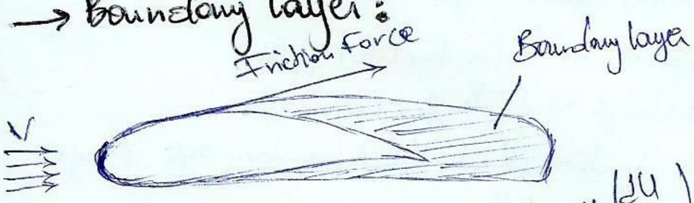


Inviscid flow (No frict°) No drag

Viscous flow (friction) Finite drag.

in real life the flow at the surface adheres to the surface because of friction between the gas and the solid material : Right at the surface the velocity is zero.

→ Boundary layer :



- In the vicinity of the surface there is a thin region of retarded flow : the boundary layer.
- The pressure through the boundary layer in a direction perpendicular to the surface is constant.
- Inside the boundary layer we cannot apply bernoulli's (isentropic flow)

→ Shear stress can be written as: $\tau_w = \mu \left(\frac{dU}{dy}\right)_{y=0}$

↳ Skin friction drag.

↳ μ = absolute viscosity coef or viscosity

Air at standard sea level : $\mu = 1.789 \times 10^{-5}$ [kg/ms]

→ Some definitions :

→ Reynolds number: $Re_x = \frac{\rho_\infty V_\infty \cdot x}{\mu_\infty} = \frac{V_\infty x}{\nu_\infty}$ (varies linearly with x)

→ laminar flow: streamlines are smooth and regular and a fluid element moves smoothly along a streamline

→ Turbulent flow : streamlines breakup and a fluid element moves in a random irregular way.

→ laminar Boundary layer , boundary layer thickness :

→ Consider flat plate flow. What is boundary layer thickness δ and skin friction drag D_f at x?

↳ From laminar boundary ~~layer~~ theory: $\delta = \frac{5.2x}{\sqrt{Re_x}}$;

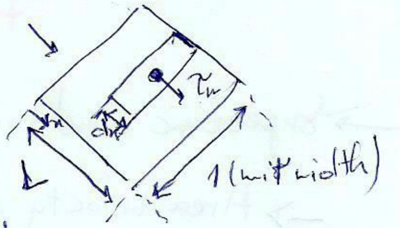
↳ Thus δ is proportional to : $\sqrt{x}$ (parabolically)

→ Laminar Boundary layer, skin friction drag:

→ Total force = total pressure force + total friction force.

↳ total friction force on element dx is: $\tau_w(x)\,dx \cdot 1 = \tau_w(x)\,dx$

↳ Total skin friction drag is: $D_f = \int_0^L \tau_w\,dx$



→ For the skin friction coefficient we find from laminar boundary layer theory:

$$C_{f_x} = \frac{\tau_w}{\frac{1}{2}\rho_\infty V_\infty^2} = \frac{\tau_w}{q_\infty} = \frac{0{,}664}{\sqrt{Re_x}} \quad \text{[dimensionless]}$$

↳ Thus C_{f_x} and τ_w decrease as $\sqrt{x}$ increases ⟺ the skin friction at the begining of the plate is larger than near the trailing edge.

→ To calculate the total aerodynamic force we must integrate.

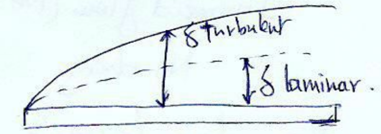
↳ $D_f = \int_0^L C_{f_x} \cdot q_\infty \cdot dx = 0{,}664\, q_\infty \int_0^L \frac{dx}{\sqrt{Re_x}} = \frac{0{,}664\, q_\infty}{\sqrt{V_\infty/\nu}} \int_0^L \frac{1}{\sqrt{x}}\,dx$ $\left(\text{or } \int \frac{dx}{\sqrt{x}} = 2\sqrt{x}\right)$

$$= \frac{0{,}664\, q_\infty}{\sqrt{V_\infty/\nu}} \times 2\sqrt{L} = \frac{1.328\, q_\infty L}{\sqrt{V_\infty/\nu}}$$

↳ We define total skin friction drag coef as: $C_f = \frac{D_f}{q_\infty S}$; thus:

↳ $C_f = \frac{1.328 \cdot L}{\sqrt{Re_L} \cdot S} = \frac{1.328 \cdot L}{\sqrt{Re_L}\; L \cdot 1} \Rightarrow C_f = \frac{1.328}{\sqrt{Re_L}}$

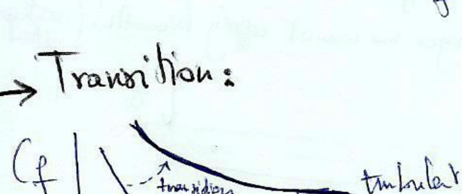
→ Results for a turbulent boundary layer:

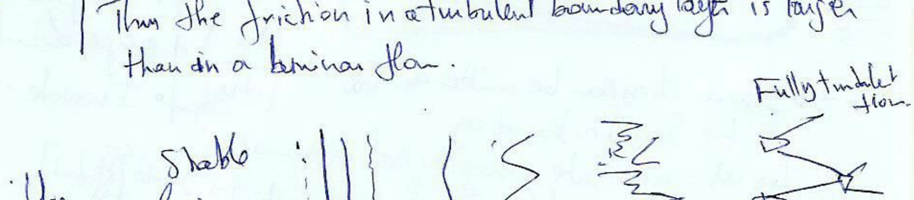


→ Due to the action of turbulence, no exact sol$^{\underline{n}}$ for turbulent boundary layers.

→ From experiments: $\delta = \frac{0.37x}{Re_x^{0{,}2}}$, $C_f = \frac{0.074}{Re_L^{0.2}}$ | C_f varies as $L^{-1/5}$ for turbulent flow while it changes as $L^{-1/2}$ for laminar flow. Thus the friction in a turbulent boundary layer is larger than in a laminar flow.

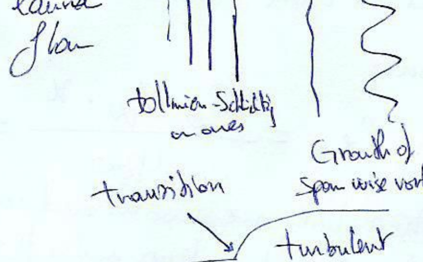
→ Transition:

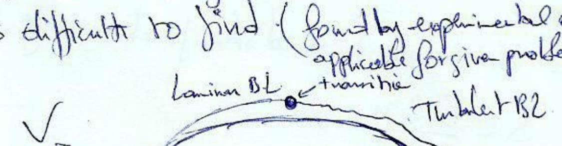




→ Skin friction: $\tau = \mu \frac{\partial u}{\partial y}$

→ The critical Reynolds number at which transition occurs is difficult to find (found by experimental data applicable for given problem).

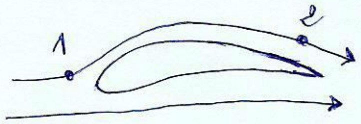




| Laminar BL: thin, low friction drag.
turbulent BL: thick, high friction drag.

→ Bernoulli's law for inviscid & incompressible flow (gravity neglected)

- Integrating Euler eq $(dp + \rho V dV = 0)$ along streamline btw pt 1 and pt 2:



$$\int_{P_1}^{P_2} dp + \int_{V_1}^{V_2} \rho V dV = 0 \Leftrightarrow \left[P\right]_{P_1}^{P_2} + \left[\rho \frac{V^2}{2}\right]_{V_1}^{V_2} = 0$$

$$\Leftrightarrow P_2 - P_1 + \rho \frac{V_2^2}{2} - \rho \frac{V_1^2}{2} = 0 \Leftrightarrow P_2 + \rho \frac{V_2^2}{2} = P_1 + \rho \frac{V_1^2}{2}$$

$$\Longrightarrow \underbrace{P}_{\text{static pressure}} + \underbrace{\tfrac{1}{2} \rho V^2}_{\text{Dynamic pressure}} = \underbrace{\text{constant}}_{\text{total pressure } (P_t)} \text{ along streamline.} \quad [N/m^2] \quad (\text{Bernoulli's eq})$$

→ Euler eq and Bernoulli's eq is in fact $F = ma$ (Newton) applied to fluid dynamics.

+ Materials, Supersonic & high speed env +

→ What is a material: | • Approximated by: "substances" and "Matter"
| • Having specific properties, but without shape

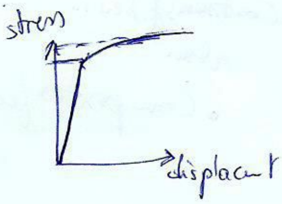
→ Structures & materials:
- Structures are made of these materials or "Substances":

+ Materials → Semi-finished parts → Structural element → Structure
sheet / profile / fibers — Stringer / Beam, Sandwich

→ Materials OVERVIEW:
- Most important materials for Aerospace applications | Metal Alloys
  Composites: Composed materials (fibers, resin) Metal
- Structurally not relevant: | Pure polymers: properties not good enough (strength, stiffness...)
  Ceramics: too brittle (desintegrates easily)

→ Metals:
+ Ore $\xrightarrow{\text{retrieve, purify, Alloy}}$ Semi-finished $\xrightarrow[\text{process}]{\text{Manufacturing}}$ final part | Manufacturing processes:
Fe: Blast furnaces
Al: Electrolysis.

→ Metals & Metal Alloys:
+ Characteristics:
- Isotropic: the material properties are independent from the direction
- Metal to be strengthened (Alloying, heat treatment)
- Plastic behavior & Melting (recycling, welding)
- Good processibility
- low cost.



+ Huge diversity in <u>tension</u> properties: init slope strength = init resistance to bending. = ↙ for stiffness measuring

Metal	Density [Kg/dm³]	E-modulus [KN/mm²]	yield strength [N/mm²]	Failure strength [N/mm²]	Max strain [%]
Carbon Steel	7.8	207 26,5	375 48	590 76	28
HS steel	7.8	207 26,5	1620 208	1760 226	12
pure aluminium	2.7	69 25,5	34 13	90 33	40
Al-2024-alloy	2.8	72 25,7	325 116	470 168	20
pure titanium	4.5	103 22,9	170 38	240 53	30
Ti-6Al-4V alloy	4.5	114 25,3	830 184	900 200	14

[in black at table]

+ Stress = $\frac{\text{Force}}{\text{cross section}}$ | specific stress = $\frac{\text{stress}}{\text{density}}$ (For tension <u>only</u>)
| We use specific stress because we have to take density into consideration for flight

→ Polymers:

+ As pure materials: Structurally not interesting

+ Macro-molecular substances: Two major types:
- Thermoplastics: softening → reversible, one component.
- Thermoset: curing irreversible, often more components.
Once cured, shape can't be changed anymore.

+ Characteristics:
- Isotropic
- low strength & stiffness.
- Huge variety.
- Plastic flow & Melting (recycle, welding)
- Good processibility.
- low costs (often).

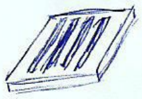
→ Composites:
+ Fiber reinforced polymers:
- polymers + fibers
- Fibers: glass, carbon, aramid, Dyneema.
- Short, long, "continuous" fibers.

+ Hybrid materials:
- Glare: composite and metal layers.

+ Have different weave styles.

∘ Principles of composite materials ("continuous" fibers):

+ fibers (strong & stiff) embedded in resin (support & protect)
↳ in one direction only ⟹ anisotropic (direction dependent) behavior.

Resin →  || 

continuous fibers in resin.

Layers (laminate)

∘ Composites (cont.):
- Anisotropic: lower weight, no overdesign
- Layered structure (laminate).
- High strength and stiffness.
- Low density but often costly.
- No plasticity
- Good processibility
∘ prepregs
∘ Draping in moulds, curing.

∘ Huge diversity in tension properties.

Material	Density [kg/dm³]	E-modulus [kN/mm²]	yield strength	Failure strength	Maximum strain
Epoxy (TS)	1.25	2.4 1.9	— —	48 60	4.5
PEEK (TP)	1.31	1.1 0.8	91 69	100 76	75
E glass epoxy UD 60%	2.10	45 21	— —	1020 486	2.3
HM carbon epoxy UD 60%	1,7	220 129	— —	760 447	0.3
Al-2024-Alloy	2.8	72 25.7	325 116	470 168	20
Ti-6Al-4V alloy	4,5	114 25.3	830 184	900 200	14

→ Link between Materials & structures:

• Load carrying capacity of a structure depends on:
+ Design, shape
+ Materials
+ Production techniques

↳ Not every (D, M, P) random combination is possible.
↳ There's an interaction

performance (concept)

+ Manufacturing techniques:
∘ Casting | Machining | Forming | Forging | Joining

+ Design | shape:
∘ Struct level | part level
Tubes & truss struct | flat
stiffened skin stru. | single curved
Sandwich struct | double curved

+ Materials: metal alloys | polymers | Fiber reinforced polymers | ceramics | hybrid material.

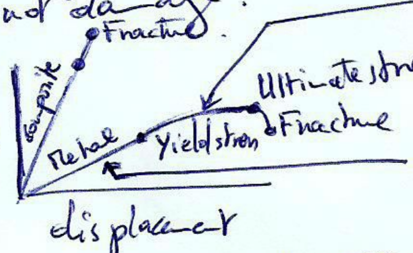
→ loads :

- Limit load : load experienced once in a lifetime ; no remaining damage allowed
- Ultimate load : limit load x safety factor ; failure allowed after 3 seconds

→ Failure behavior materials :

- Within the limits of load diagram , material should not fail.
  - metal should not yield.
  - Composite should not damage.

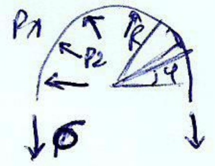
- Stress - strain diagram : Force



plastic zone : permanent deformation

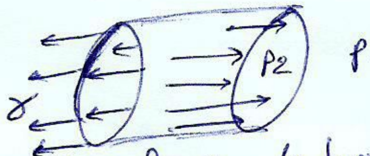
elastic zone : material return to init shape.

→ Pressure Cabin :



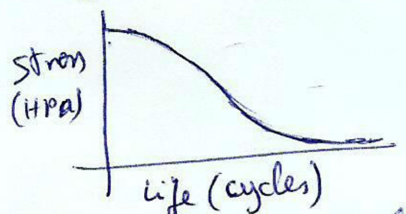
- $\Delta p = P_2 - P_1$; $\sigma_{cir} = \Delta p \cdot R \cdot \frac{1}{t}$

- $\sigma_{cir} / \sigma_{long} = 2$ / $\sigma_{long} = \Delta p \cdot R / 2t$



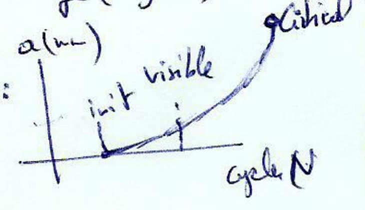
→ Fatigue : Repetitive force smaller than breaking force destroys the part.

- SN curve :



- the more cycles, the poorer the material the less force it can take because of fatigue

- Crack growth curve :



- fatigue : crack initiation / crack growth.
- Two limits : visibility limit (detection) / criticality limit (failure).
- inspection intervals : 3 times btw visible and critical.

→ Metal - Manufacturing: | • Liquid : casting
| • Solid high T° : forging
| • Solid Room T° : forming (sheet); maching.
| • Assembly : Joining.

↳ Casting : Solid ⇒ liquid + injection molding.
↳ Maching : Solid + Cutting.
↳ Forming : Sheet + stress.
↳ Joining : solids + welding...

prepreg = pre-impregnated.

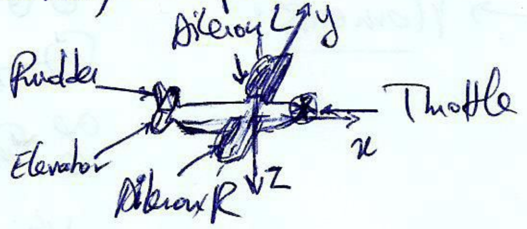
→ Composites : | lay-up and curing.
| forming : press forming (with heated material to soften it) for thermoplastic mat.
| filament winding / tape laying.

→ Composites vs metals :

Metals	Composites
+ Plastic behavior / damage tolerant / - Joining.	+ High spec. strength & stiffness / low weight.
+ Cheap mat. / easy proc / - Labor intensive	+ High integration possible / expensive mat → compensated by production.

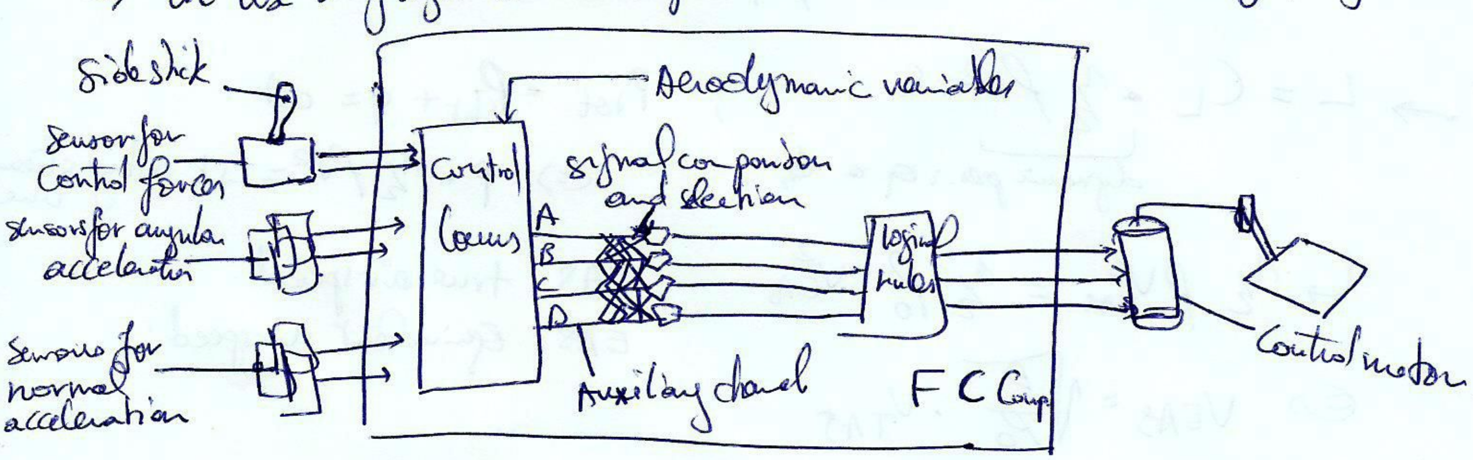
→ In total, there 4 degrees of freedom (control) in a plane:

- Elevator: for pitch.
- Rudder: for yaw
- Ailerons: for roll
- Throttle: for forward acceleration

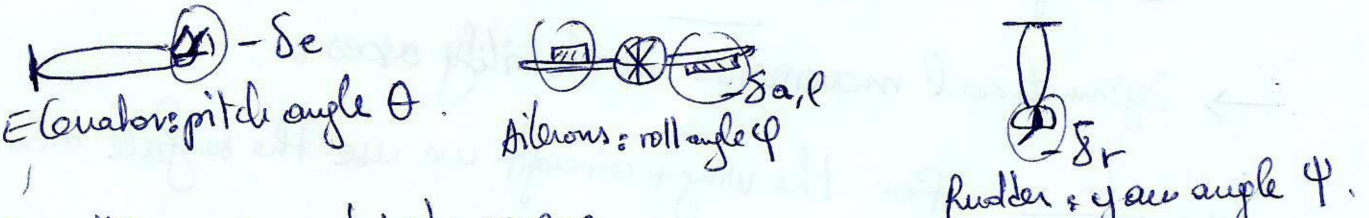


→ the aircraft can move in 3 directions and has 3 angles around the 3 axes ⟹ with 4 controls you can pilot an aircraft with 6 degrees of freedom.

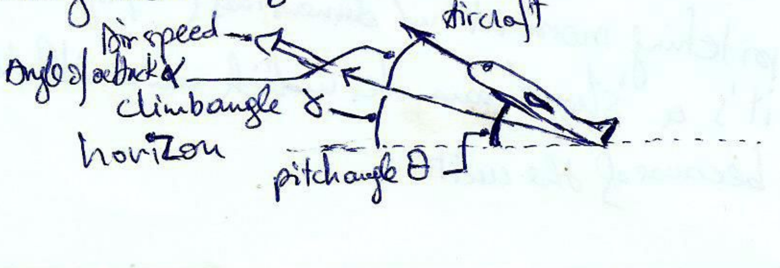
→ We use a flight control system to maintain stability (by wire (elec)).



→ Control surfaces and rotations:
↳ sign convention: negative deflections → positive a/c response around it's primary axis.



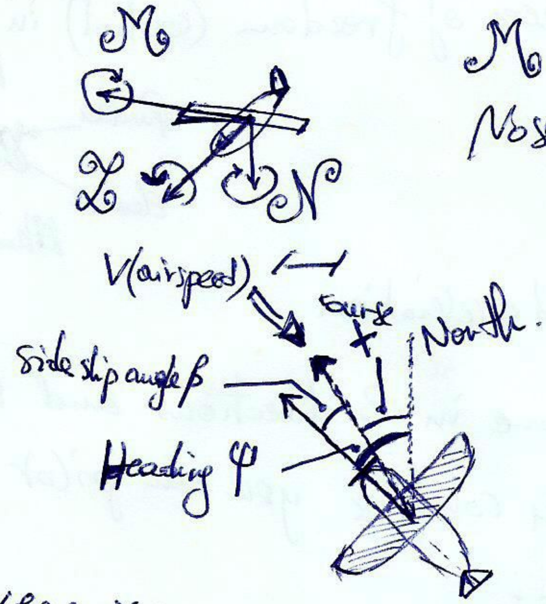
→ Stability axes and body axes:
↳ Stability: x_s-axis is attached to velocity.
↳ Body axes: x_b-axis is fixed to aircraft.



$\models$ ⟹ $\alpha + \gamma = \theta$

- the pitch angle indicates the pitch attitude of the airfoil.
- the angle of attack: the angle btw the airspeed and the direction of airfoil.

→ Moments:



M: pitching moment.
Nose up = positive.

→ Geodetic axes: x_g-axis is attached to North and Horizon

→ $L = C_L \cdot \underbrace{\frac{1}{2} \rho V^2}_{\text{dynamic pres.}: q} \cdot S$; $P_{tot} = P_{stat} + q = cst$.

$\Leftrightarrow p + \frac{1}{2}\rho V^2 = cst$ along stream line

↳ $\frac{1}{2} \rho V_{TAS}^2 = \frac{1}{2} \rho_0 V_{EAS}^2$; TAS: true airspeed. EAS: Equivalent airspeed.

$\Leftrightarrow V_{EAS} = \sqrt{\frac{\rho}{\rho_0}} \cdot V_{TAS}$

- Indicated airspeed (in a plane) = EAS ⇔ (IAS = EAS).

→ C_L captures a shape that's why it doesn't have any units.

→ Symmetrical movements in stability axes:

$C_L = \frac{L}{\frac{1}{2}\rho V^2 S}$: For the wing + aircraft we use the surface area of the wing S.

$C_{L_H} = \frac{L_H}{\frac{1}{2}\rho V^2 S_H}$: For the tail we use the surface of the tail: S_H.

$C_m = \frac{M}{\frac{1}{2}\rho V^2 S \bar{c}}$: it's a 'shape' parameter which varies with the angle of attack

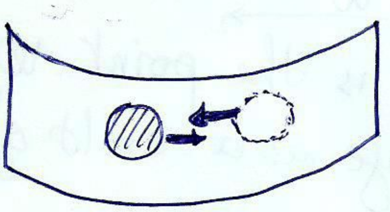
M ← pitching moment w/ dimensions (w/ influence of ρ, V, S)

$\bar{c}$ ← because of the unit.

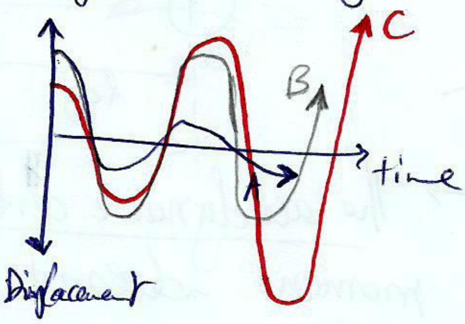
chord

→ Static stability :

- the product of the two forces
  - positive ⇒ unstable
  - negative ⇒ stable



→ Dynamic stability :



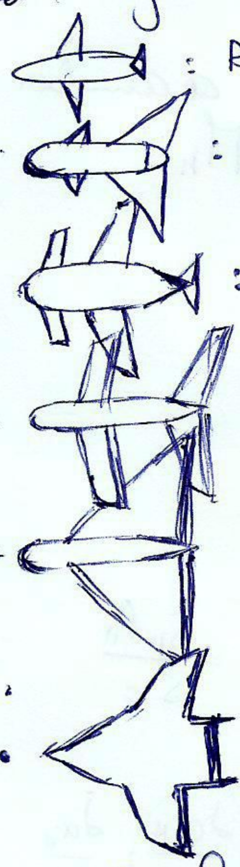
A : Damped Oscillation
↳ Positive static / positive dynamic

B : Undamped oscillation
↳ Positive static / neutral Dyn.

C : Divergent oscillation
↳ Positive static / Negative Dyn.

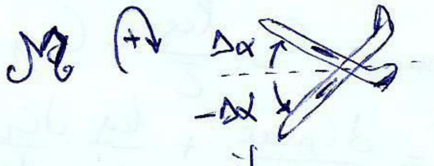
→ Harder to judge than static stability.

→ Tail-wing configuration :



- : Aft tail
- : tail first
- : Three surfaces.
- : tandem wing.
- : tailless.
- : flying wing

→ longitudinal static stability :

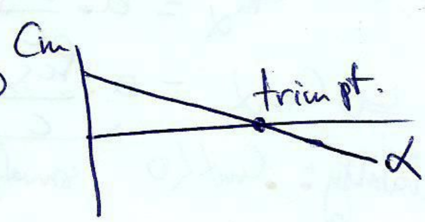


if $\Delta\alpha > 0 \Rightarrow \Delta C_m < 0$

if $\Delta\alpha < 0 \Rightarrow \Delta C_m > 0$

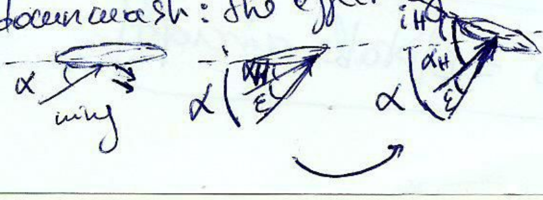
$\Bigg\} \Rightarrow \frac{\Delta C_m}{\Delta\alpha} < 0$ if the plane is long. stat. stable.

$\hookrightarrow \boxed{\frac{dC_m}{d\alpha} < 0} \quad \left(\frac{dC_m}{d\alpha} = C_{m_\alpha}\right) \; (=)$



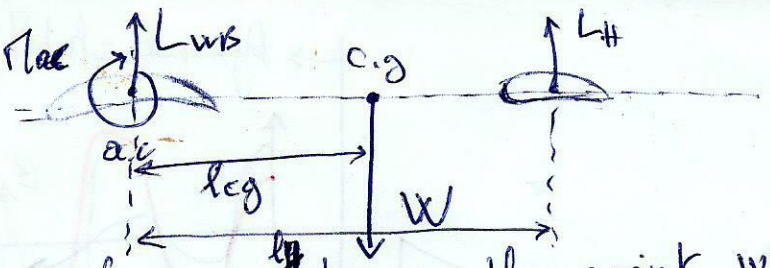
→ For the tail and downwash consideration : (...)

- downwash : the effect of the wing and air flow direction at the tail.



$\Rightarrow \alpha_H = \alpha - \varepsilon + i_H \Rightarrow \left(\frac{d\alpha_H}{d\alpha} = 1 - \frac{d\varepsilon}{d\alpha}\right) \quad \left(\frac{di_H}{d\alpha} = 0\right)$

(7)



→ The <u>aerodynamic center</u> is the point where the aerodynamic moment doesn't change as a result of change of angle of attack.

$$\Leftrightarrow \frac{dM_{ac}}{d\alpha} = 0$$

→ Equilibrium : $W = L_{WB} + L_H = L$.

→ The moment of an air foil happens when there's a camber in it. So in a symetrical air foil $\frac{dM_{ac}}{d\alpha} = 0$. there's no M_H.

$$\rightarrow M_{tot} = M_{ac} + L_{WB} \cdot l_{cg} - L_H (l_H - l_{cg}).$$

$$= M_{ac} + L_{WB}\, l_{cg} - L_H\, l_H + L_H \cdot l_{cg}$$

$$= M_{ac} + L \cdot l_{cg} - L_H\, l_H$$

$$\hookrightarrow \frac{M_{tot}}{\frac{1}{2}\rho v^2 S\bar{c}} = \frac{M_{ac}}{\frac{1}{2}\rho v^2 S\bar{c}} + \frac{L \cdot l_{cg}}{\frac{1}{2}\rho v^2 S\bar{c}} - \frac{L_H\, l_H}{\frac{1}{2}\rho v^2 S\bar{c}}$$

$$\Leftrightarrow C_m = C_{m_{ac}} + C_L \cdot \frac{l_{cg}}{\bar{c}} - C_{L_H} \cdot \frac{S_H}{S} \cdot \frac{l_H}{\bar{c}} \rightarrow V_H = \frac{S_H\, l_H}{S\, c}$$

$$\Leftrightarrow C_m = C_{m_{ac}} + C_L \cdot \frac{l_{cg}}{\bar{c}} - C_{L_H} \cdot V_H.$$

$$\hookrightarrow \frac{dC_m}{d\alpha} = \underbrace{\frac{dC_{m_{ac}}}{d\alpha}}_{=0} + \frac{l_{cg}}{c}\frac{dC_L}{d\alpha} - V_H \cdot \frac{dC_{L_H}}{d\alpha} \rightarrow \underbrace{\frac{dC_{L_H}}{d\alpha_H}}_{a_t} \cdot \frac{d\alpha_H}{d\alpha}$$

$$\Leftrightarrow C_{m_\alpha} = a \cdot \frac{l_{cg}}{c} - a_t \cdot V_H \cdot \frac{d\alpha_H}{d\alpha}.$$

$$\hookrightarrow C_{m_\alpha} = \underbrace{a \cdot \frac{l_{cg}}{c}}_{\text{small}} - \underbrace{a_t \cdot V_H \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right)}_{\text{large}}. \qquad \frac{d\varepsilon}{d\alpha} \simeq 0.1$$

for stability : $C_{m_\alpha} < 0$

neutral stability : $C_{m_\alpha} = 0 \Leftrightarrow a\frac{l_{cg}}{c} = a_t V_H \left(1 - \frac{d\varepsilon}{d\alpha}\right).$

$$\Leftrightarrow \frac{l_{cg}}{c} = \frac{a_t}{a} V_H \left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

$l_{cg} = l_{np}$ neutral point

$\mapsto \boxed{l_{cg} < l_{np}} \Rightarrow \boxed{\text{Stable aircraft}}$

→ Why do we use these cambered profiles, why not simple symmetrical ones?

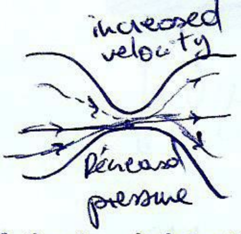
↳ Answer: For horizontal flight (cruise) an inclined symmetrical wing profile generates more drag = fuel inefficient.

→ Airfoil description: NACA (National Advisory committee for aeronautics [US])

→ 4 numbered nomenclature to describe:

• 2412 (NACA-2412) means: 2% camber (of chord length) at 0,4 of the chord (from LE (leading edge)); and 12% thickness/chord ratio (or 0,12).

→ Generation of lift: Bernoulli:



→ Sum of static and dynamic pressure remains constant:

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2 \quad \text{or} \quad P + \frac{1}{2}\rho V^2 = cte \text{ (along a streamline)}$$

→ Higher speed = lower static pressure

→ If we have lower speed below airfoil and higher airspeed over the airfoil
⇔ higher pressure below // and lower pressure over //

↳ which leads to upwards pressure which causes lift.
lift

↳ All of the above is for incompressable air flows.

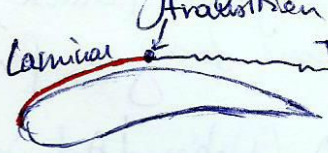
→ Lift by pressure difference

→ Airfoils curvature of the upper surface longer than for lower surfaces. And the angle of attack improves lift (have influence on pressure diff)

→ symmitrical Airfoil doesn't generate lift at 0° angle of attack contrary to non-symitrical Airfoil.

→ Coef. (C_L) has a maximum (seperation of air flow)

→ Airflow: | Laminar • Boundary layer: thin, low friction
| Turbulent • Boundary layer: thick, high friction (but no lift).



→ Maximise laminar flow because of low drag.

normal Angle of Attack

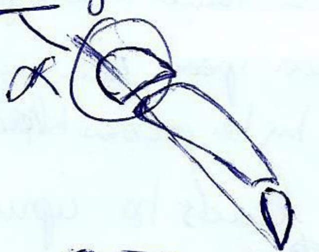
← No contact btw flow and Airfoil: not good

very high attack angle

→ To be able to fly at low airspeeds, we need to increase: critical α, maximum $C_{L max}$, wing Area S.

↳ α: We use Flaps and slots for increase in α critical.

Plain



Split

Droop snoot

→ Wing surfaces / Areas: Many different wings possible using these variables:

- Wing span
- Wing surface
- Taper
- Sweep angle
- Dihedral
- Chord (root & tip)
- Winglet

→ **Drag:**

$$+ \boxed{D = C_D \times \left(\tfrac{1}{2} \rho V^2\right) \times S}$$

- C_D (Drag coef) consists of: | profile drag (result of pressure & friction forces) | parasitic drag.
- ρ (air density): depends on alt.
- V (air speed) and S (wing area) are design parameters.

—

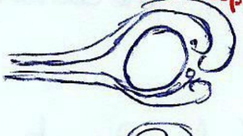
→ Drag Origins:

- Skin friction
- Pressure drag
- Wave drag (at transonic & supersonic speeds)
- Parasitic drag (no lift devices like fuselage, engines, ...)

↳ Drag: $C_D = \underset{\uparrow \, \text{cte}}{C_{D_0}} + C_{D_i}$ ← induced drag

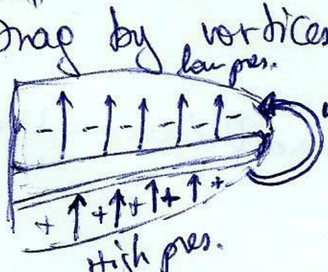
$\Leftrightarrow C_D = C_{D_0} + \dfrac{C_L^2}{\pi e AR}$

↳ Relative contributions of form drag (pressure drag) and skin drag to the total drag.

shape	Form drag	skin drag	C_D
	0%	100%	–
	~10%	~90%	0,12
	~90%	~10%	–
	100%	0%	2

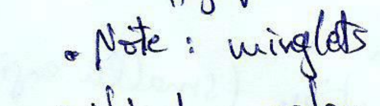
↳ Difference btw 2D & 3D study approach:

- Drag by vortices caused by pressure difference over the wing: vortex at the side bc high pres air wants to restore, causing drag.



- Note: winglets reduce fuel consumption (6% - 7% ...), they oppose the vortices.

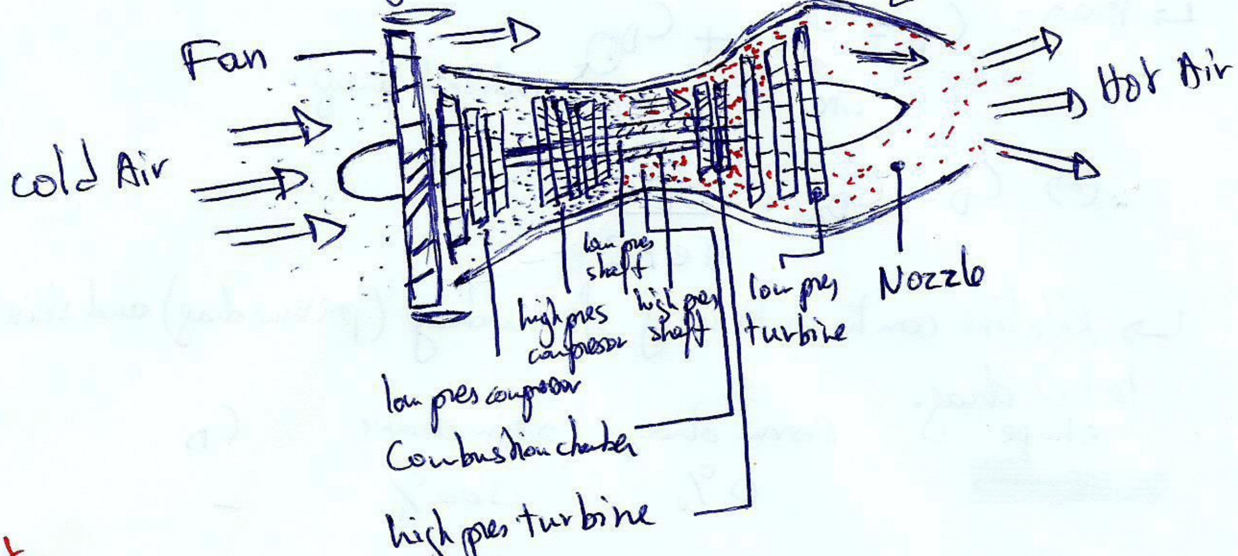
↳ Lift drag polar



- at each degree of attack angle we plot C_L and C_D.
- with the max slope we get maximum C_L/C_D ratio, means, we get the best angle of attack for max lift and min drag (and low fuel consumption).
- C_L/C_D is an important design parameter.

→ Thrust :

- To maintain cst speed : T = D .
- Engine types : • propeller engine • Turboprop engine • Turbojet • Ramjet
  • Piston engine • Jet engine • Turbofan



→ Weight :

- Aircraft empty weight :

↳ Structure : Wing , Horizontal tail, vertical tail , Fuselage , landing gear , surface controls , propulsion system , APU .

↳ Systems : Instruments & navigation , hydraulics & pneumatics , Electrical systems , Electronics , Furnishings , Airconditionning and anti-ICe .

↳ Crew and flight attendants .

↳ Operating items

- payload
- Fuel .

• Minimize weight of aircraft struct/sys. & fuel to maximize payload.

→ Snow Ball effect :

→ Reducing the weight of struct / systems /— result in overall lower weight of aircraft

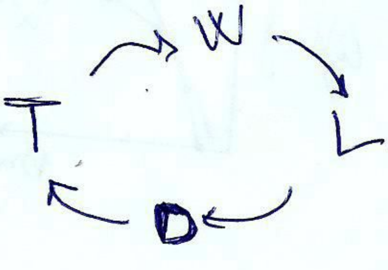
↳ which results in less required Lift .

↳ which results in a smaller wing .

↳ which results in drag reduction

↳ which results in Thrust reduction (smaller engines) .

- Both the smaller wing and smaller engines will result in less weight and the cycle can start again .



+ it's assumed that the weight for fuel & payload doesn't change .

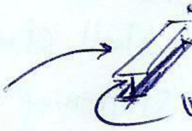
– Structural concepts –

→ What is a structure:
- features: + Many elements + Coherence + Different materials.
  + Several functions + Joints
- functions of a structure:
  + Carrying of the loads (dominant) + protection + Framework to attach other systems.
- Relationship between type of structure and material:

+ period	+ structure type	+ Materials
1932 – today	stiffened shell struct.	Aluminium
1948 – today	Pressure cabin	Improved Al-alloys
1980s – today	Composite struct.	Carbon fibers.

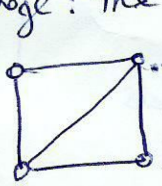
- Difference btw strength & stiffness:
  - strength: Related to the maximum loads a struct. of material can take.
  - stiffness: How far a mat. of struct can deform during certain loads, if there's little deformation with a load then it's a very stiff/rigid material, if large deformation with same load then the material is flexible (unstiff).

- Anatomy of a structure:
  - A structure is an Assembly of structural elements
  - Each element participates in (some of) the functions of the struct.
  - Structure has coherence.
  - Structural elements are joined together
  - Most structural elements are derivative of a beam



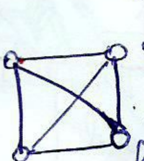
→ FROM TRUSS TO BEAM:
- What happens when 2 diagonal rods are used?
  + The structure becomes:
    - More difficult to assemble (no hinge at crossing)
    - More difficult to calculate
    - heavier? X
  + The one advantage: The struct has some 'reserve' (one rod may fail) – failsafe strut.

safe life :



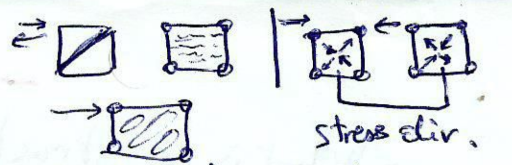
- Each element strong enough to stay intact for the entire life cycle
- thicker → stronger
  ↳ heavier

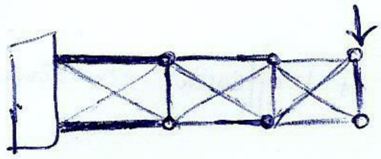
Fail safe :



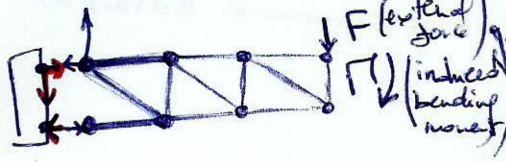
- One element may fail: other elements strong enough to stay intact for limited time; inspection required.
- More elements, but overall lighter struct.

+ The rod can be replaced by a thin sheet.



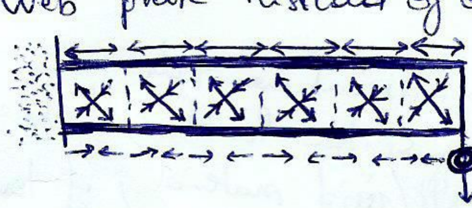


| "Wire braced" structure | combination of rods & wires.
· Increasing thickness of the rods to the left: the moment grows larger when we go from right to left because the distance decreases.

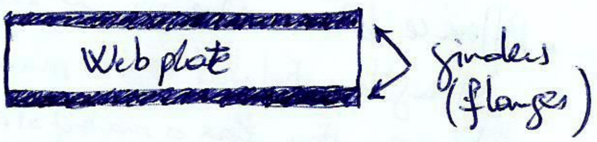


· The vertical rods & wires do not: the force is transferred to the wall via vertical elements and wires, and this force is constant.

· Truss applies force to the supports (red arrows); support reacts on Truss for equilibrium (blue arrows)
· Truss can be replaced by sheet metal.
· Web plate instead of diagonal tubes. | web plates - shear forces.
| girders - tension and compression forces.



simplified:

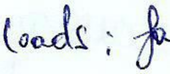


→ When applying high forces on the structure, buckling starts. Compression forces cause local buckling of sheet.

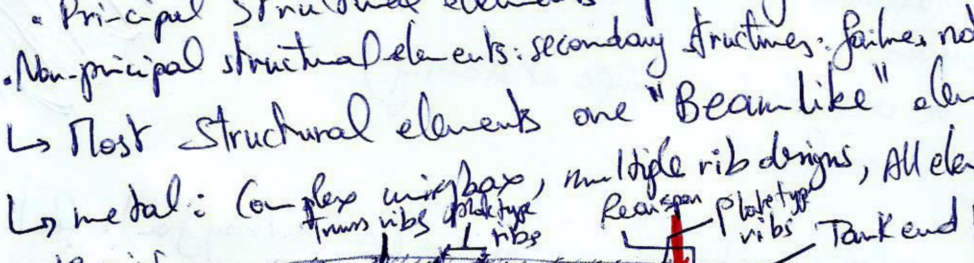
| Elastic buckling is no failure, only reduced compression load capacities, tensile forces are fully carried.

→ Shell structures:
- Shell structure: load bearing thin sheet material (incl. stressed skin), with stiffening elements.
- Monocoque: structure consisting of only a load bearing skin
  ↳ Semi coque (semi-monocoque): with some supporting elements.

→ Principle structural elements: (PSE)
- Principal structural elements: primary structures: carry loads: failure is catastrophic
- Non-principal structural elements: secondary structures: failure not catastrophic (eg. fairings, hatches.)

↳ Most structural elements are "Beam like" elements  > girders.

↳ All elements together: load path.
↳ metal: Complex wingbox, multiple rib designs,



| We can transport a vertical force through the root of the wing to make equilibrium (with web plates), same with bending moment or compression or tension (all loads have to have a load path.)