SNARKs from Hash Functions

Nick Spooner

Boston University (➡️ Warwick)

In the Beginning:

SNARKs from PCPs

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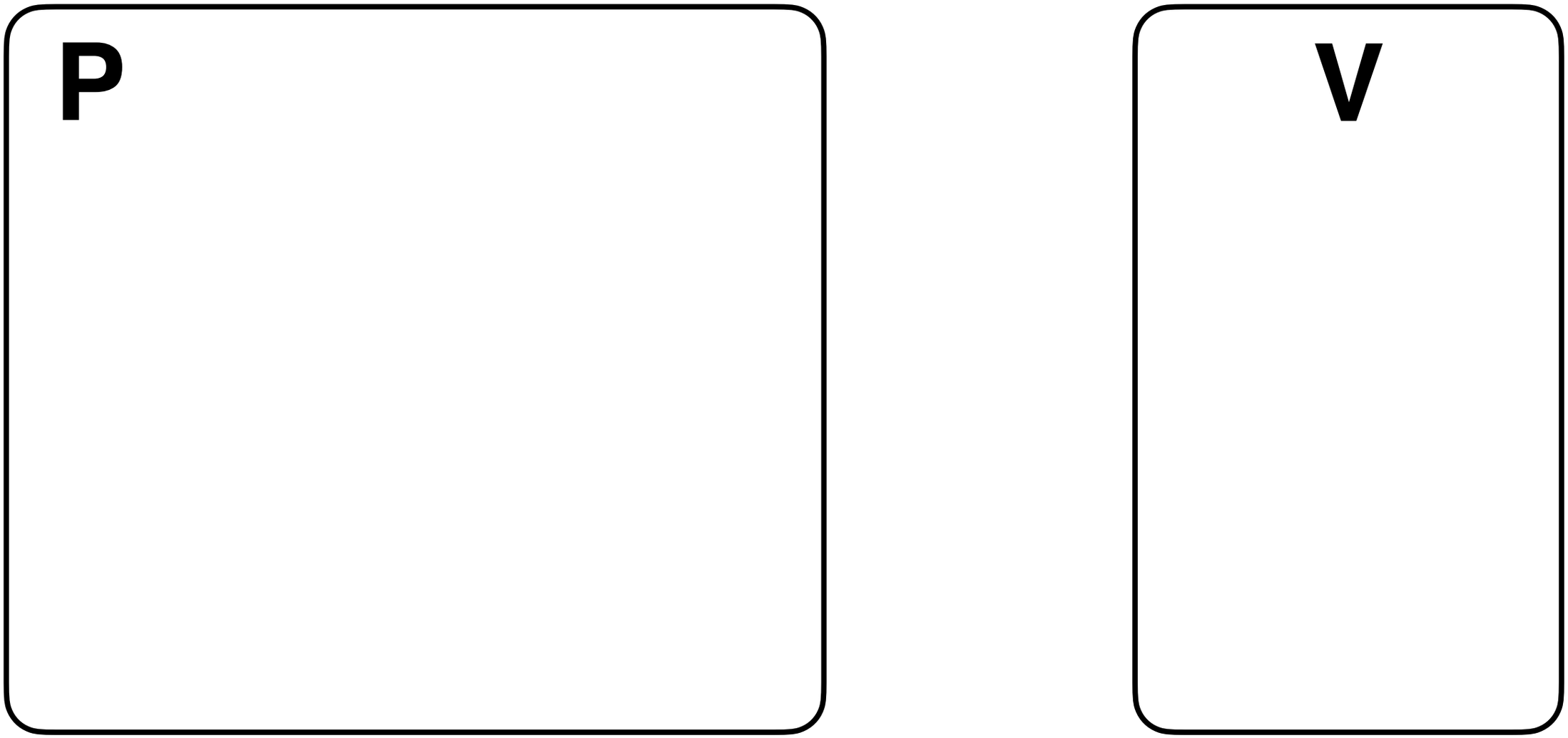

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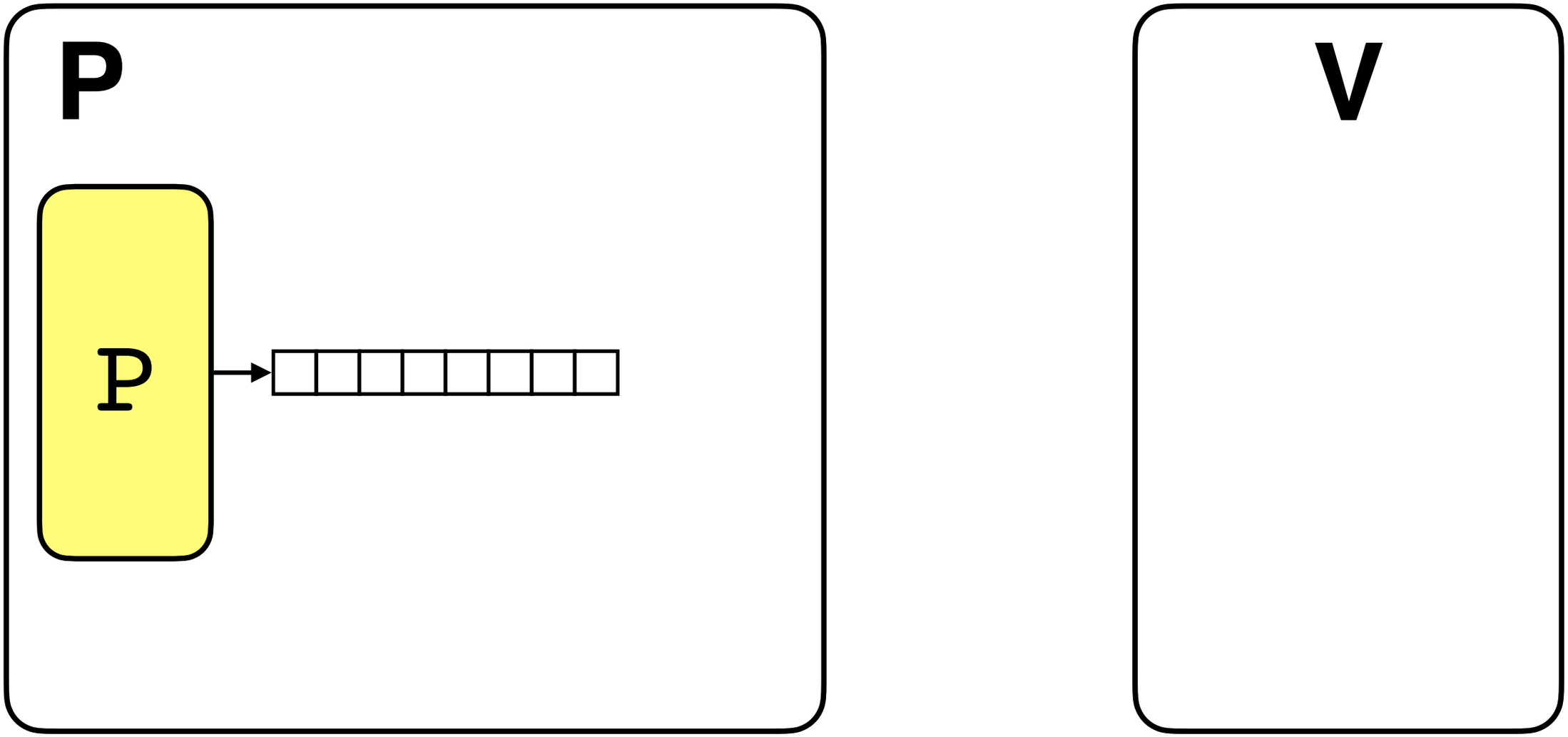

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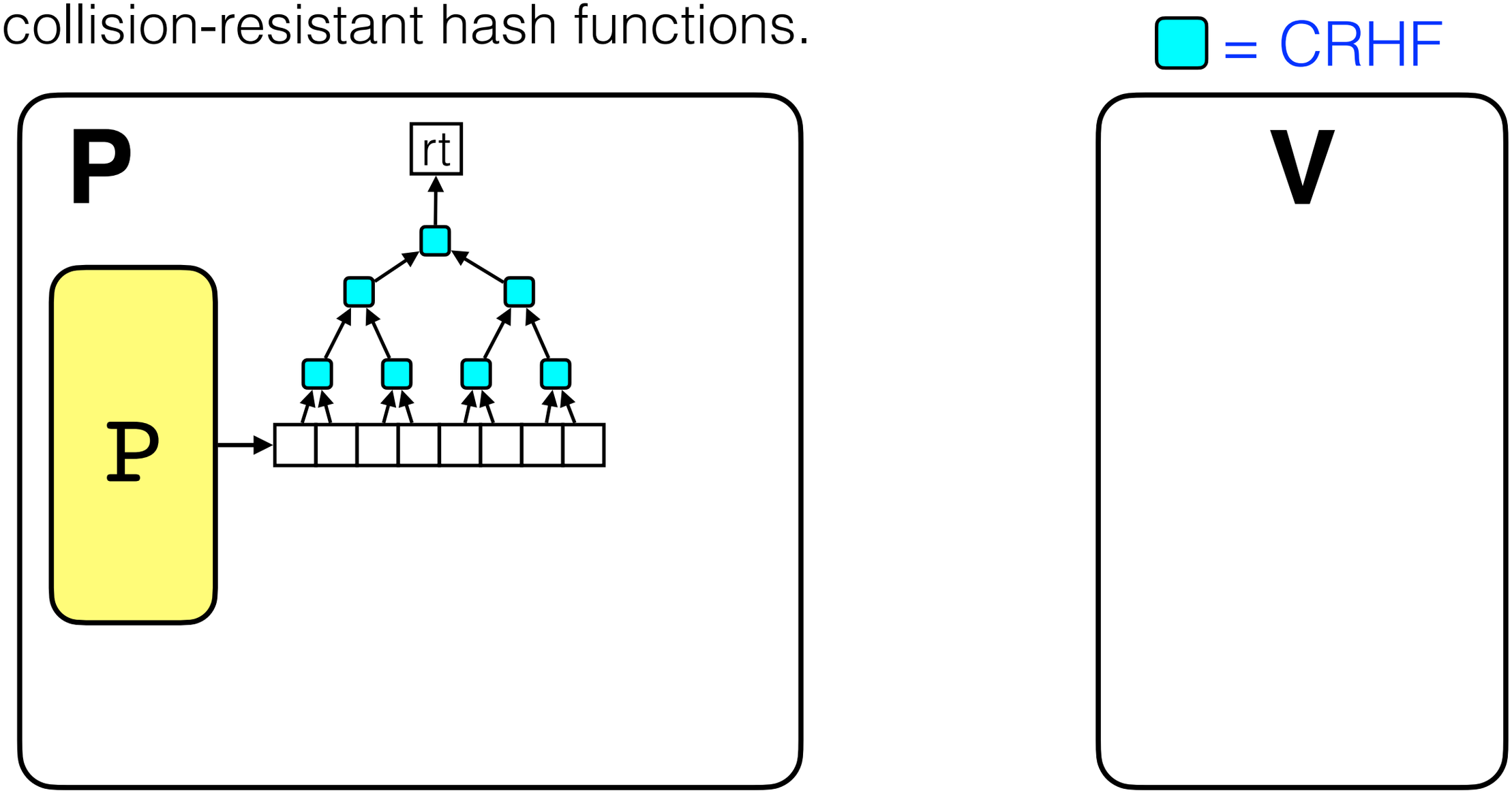

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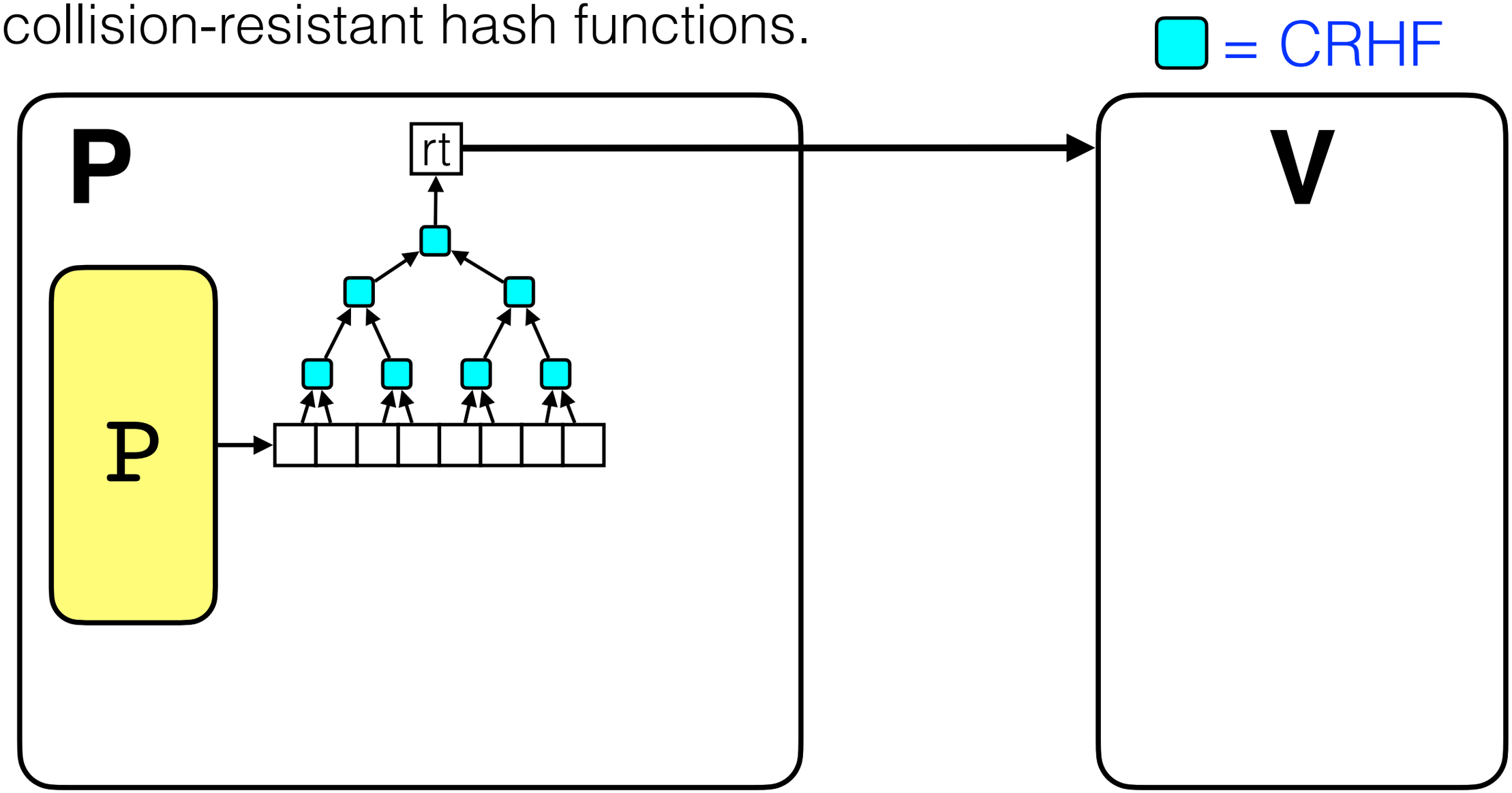

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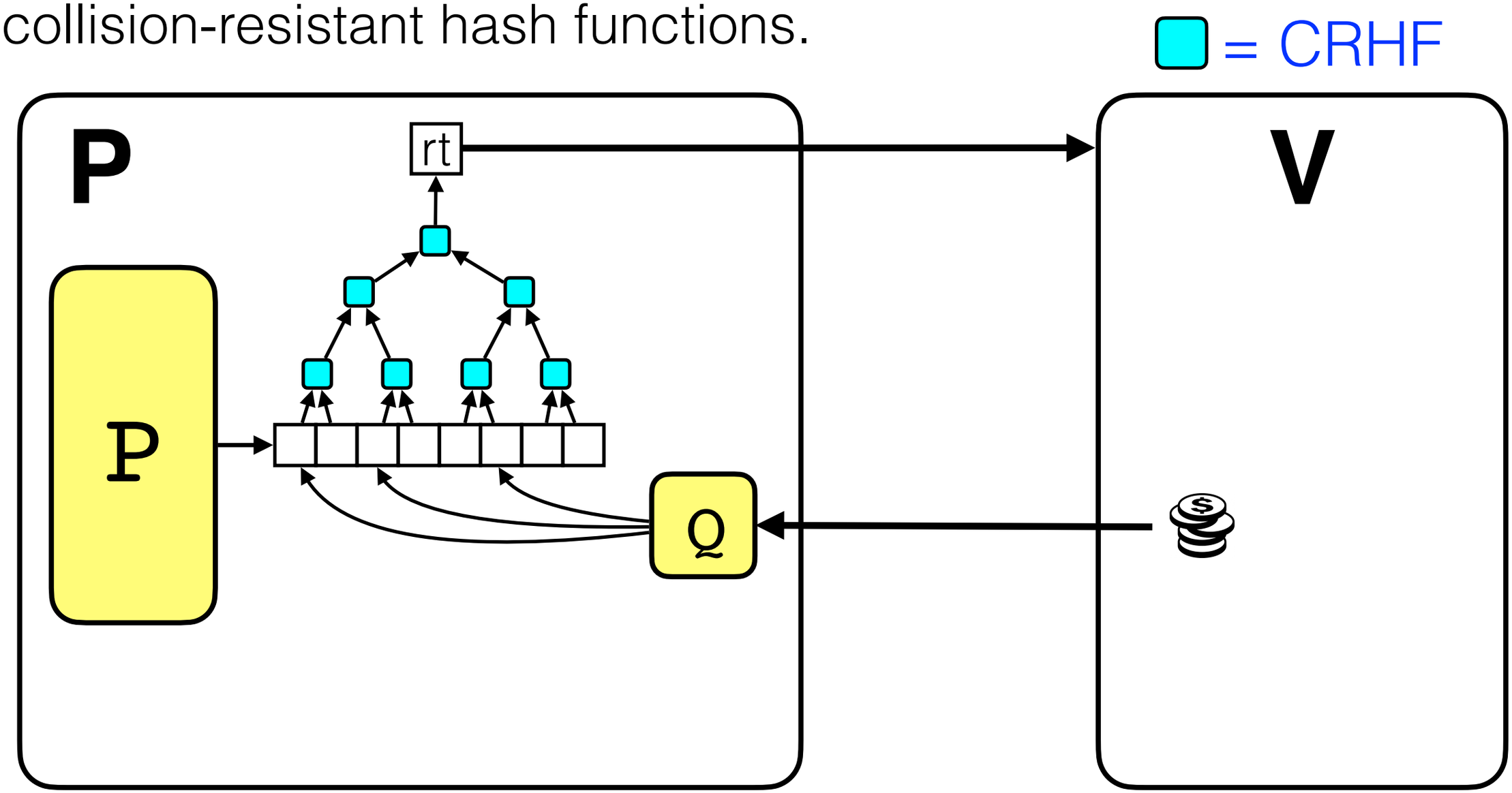

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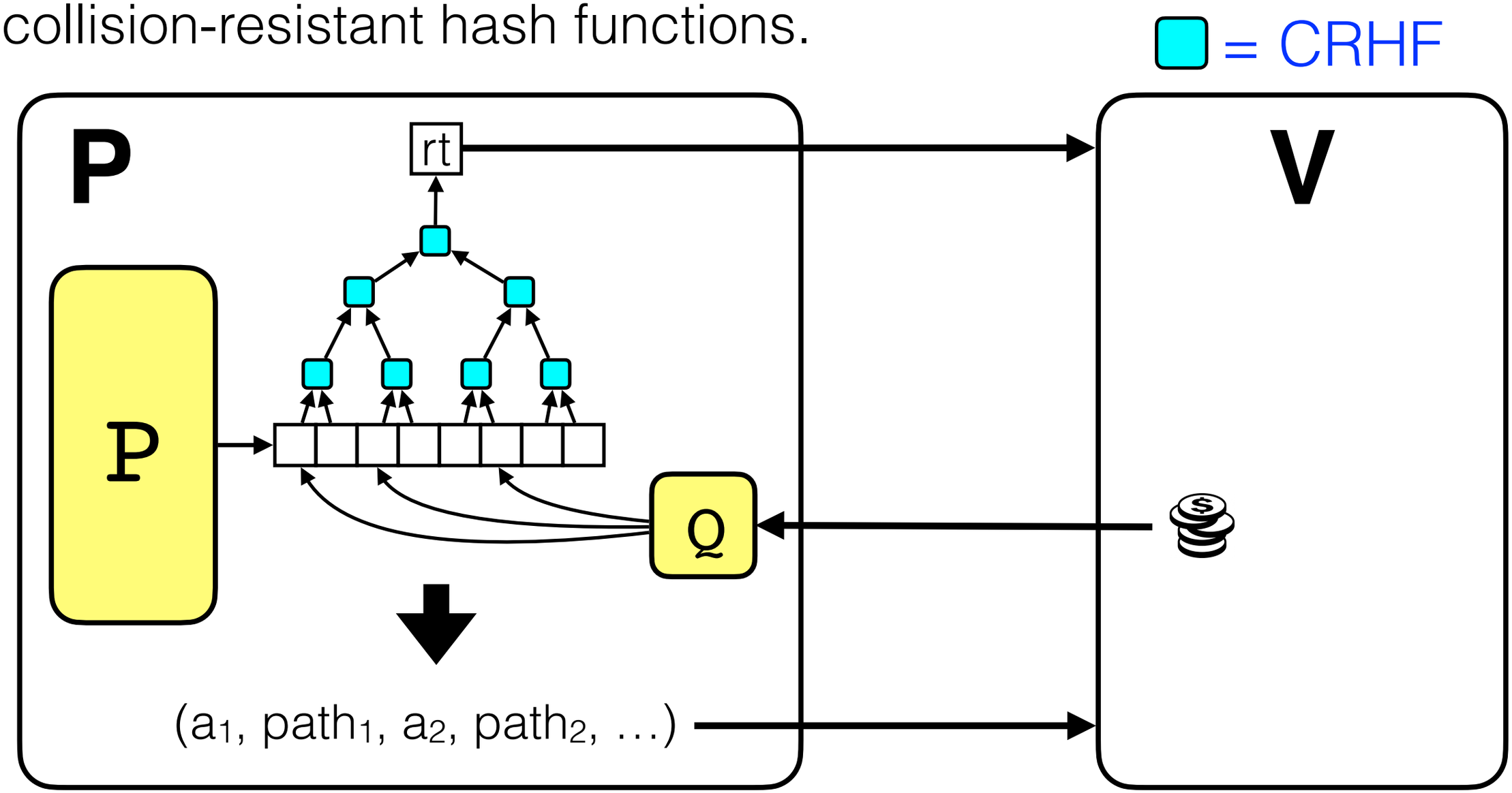

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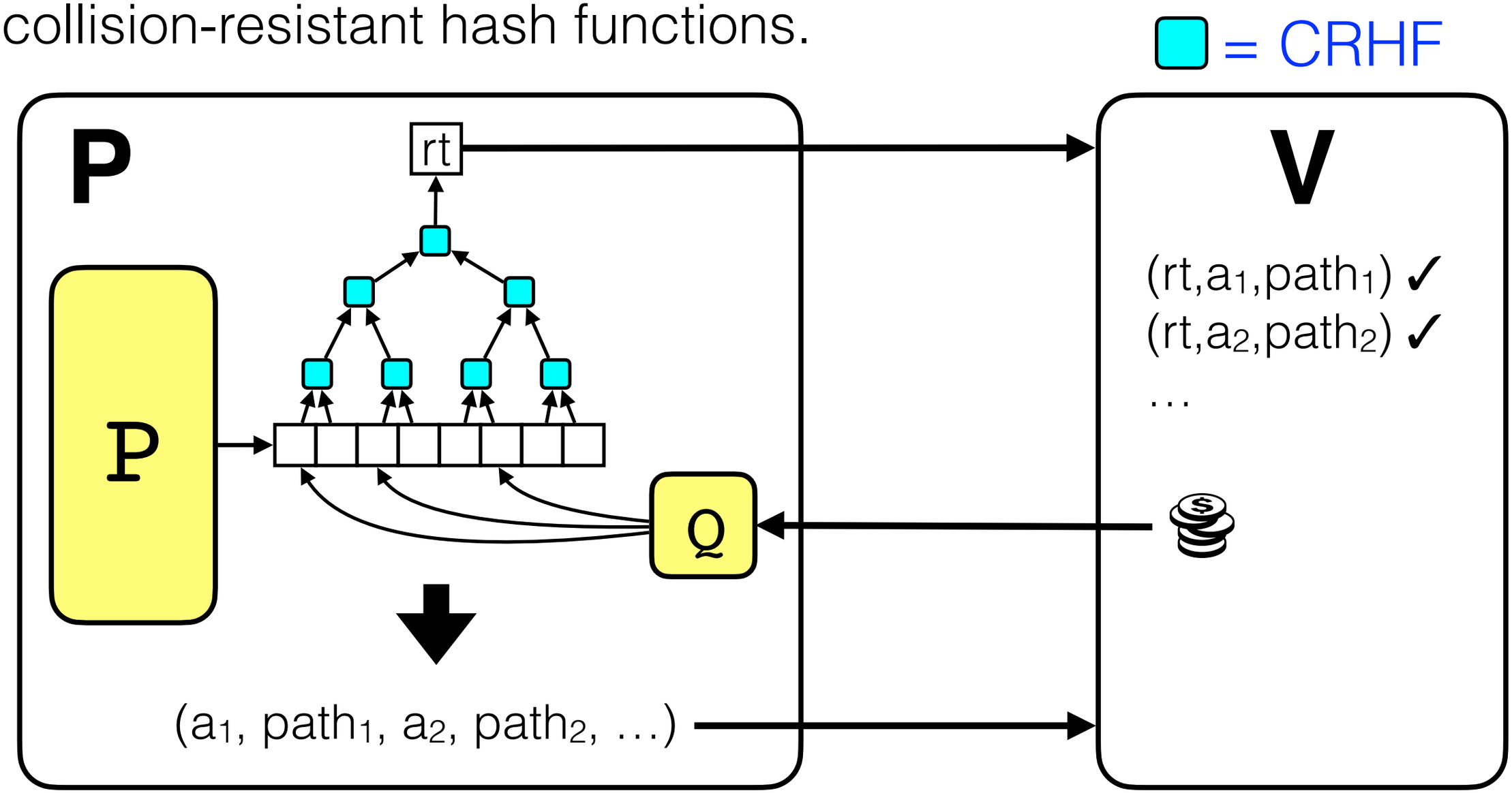

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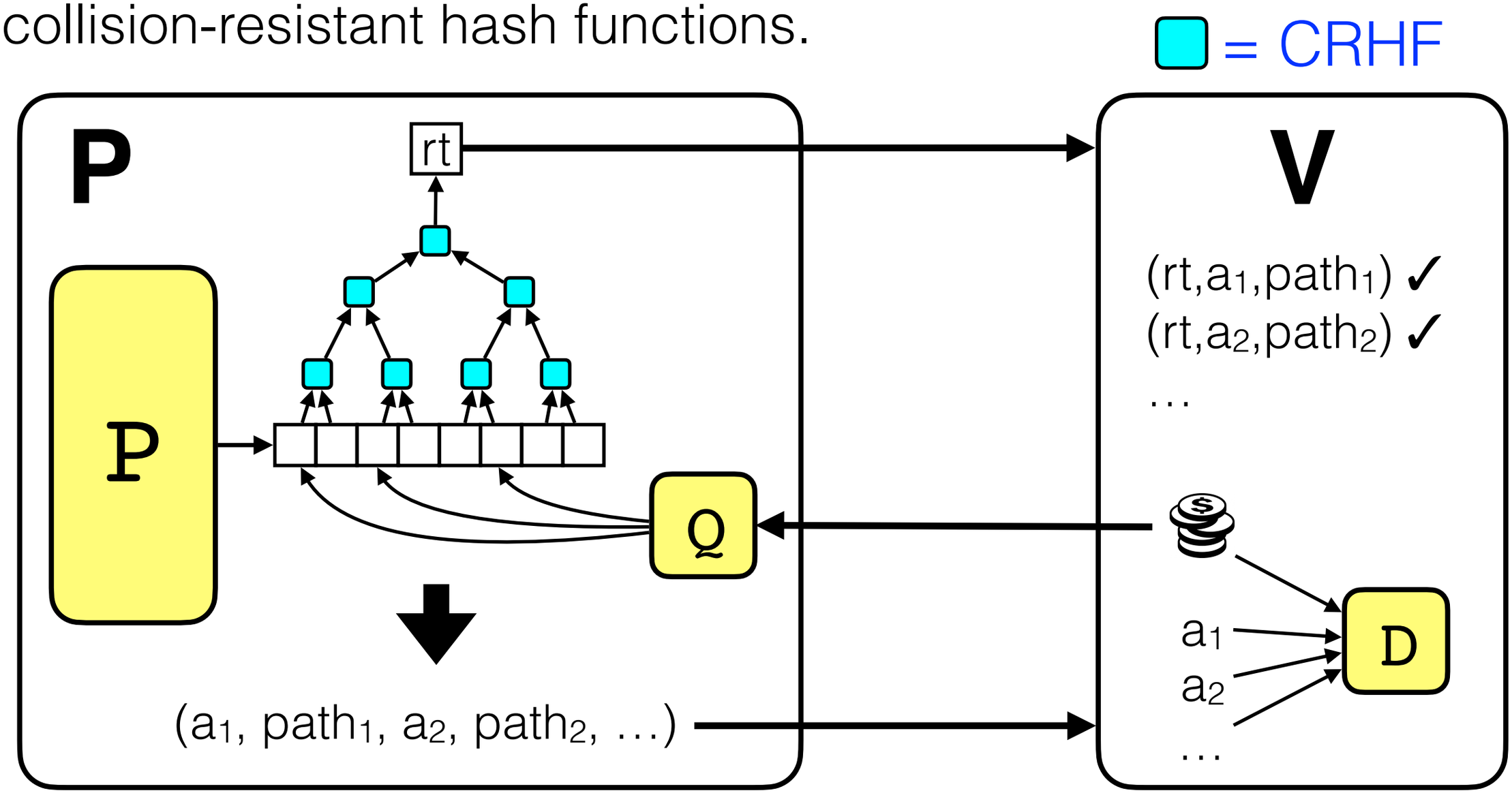

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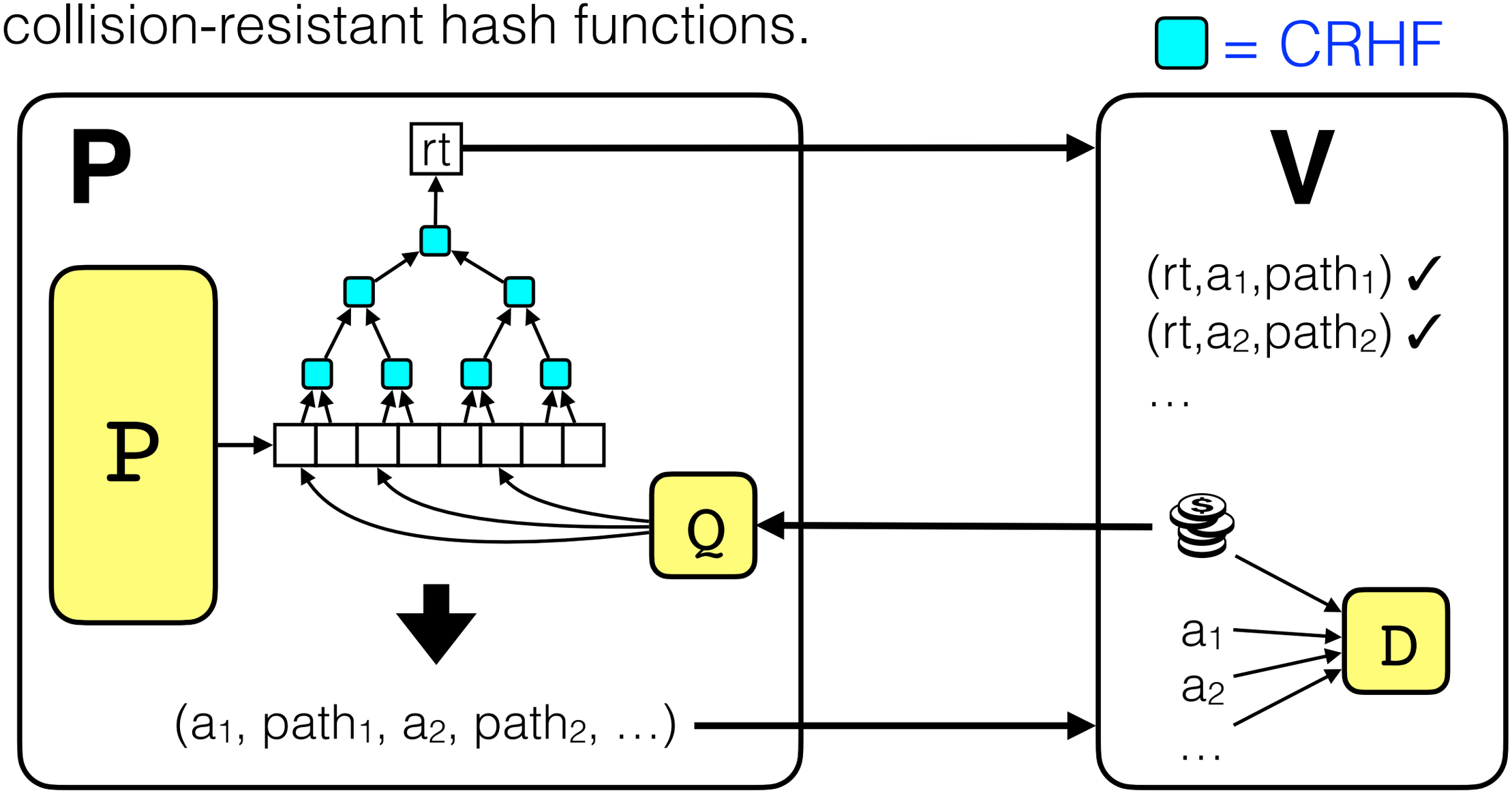

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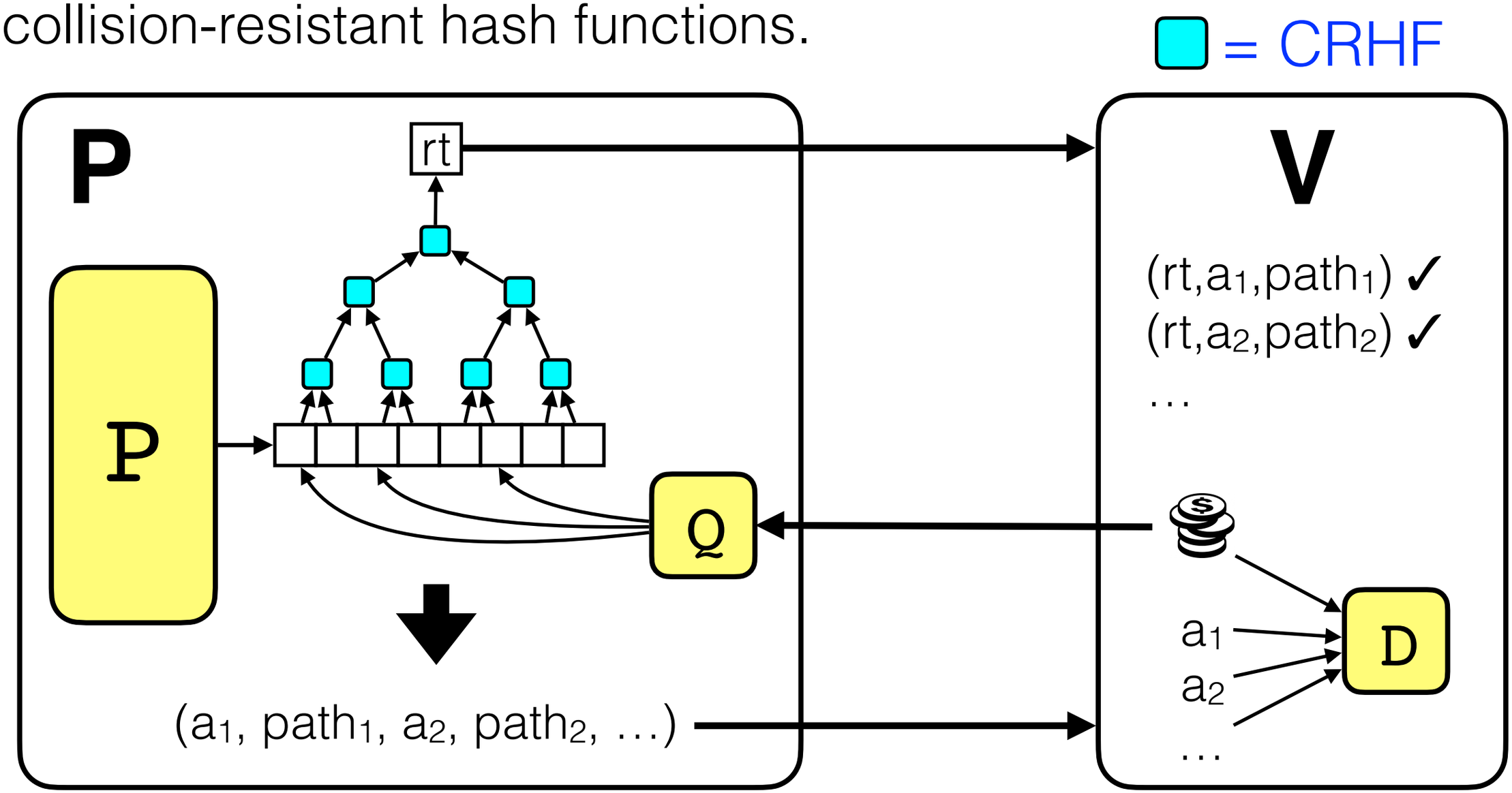

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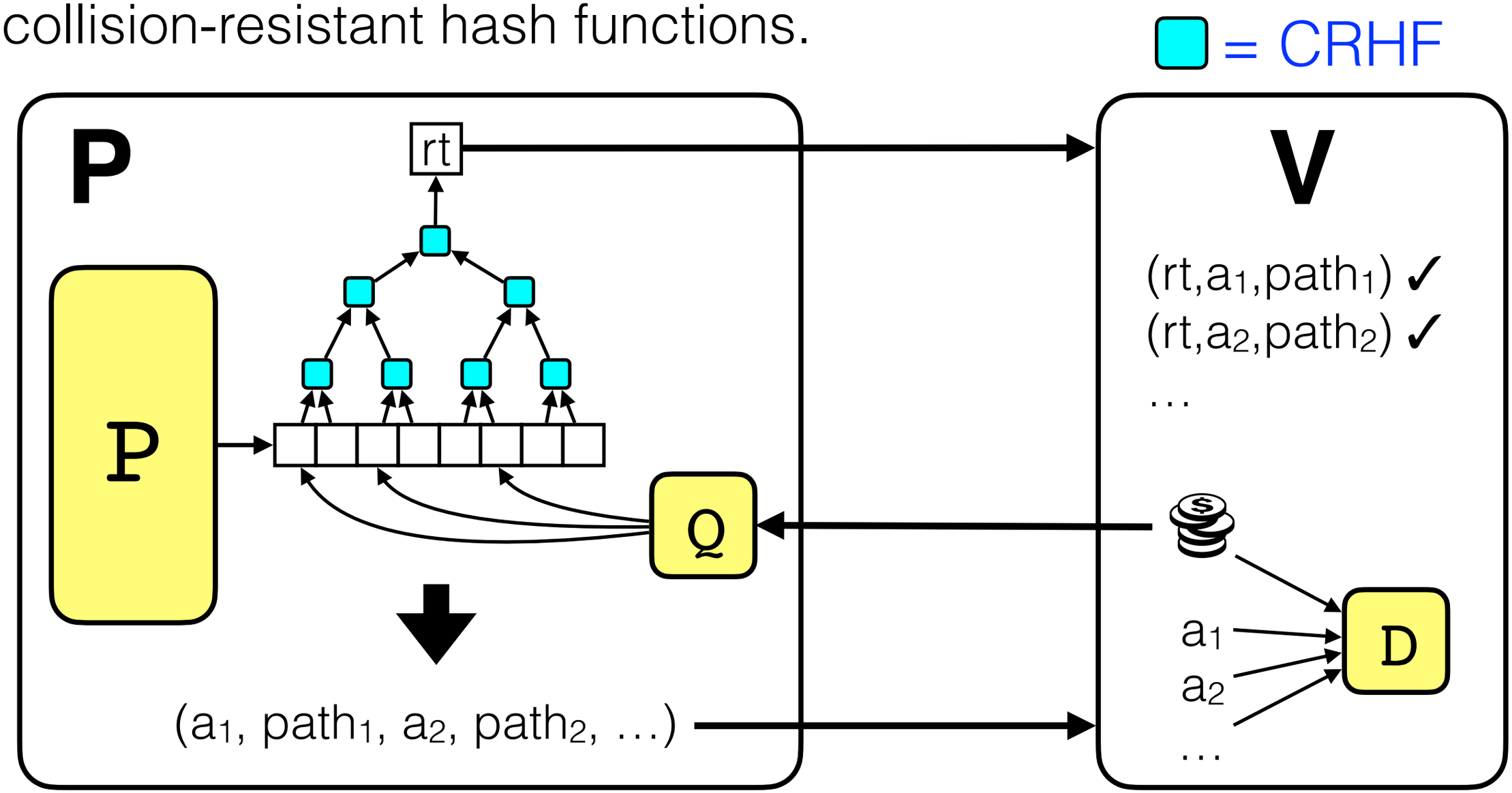

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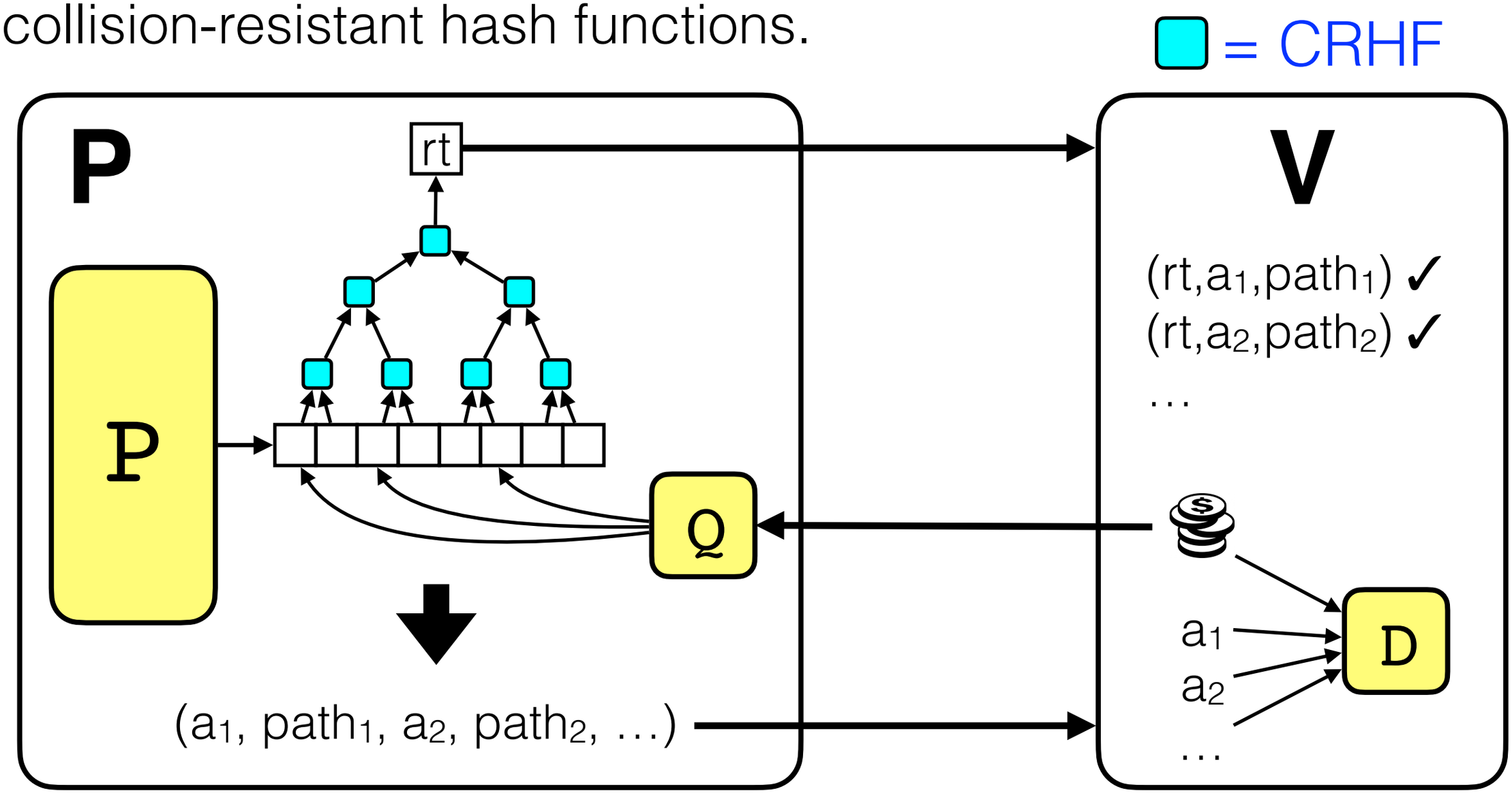

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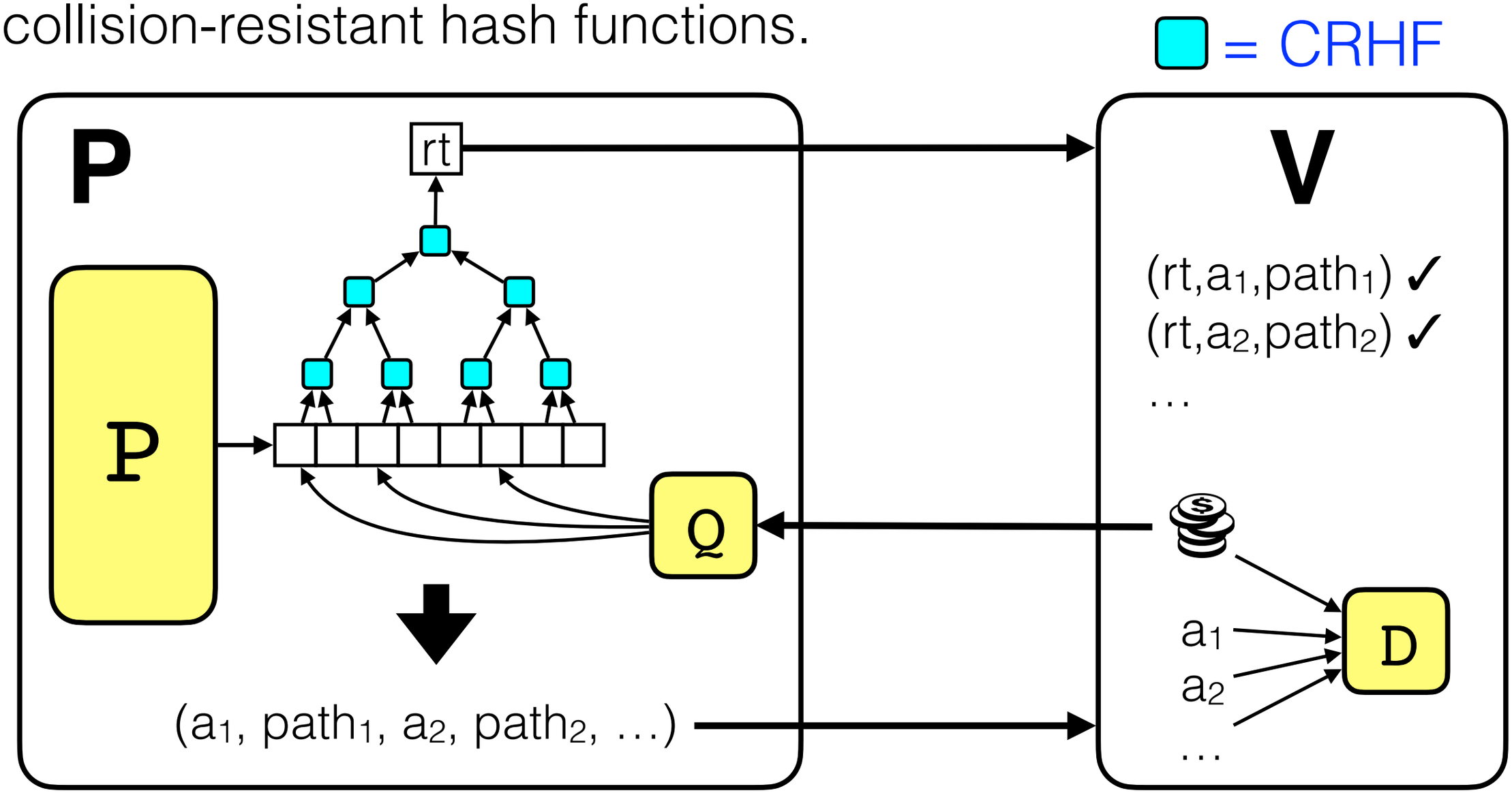

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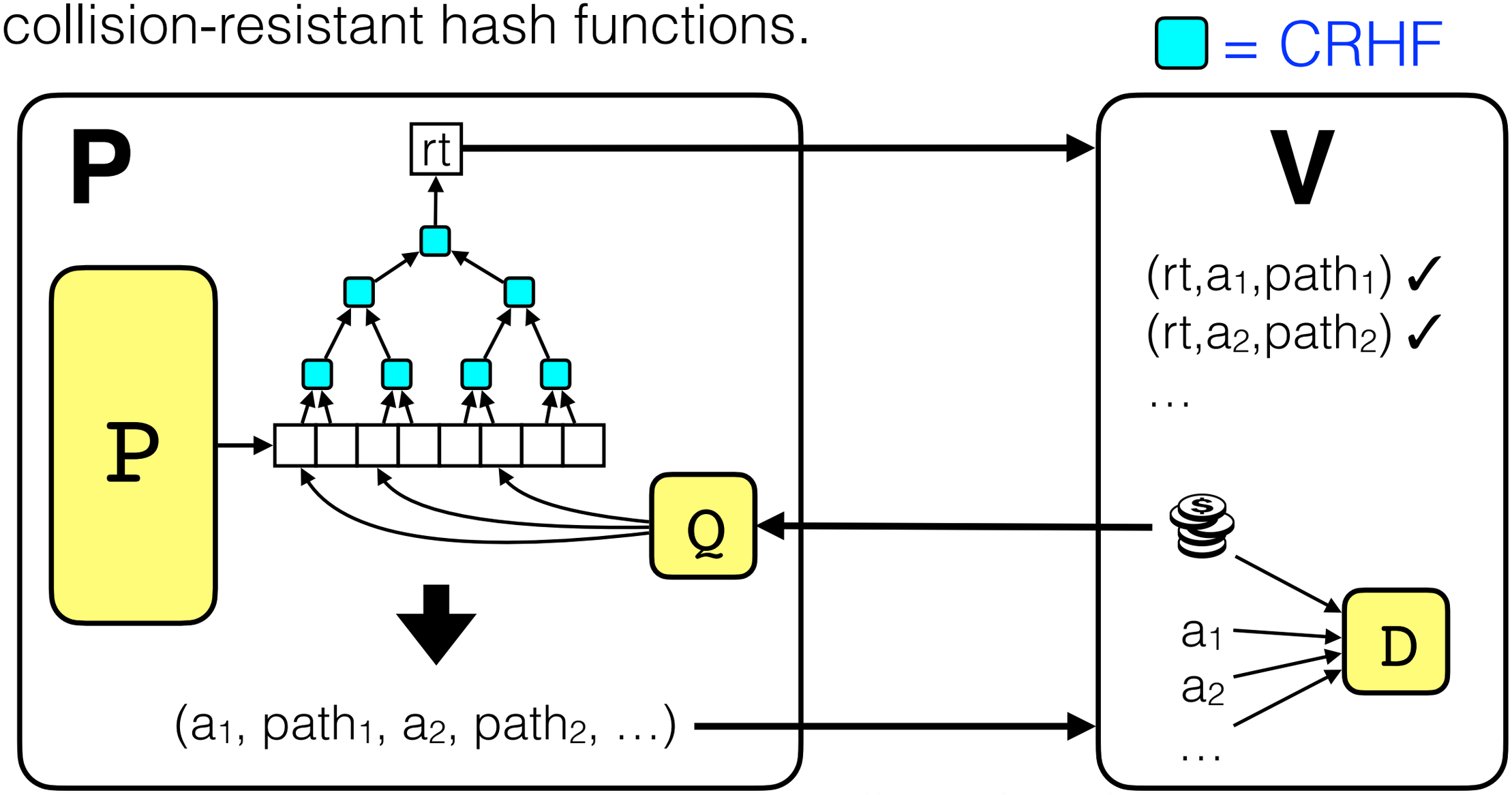

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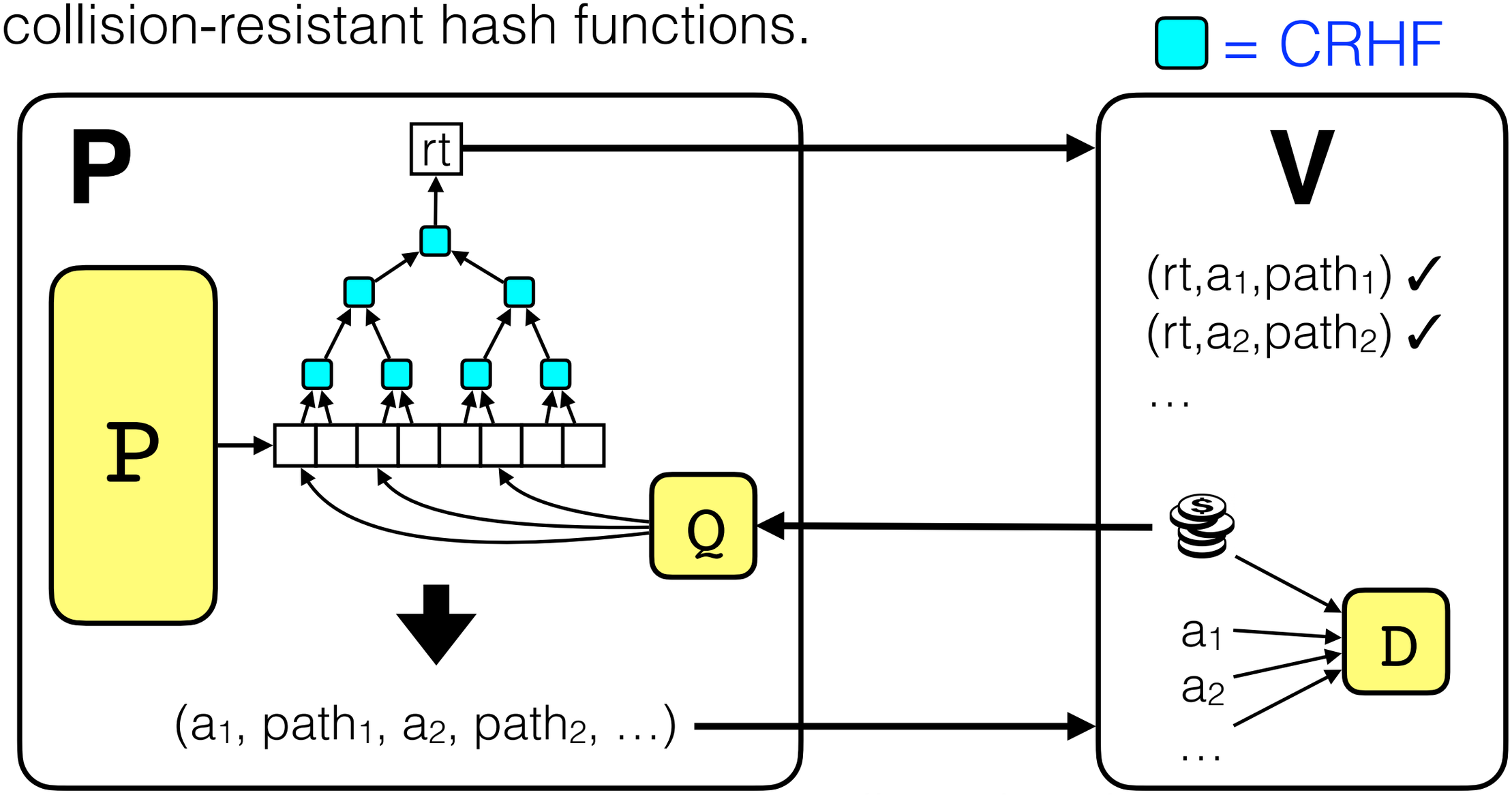

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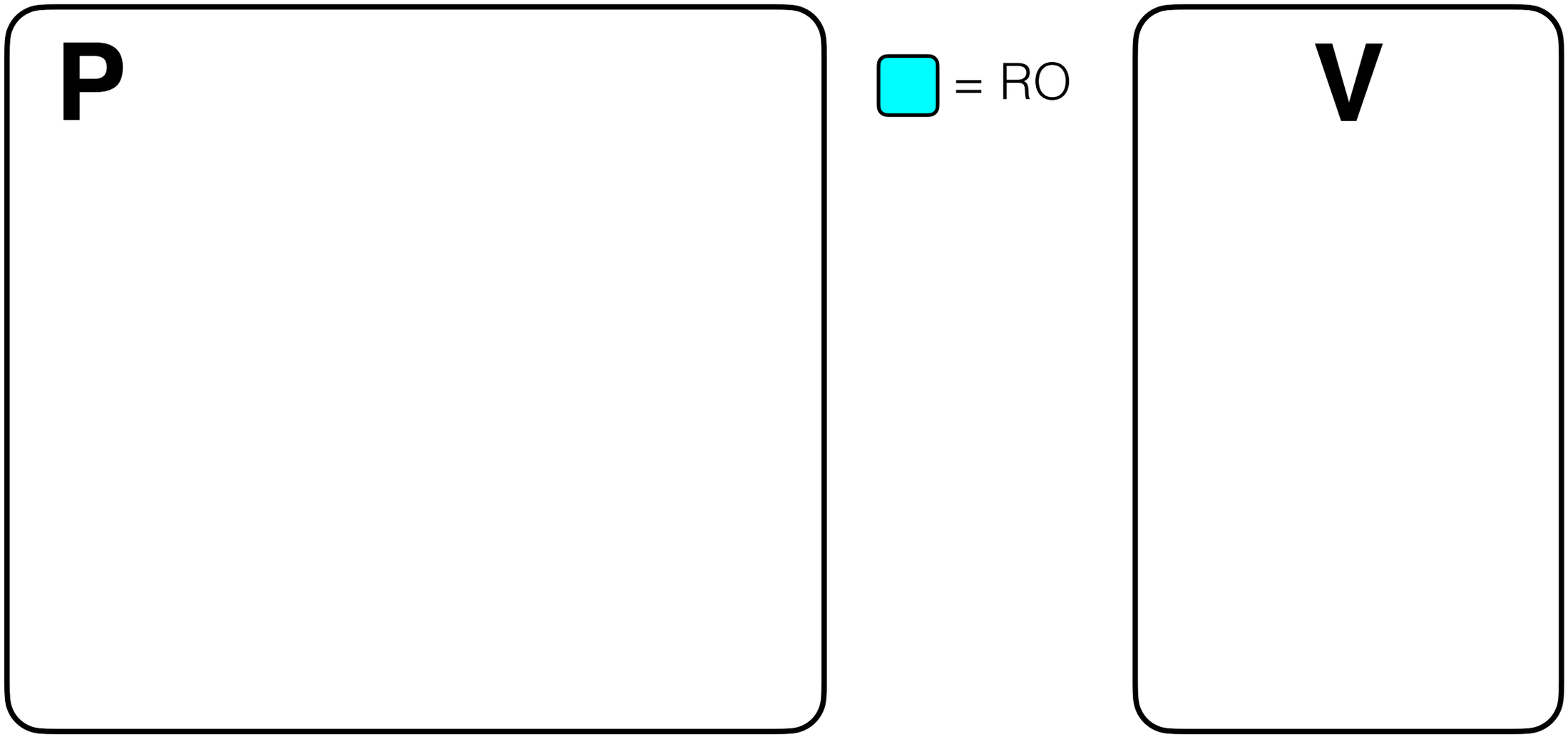

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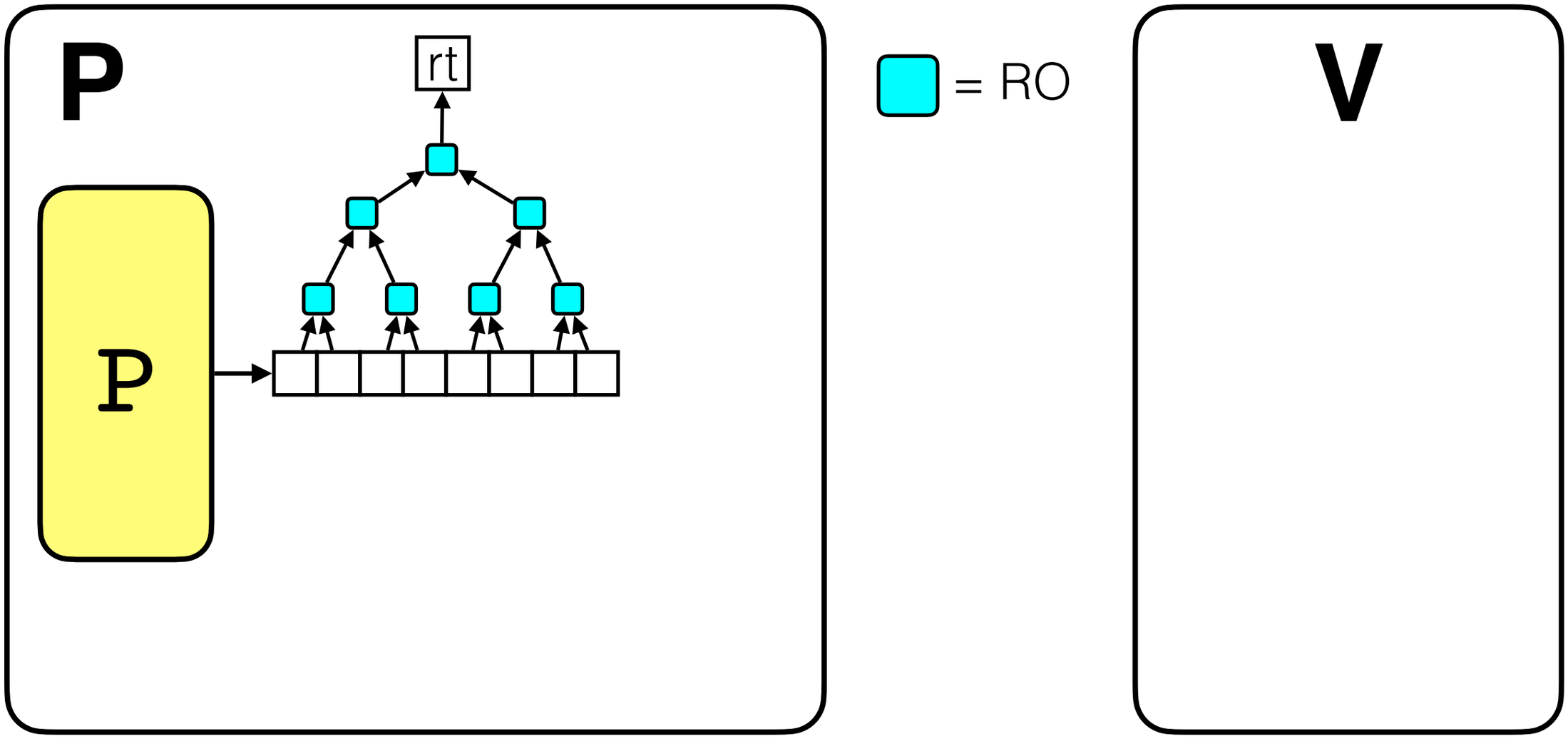

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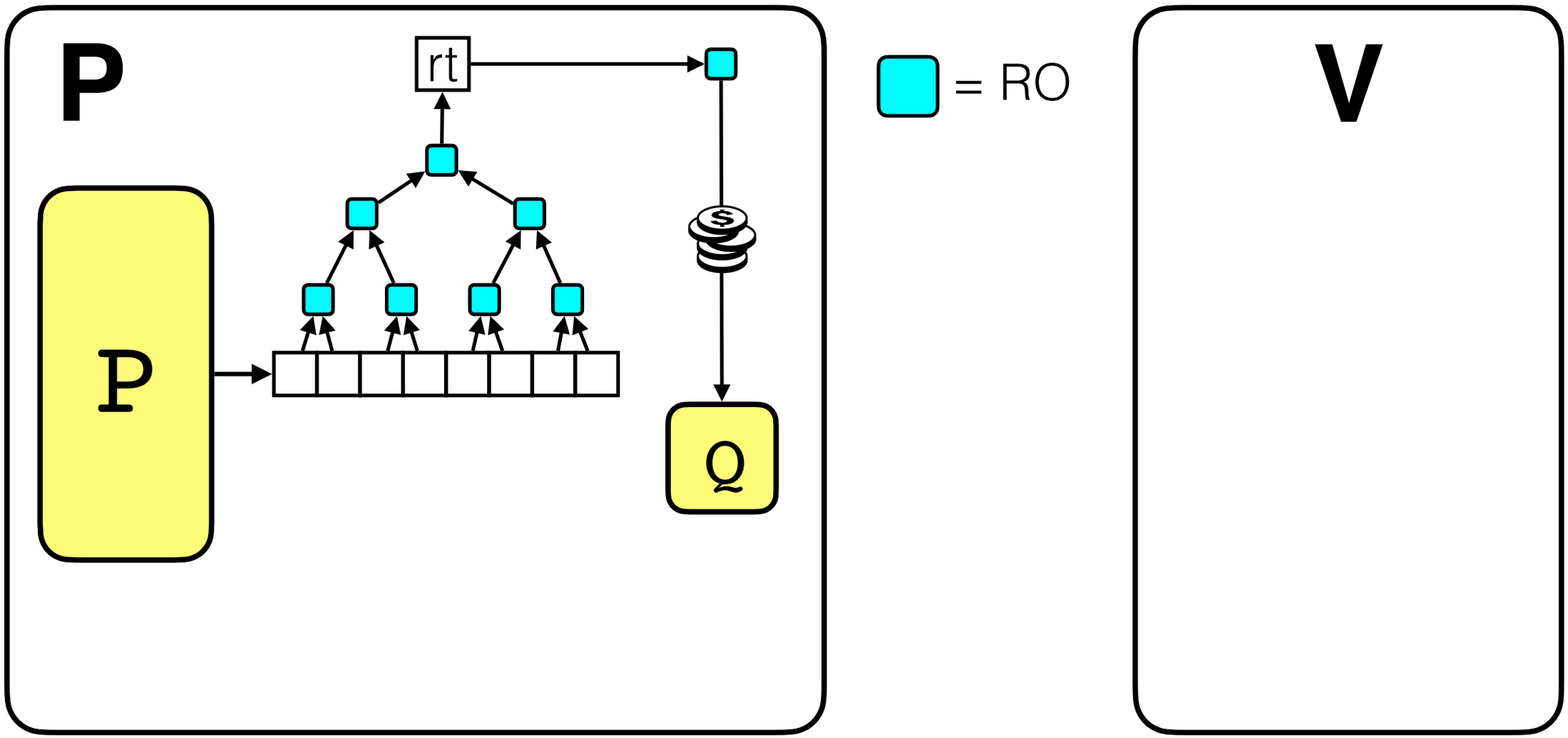

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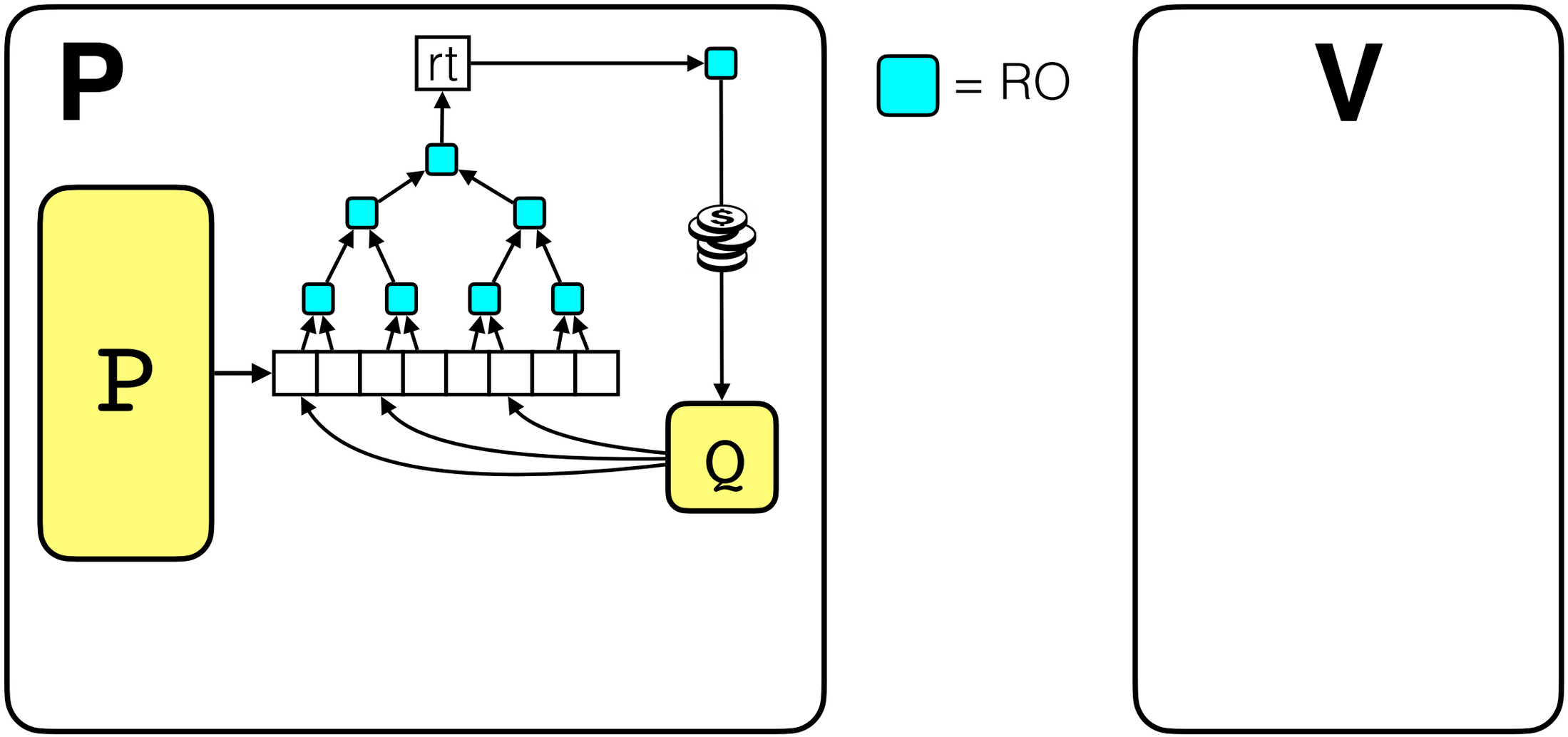

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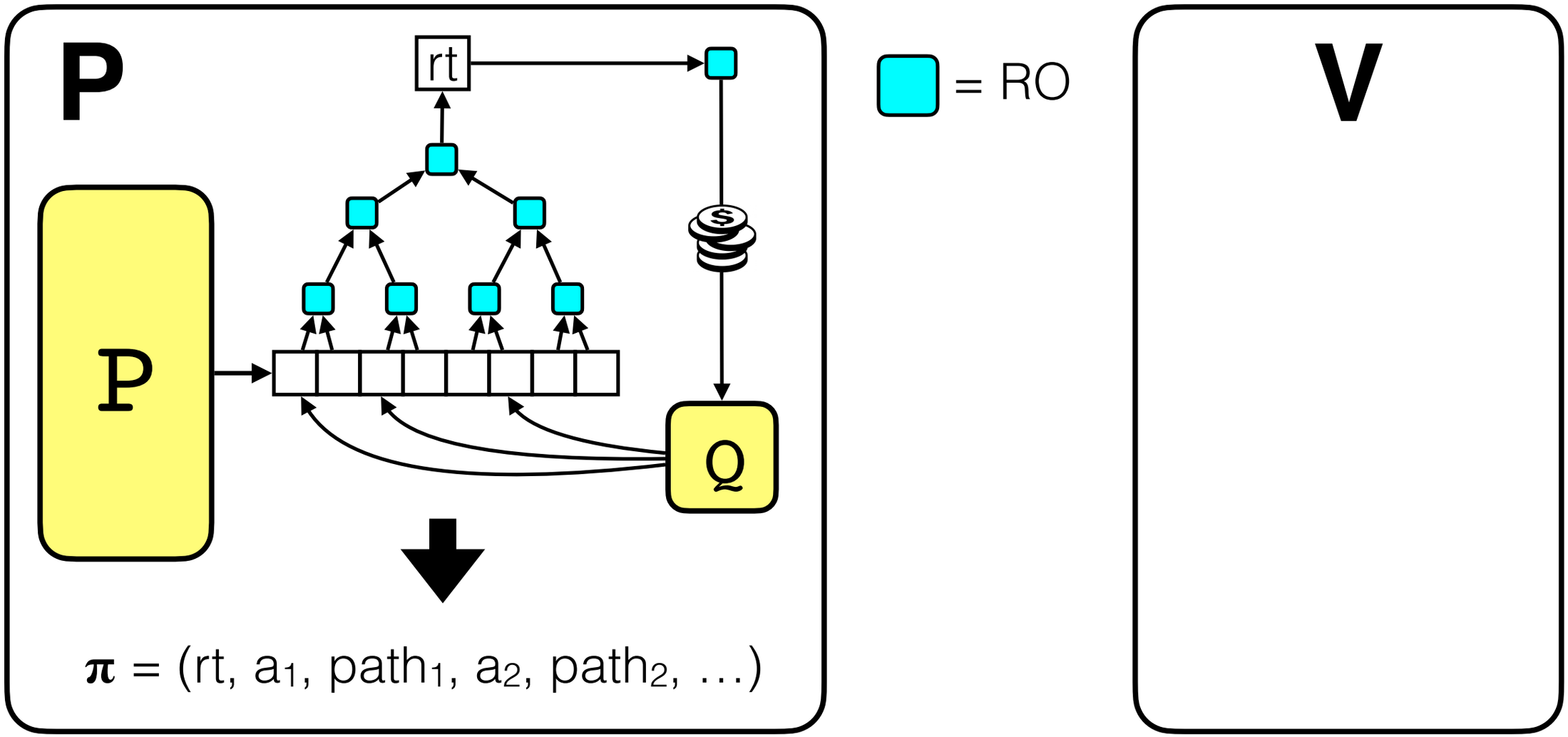

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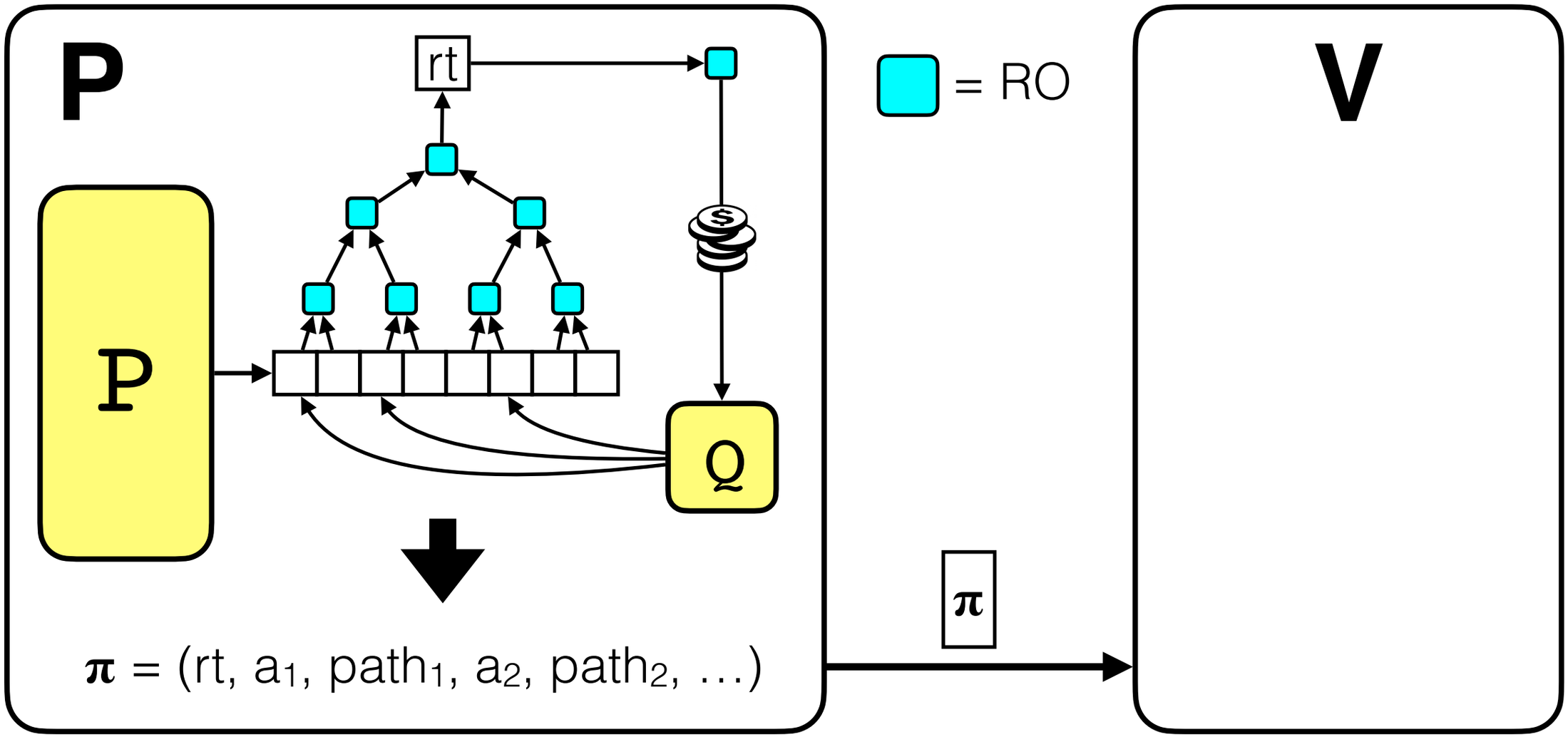

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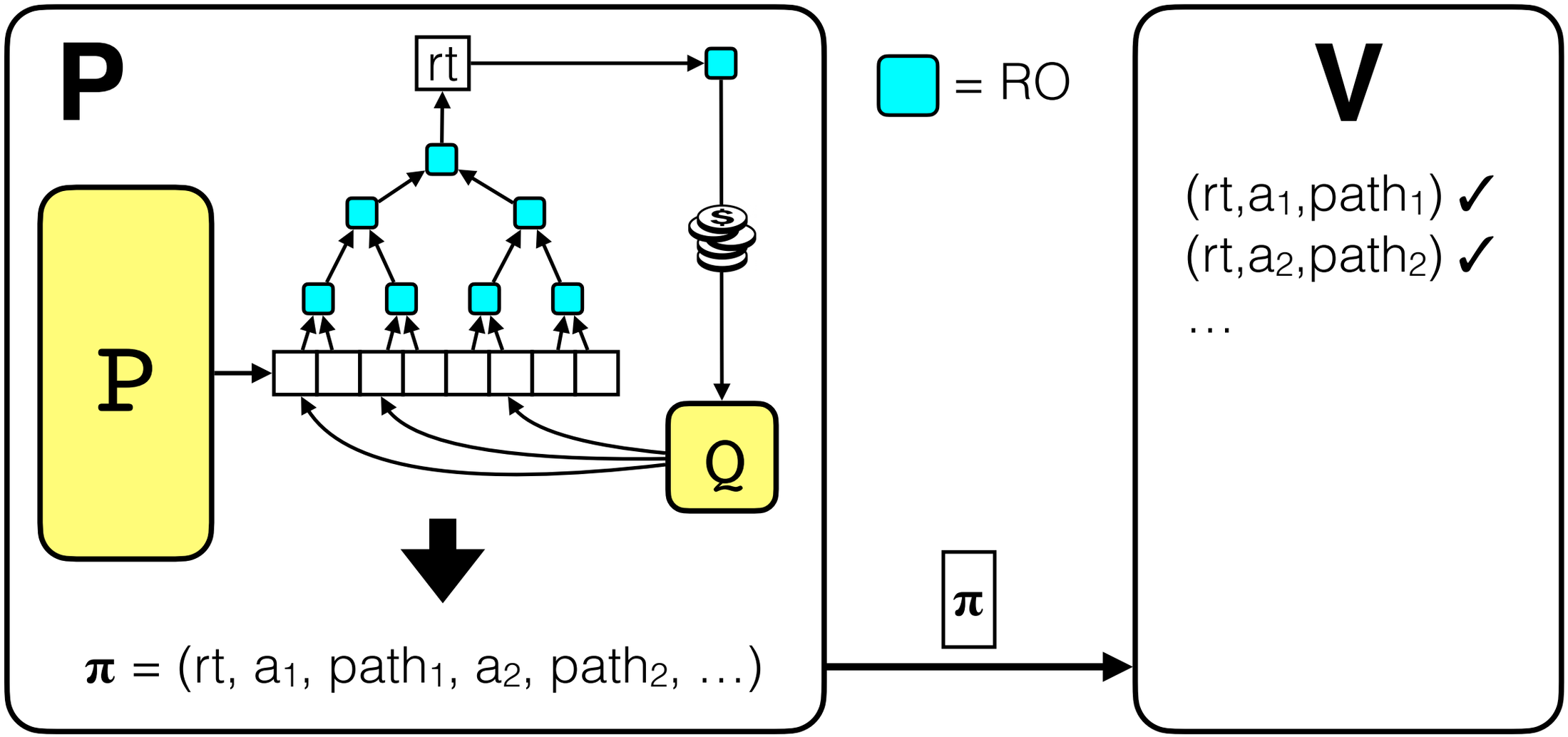

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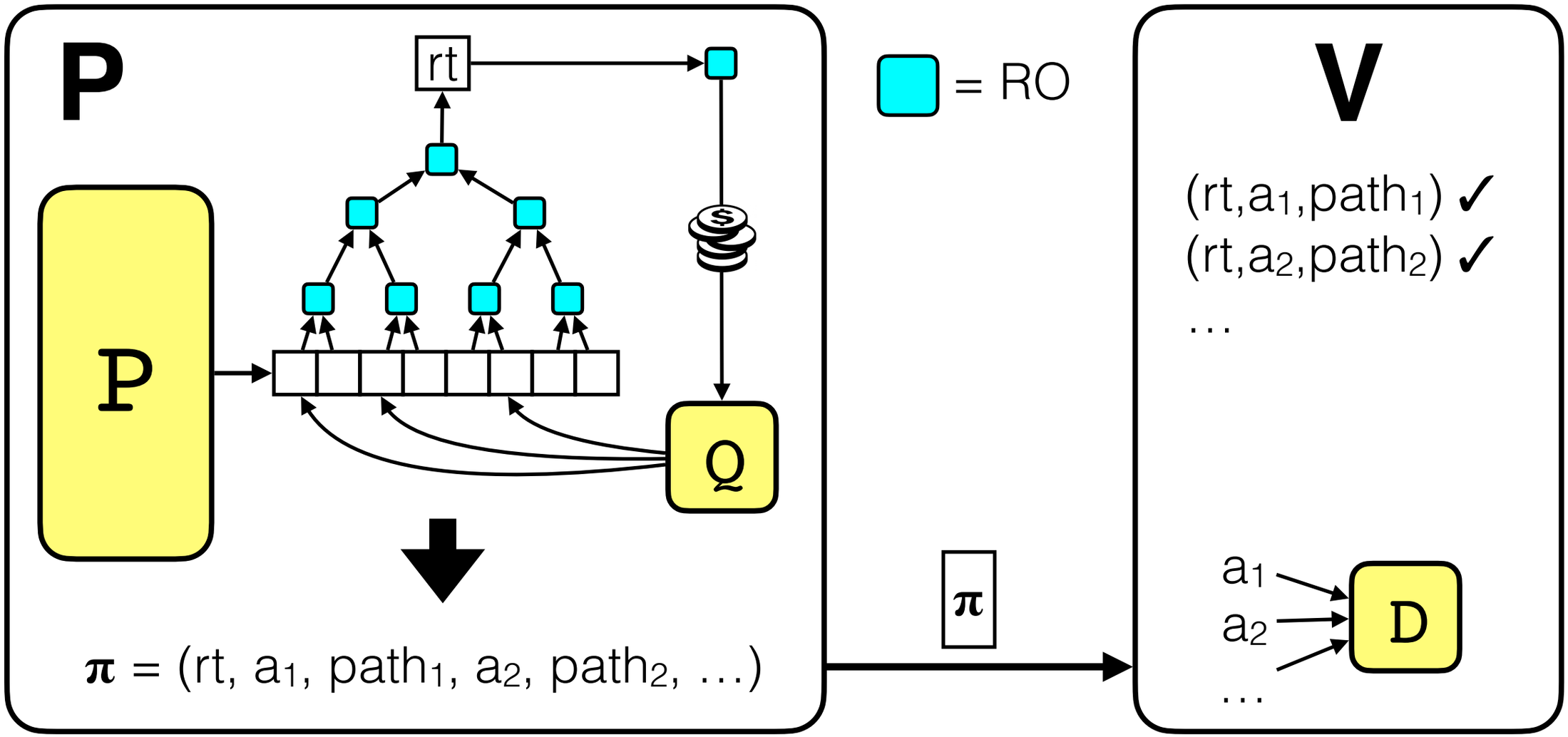

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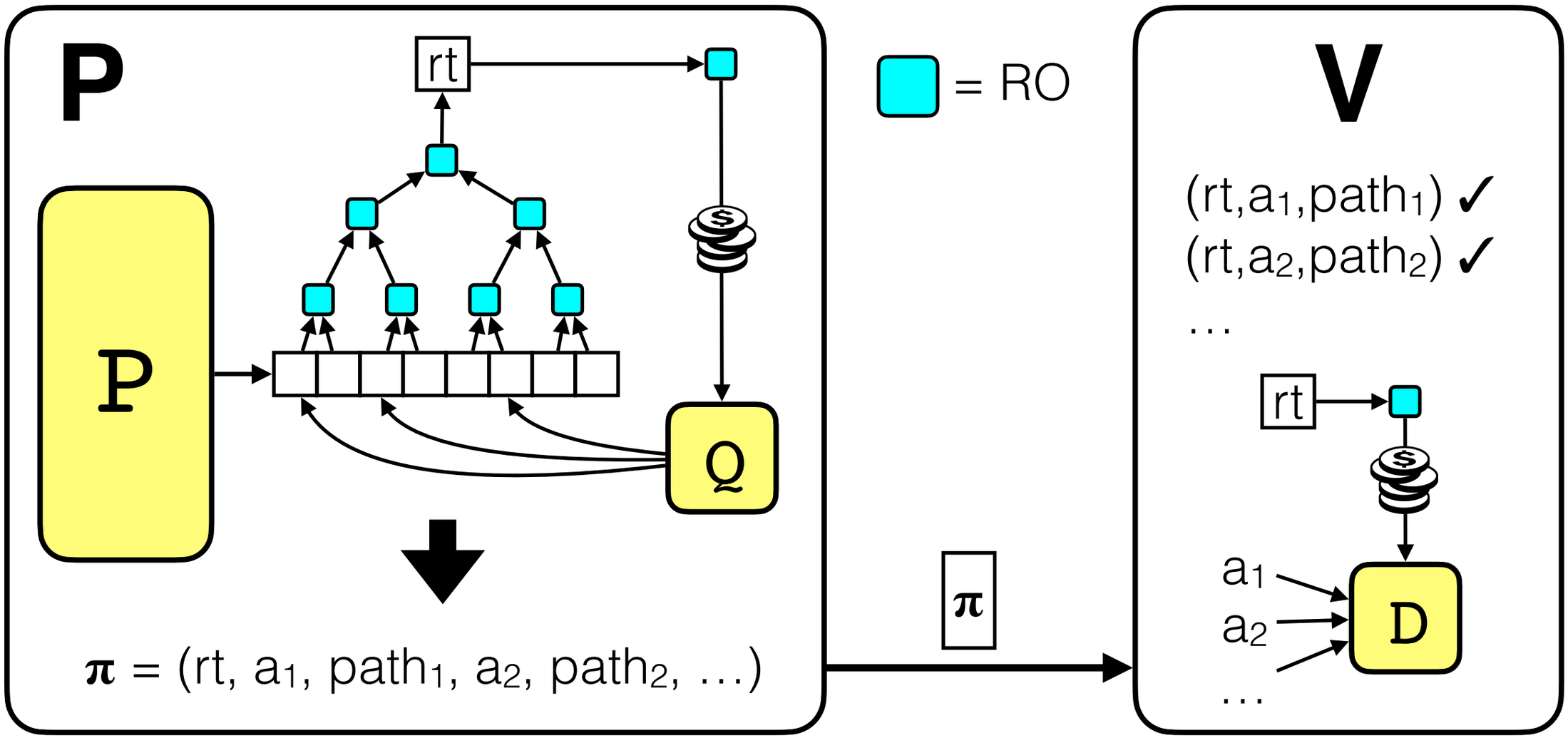

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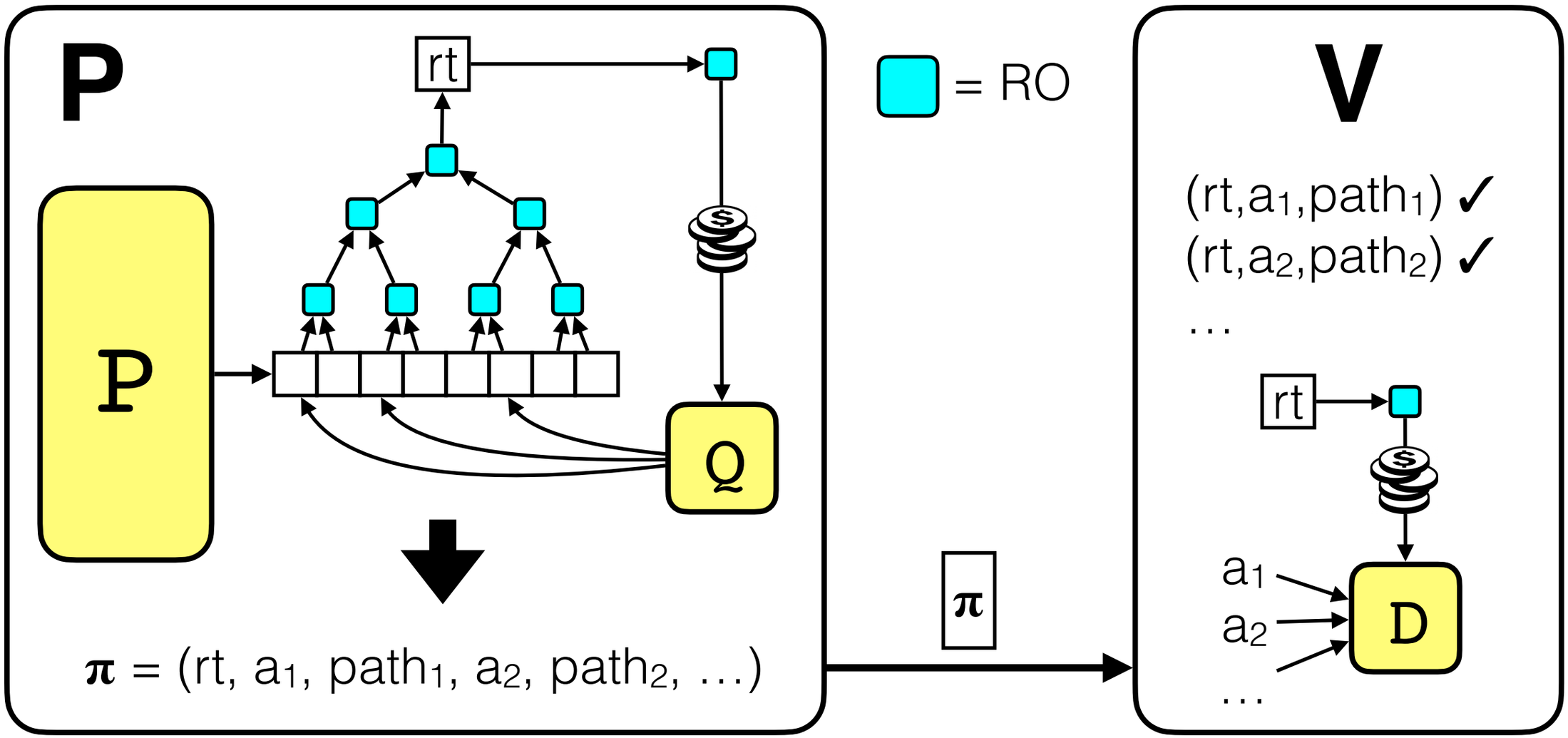

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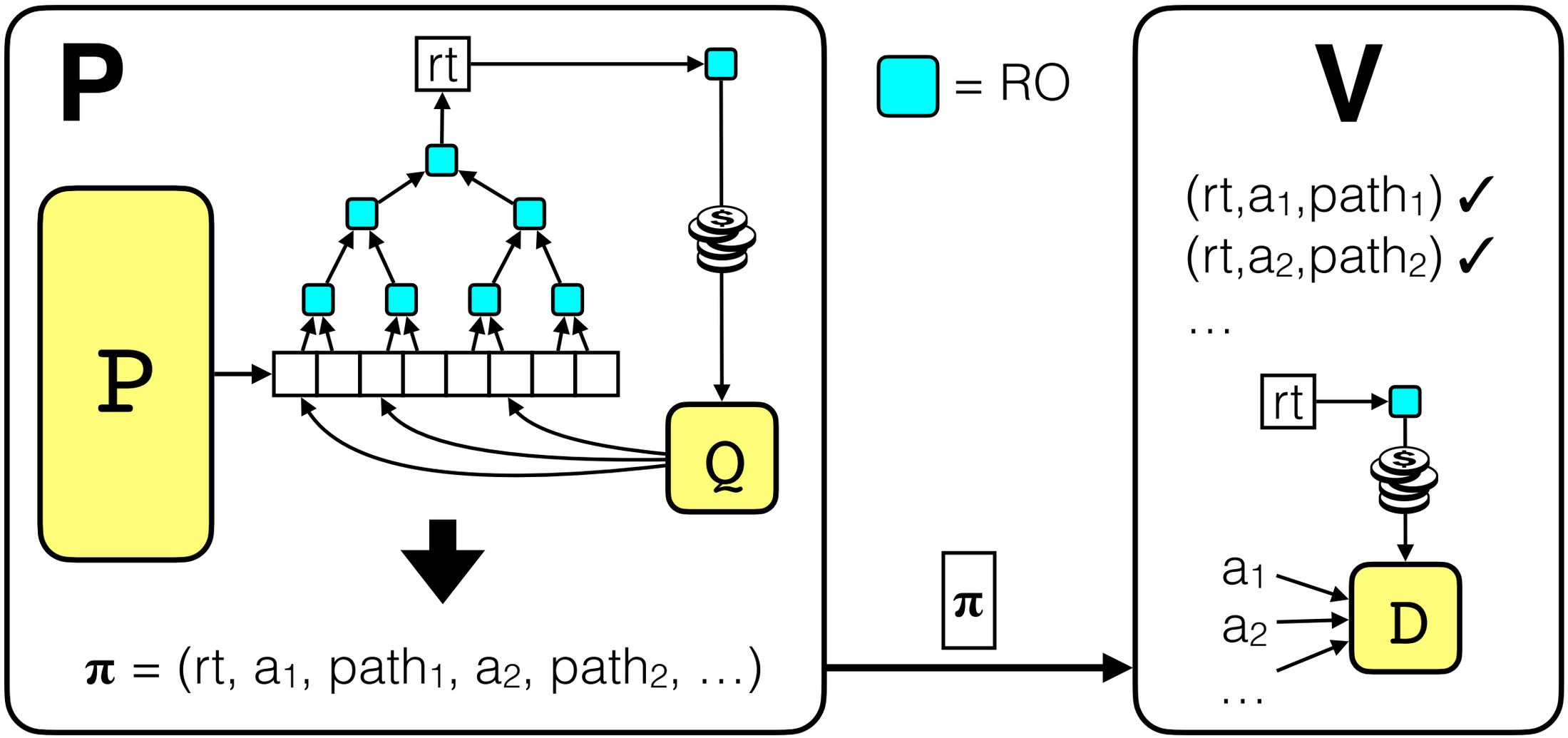

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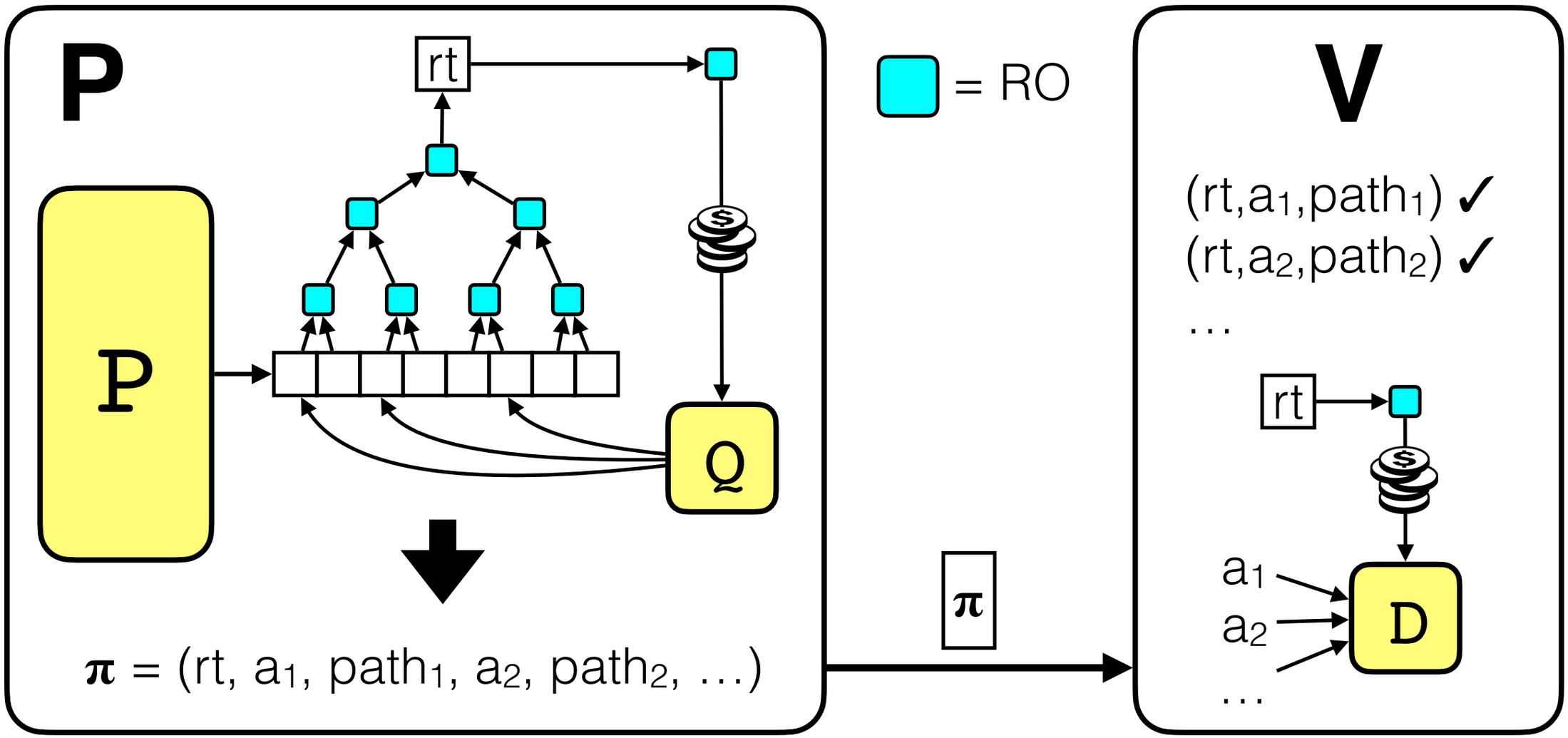

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adversary can attack PCP! needs better PCP soundness than Kilian, less efficient

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What do we do?

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Alessandro's talk: known IOPs are more efficient than known PCPs

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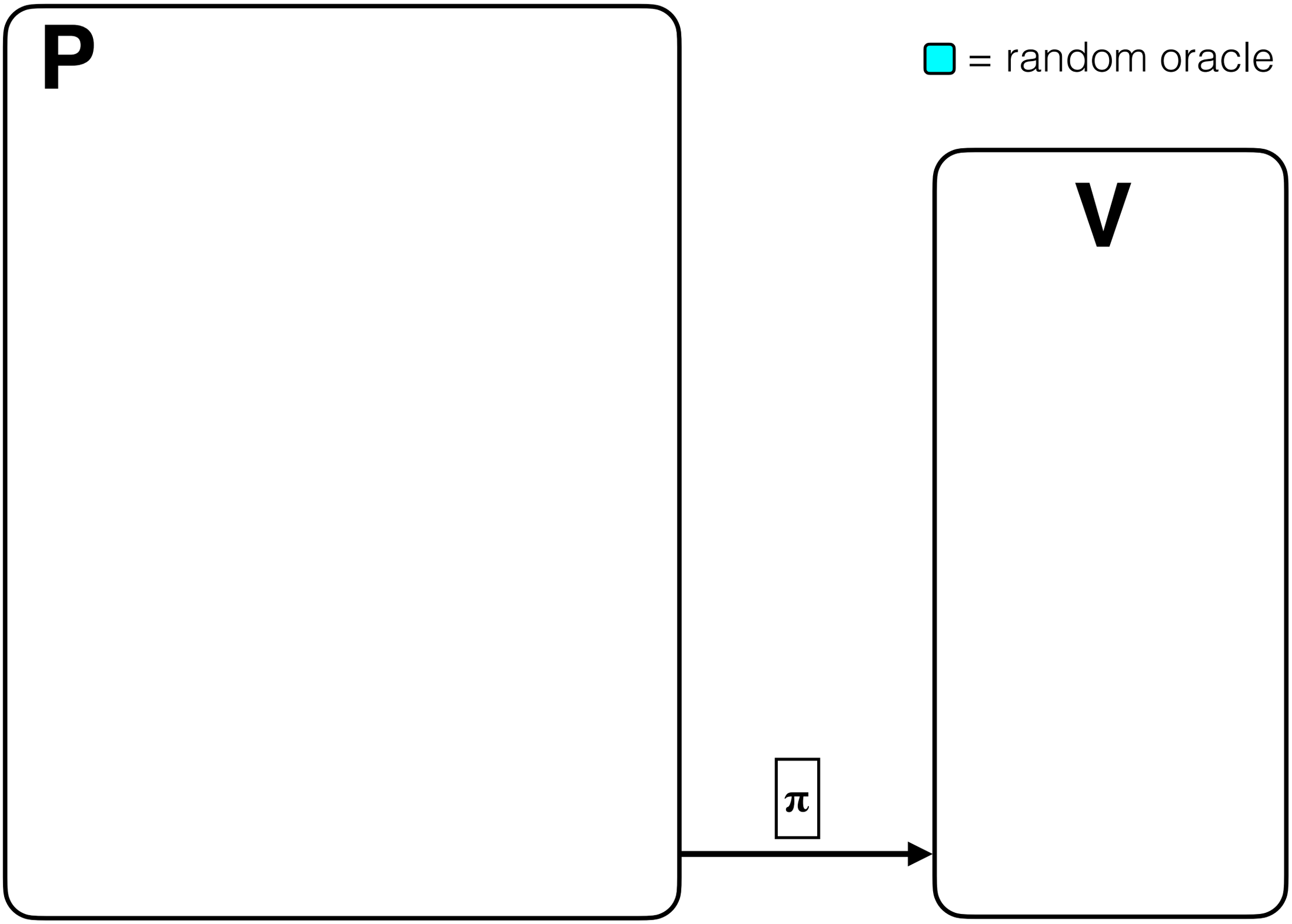

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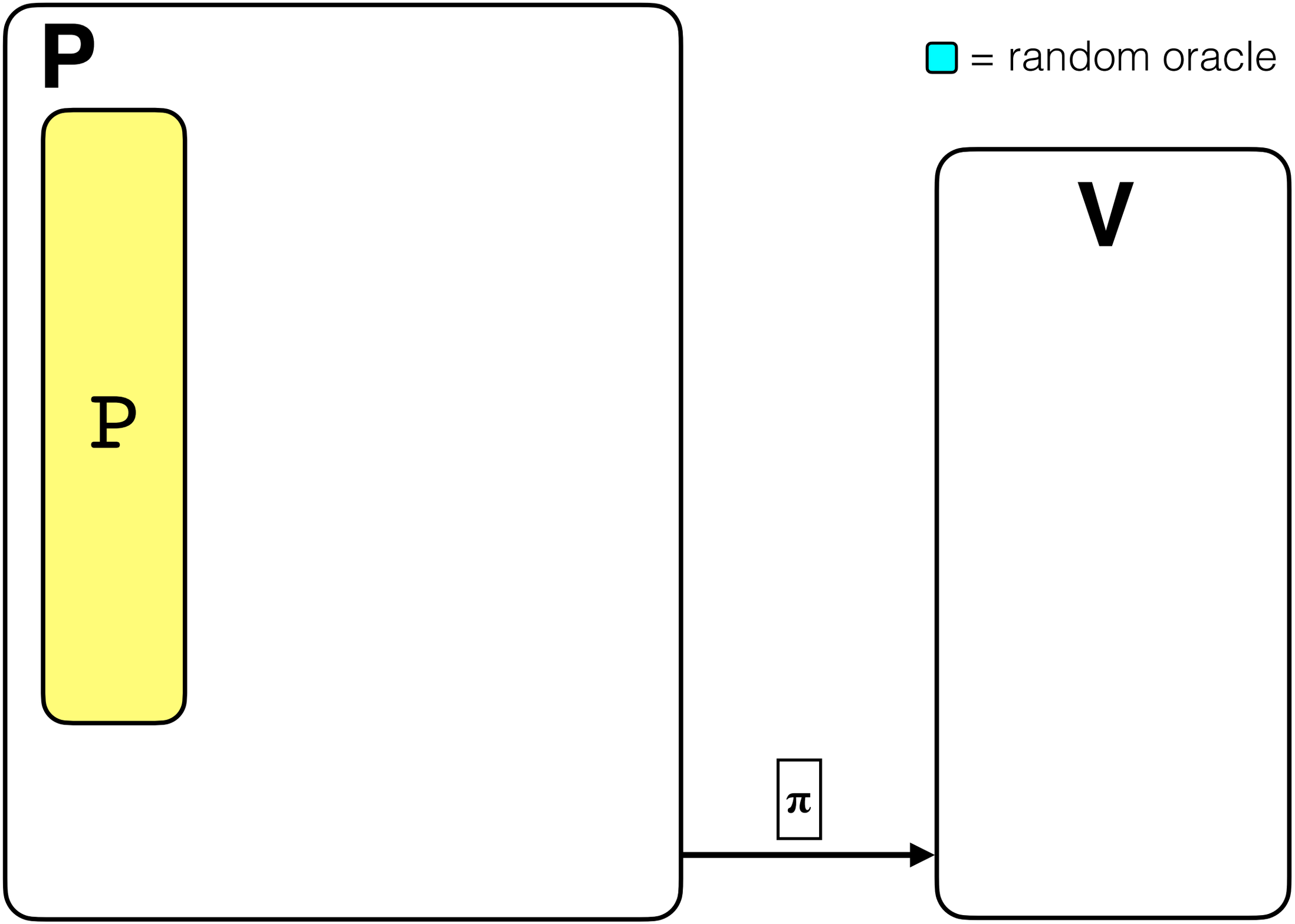

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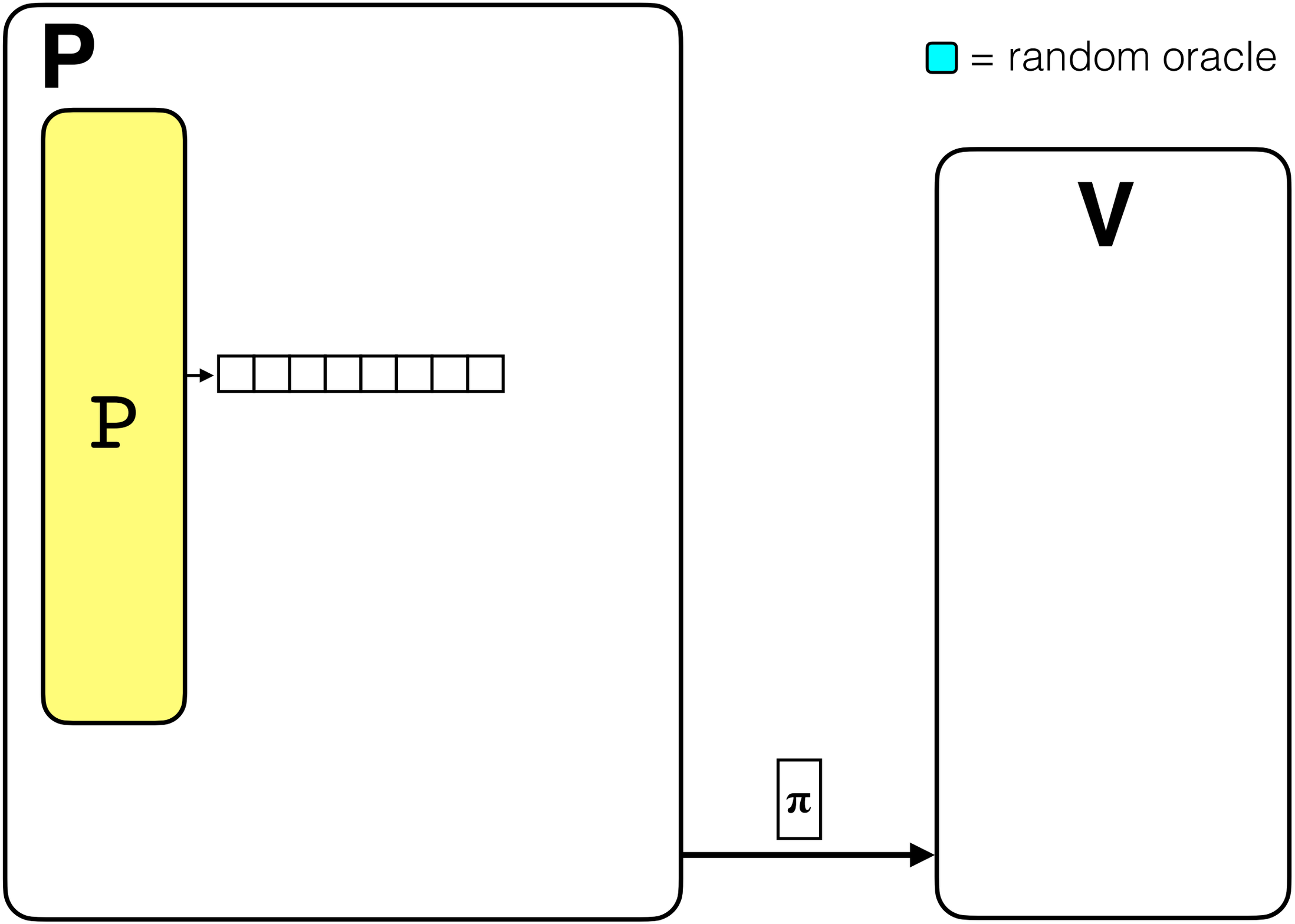

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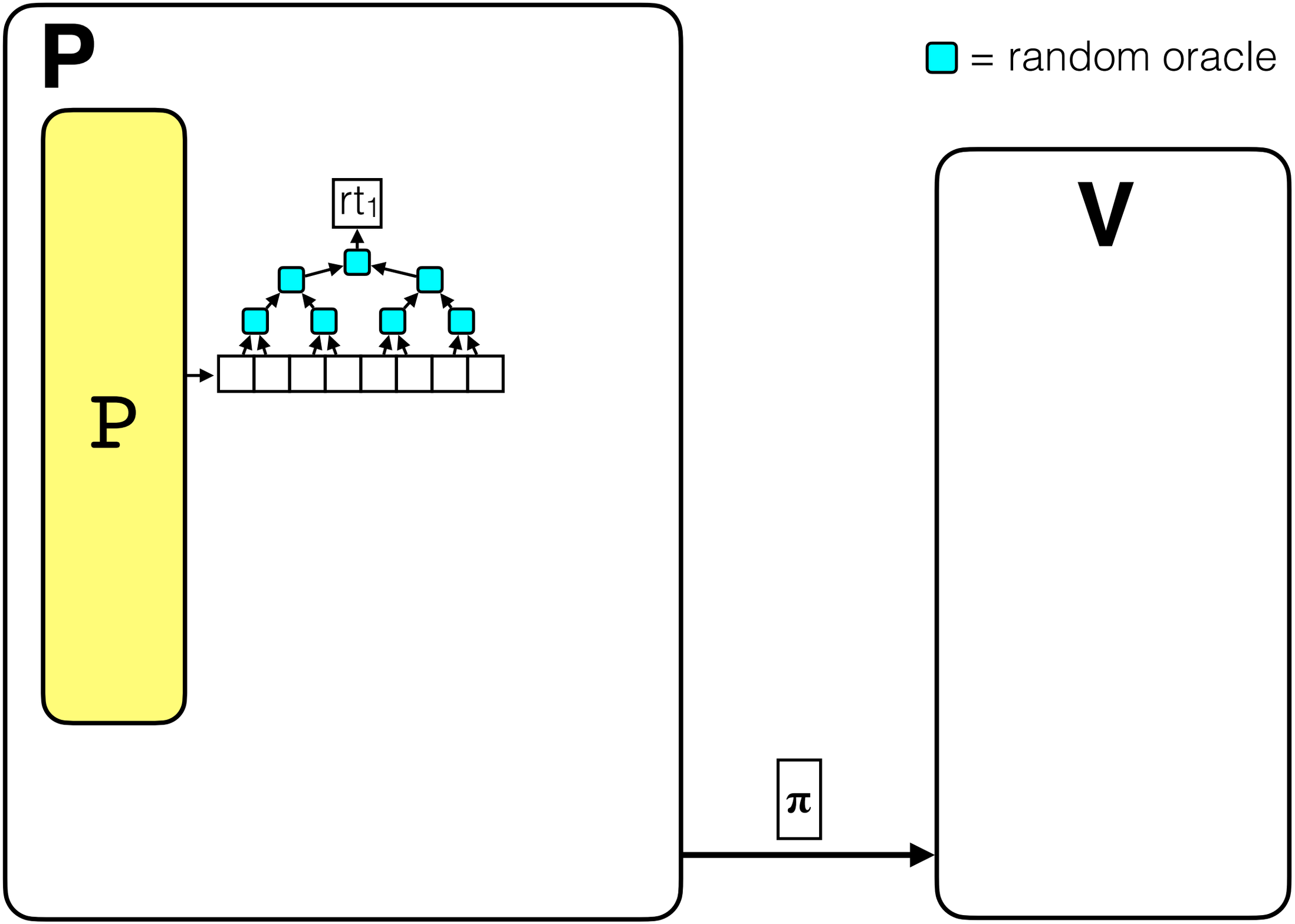

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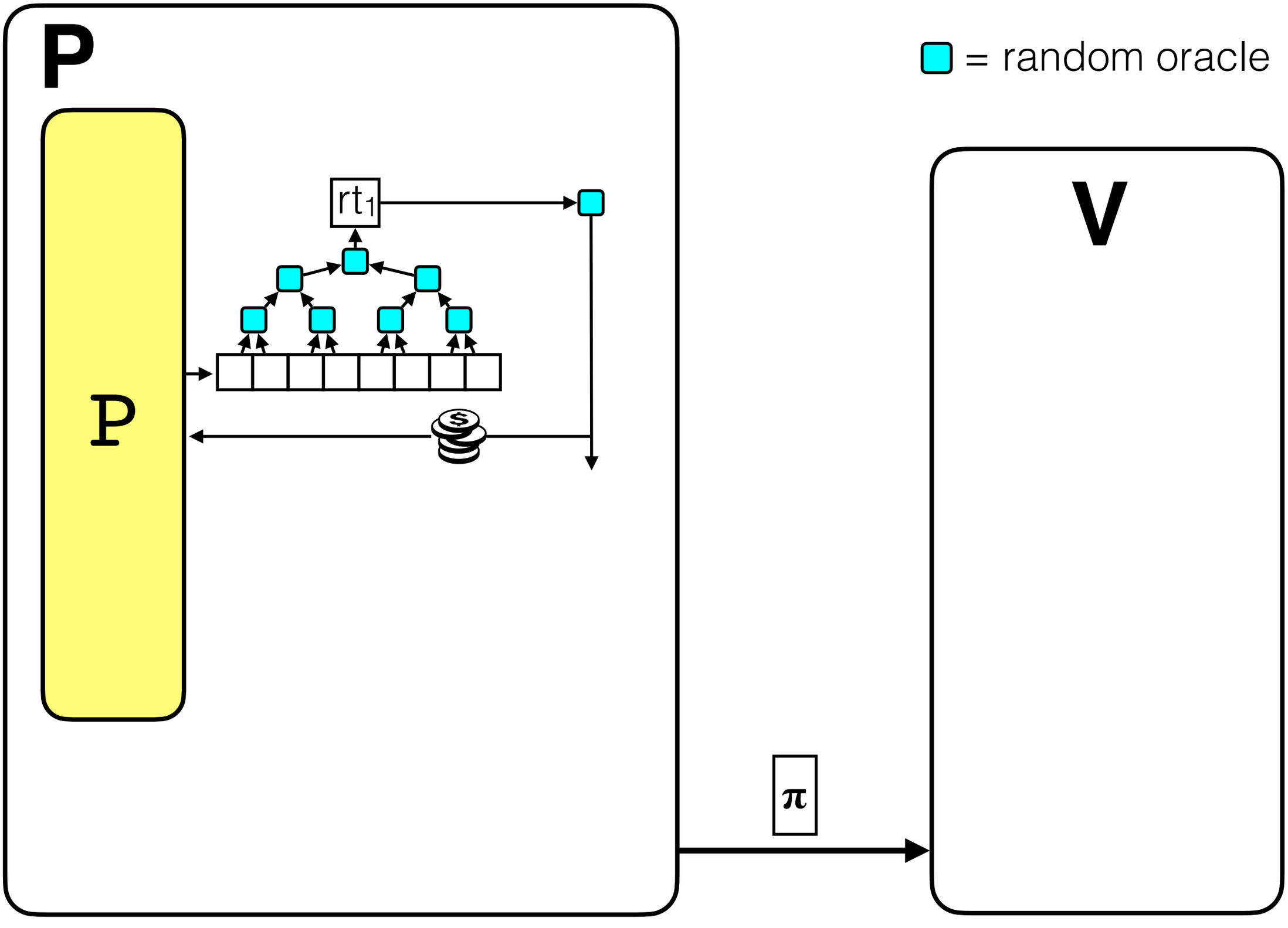

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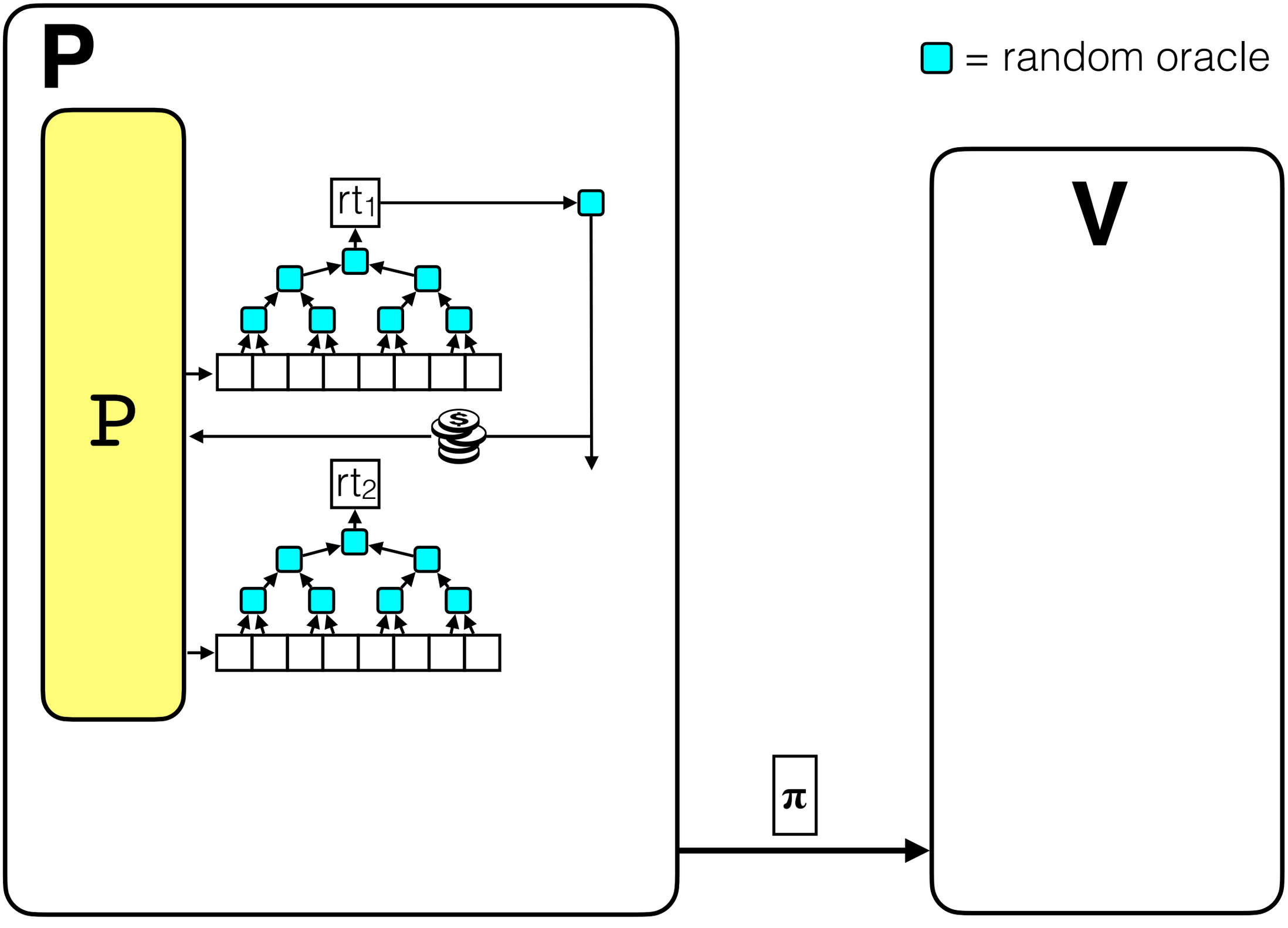

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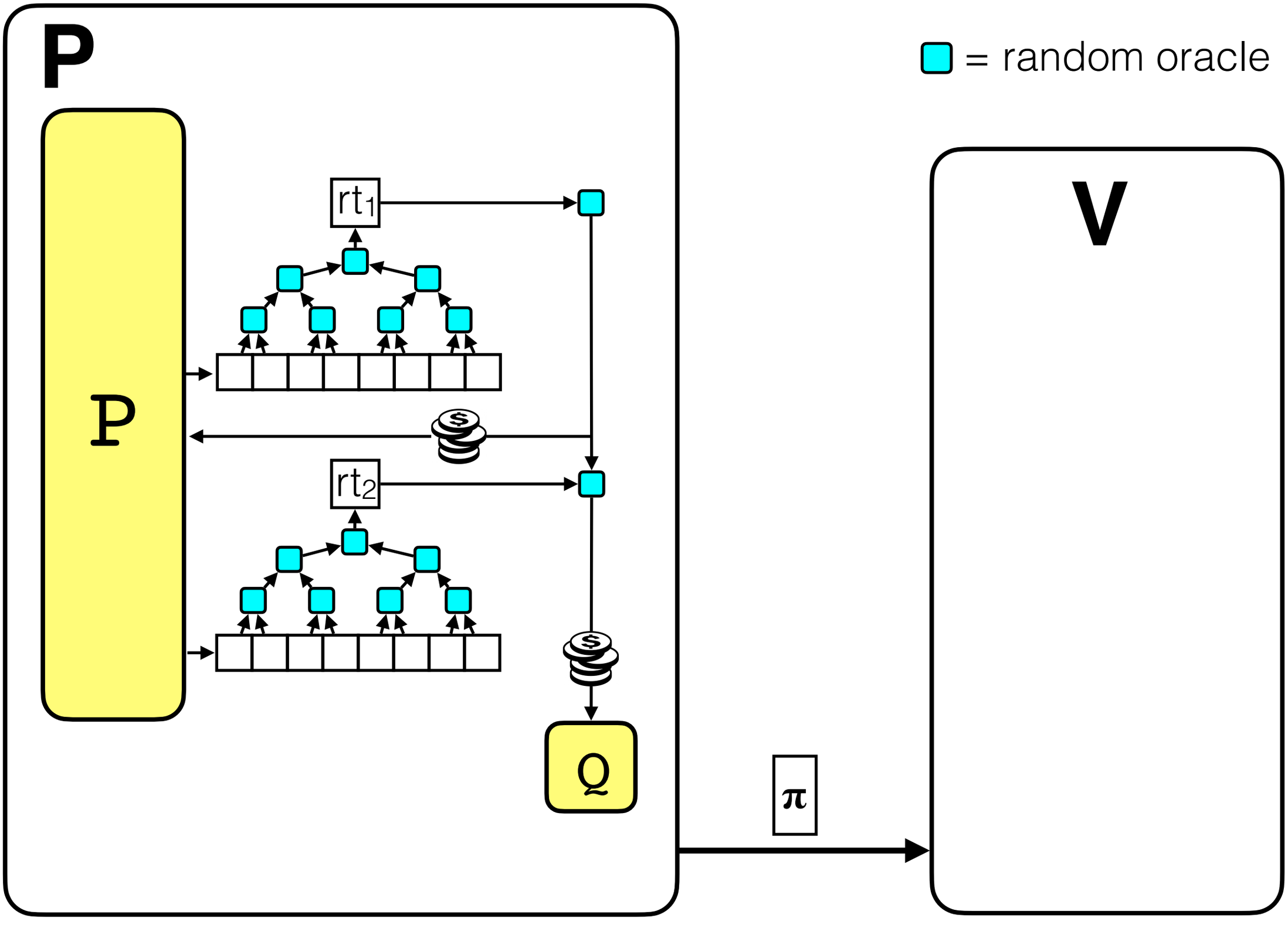

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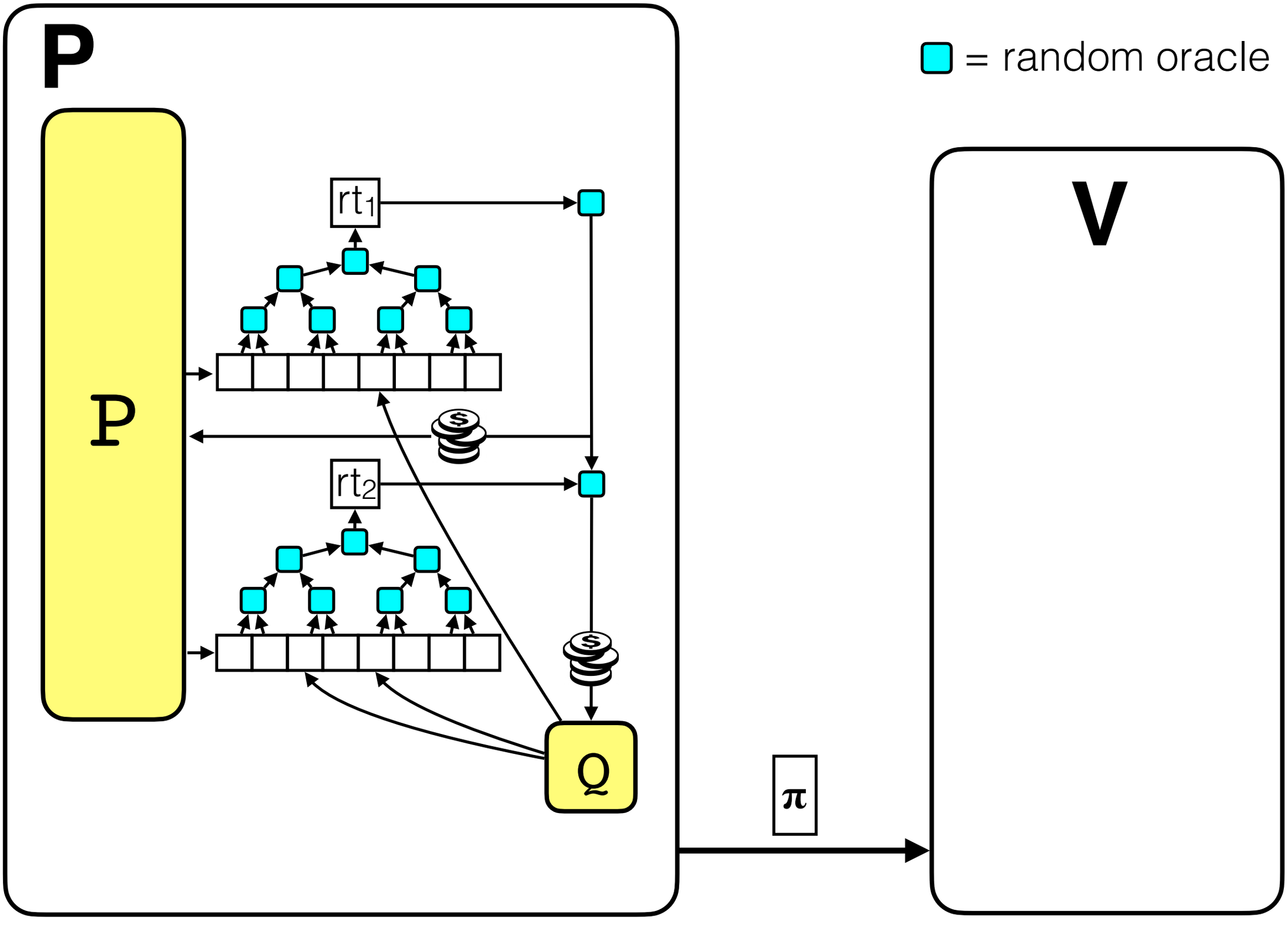

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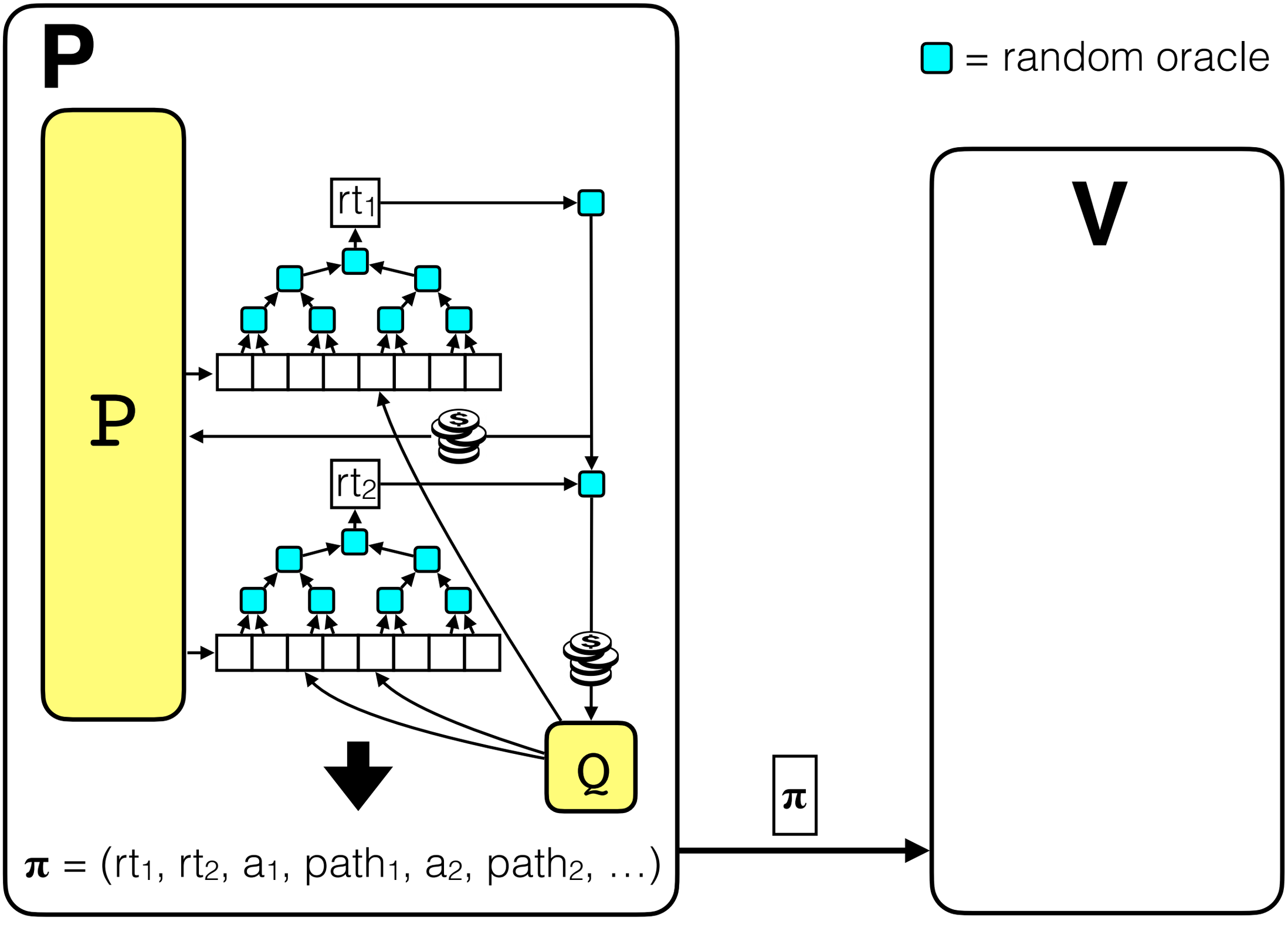

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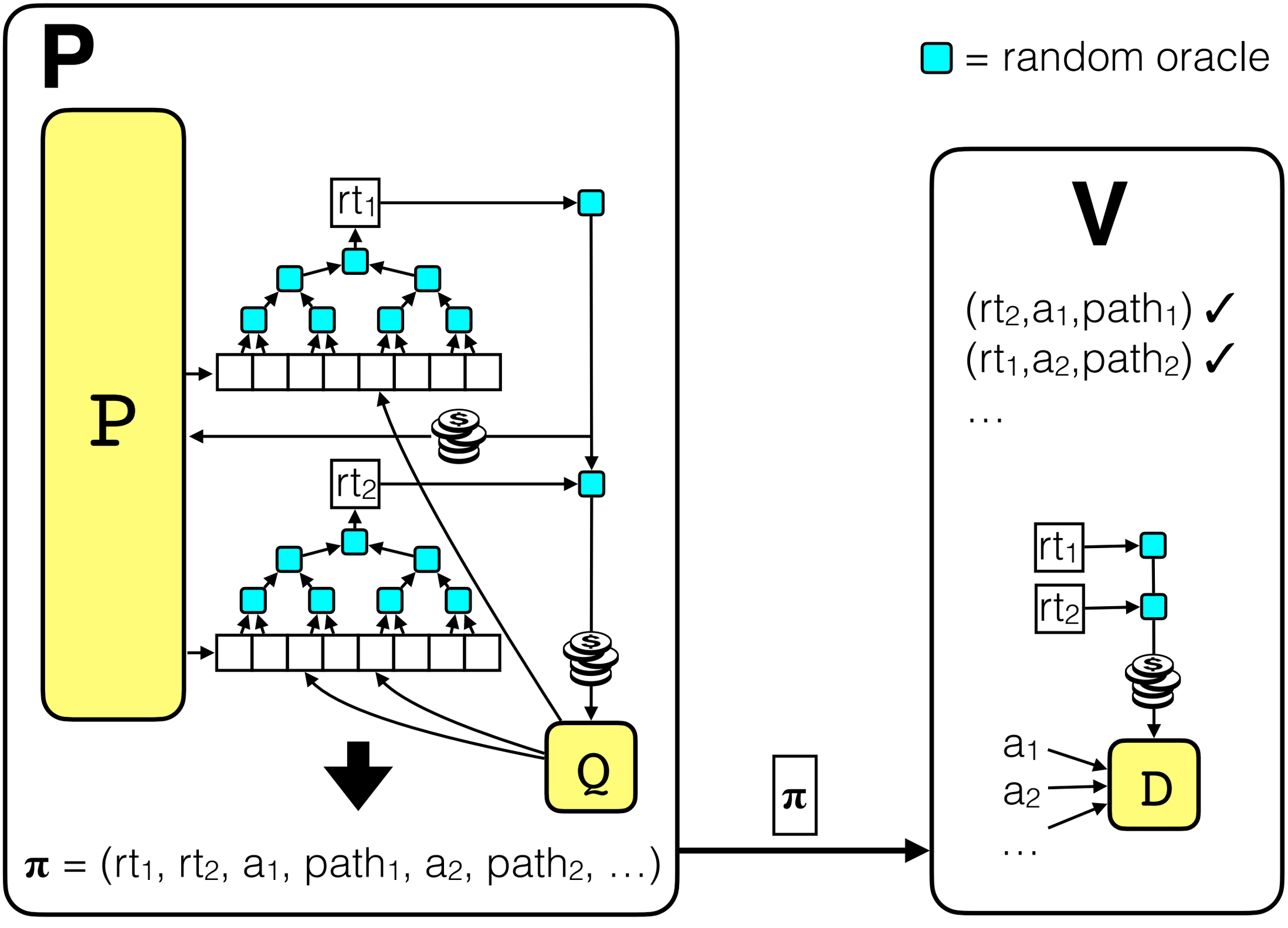

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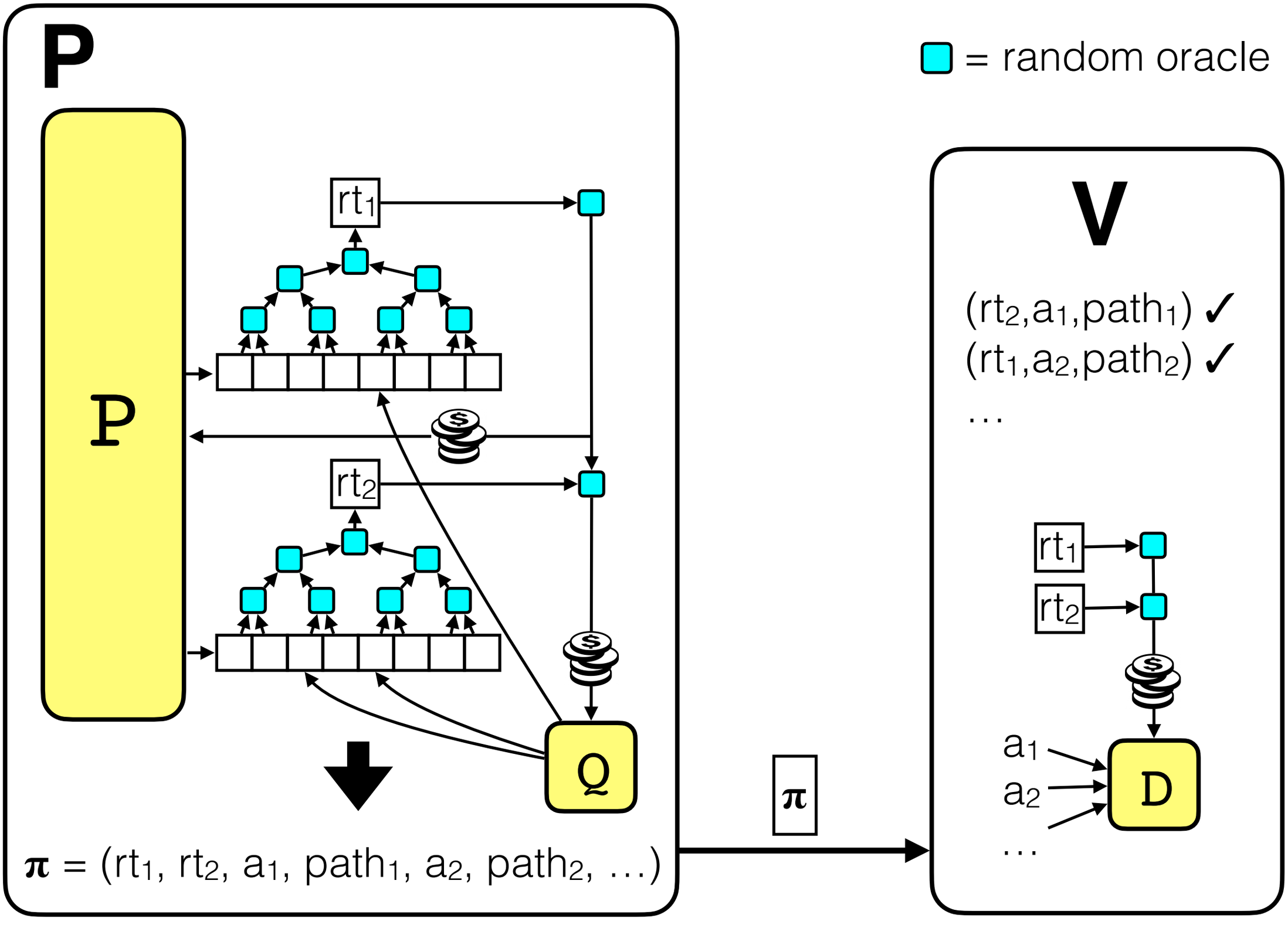

Interaction ➡️ more efficient *non-interactive* arguments

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Lemma: The transformation preserves ZK and PoK:
if IOP is ZK and PoK then you get a zkSNARK.

What about **soundness**?

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Theorem [BCS16]:
SNARG soundness = IOP **state restoration soundness**

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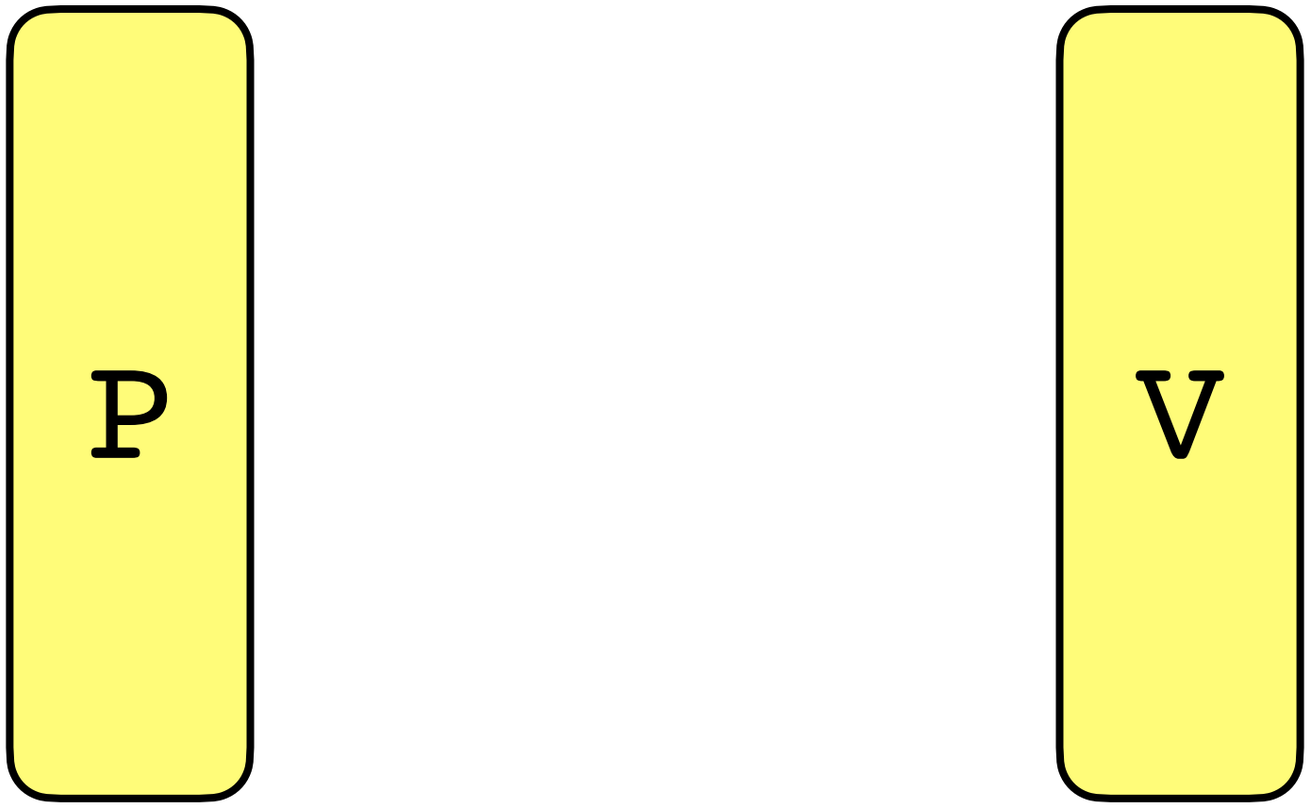

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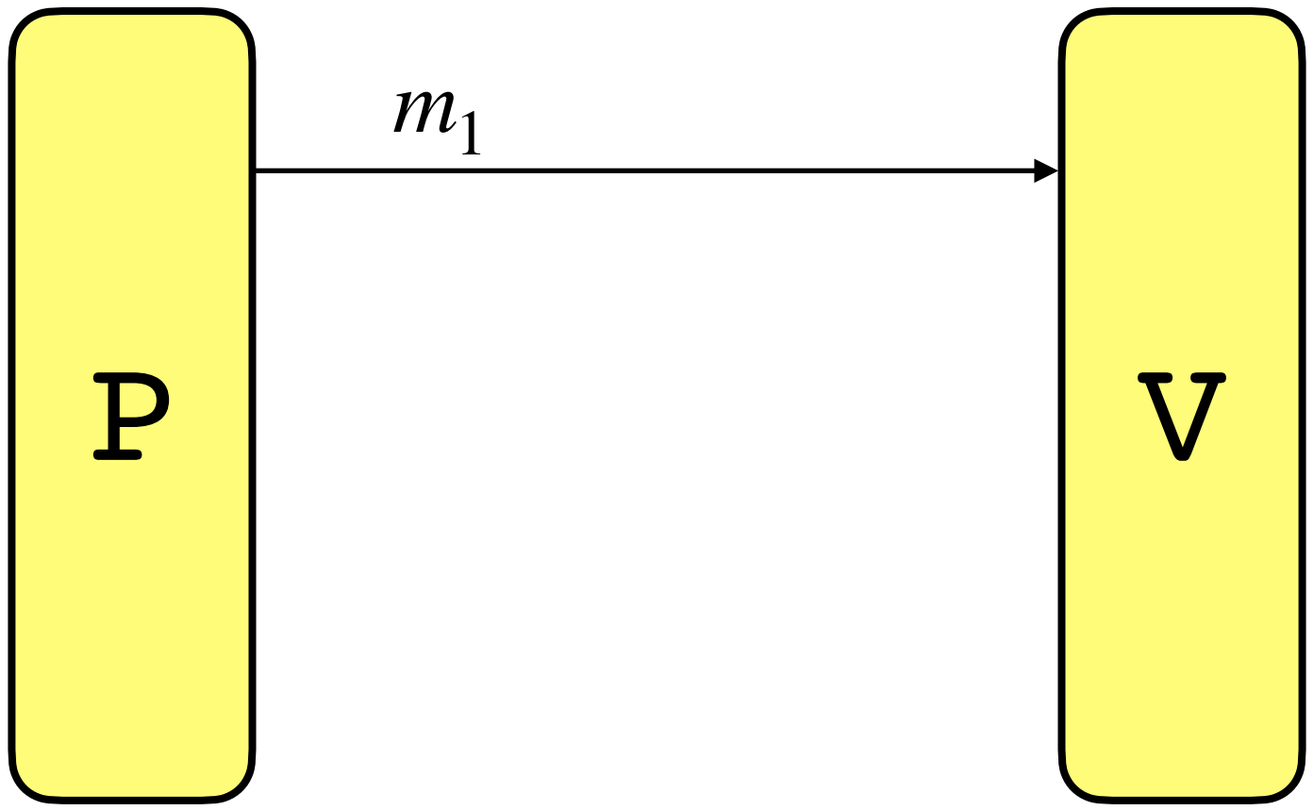

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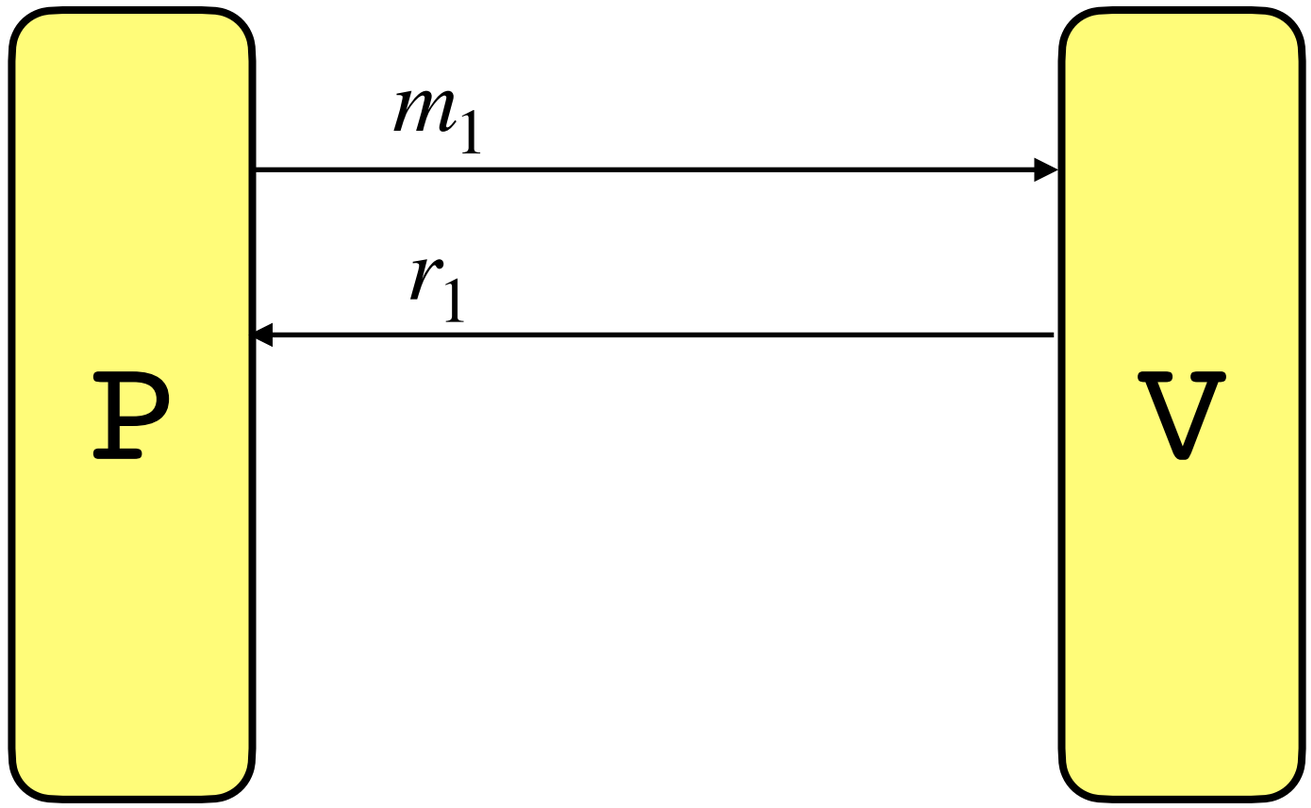

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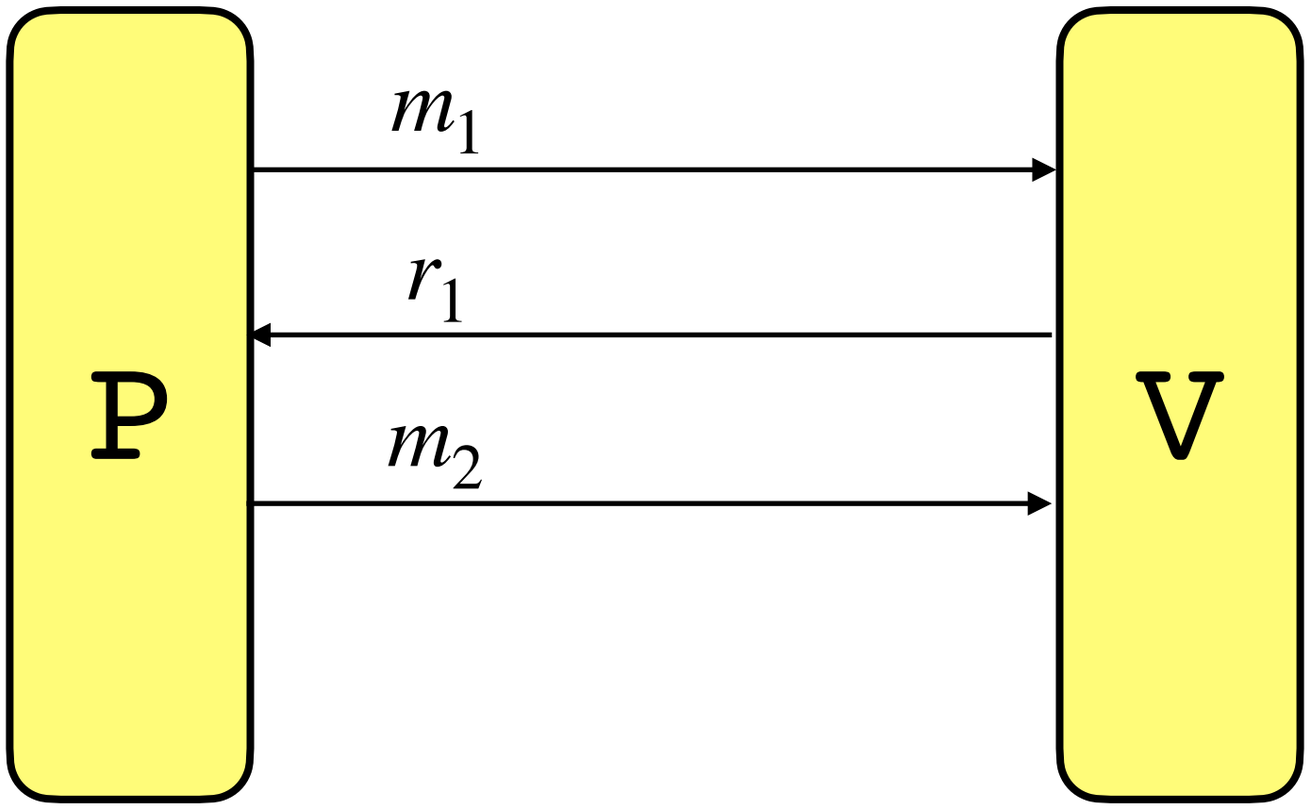

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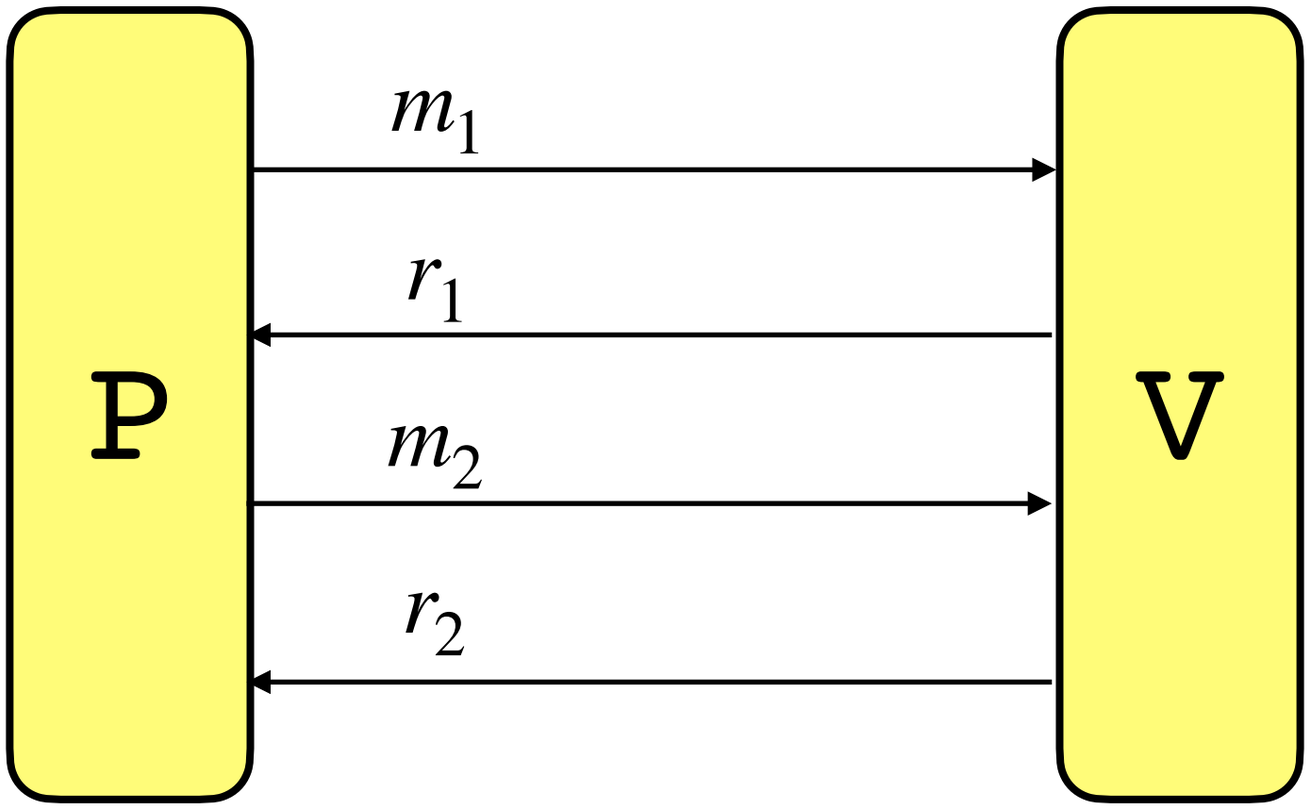

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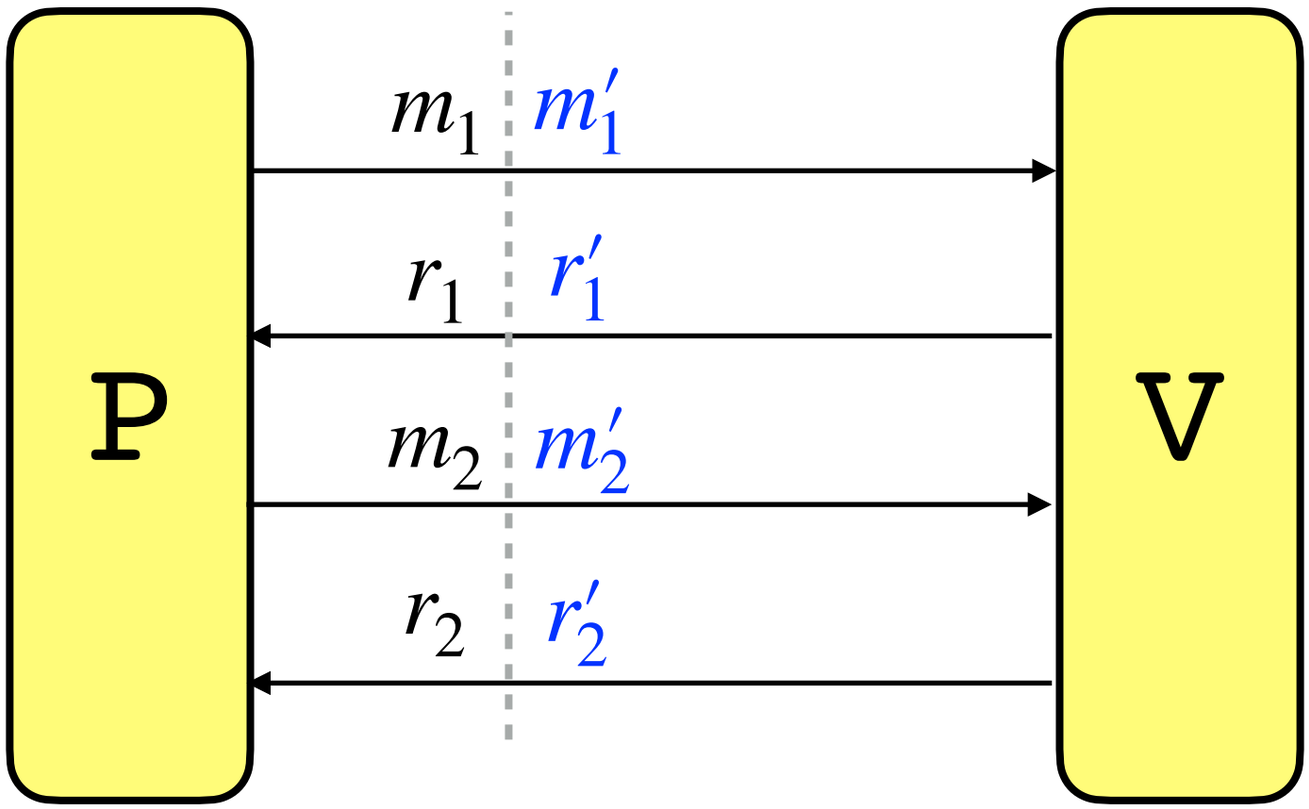

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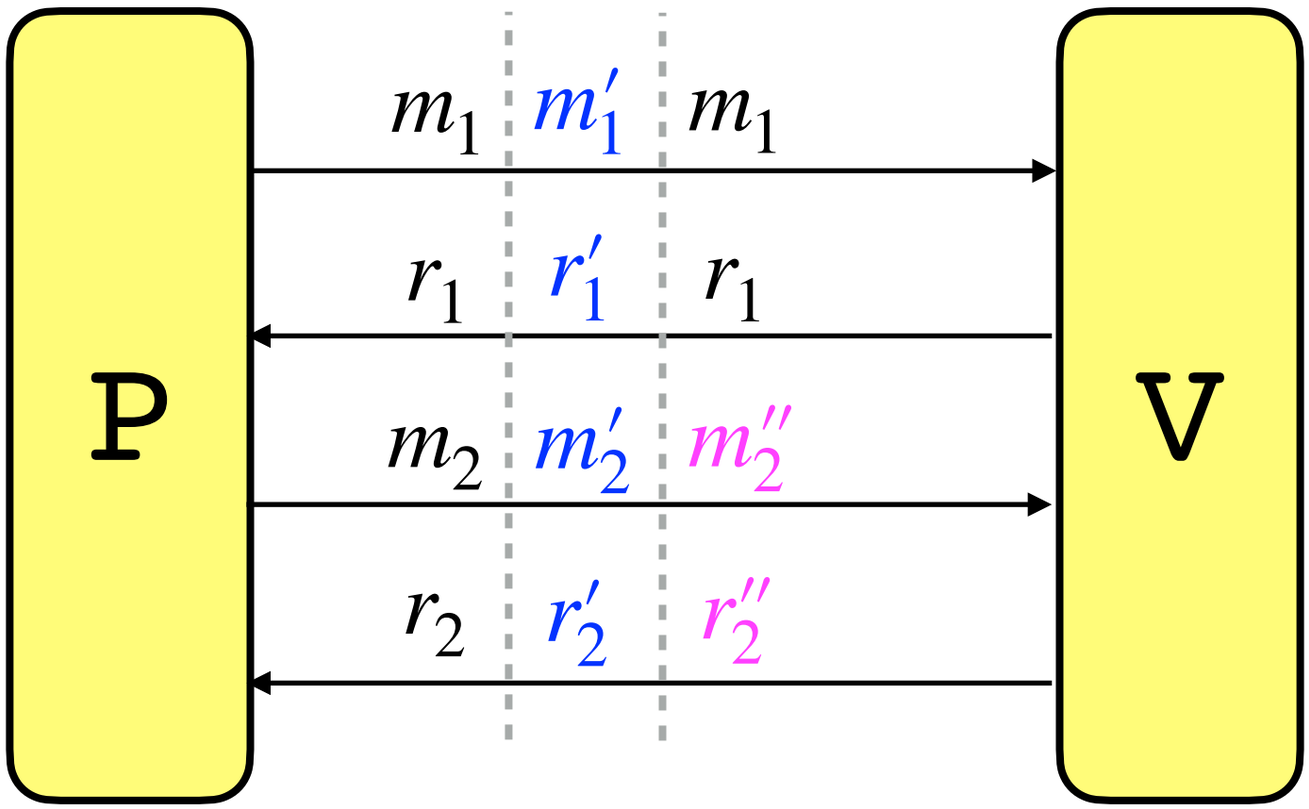

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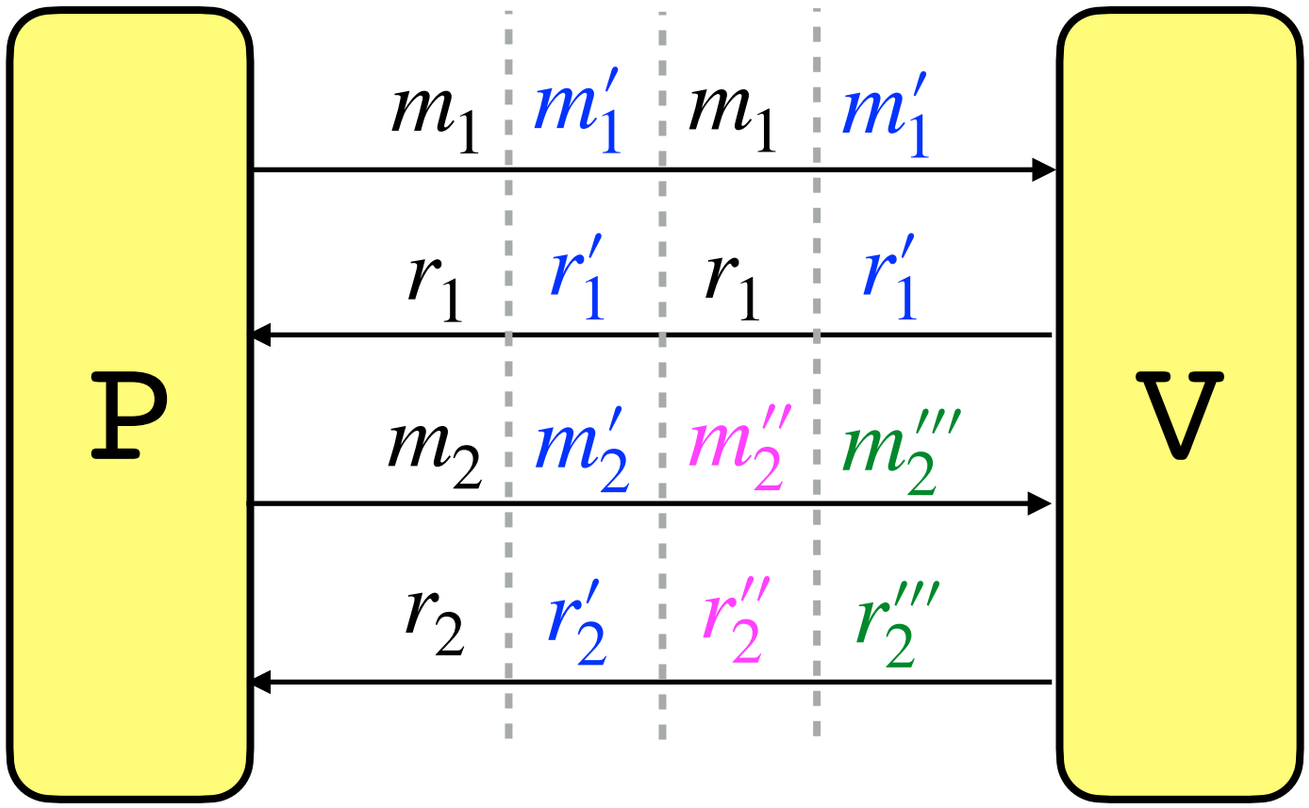

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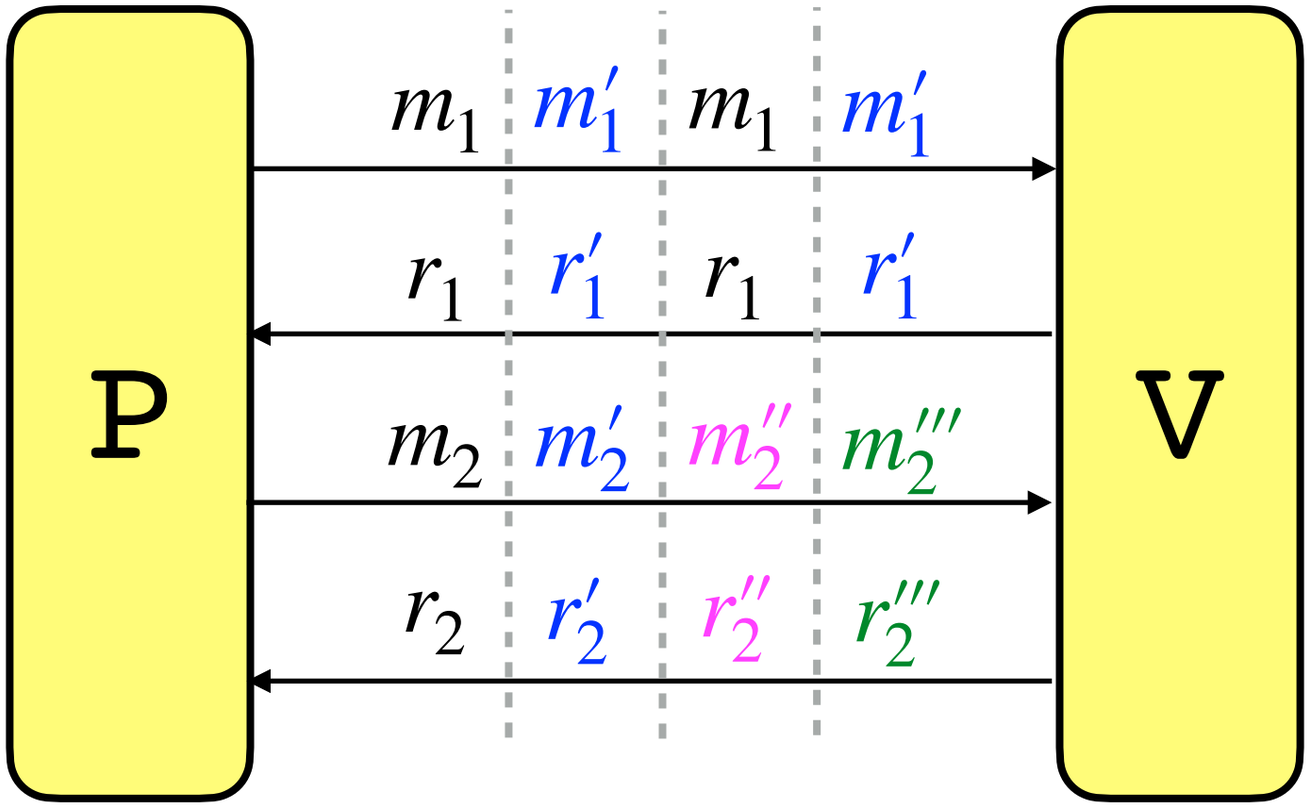

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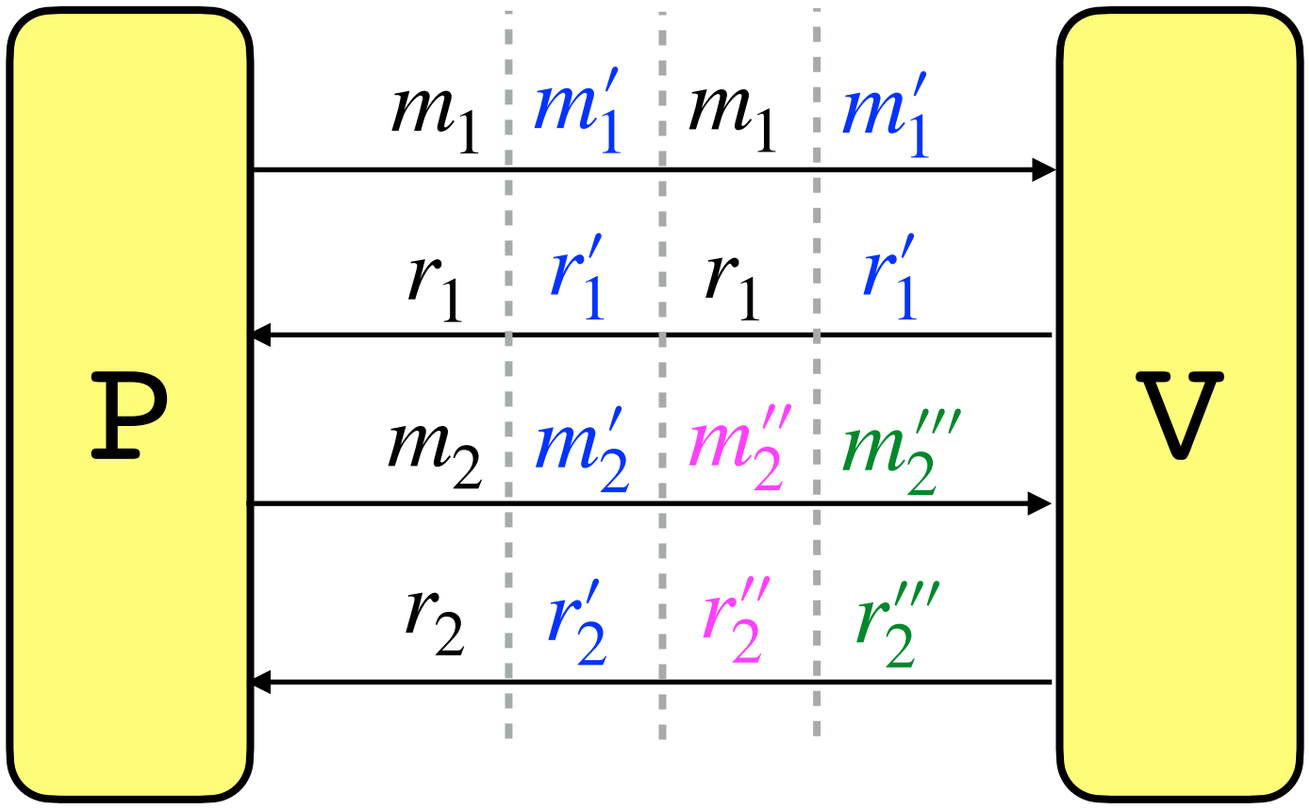

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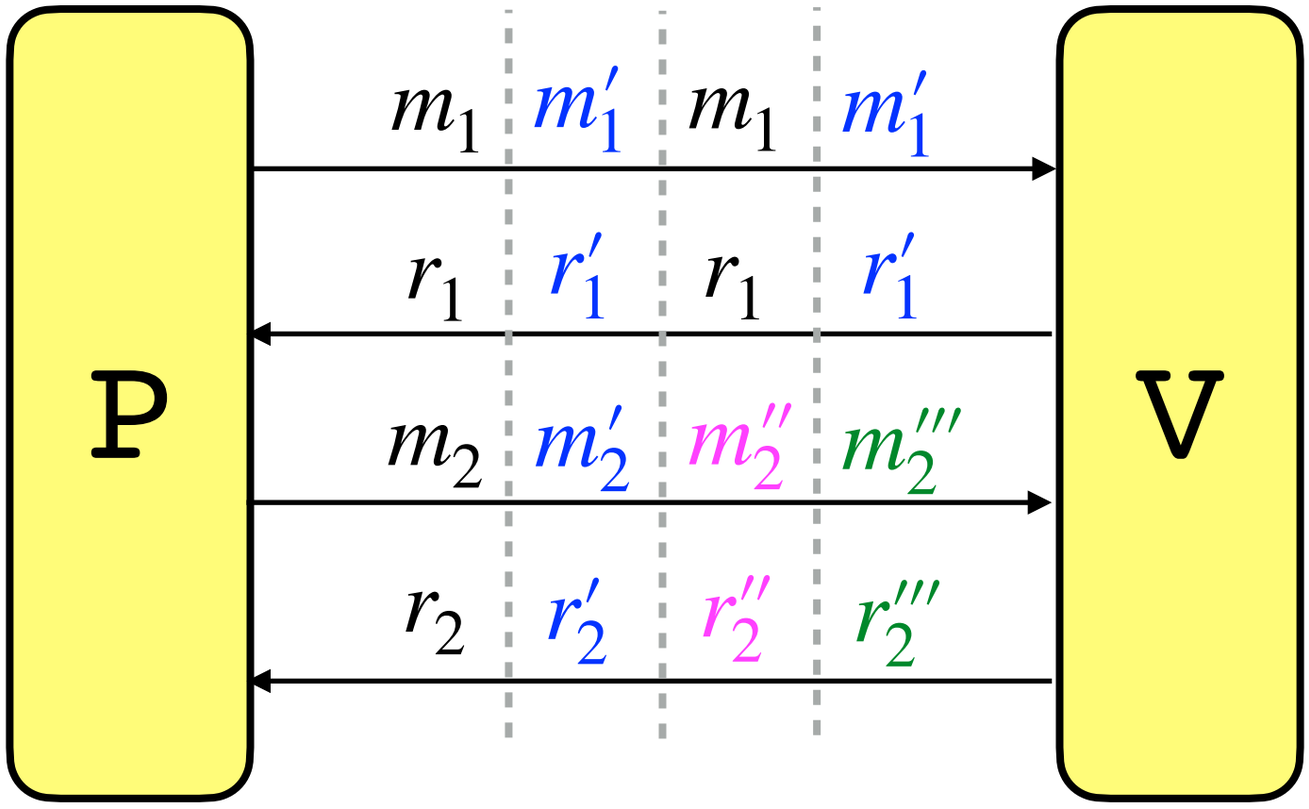

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Thankfully, widely-used protocols have good SR soundness!
(**but** it is still important to prove it [AFK21])

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GOOD

BAD

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- random oracles don't exist*

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recall: today IOPs are much more efficient than PCPs!

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asymptotic
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asymptotic improvements in IOP protocols ⟶ concrete improvements in IOP protocols

⟵

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FRONTIER	**10 kB? 1 kB?**	**???**

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[CY21]: can do a bit better than $O(\kappa^2)$ by optimizing Micali

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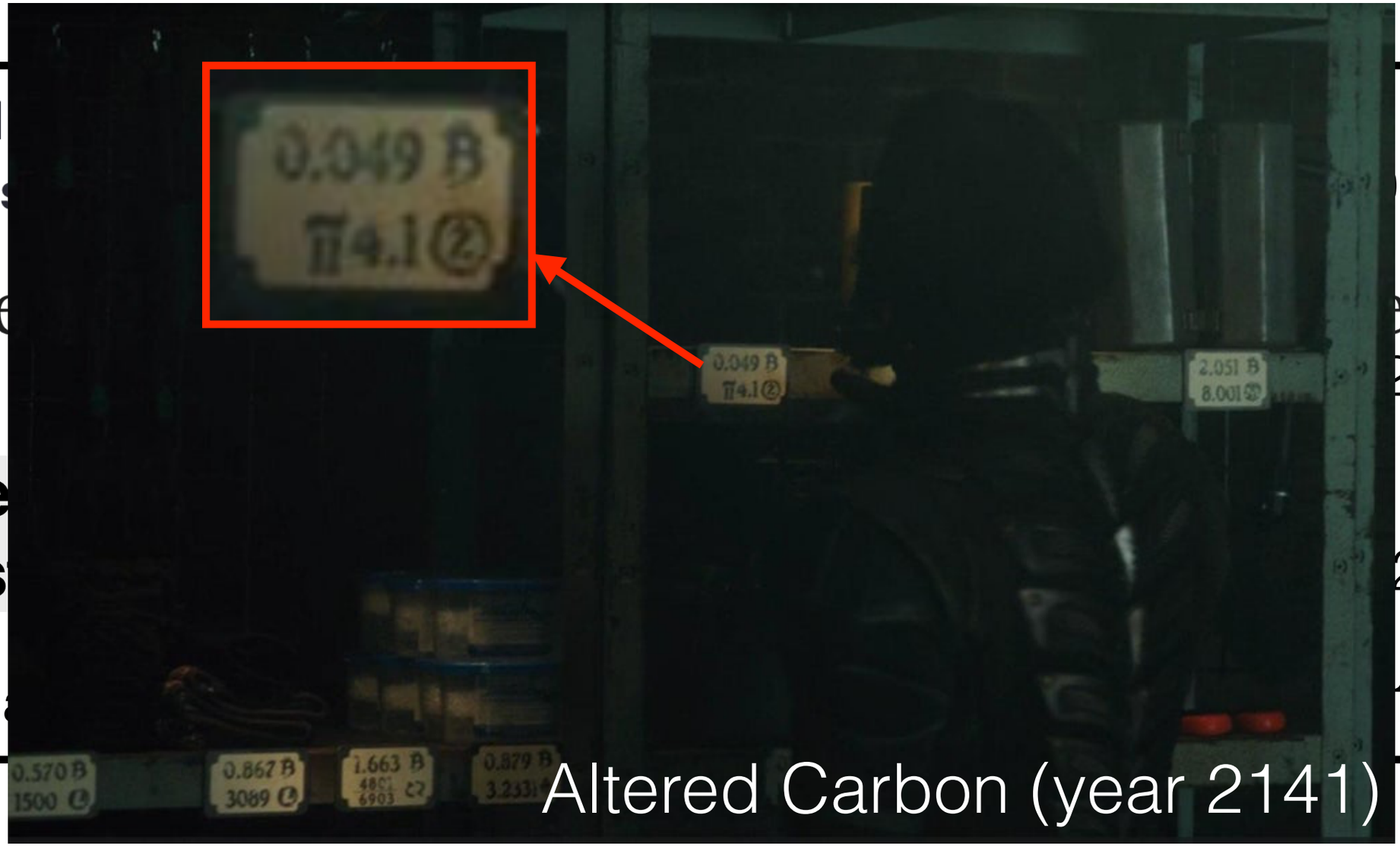
NIST's call for post-quantum cryptography **CLOSED** in 2017.11.30.

Post-Quantum Security

Altered Carbon (year 2141)

Post-Quantum Security



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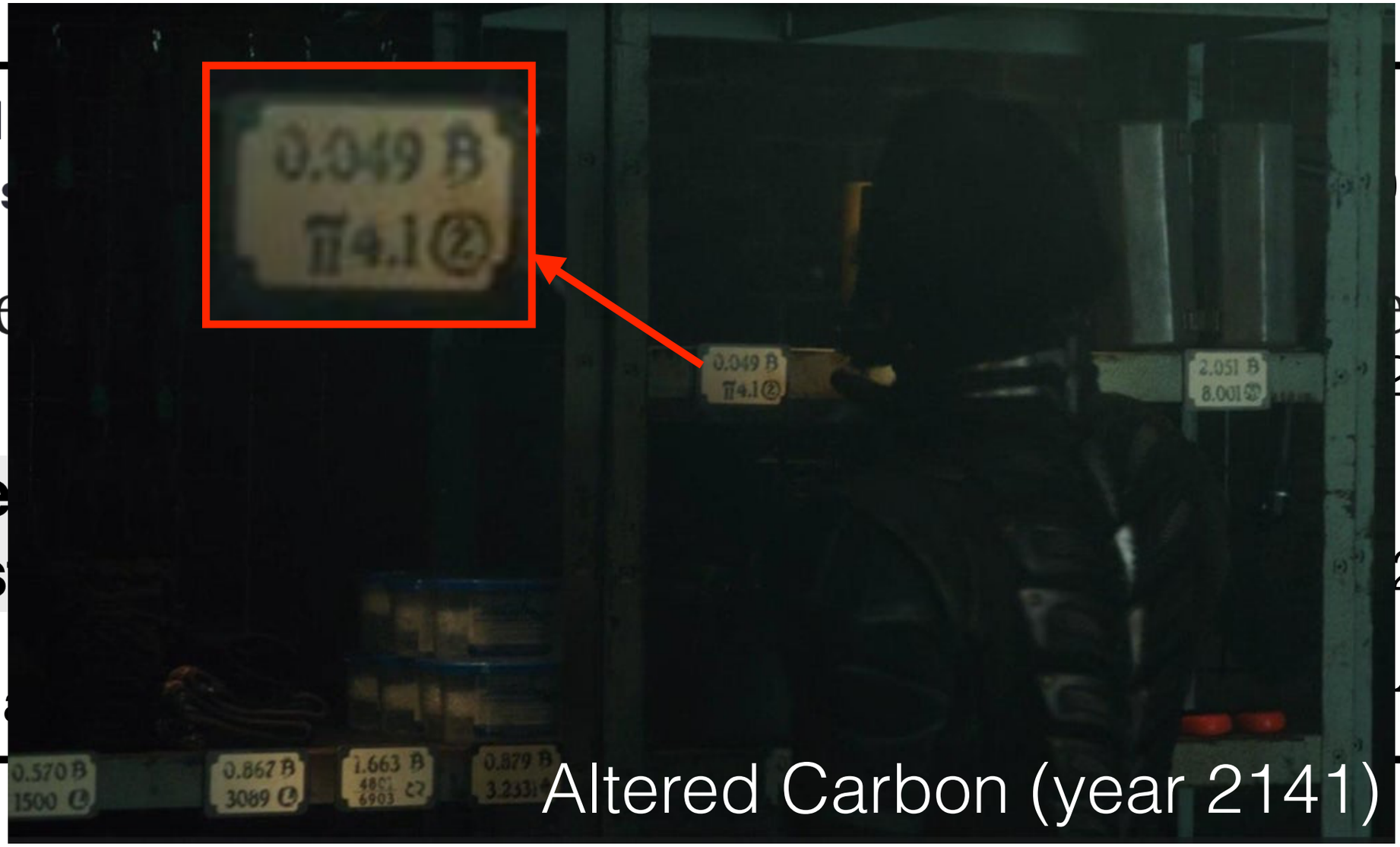

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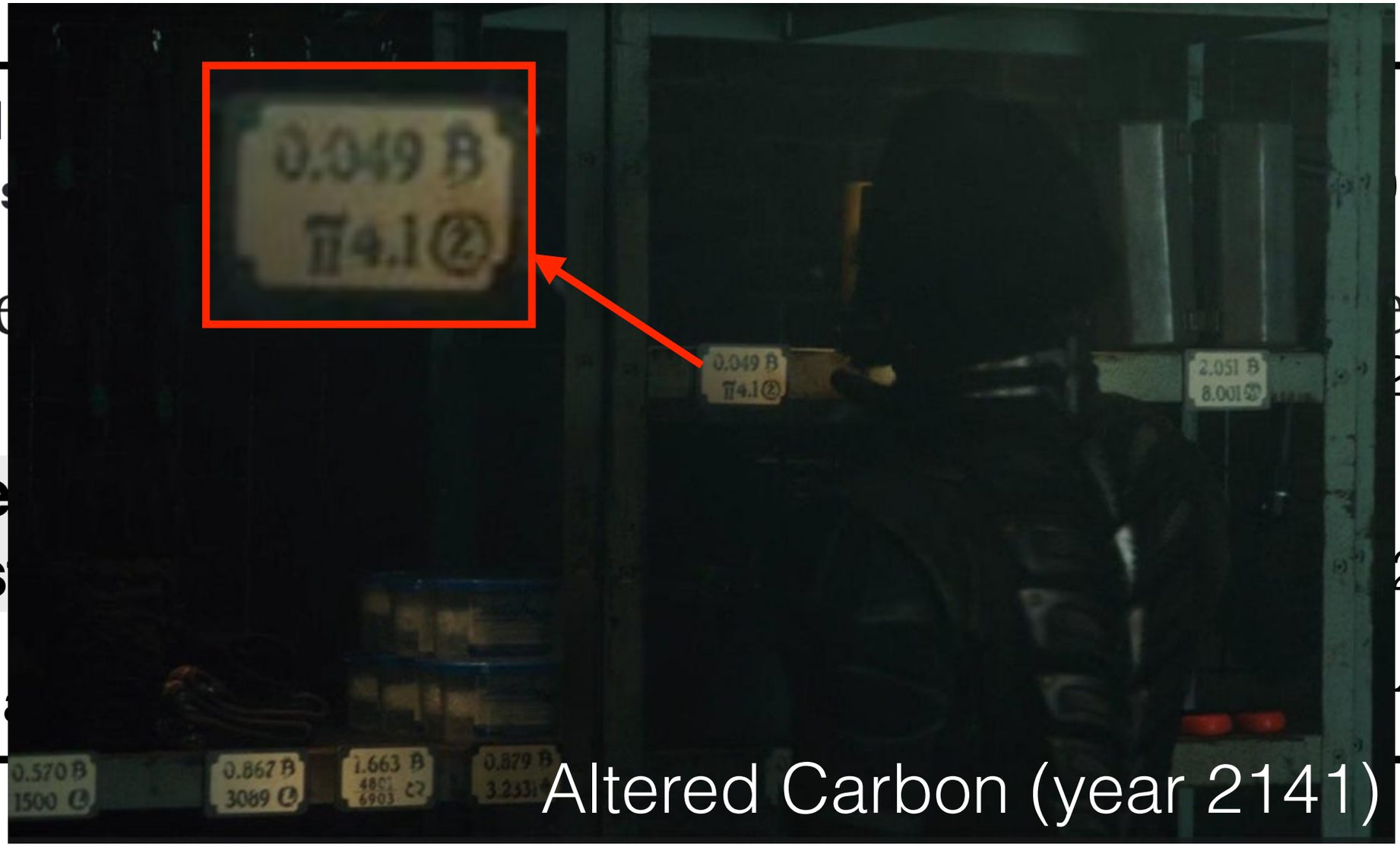

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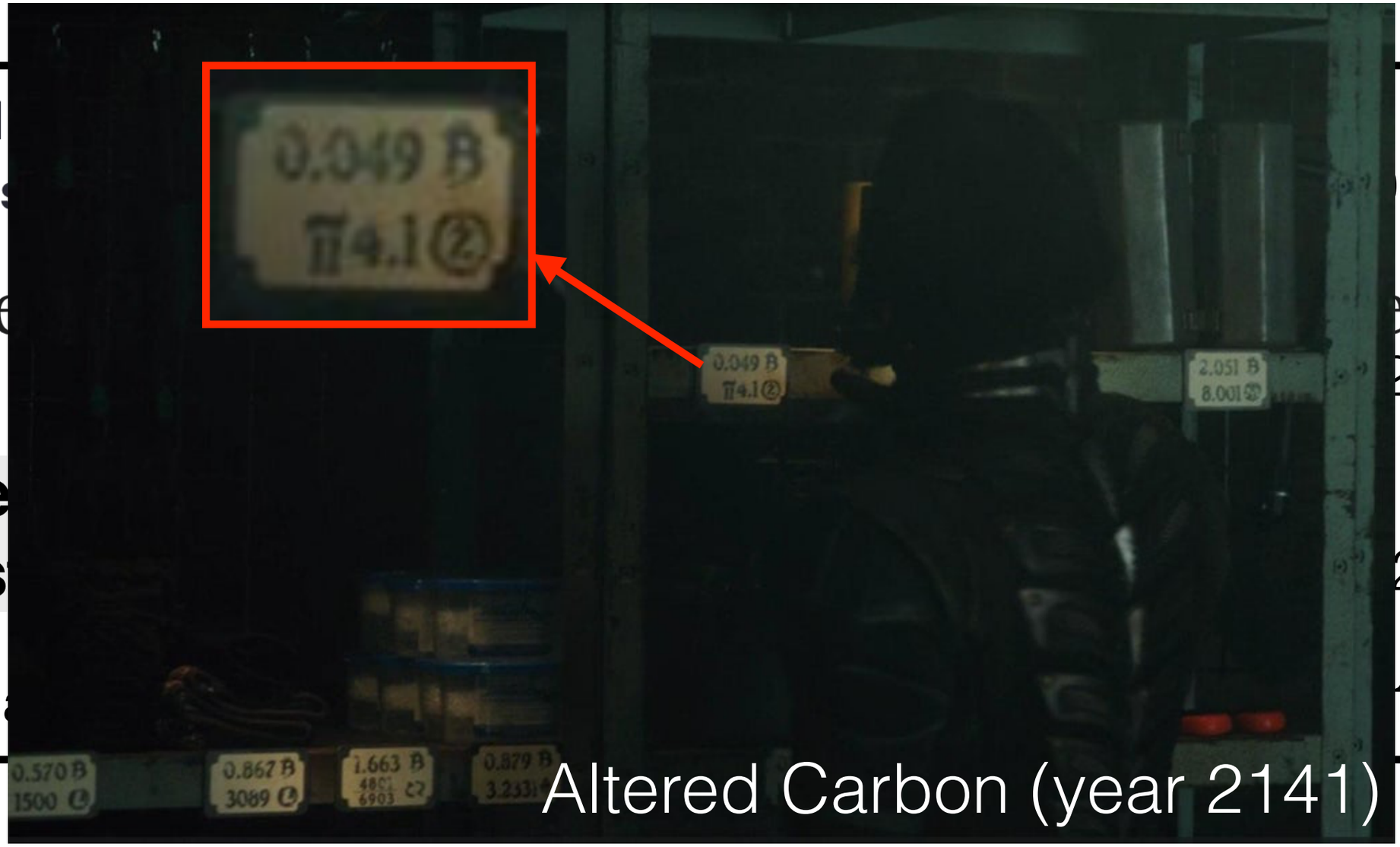

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All constructions presented in this talk are post-quantum secure!

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Many open questions: security of Kilian for IOPs, security from “plain” CRHFs, security of “lattice Bulletproofs”, more efficient lattice-based constructions…

2018: “plausibly” post-quantum

2021: *provably* post-quantum

Theorem [CMS19]: The [BCS16] transformation is secure in the quantum random oracle model (QROM)

QROM models **superposition access** to hash functions

Caveat: IOP must satisfy **round-by-round** soundness, stronger than SR

Theorem [CMSZ21]: Kilian’s interactive protocol (for PCPs) is post-quantum secure assuming QLWE.

This is highly non-trivial to show: classical proof is by **rewinding** which is **impossible** for quantum adversaries

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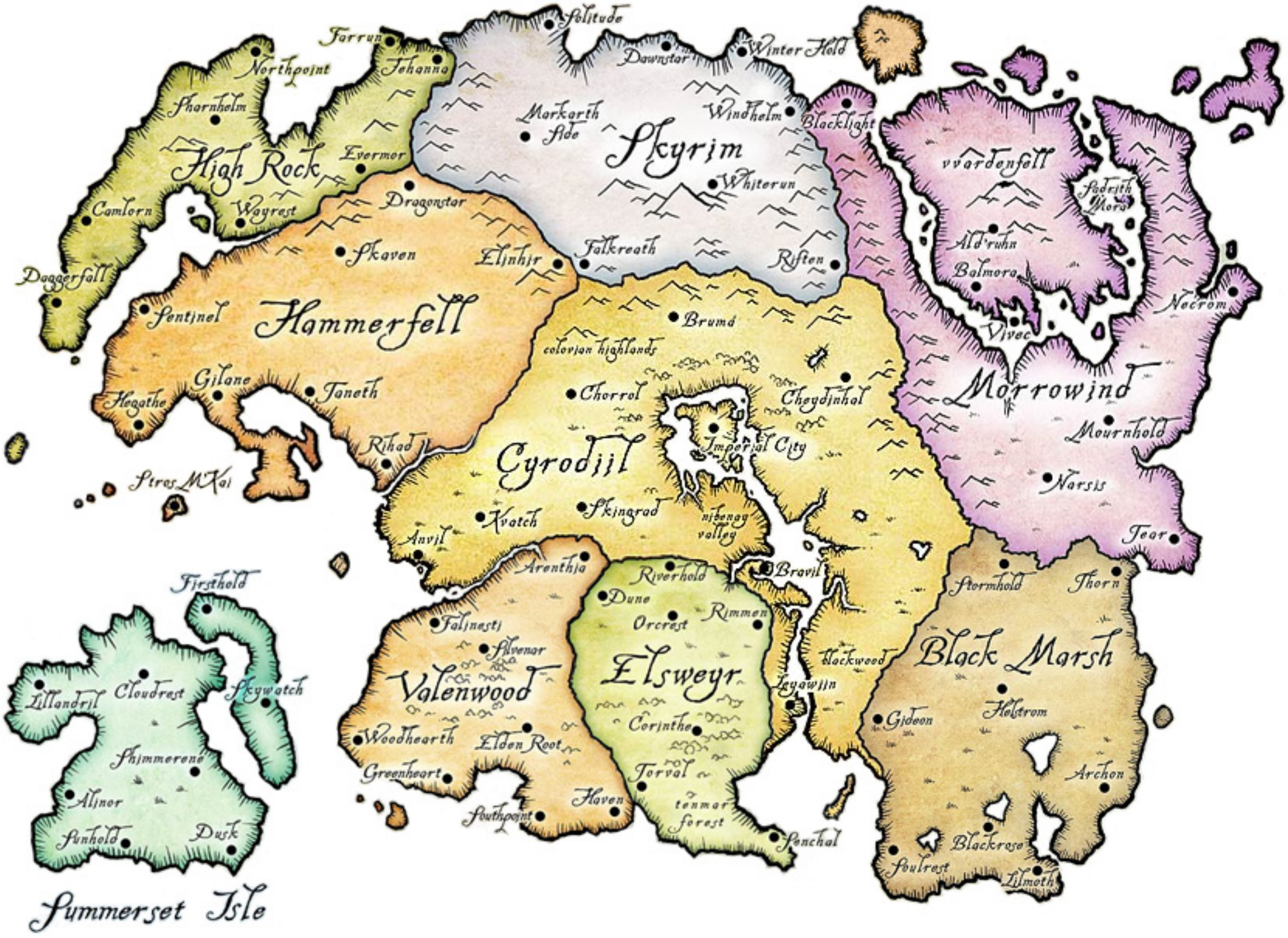
Many open questions: security of Kilian for IOPs, security from “plain” CRHFs, security of “lattice Bulletproofs”, more efficient lattice-based constructions…

Very active research area (talk to me if interested)

Conclusion: The World of Hash-Based SNARKs



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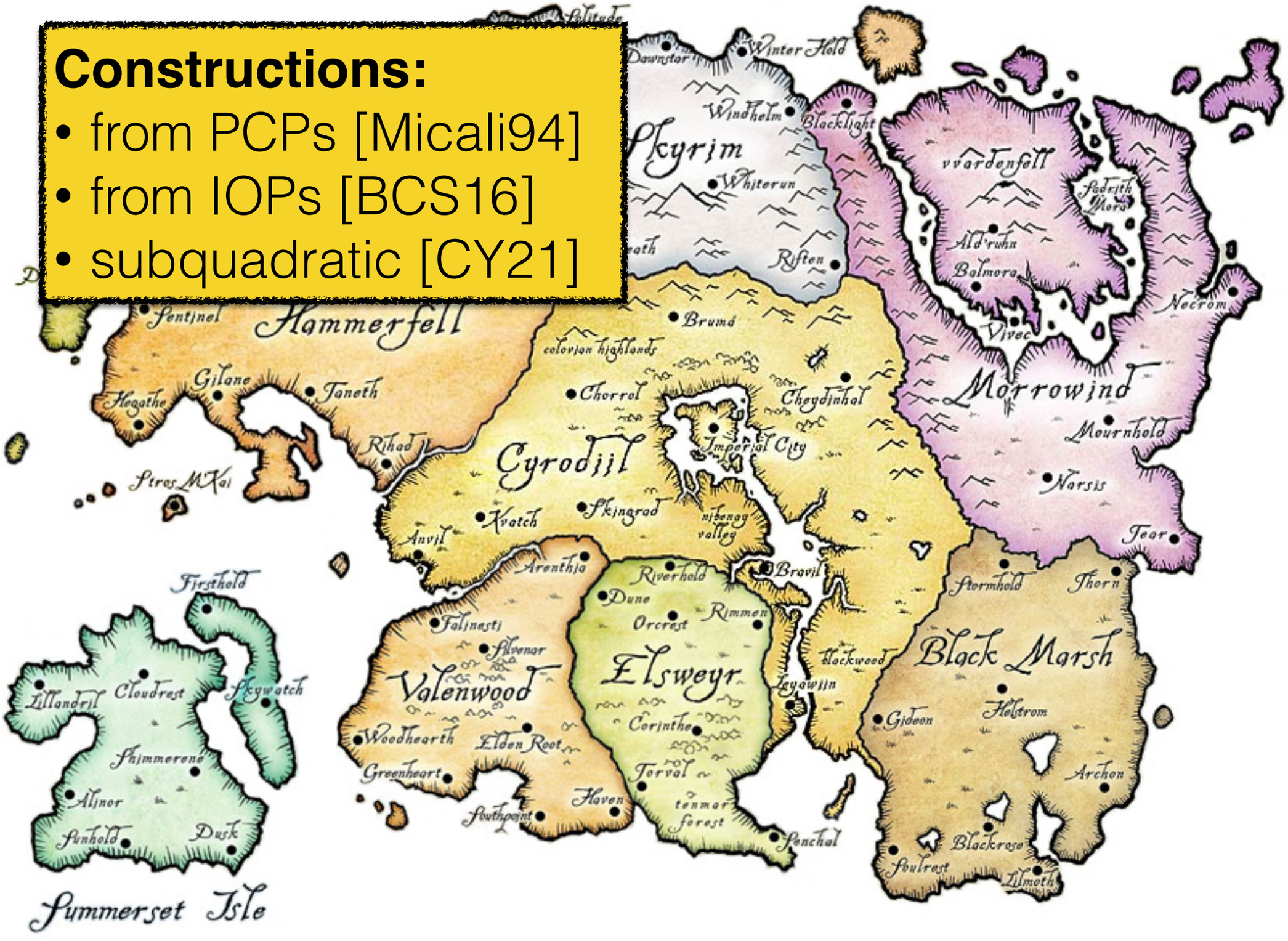
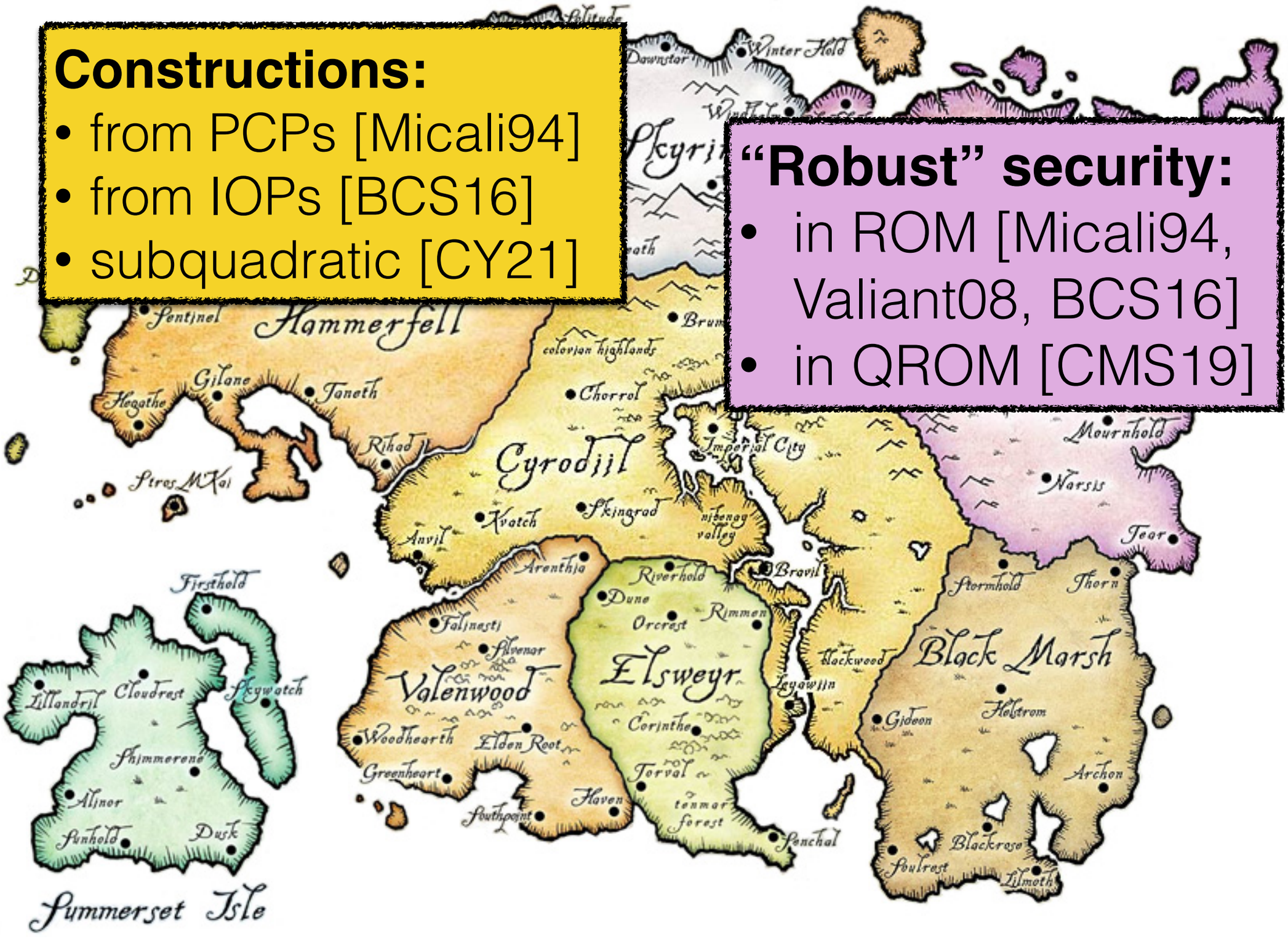


Conclusion: The World of Hash-Based SNARKs

Constructions:
- from PCPs [Micali94]
- from IOPs [BCS16]
- subquadratic [CY21]

"Robust" security:
- in ROM [Micali94, Valiant08, BCS16]
- in QROM [CMS19]



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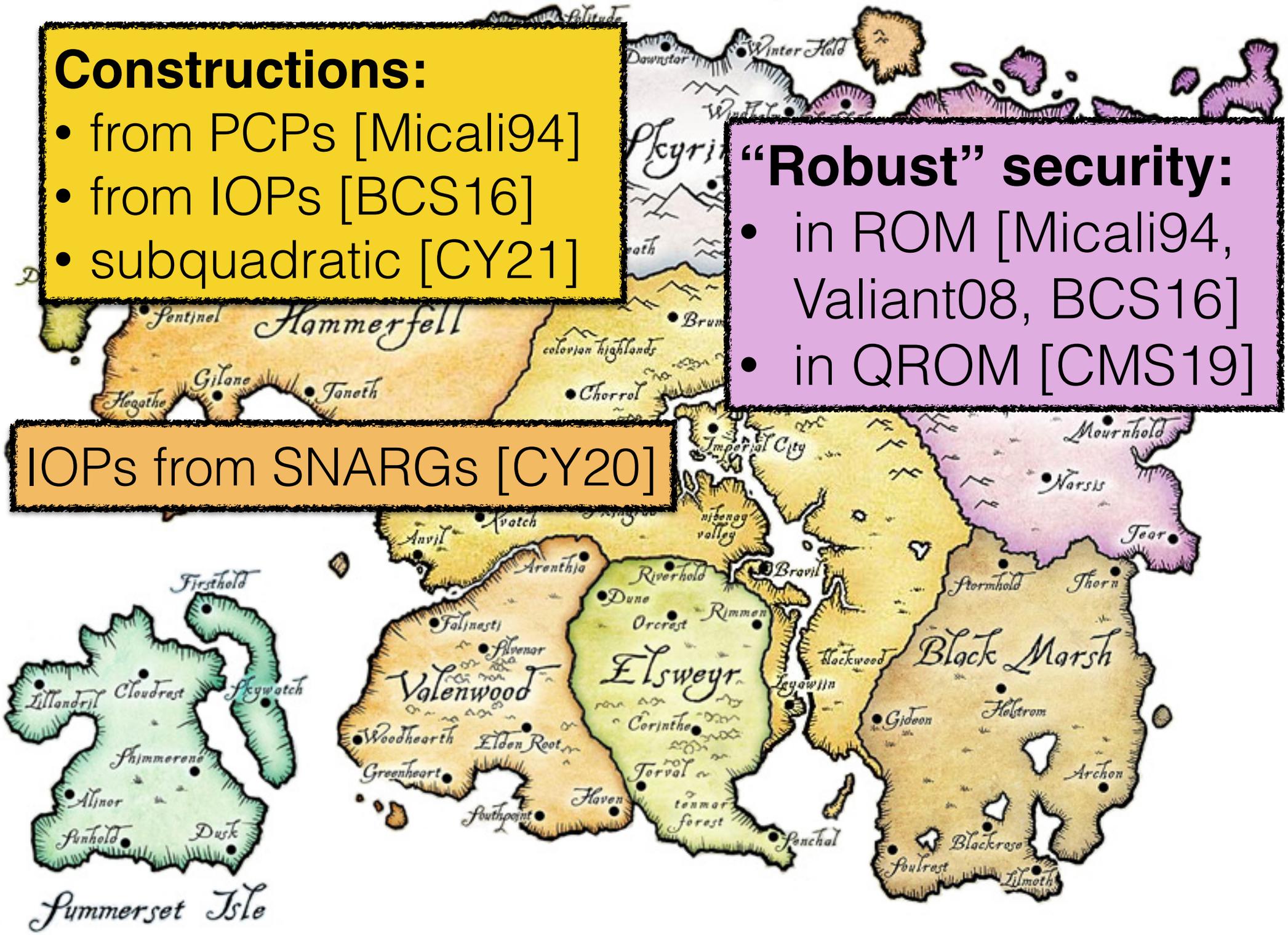

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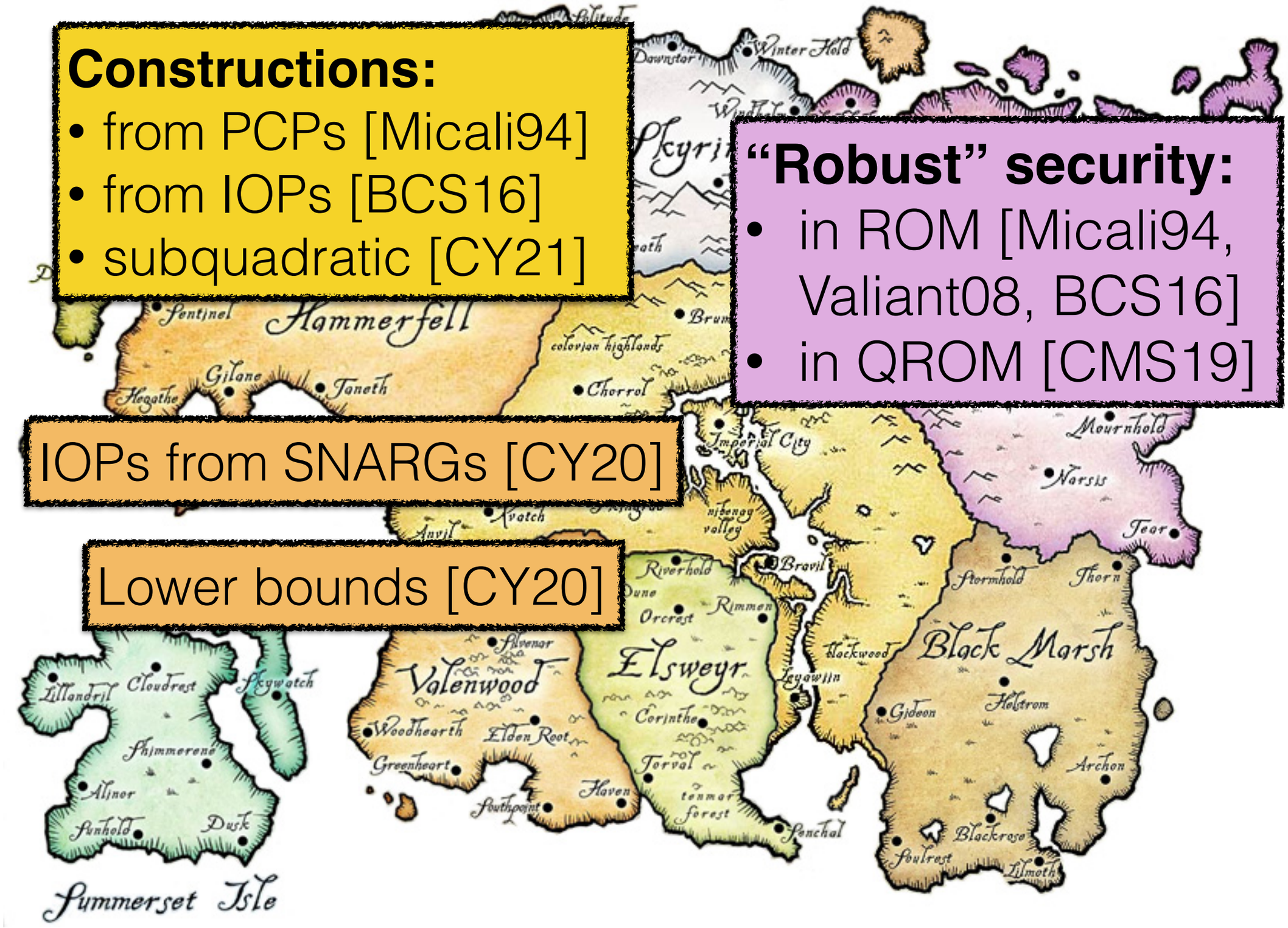
IOPs from SNARGs [CY20]

Lower bounds [CY20]



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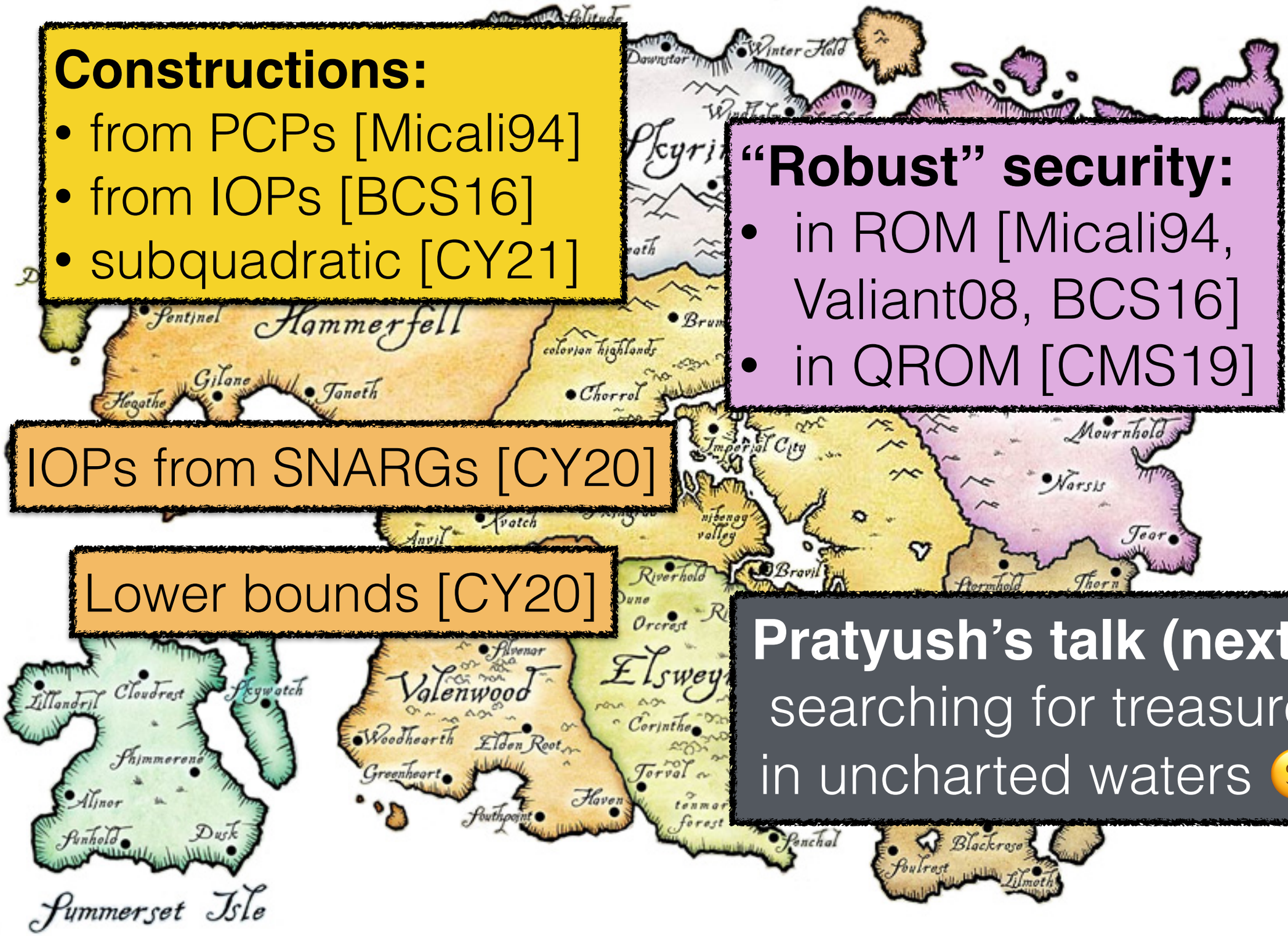

Thanks!