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Abstract

Walther Gerlach and Otto Stern in 1922 demonstrated that a beam of neutral silver atoms was split into two beams by passage through an inhomogenous magnetic field [1], and that the magnetic moment of a neutral silver atom is one Bohr magneton [2]. This was immediately accepted as compelling evidence for Bohr's quantum theory. The splitting is now interpreted as due to electron spin (postulated in 1925), in accordance with quantum mechanics (also initially developed in 1925). Potassium is used rather than silver primarily due to its lower melting point.

The Original Experiment

Stern and Gerlach generated a beam of neutral silver atoms by evaporating silver from an oven in a vacuum. The beam of atoms was collimated by passing through two slits and sent through a region with a large non-uniform magnetic field.

A magnetic field non-uniformity $\frac{\partial B_z}{\partial z}$ produces a force $\frac{\partial B_z}{\partial z}\mu_z$ on a magnetic moment where μ_z is the component of the magnetic moment μ in the z direction.

The silver atoms were thus deflected and allowed to strike a cold metallic plate. After about 8-10 hours the number of condensed silver atoms was large enough to show a visible trace. The trace showed 2 marks showing that the silver atoms had 2 possible values of μ_z.

This would not have been expected with classical physics, which predicts that the z component of μ would have been

$$\mu_z = \mu \cos\theta$$

where $\cos\theta$ could have all values from -1.0 to $+1.0$.

This Experiment

We include two improvements to the original experiment that allow it to be done more easily.

1. Potassium is used instead of silver because:
   (a) it is easier to evaporate (melting points: K: 63.71°C; Ag: 961.93°C, according to Physics 407/707 CRC Handbook)
   (b) The low temperature means that the potassium atoms are moving at a lower speed and are more easily deflected.
   (c) potassium is easier to ionize than silver and is thus easier to detect electrically.
2. A hot wire detector is used instead of a cold metallic plate. The hot wire ionizes a fraction of the potassium atoms which strike it. The positive ions are then collected on a nearby negative electrode and the small ionization current is measured by an electrometer. This allows modern electronics to be used, which have high sensitivity for small currents and fast reaction time.

Bohr's Quantum Theory (1913)

Bohr postulated that electrons move around the nucleus in circular orbits, and that for some reason only orbits with angular momentum equal to an integer multiple of $\hbar$ were allowed.[3] This explained the observed line spectrum of hydrogen. The magnetic moment $\mu = IA$ of the lowest-angular-momentum orbit would then be $\mu = e\hbar/2m_e = 9.274 \times 10^{-24} J/T$. This quantity is now called the Bohr magneton.

Sommerfeld generalized this by assuming the actual quantization condition was that the action integral for a closed orbit was quantized [4]

$$\oint_{H(p,q)=E} p_i dq_i = n_i \hbar \ , \tag{1}$$

where p_i and q_i are the generalized momenta and coordinates of the system in the Hamiltonian formulation of mechanics. This has the consequence that each spatial component of the angular momentum would be quantized in units of $\hbar$ ("space quantization"), and thus each spatial component of the the magnetic moment would be quantized in units of the Bohr magneton.

This suggested to Stern a decisive test of the Bohr-Sommerfeld theory: send a beam of atoms through a magnetic field gradient. If the beam was observed to split into multiple components, rather than to broaden, this would present convincing evidence the Bohr-Sommerfeld theory was right. This is shown in detail below.

(The Bohr-Sommerfeld quantum theory would be superseded in 1925 by quantum mechnics, but Eq. 1 is still being studied.)

Force on Moving Magnetic Moment in Inhomogenous Magnetic Field

Consider a neutral atom with a magnetic moment $\vec{\mu}$ moving with speed v in the x-direction through a region in which there is a strong magnetic field $\vec{B}$ in the z-direction that also has a gradient in the z-direction. The field exerts a deflecting force on the atom that is proportional to the gradient:

$$\begin{aligned}
\vec{F} &= -\vec{\nabla}(\text{potential of magnetic moment in field } \vec{B}) \\
&= -\vec{\nabla}(-\vec{\mu} \cdot \vec{B}) \\
F_z &= -\frac{\partial}{\partial z}(-\mu_z B_z) = \mu_z \frac{\partial B_z}{\partial z}.
\end{aligned}$$

z Displacement

The force will act on the atom for a time $t = d_1/v$, where d_1 is the distance travelled by the atom in the magnetic field. The acceleration along the z-axis (the direction of the magnetic field gradient; perpendicular to v) will be

$$a_z = \frac{\mu_z \partial B_z/\partial z}{M}$$

where M is the atom's mass. As the atom leaves the region of the field, the z-component of its velocity v_z and deflection ${s_z}'$ in the z-direction will be:

$$\begin{aligned} v_z &= a_z t \\ &= \frac{\mu_z \partial B_z/\partial z \; d_1}{Mv} \end{aligned}$$

and

$${s_z}' = a_z t^2/2 = \frac{\mu_z \partial B_z/\partial z \, d_1^2}{2Mv^2} \, .$$

From that point to the detector, v_z remains constant, so that the deflection s at the detector is:

$$\begin{aligned} s &= v_z d_2/v + {s_z}' \\ &= \mu_z \frac{\partial B_z}{\partial z} \frac{[d_1^2 + 2d_1 d_2]}{2Mv^2} \end{aligned}$$

where d_2 is the distance between the magnet exit and the detector. For fixed v, s is proportional to μ_z.

However, atoms issuing from an oven at absolute temperature T do not all have the same v. The number of atoms leaving the oven per unit time with speeds between v and $v + dv$ is:

$$I(v)\, dv = 2I_0 (v/\alpha)^3 \;\; e^{-(v/\alpha)^2} \;\; d(v/\alpha), \tag{2}$$

where I_0 is the total number of atoms leaving the oven, $\alpha = \sqrt{2kT/M}$ is the most probable speed of atoms in the oven (but not in the beam), and k is the Boltzmann constant.

Let s_α be the deflection of an atom of speed α. Then

$$s/s_\alpha = (\alpha/v)^2,$$

and changing variables in Eq. 2 from v/α to s/s_α yields the following relation for the number of atoms $I(s)$ that are deflected by an amount between s and $s + ds$, if all have the same moment μ_z:

$$I(s)\,ds = I_0 \frac{s_\alpha^2}{s^3} e^{-(s_\alpha/s)} ds$$

where the total number of atoms is I_0. $I(s)$ has a maximum at $s_{max} = s_\alpha/3$.

If the undeflected beam has a finite spatial width, the spatial profile of the beam must be convolved with the deflection $I(s)$ given above to determine the predicted spatial distribution of atoms at the detector. Let the spatial profile of the undeflected beam be a gaussian with half-width a. An example of the predicted spatial distribution of atoms for $a = s_\alpha/10$ is shown in Fig. 1. Note the non-zero number of atoms with $s = 0$; as a increases the number of such atoms will increase. Each value of μ_B gives rise to a pattern similar to Fig. 1, but with a different value for s_α and therefore for s_{max}.

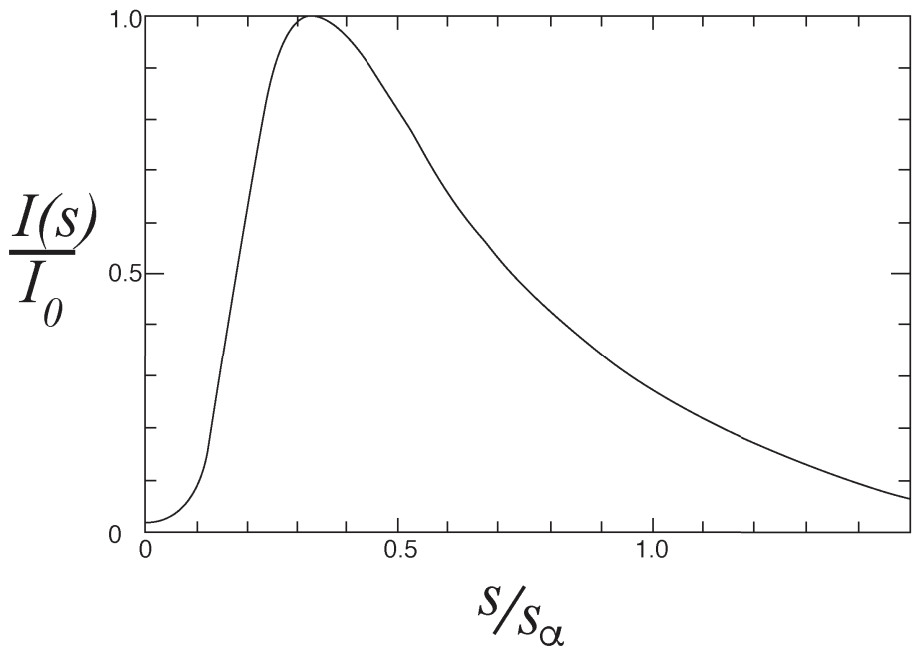

Figure 1: Deflection pattern for atoms having a unique value of μ_z. Undeflected beam width, a, is $s_\alpha/10$.

Summary of Predictions of z direction distributions

1. The classical (pre-quantum theory) predicts a uniform distribution of μ_z between the limits. This would give rise to a roughly bell-shaped curve centered at $s = 0$. The larger the magnetic field gradient, the wider the curve.

2. If μ_z could only take on N evenly-spaced discrete values, one would expect to find N distinct peaks, assuming the field gradient is large enough. The increment $\Delta\mu_z$ between successive allowed values of μ_z would be proportional to the separation between the peaks.

The Apparatus: Potassium Oven

The oven is shaped roughly like a cube 3 cm on a side. The oven is thermally isolated on 3 mounts and is heated by a nichrome cartridge heater. The heater is powered by a variable AC source. At 35 V rms the oven dissipates about 3 Watts, and equilibrates to a temperature of about 105° C, which is above the 63.71° C melting point of potassium. Under these conditions a beam of potassium atoms exits the oven through a small hole covered by two steel precision slits. The heater has been carefully located with respect to the slits so that the slit area is kept hotter than the rest of the oven and therefore remains clean. The potassium beam intensity depends upon the vapor pressure of the potassium, which depends critically upon the temperature: it is 10^{-6} Torr at 63 °C and 10^{-3} Torr at 161 °C.

Vacuum System

A pressure of less than 2×10^{-6} mm Hg (Torr) is necessary for a good signal since the potassium atoms can be easily scattered. The system is pumped by a turbomolecular pump which vents to a mechanical forepump. System vacuum is monitored by two hot-filament ionization gauges, one on either side of the magnet box, while foreline pressure is monitored by a thermocouple gauge. Migration of unwanted potassium through the magnet box to the detector is reduced by additional slits and by running the oven at low temperature. Tempered pyrex glass pipe forms the major part of the vacuum system and allows visual monitoring of the position of the beam gate, the temperature and position of the detector, and possible accidental contamination of the system by potassium or pump vapors.

Beam Gate

The elegant beam gate is attached to the long vertical baffle seen in Fig. 2. in the cross tube. It is able to pivot and stay in position, either open or closed, since its center of gravity is higher than the pivot point. The gate is moved from one position to another by a hand-held magnet outside the vacuum system, and rests against the glass in both positions, either fully open or fully closed. When the gate is closed, the oven end of the system is well separated from the detector end of the system.

The overall layout of the potassium beam path is shown in Fig. 2.

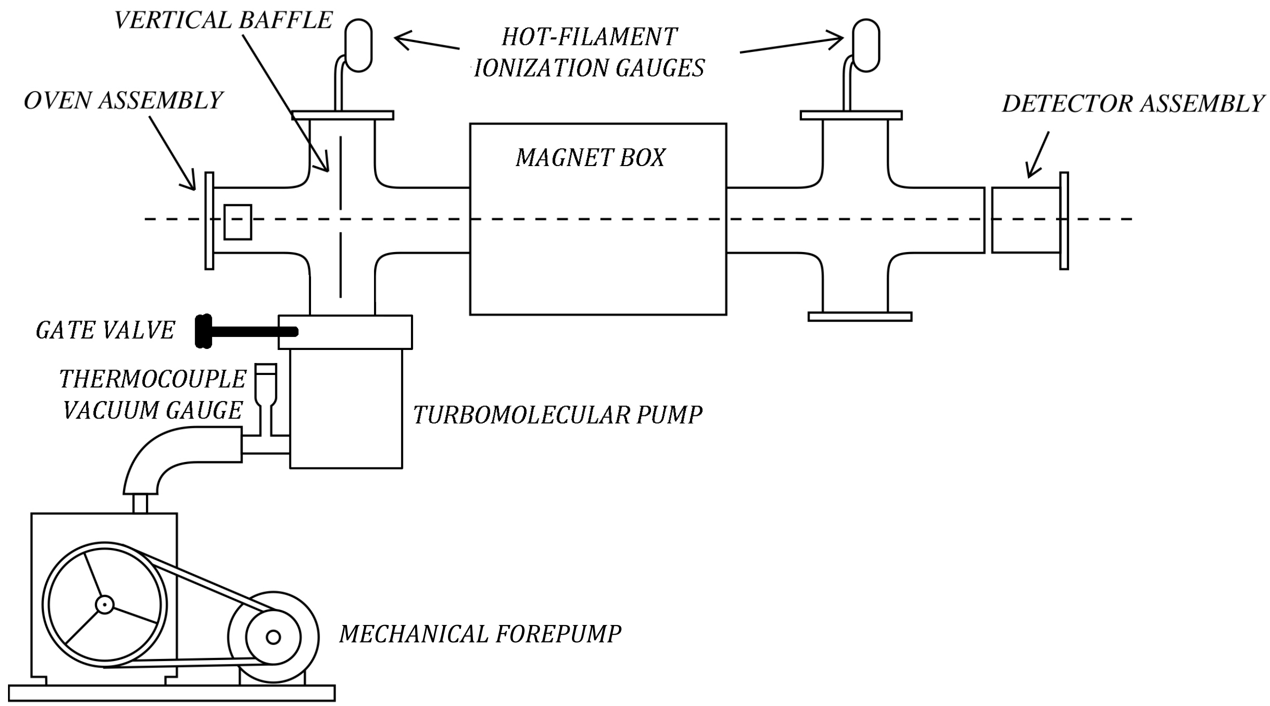

Figure 2: Overall layout of apparatus.

Hot-filament Ionization Gauges

A hot-filament ionization gauge contains a collector, a grid, and two filaments, one of which is a spare. The filament is heated to incandescence, in which condition it emits electrons (thermionic emission). The grid is biased positive relative to the filament so that electrons are attracted to the grid. Some of the electrons collide with gas molecules and form positive ions. These are collected by the collector (at the center of the gauge), which is kept at a negative potential. The ratio of the collector current to the grid current is proportional to the gas pressure for pressures below about 10^{-4} Torr. The gauge will burn out if operated at higher pressures

Nonuniform Magnetic Field

A cross section of the pole tips of the magnet looking down the beam path is shown in Fig. 3. The beam passes through the small hole in the center. The pole tips are designed to produce in the region of the hole a magnetic field in the horizontal direction with a gradient also in the horizontal direction.

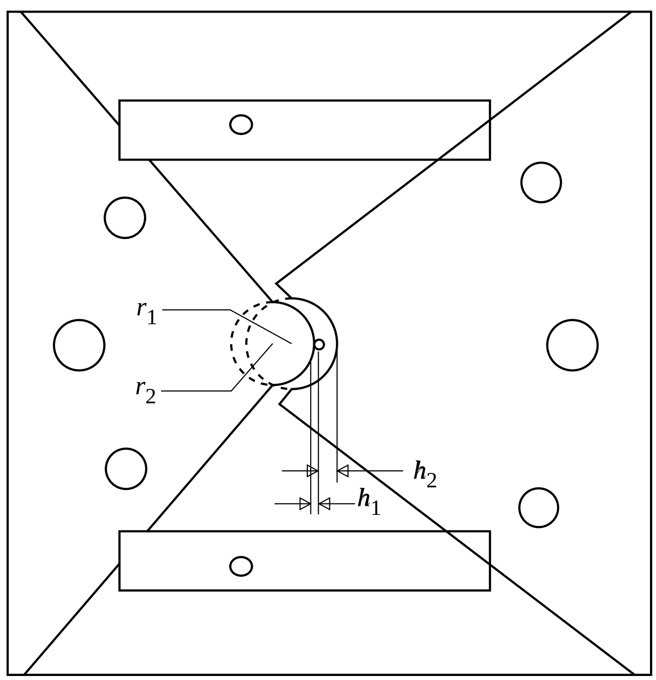

Figure 3: The extension of the concave circle (r_1 = .25") intersects the ends of the vertical diameter of the convex circle (r_2 = .218"). Beam atoms must pass through a small hole (diameter 0.078") located h_1 = .055" and h_2 = .100" relative to the circle perimeters. A slit on the oven side of the magnet is not shown.

The iron pole tips are magnetic equipotentials of the magnetic field that would be present if the pole tips were replaced by two current-carrying wires located at the intersection points of the circles (verifying the field gradient produced by these imaginary wires is a good Physics 322 exercise). The pole tips are mounted between the faces of an electromagnet. An electromagnet capable of producing 0.4 Tesla (4000 gauss) across a .750 inch (1.91 cm) gap with a face 4 inches (10.16 cm) in diameter will produce good results (this corresponds to about 7000 ampere turns). When used with such an electromagnet, these poles produce a gradient $\vec{\nabla} B$ of a little over 100 Tesla/meter in the region through which the beam passes.

Hot Wire Detector

The detector is a hot pure tungsten filament wire, .005 inch in diameter, surrounded by a collecting cylinder which has a narrow slit aperture to admit the potassium atoms. The wire is heated by a adjustable constant-current power supply, and is biased positive with respect to the cylinder.

Electrons continuously boil off the hot wire ("thermionic emission"), and potassium atoms that pass nearby are ionized. The positive potassium ions are then attracted to the the cylinder, resulting in a current on the order of pA. This current is detected by an electrometer. The magnitude of the current is dependent on the filament temperature, which is strongly dependent on filament current.

The detector function is not unlike the function of the ion gauges mentioned earlier. If the filament temperature is increased, the detector assembly heats up, and potassium atoms are desorbed from warming surfaces, resulting in an increased density of potassium atoms in the detector. This results in a higher detector current. After a while at constant temperature (and no beam) some of the potassium atoms are pumped out of the detector, and the detector current falls.

The detector can be moved laterally, by means of a micrometer, from outside the system; its position is indicated by the scale on the micrometer drive. The micrometer shaft passes through an O-ring into the vacuum chamber. DO NOT EXCEED a micrometer setting of 325 mils. The micrometer screw rod forms part of the seal in maintaining a good vacuum. If the rod is pulled out farther than 350 mils the rod could slip out of the O-ring and this could destroy the vacuum and may damage the turbo pump.

POTASSIUM BEAM PATH

What does the Stern-Gerlach apparatus look like from the inside?

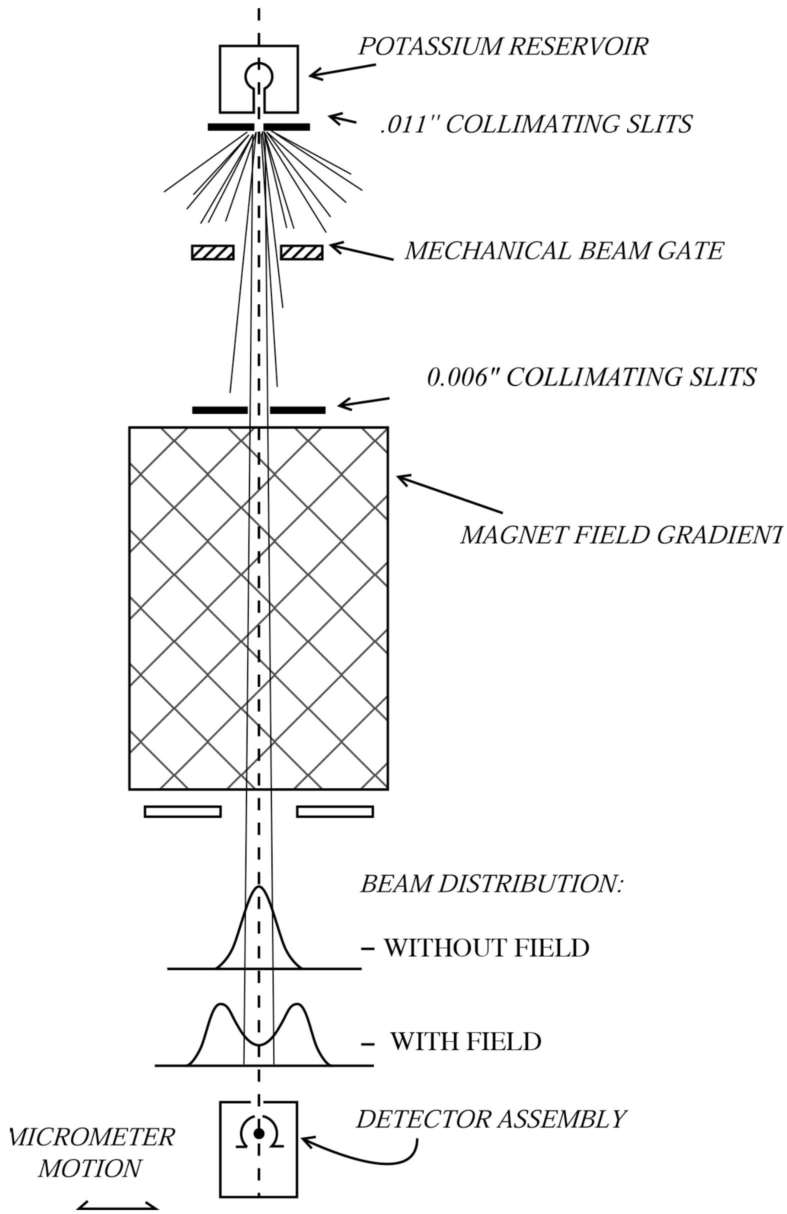

Figure 4: Beam path, showing collimating slits.

Vacuum Operations Checklist

The instructors have prepared the experiment so that it should be ready for you to operate after you have gone through the following checklist. **Before going through this checklist, please read the "Cautions" and "Procedure" below.**

1. **Confirm pumps are operating**. The turbo pump must always be on. It spins at tens of thousands of RPM, but is so quiet you can't tell by listening that it is on. Look/listen for the digits on the hours indicator of the pump controller to change. The pressure will rise rapidly if the turbo pump is off. The mechanical forepump must operate **continuously** to vent the turbo pump. It makes a whirring noise when it is operating. The rubber hose from the forepump to the turbo pump must be free from crimps or sharp bends that would restrict the flow of gas inside.

2. **Check base vacuum pressure**. Make sure both ion gauge controllers are set to "Low" and the multiplier is set to the -6 scale. Turn on both gauges. The displayed pressure, which is approximately 10 times lower than the actual pressure, should be below 1×10^{-6} Torr. If so, set the emission to "Hi" (or "Normal"). The actual pressure should then be displayed (the pressure gauge meter reads correctly for 8 mA grid current). It is not unusual for the pressure to decrease for a few minutes after the gauge is turned on, as material that has condensed in the gauge warms up and evaporates away. The actual pressure should soon be below 2×10^{-6} Torr; if it is, record it; if it isn't, call an instructor. When uncontaminated the present system will maintain a vacuum better than $1.0{\times}10^{-6}$ Torr with the oven off. Due to the finite pumping speed through the pole piece box, the pressure at the detector end will be slightly higher than the pressure in the oven end.

3. Once the pressure is deemed satisfactory, turn the emission back to "Low". This reduces the grid current by a factor of 10, which greatly increases the filament lifetime. Keep in mind the displayed pressure will now be a factor of 10 lower than the actual pressure.

4. **Turn on oven**. It is important that the oven is allowed to reach a steady temperature between 120-125°C before starting to take data.

The temperature of the oven is monitored by an iron-constantan thermocouple which passes through the oven mounting face. The thermocouple is connected to a voltmeter calibrated in °C. Heat the oven at 75 V rms to an operating temperature of about 120°C. Back off the heater voltage to $\sim$ 45 V when close to 120° so you don't overshoot. Wait for the temperature to stabilize at about 120°. It is very important to make the actual measurements with a stable oven temperature.

5. **Turn on detector**. The detector assembly is surrounded by heater tape wrapped in aluminum foil so that it can be continuously baked to help prevent potassium build-up in the detector. Verify that the detector bake temperature is about 100°C. Turn on the filament bias power supply to 12.5 V. Turn on the detector power supply and use the voltage adjust knob (why not the current adjust knob?) to set the hot wire current to 1.2 A. At this current the hot wire glows orange (much hotter than 100°C). Any built-up potassium in the detector volume is heated and evaporates, causing a large background signal that decreases as the potassium is pumped away. Wait at least ten minutes after turn-on (the longer the better) for this background to subside. Turn down the hot wire current to 0.8 A to take measurements. (If the background electrometer current (see below) is too high, the detector wire may need to be baked at $\sim$1.6 A for 10 minutes to clean the wire. Consult with the instructor if unsure.)

6. **Turn on electrometer**. Set the range of the electrometer to 1 nA full scale. Then turn on the electrometer to read + currents (the detector collects K^+ ions). You should be able to decrease the range to 1 pA full scale. If not, call an instructor.

CAUTIONS

1. The ion gauges must never be operated at $p > 1.0 \times 10^{-4}$ Torr.

2. The oven temperature must never exceed 140°C.

3. The detector bake temperature must not exceed 100°C.

4. The hot wire current must not exceed 1.6 A.

5. The micrometer must never be pulled out past a reading of 325 mils, lest it cause a sudden vacuum leak that could destroy the turbopump.

Procedure

1. Estimate the expected width of the beam at the detector using the slit information in Fig. 4. You may need to measure some distances. Choose an increment of detector translation (step size) that should allow you to get a good measurement of the beam profile.

2. Get the experiment ready to take data by following the vacuum operations checklist above.

3. Do a first scan for signal with the magnet off, in order to locate the center of the peak. The signal current at the undeflected beam peak with a good vacuum can be as high as 10 pA. Depending on the conditions of the system it may be less.

4. At the peak, see if you can reduce the detector wire current without losing appreciable signal. This will result in a longer lifetime of the detector before it becomes contaminated with potassium. The bias voltage is not critical.

5. Do a fine scan over the peak using the step size you found earlier. Every two steps take a reading with the beam gate closed in order to measure the background signal. Record your data (and uncertainties).

6. Do the same with the magnet on at $\sim$1.0 A. You should immediately notice that the signal at the previous peak position is significantly smaller. Do a complete scan to map out the new distribution.

7. Make two plots (one for each dataset), showing the signal, the background, and the uncertainties.

8. Possible additional studies include a different oven temperature and/or a different magnet setting.

Physics Analysis and Questions

1. Derive expressions for the most probable speed of the potassium atoms in the oven, the most probable speed of the atoms in the beam, and the speed of the atoms that land at s_{max}. Why are they different? Evaluate all three quantities at the oven operating temperature.

2. Calculate the magnetic field gradient from your data assuming the magnetic moment for potassium is one Bohr magneton. This will involve making some geometry measurements and deciding on a procedure for analyzing the data. We can either compute the electron magnetic moment having a value for the magnetic field gradient or compute the magnetic field gradient having a value for the electron magnetic moment. We do the latter since we have no way to measure the magnetic field gradient. The field region is in vacuum and the pole piece gap is very small.

3. Potassium atoms have a total of 19 electrons, of which 18 form closed shells, and one is in a 4s orbital. Write a paragraph explaining our understanding (via contemporary quantum mechanics) of why the magnetic moment of a potassium atom is one Bohr magneton, referencing concepts including "spin angular momentum", "orbital angular momentum", "gyromagnetic factor", and "nuclear magnetic moment".

References

[1] W. Gerlach and O. Stern, *"Der experimentelle Nachweis der Richtungsquantelung im Magnetfeld,"* Zeitschrift für Physik **9** 349 (1922).

[2] W. Gerlach and O. Stern, *"Das magnetische Moment des Silberatoms,"* Zeitschrift für Physik **9** 353 (1922).

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[4] A. Sommerfeld, "Zur Quantentheorie der Spektrallinien," Ann. Phys. **51** 1 (1916). The quantization condition itself was actually first published by W. Wilson, "The Quantum-Theory of Radiation and Line Spectra," Phil. Mag. **29** 795 (1915).