N5 Maths Crash Course

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Contents

Click on Chapter to go to the Relevant Slide

Contents

Click on Chapter to go to the Relevant Slide

Fractions & Mixed Numbers

Addition & Subtraction of Fractions

The Denominators of both fractions **must be the same** in order to add them together.

In order for the denominators to be the same, you must multiply both the numerator & the denominator to get to the LCM (Lowest Common Multiple).

Example: $\frac{1}{2} + \frac{1}{3}$ cannot add because the denominators, 2 & 3, are **not the same**.

To fix this, you can multiply to get the LCM of both denominators, in this case, 6, to get 3/6 + 2/6.

Now add them to get 6/6 and simplify it, in this case, it simplifies to 1/1 or simply 1. You can also get mixed numbers like $1\frac{1}{3}$ as an answer as well.

The Subtraction of Fractions is basically the same as Addition but instead of adding, you subtract and then simplify, if possible.

Multiplication & Division of Fractions

When Multiplying Fractions, you can simply multiply the denominators and the numerators together and simplify.

e.g $\frac{2}{3} \times \frac{1}{3} =$ **2/9**

When Dividing Fractions, you **swap** the second fractions numerator & denominator around, then multiply them together.

e.g $\frac{2}{3} / \frac{1}{3} = \frac{2}{3} \times 3/1 =$ **6/3** (2)

Remember to always Simplify (Dividing by the H.C.F (Highest Common Factor).

Adding & Subtracting Mixed Numbers

First thing you do is Change the **mixed number into Top heavy** fractions, then you change the denominators to **be the same**, add, then simplify and, if possible, change back into a mixed number.

Note, Changing to mixed number is optional unless stated otherwise.

Example: $1\frac{1}{3} + 2\frac{1}{4}$

- Change To Top Heavy Fractions: 4/3 ($1\frac{1}{3}$) + 9/4 ($2\frac{1}{4}$)
- Change Denominators: 4/3 = 16/12, 9/4 = **27/12**
- Add Fractions: 16/12 + 27/12 = **43/12**
- Simplify and Change back to mixed number: **3 7/12**

Subtracting is the same as above except Subtract instead of Adding.

Multiplying Mixed Numbers

When Multiplying Mixed Numbers, **Change them** into top heavy fractions, Make sure the denominators **are the same**, **Multiply them** together and then **Simplify** and change back to mixed number

Example: $2\frac{3}{5} \times 3\frac{1}{4}$

- Change to Top Heavy Fraction: 13/5 x 13/4
- Multiply them together: 169/20
- Simplify & Change back to Mixed Number: 8 9/20

Dividing Mixed Numbers

When Dividing Mixed Numbers,You convert it to a Top Heavy Fractions, **Swap** the second fractions' numerator and denominators around. Then **multiply** and Finally **change** the fraction back to a mixed number and **simplify** the remaining fraction

Example: 1 4/7 / 1¼

- First Change to Top Heavy Fractions: 11/7 / 5/4
- Swap the numerator & denominator for second fraction: 11/7 / ⅘
- Multiply: 11/7 x ⅘ = 44/35
- Change back to a mixed number and simplify the remaining fraction, if possible: 1 9/35

P.S Most of the time, multiplication & division do not require conversion back to mixed numbers after completion of the operation unless specified.

Questions

Add & Subtract Fractions 1: Link

Add & Subtract Fractions 2: Link

Add & Subtract Fractions 3: Link (Ans For All)

Multiplying & Dividing Fractions 1: Link (Ans)

Dividing Fractions: Link (Ans)

Multiplying Fractions: Link

Mixed Numbers & Improper Fractions: Link

Percentages

Terms

Some Terms Used in Percentages

Terms for Increase:

- Growth
- Appreciation
- Compound Interest

Terms for Decrease:

- Decay
- Depreciation

Finding the Percentage of a Number

There are a few methods that you can use for finding the percentage of a number

Method 1: Calculator Method

- Multiply the given number by the percentage as a decimal
- e.g 34% of £79 = 79 x (0.34) = £26.86 Or 114% of £35 = 35 x (1.14) = £39.90

Method 2: Divide & Multiply

- Divide by 100 then multiply by the percentage
- e.g 34% of £79 = 79/100 x 34 = £26.86 Or 114% of £35 = 35/100 x 114 = £39.90

Percentage Change

Examples of Percentage Change

$$\xrightarrow{+a\%}$$

$100\% \rightarrow (100 + a)\%$

$$\xrightarrow{-a\%}$$

$100\% \rightarrow (100 - a)\%$

Reversing Percentage Change

Some Worked Examples

1. Including VAT of 20%, a radio costs £96. Find the Original Cost of the Radio

- Step 1: Find the Percentage 100% + 20% = 120% (1.20)
- Step 2: Find the Original Cost of the Radio using the percentage

$$96 / 1.20 = 80$$

- Step 3: Answer the question

The Original Cost of the Radio was £80

Reversing Percentage Change 2

1. A camera costs £120 after a discount of 25% is applied. Find the Original Cost of the Camera.

- Step 1: Find the Percentage

  100% - 25% = 75% (0.75)

- Step 2: Find the Original Price

  120 / 0.75 = 160

- Step 3: Answer the Question

  The original cost of the camera was £160

Compound Interest Change

Appreciation & Depreciation. Some Worked Examples

1. A £240,000 house appreciates in value by 5% in 2007, appreciates 10% in 2008 and depreciates by 15& in 2009, Calculate the value of the house by the end of 2009.

- Step 1: Find all Percentages

  100% + 5% = 105% (1.05)

  100% + 10% = 110% (1.10)

  100% - 15% = 85% (0.85)

- Step 2: Find the Value of the House

  240,000 x 1.05 = 252000

  252,000 x 1.10 = 277,200

  277,200 x 0.85 = 235620

- Step 3: Answer the Question

  The Value of the House by the end of 2009 is £235,620

Compound Interest Change 2

1. Calculate the compound interest on £12,000 invested at 5% p.a (Per Annum) for 3 years

- Step 1: Find the Percentage

  100% + 5% = 105% (1.05)

- Step 2: Calculate the Final Total

  $12000 \times (1.05)^3 = 13891.50$

  13891.50 - 12000 = 1891.50

- Step 3: Answer the Question

  The Compound Interest is £1,891.50

Compound Interest 3

1. A £15,000 car is resold for £12,000. Find the percentage loss

- Step 1: Find the Percentage

Loss = 15000 - 12000 = 3000

3000/15000 x 100% = 20%

- Step 2: Answer the Question

The Percentage Loss is 20%.

Questions

Percentage 1: Link (Ans)

Mixed Percentages 1: Link

Mixed Percentages 2: Link

Past Paper Percentages: Link

Reverse Percentages: Link

Compound Interest: Link (Ans)

Percentages (No Calc): Link

Percentages (No Calc) 2: Link

Surds

What are Surds

Rational Numbers can be Written as a division of two integers (Whole Numbers) where as Irrational Numbers cannot be written as a division of two integers.

Surds are Irrational Numbers, so they are written as **irrational roots**.

For Example: $\sqrt{2}$, $\sqrt{5/9}$ are **surds**

However $\sqrt{25}$, $\sqrt{4/9}$ are **not surds** as they are 5 and $\frac{2}{3}$ respectively.

Simplifying Surds

Some Rules: $\sqrt{mn} = \sqrt{m} \times \sqrt{n}$

$\sqrt{m/n} = \sqrt{m}/\sqrt{n}$

Some Worked Examples:

1. Simplify $\sqrt{24} \times \sqrt{3}$

- $\sqrt{24} \times \sqrt{3}$
- $\sqrt{36} \times \sqrt{2}$ (36 is largest square number which is a factor of 72)
- $6 \times \sqrt{2}$
- **$6\sqrt{2}$**

Simplifying Surds 2

2. Simplify $\sqrt{72} + \sqrt{48} - \sqrt{50}$

- $\sqrt{72} + \sqrt{48} - \sqrt{50}$
- $(\sqrt{36} \times \sqrt{2}) + (\sqrt{16} \times \sqrt{3}) - (\sqrt{25} \times \sqrt{2})$
- $6\sqrt{2} + 4\sqrt{3} - 5\sqrt{2}$
- $6\sqrt{2} - 5\sqrt{2} + 4\sqrt{3}$
- **$\sqrt{2} + 4\sqrt{3}$**

3. Remove the brackets and fully simplify: $(\sqrt{3} - \sqrt{2})^2$

- $(\sqrt{3} - \sqrt{2})(\sqrt{3} - \sqrt{2})$
- $\sqrt{3}(\sqrt{3} - \sqrt{2}) - \sqrt{2}(\sqrt{3} - \sqrt{2})$
- $\sqrt{9} - \sqrt{6} - \sqrt{6} + \sqrt{4}$
- $3 - \sqrt{6} - \sqrt{6} + 2$
- **$5 - 2\sqrt{6}$**

Simplifying Surds 3

Remove the brackets and fully simplify: $(3\sqrt{2} + 2)(3\sqrt{2} - 2)$

- $(3\sqrt{2} + 2)(3\sqrt{2} - 2)$
- $3\sqrt{2}(3\sqrt{2} - 2) + 2(3\sqrt{2} - 2)$
- $9\sqrt{4} - 6\sqrt{2} + 6\sqrt{2} - 4$
- $18 - 6\sqrt{2} + 6\sqrt{2} - 4$
- **14**

Rationalising the Denominators

Removing Surds From the Denominator

Express with a rational Denominator (All Questions are Fractions):

1. $4 / \sqrt{6}$
   - $4 / \sqrt{6}$
   - $4 \times \sqrt{6} / \sqrt{6} \times \sqrt{6}$
   - $4\sqrt{6} / 6$
   - **$2\sqrt{6} / 3$**

2. $\sqrt{3} / 3\sqrt{2}$
   - $\sqrt{3} / 3\sqrt{2}$
   - $\sqrt{3} \times \sqrt{2} / 3\sqrt{2} \times \sqrt{2}$
   - $\sqrt{6} / 3 \times \sqrt{4}$
   - **$\sqrt{6} / 6$**

Questions

Surds 1: Link (Ans)

Surds 2 (Q7 and above): Link (Ans)

Further Surds: Link (Ans)

Indices

Indices

Base → a^n ← Index or Exponent

Indices Rules: Require the same base

$a^m \times a^n = a^{m+n}$

$a^m / a^n = a^{m-n}$

$(a^m)^n = a^{mn}$

$(ab)^n = a^n b^n$

$a^0 = 1$

$a^1 = a$

$a^{m/n} = n\sqrt{a^m} = (n\sqrt{a})^m$

$1/a^p = a^{-p}$

Indices Examples

1. $w^2 \times w^5 / w^3$ (Fraction)
 - w^7 / w^3
 - **w^4**

1. $(3^5)^2 = 3^{10}$

2. $(2a^3b)^2$
 - $2^2a^6b^2$
 - $4a^6b^2$

Indices Examples 2

1. $5^0 = 1$
2. $5^1 = 5$
3. $8^{4/3}$
   - $(\sqrt[3]{8})^4$
   - 2^4
   - 16
4. $8^{-4/3}$
   - $1/8^{4/3}$
   - $1/16$

Questions

Further Indices: Link

Fractional Indices: Link

Indices: Link

Surds & Indices: Link (Ans)

Scientific Notation

Scientific Notation (Sci Not) is Written in the form a $x10^n$

Used to simplify large numbers into easier to manage ones. e.g 3000000000 to $3x10^9$

1. 32800
   - $3.28 \times 10 \times 10 \times 10 \times 10$
   - **$3.28x10^4$**

2. 0.000328
   - $3.28 / 10 / 10 / 10 / 10$
   - $3.28x10^4$
   - **$3.28x10^{-4}$**

Questions

Sci Not 1: Link

Sci Not 2: Link

Sci Not 3: Link

Sci Not 4: Link

Sci Not (Calc): Link

Significant Figures

Significant Figures indicate the accuracy of a **measurement**.

e.g 3400 cm

- 34m
- 0.034km

Same Measurement, same accuracy, each 2 significant figures.
Count the number of figures used, but not zeros at the end of a whole number or zeros at the start of a decimal.

Rounding: 5713.4 → 5700 to 2 s.f

- 0.057134 → 0.057 to 2 s.f

Significant Figures

Some Worked Examples

1. One **milligram** of Hydrogen Gas contains 2.987×10^{20} molecules.
   - $5000 \times 2.987 \times 10^{20}$ (Learn to enter sci not in calc using appropriate buttons)
   - 1.4935×10^{24}
   - **1.49×10^{24} molecules**

2. The total mass of argon in a flask is 4.15×10^{-2} grams, the mass of a single atom of argon is 6.63×10^{-23} grams. Find, correct to 3 s.f, the number of argon atoms in the flask.
   - $4.15 \times 10^{-2} / 6.63 \times 10^{-23}$ (Use ‘(-)’ button for a minus on calc)
   - 6.259×10^{20} (Divide top by bottom and write unrounded answer)
   - 6.26×10^{20} atoms (Write rounded answer)

Questions

S.F 1: Link

S.F 2: Link

S.F 3: Link

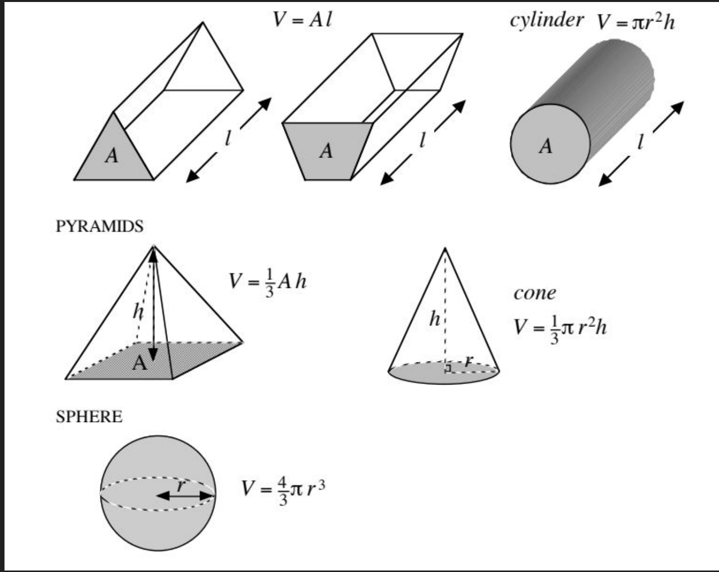
S.F Mini 1: Link (Ans)

Volume

Volume

Prisms: A solid with the same cross-section throughout its length.

Length ‘l’ is at right angles to the area A.



Volume

Describe the effect on the volume of a cylinder of:

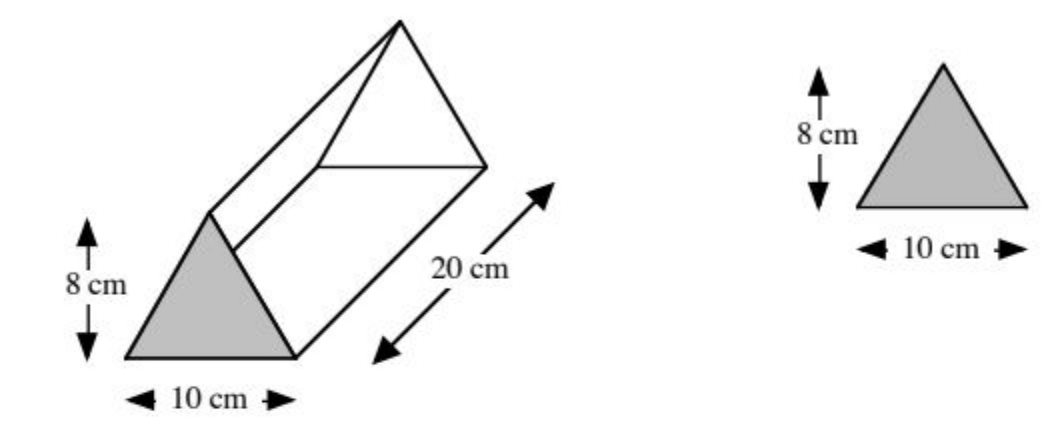
1. Trebling the radius (3)
 - $V = \pi(3r)^2h$
 - $\pi 9r^2h$
 - $9\pi r^2h$
 - **9 times bigger**
2. Doubling the Radius and halving the height
 - $V = \pi(2r)^2(\frac{1}{2}h)$
 - $\pi 4r^2 \times \frac{1}{2}h$
 - $2\pi r^2h$
 - **2 times bigger**

Volume

Some Worked Examples

1. Calculate the Volume

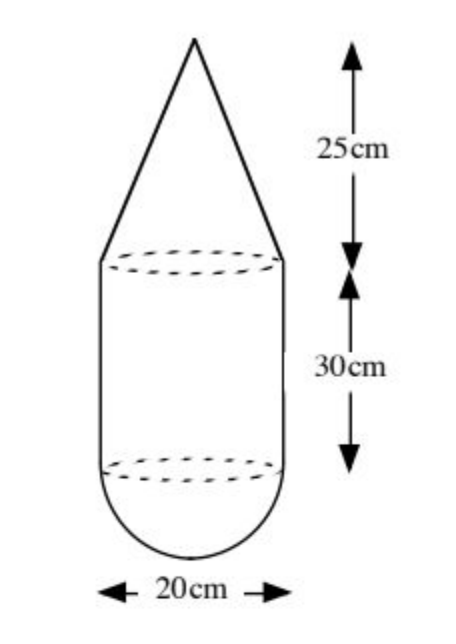
- $A = \frac{1}{2}bh$
- $10 \times 8 / 2$
- $40cm^2$
- $V = Al$
- 40×20
- $800cm^3$



Volume

Some more Worked Examples

1. Calculate the volume correct to **3 s.f**
- Radius = 20cm / 2 = 10cm
- $V = ⅓\pi r^2 h$
- ⅓ x π x 10 x 10 x 25
- $2617.993cm^3$
- $V = \pi r^2 h$
- π x 10 x 10 x 30
- $9424.777cm^3$
- $V = \pi r^2 h$
- 4/3 x π x 10 x 10 x 10 / 2
- $2094.395cm^3$
- Total Volume = 2617.993 + 9424.777 + 2094.395
- Total Volume = 14137.166
- $14100cm^3$



Questions

Vol Cuboids: Link

Vol Prisms: Link

Vol Cylinders: Link

Vol Cylinders 2: Link

Vol Pyramids: Link

Vol Spheres: Link

Revision: Link

Mixed Qs Past Paper: Link

Pythagorean Theorem

Pythagorean Theorem

The Equation for a Right Angled Triangle: $c^2 = a^2 + b^2$

The **Adjacent** (a) is always adjacent to the right angle

The **Opposite** (b) is always opposite the adjacent

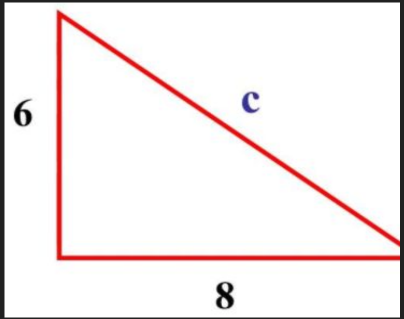
The **Hypotenuse** (c) is always the **opposite** the right angle and is the **longest** of the three

The Equation can be rearranged to find either the **opposite** or the **adjacent**:

$a^2 = c^2 - b^2$

Pythagorean Theorem

Some Worked Examples



1. Find C

- $c^2 = a^2 + b^2$ (Write out formula)
- $c^2 = 6^2 + 8^2$ (Sub in actual numbers)
- $c^2 = 36 + 64$ (Square numbers)
- $c^2 = 100$
- $c = \sqrt{100}$ (Square root to find actual length)
- $c = 10$ cm

Opposite of Pythagorean Theorem

Once you know how to find the length of a missing side, you can figure out if a triangle has a right angle or not.

IF: $c^2 = a^2 + b^2$

THEN: Triangle = Right Angled

Example

1. Show that ΔABC is right angled
   - $AB^2 + BC^2 = 8^2 + 6^2 = 100$
   - $AC^2 = 10^2 = 100$
   - Since $AB^2 + BC^2 = AC^2$
   - "By the converse of Pythagoras, ΔABC is right angled at B (i.e $\angle ABC = 90^o$)"

Pythagoras in Circles

Questions

Finding Hypotenuse: Link

Application: Link

Other Sides: Link

Extension: Link

Mixed Qs: Link

Random Qs: Link

Past Paper Qs: Link

Algebraic Equations

Removing Brackets

In order to remove brackets from an equation, you must “multiply them out”

Example

1. $3x(2x - y + 7)$
- **$6x^2 - 3xy + 21x$**
- (Working)
- $3x \times 2x = 6x^2$
- $3x \times -y = -3xy$
- $3x \times 7 = 21x$

Algebraic Equations

Some Worked Examples (Single Bracket)

1. $-2(3t + 5)$

- $-2 \times 3t = -6t$
- $-2 \times 5 = -10$
- **$-6t - 10$**

2. $-3w(w^2 - 4)$

- $-3w \times w^2 = -3w^3$
- $-3w \times -4 = 12w$
- **$-3w^3 + 12w$**

Algebraic Equations

1. $2t(3 - t) + 5t^2$
   - $6t - 2t^2 + 5t^2$
   - $6t + 3t^2$

2. $5 - 3(n - 2)$
   - $6t - 2t^2 + 5t^2$
   - $6t + 3t^2$

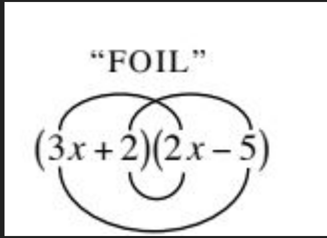
Algebraic Equations

When you have 2 brackets, you can “multiply out” by multiplying both brackets together

Example

1. $(3x + 2)(2x - 5)$
   - $3x(2x - 5) + 2(2x - 5)$
   - $6x^2 - 15x + 4x - 10$
   - $6x^2 - 11x - 10$

FOIL Method



Algebraic Equations

Some Worked Examples

1. $(2t - 3)^2$
   - $(2t - 3)(2t - 3)$
   - $2t(2t - 3) - 3(2t - 3)$
   - $4t^2 - 6t - 6t + 9$
   - $4t^2 - 12t + 9$
2. $(w + 2)(w^2 - 3w + 5)$
   - $w(w^2 - 3w + 5) + 2(w^2 - 3w + 5)$
   - $w^3 - 3w^2 + 2w^2 + 5w - 6w + 10$
   - $W^3 - w^2 - w + 10$

Factorising Algebraic Equations

Common Factors

Highest Common Factors are used to write expressions in a fully factored form.

Factorise Fully:

$4a - 2a^2$

$= 2a(2 - a)$ "$2a \times 2 - 2a \times a$ (using $HCF(4a, 2a^2) = 2a$)"

Note: the following answers are factorised but not **fully** factorised

- $2(2a - a^2)$
- $a(4 - 2a)$

Difference of Two Squares

Factorise Fully

1. $4x^2 - 9$
   - $(2x)^2 - 3^2$
   - $(2x + 3)(2x - 3)$

2. $4x^2 - 36$
   - $4(x^2 - 9)$
   - $4(x + 3)(x - 3)$

Note $(2x + 6)(2x - 6)$ is factorised but not fully factorised.

Trinomials

$x^2 + \mathrm{b}x + \mathrm{c} = (x + ?)(x + ?)$

The missing numbers are: **A pair of factors of c that sum to b**

Some Worked Examples

1. $x^2 + 5x + 6$
   - 1 x 6 = 2 x 3 = 6
   - 2 + 3 = 5
   - Use + 2 and + 3
   - $(x + 2)(x + 3)$

Trinomials

1. x^2 - 5x + 6
   - -1, -6 or -2, -3
   - -2 + (-3) = -5
   - Use -2 and -3
   - (x - 2)(x - 3)
2. x^2 - 5x - 6
   - -1, 6 or 1, -6 or -2, 3 or 2, -3
   - 1 + (-6) = -5
   - Use +1 and -6
   - (x + 1)(x - 6)

Questions

Inequality Equations

Equations

You can solve inequalities as shown below

1. x + a = b
   - x = b - a
2. x - a = b
   - x = b + a
3. ax = b
   - x = b/a
4. x/a = b
   - x = ab

- Solve 5x - 4 = 2x - 19
  - 3x - 4 = -19 (Subtract 2x from both sides)
  - 3x = -15 (add 4 to each side)
  - x = -5 (Divide each side by 3)

Equations with Brackets

In order to solve inequalities with brackets, you must first multiply out the brackets and simplify

1. Solve $(4x + 3)(x - 2) = (2x - 3)^2$

- $4x^2 - 5x - 6 = 4x^2 - 12x + 9$ (Remove brackets, fully simplify)
- $-5x - 6 = -12x + 9$ (Subtract $4x^2$ from both sides)
- $7x - 6 = 9$ (Add $12x$ to both sides)
- $7x = 15$ (add 6 to both sides)
- $x = 15/7$ (Divide both sides by 7)

Equations with fractions

In order to solve an equation with fractions, remove first, multiplying by the LCM of the denominators

1. Solve $\frac{1}{2}(x + 3) + \frac{1}{3}x = 1$
- $3/6(x + 3) + 2x = 6$ (Write with common denominators)
- $3(x + 3) + 2x = 6$ (Both sides x6 to remove fractions)
- $3x + 9 + 2x = 6$
- $5x = -3$
- $x = -\frac{3}{5}$

Inequalities

Follow the same rules for equations except when multiplying or dividing by a negative number. Reverse the direction of the inequality sign

1. $-ax > b$
   - $x < b/-a$
2. $x/-a > b$
   - $x < -ab$

Examples

1. Solve $8 + 3x > 2$
   - $3x > -6$
   - $x > -6/3$ (Divided each side by 3, notice sign unchanged)
   - $x > -2$
2. Solve $8 - 3x > 2$
   - $-3x > -6$ (Subtract 8 from both sides)
   - $x < -6/-3$ (Divided both sides by -3, notice sign changed)
   - $x < 2$ (simplified)

Questions: [Link]()

Straight Line Equations

Gradient

Formula for gradient: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Top Row: Vertical Change (Y Axis)

Bottom Row: Horizontal Change (X Axis)

Example

Find the Gradient of line (1,4)(2,3)

$m = y_2 - y_1 / x_2 - x_1$

m = 3 - 4 / 2 - 1

m = -1 / 1

Questions

Gradient 1: Link

Gradient 2: Link

Equation of a straight line

Formula for a straight line: $y = mx + c$

Gradient (m), Y-Intercept (c) & Straight Line (y)

The Y-Intercept is the point the line goes through on the y axis

Questions: [Link]

Finding the equation of a straight line

In order to find the equation of a straight line, you must first find the gradient using the formula

$m = y_2 - y_1 / x_2 - x_1$ (y = Vertical values, x = horizontal values)

Then you must find the Y-Intercept as shown below:

Points (3,10)(7,18)

m = 18-10/7-3

m = 8/4 = 2

$y = mx + c$ (Write out formula)

10 = (2)(3) + c (Sub in numbers)

10 = 6 + c (c is just 10 - 6)

c = 4

$y = 2x + 4$ (Reconstruct Formula)

Questions

Finding Equation of Straight Lines: Link

General Equation of a straight line: Link

Simultaneous Equations

Simultaneous Equations

Very simple to do

If the signs are different (e.g + and -) then add them together, if the signs are the same (e.g + and +) then subtract them.

Example

1. $4x + 3y = 5$

- $5x - 2y = 12$
- $8x + 6y = 10$ (Multiply to make the y's the same)
- $15x - 6y = 36$
- $23x = 46$ (Divide non x value by x value to get the value of 1x)
- $x = 2$
- $4x + 3y = 5$
- 4(2) + 3y = 5 (Sub in the x value)
- 8 + 3y = 5
- 3y = -3
- y = -1 (Divide by number to get y on its own)
- x = 2, y = -1 (Write out the x and y values separately)

Questions

Simultaneous Equations 1: Link

Simultaneous Equations 2: Link

Application: Link

Changing the Subject of a Formula

Changing the subject of a formula

The subject of a formula is the letter on its own on one side of the equals sign.

e.g $\boldsymbol{x} = 3 + y$ or $12 - x = \mathbf{y}$

Example: Change the subject

1. a + b = c (c)

- c = a - b

A simple rule for this is as follows:

Change the side, Change the sign

Changing the subject of a formula

Some worked examples

1. p = q + r (q)
   - p - r = q

2. h = m/n (Or h/1 = m/n) (m)
   - m = hn (hxn) (Or hn = m)

3. v = rs (s)
   - s = v/r

4. $2x$ + y = w (x)
   - w - y = $2x$
   - x = w-y/2

5. 7 = c - x
   - 7 - c = -x
   - -7 - c = x

Changing the subject of a formula

Some more worked examples

1. $p = a(x + n)$ (x)
- $p = ax + an$
- $ax = p - an$
- $x = p - an/a$

2. $T = 1/5k^2h$ (k)
- $5T = k^2h$
- $k^2 = 5T/h$
- $\sqrt{}5T/h = k$

3. $vtu/w = bw/6$
- $6v = 6w^2 - 6u$
- $v = bw^2 - 6u/6$

Questions

Changing the Subject 1: Link
Changing the Subject 2: Link

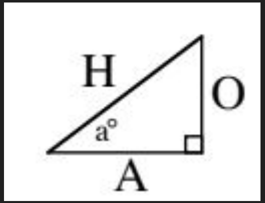
SOHCAHTOA

SOHCAHTOA

SOH - CAH - TOA

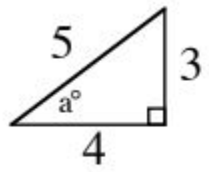
The sides of a right angle triangle are labelled as follows

- **Opposite**: Opposite to angle a°
- **Adjacent**: Next to the angle a°
- **Hypotenuse**: Opposite the right angle



The ratios of sides O/H, A/H and O/A have **values** which depend on the s**ize of angle a°**.

For example:



$$S = \frac{O}{H} \qquad C = \frac{A}{H} \qquad T = \frac{O}{A}$$

$$\sin a^\circ = \frac{3}{5} \qquad \cos a^\circ = \frac{4}{5} \qquad \tan a^\circ = \frac{3}{4}$$

Trigonometry

The trig function acts on an **angle to produce the value of the ratio**.

The inverse trig function acts on the **value of a ratio to produce the angle**.

For example

- $\sin 30^{o} = 0.5$
- $\sin^{-1} 0.5 = 30^{o}$

Accuracy

Rounding the **angle** or the **value** in a calculation can result in **significant errors**.

e.g

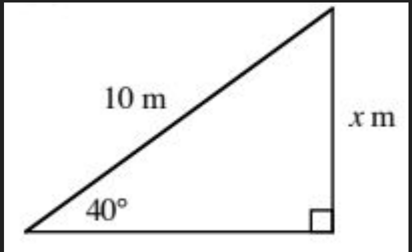
- $100 \times \tan(69.5^{o}) = 267.462\ldots \approx 267 \rightarrow 100 \times \tan(70^{o}) = 274.747\ldots \approx 275$
- $\tan^{-1} 2.747 = 69.996\ldots \approx 70.0 \rightarrow \tan^{-1} 2.7 = 69.676\ldots \approx 69.7$

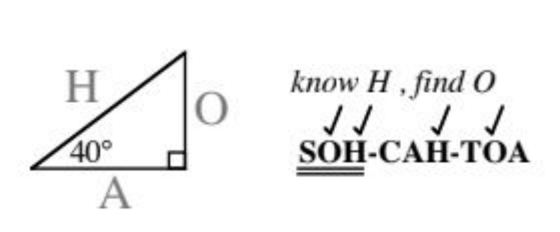
Trigonometry

Finding an unknown side

1. Find x

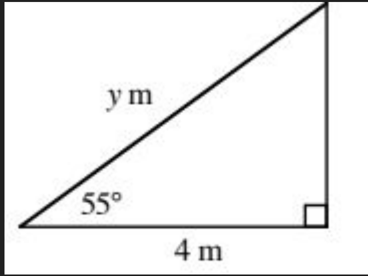
- $\sin(40) = x/10$
- $x = 10 \times \sin(40)$
- $x = 6.427$
- $x = 6.4m$

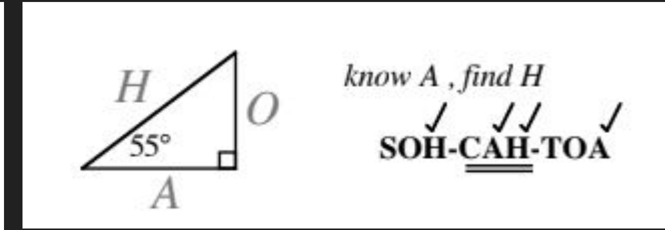




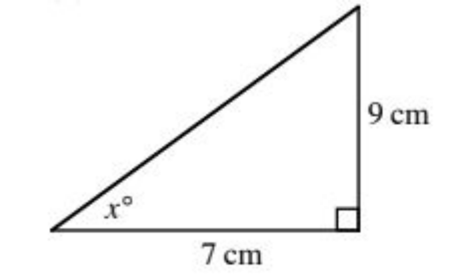
Trigonometry

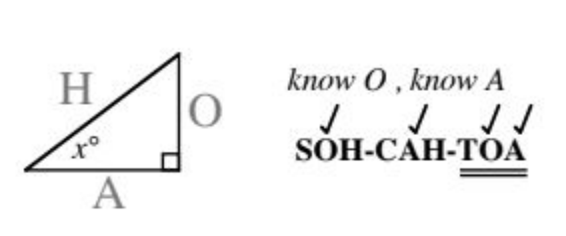
1. Find y
- $\cos(55^o) = 4/y$
- $y = 4/\cos(55^o)$
- $y = 6.973$
- $y = 7.0m$





2. Find x (Finding an unknown angle)
- $\tan x^o = 9/7$
- $x = \tan^{-1}(9/7)$
- $x = 52.125$
- $x = 52.1^o$





Questions

Calculate Missing Side 1 (Tangent): Link

Missing Side 2 (Tangent): Link

Missing Side 3 (Tangent): Link

Missing Side 4 (Sine): Link

Missing Side 5 (Sine): Link

Missing Side 6 (Cosine): Link

Missing Side 7 (Cosine): Link

Missing Angle 1 (Tangent): Link

Missing Angle 2 (Tangent): Link

Missing Angle 3 (Sine): Link

Mixed Calculations (SOHCAHTOA): Link

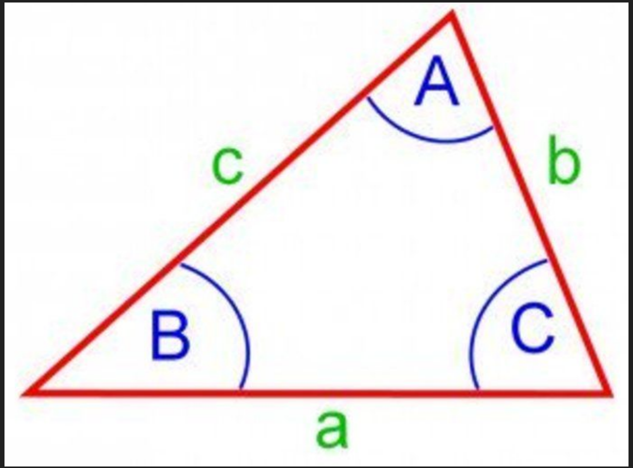
Trigonometry

Area of a Triangle

Area = ½ abSinC

NOTE:

- Capital Letters are for Angles
- Lowercase Letters are for Sides



Use formula when you have opposite sides & opposite angles

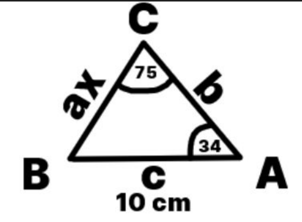
Sine Rule

The sine rule can be used to find a missing side or a missing angle

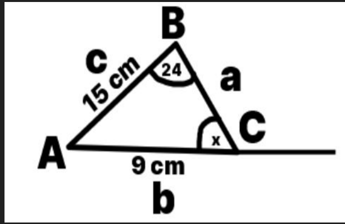
$$\frac{a}{\text{sine } A} = \frac{b}{\text{sine } B} = \frac{c}{\text{sine } C}$$

$$\frac{\text{sine } A}{a} = \frac{\text{sine } B}{b} = \frac{\text{sine } C}{c}$$

Example



- a/sinA = b/sinB = c/sinC
- a/sin(34) = 10/sin(75)
- a = 10 x sin(34) / sin(75)



NOTE: For obtuse angles, you must find the angle and take it away from 180 to find the answer

- sinA/a = sinB/b = sinC/c
- sin24/9 = sinC/15
- 15 x sin(24)/9 = sinC
- $\sin^{-1}$(15 x sin(24)/9)
- 43º
- 180 - 43
- 137º

Use formula when you have all sides & no angles or when you have 2 sides & included angle

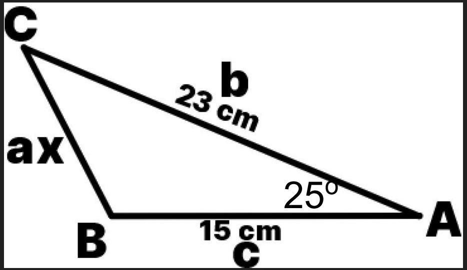
Cosine Rule

The cosine rule can be used to find a missing side or angle of a triangle

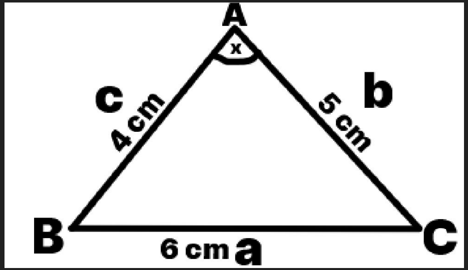
Formula: $a^2 = b^2 + c^2 - 2bc \, CosA$

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

Example



- $a^2 = b^2 + c^2 - 2bc \cos A$
- $= 23^2 + 15^2 \; 2(23)(15) \cos(25)$
- = sqrt(128.647)
- = 11.3 cm



- $\cos A = b^2 + c^2 - a^2/2bc$
- $= 4^2 + 5^2 - 6^2/2(5)(4)$
- $\cos^{-1}(4^2 + 5^2 - 6^2/2(5)(4))$
- 83°

Questions

Area of a Triangle: Link

Sine & Cosine Rule: Link

Sine Rule: Link (Ans)

Cosine Rule: Link (Ans)

Random Qs (Trig Rules): Link

Arcs & Sectors

Arc Length

An arc is a fraction of the circumference of a circle. It can be calculated using the following formula:

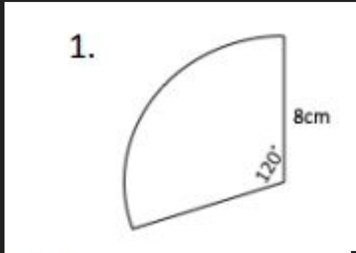
$$\frac{Angle}{360^\circ} \times \pi d$$

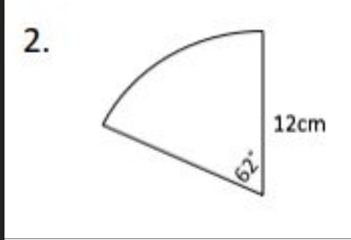
(Angle is θ)

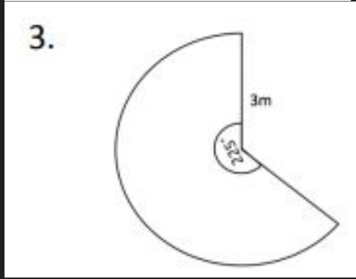
Arc Length

Examples

1. AoL = Angle/360 x (π x Diameter)
   - = 120/360 x (π x 16)
   - = 16.8 cm
2. AoL = Angle/360 x (π x Diameter)
   - = 62/360 x (π x 24)
   - = 13.0 cm
3. AoL = Angle/360 x (π x Diameter)
   - = 225/360 x (π x 6)
   - 11.8 m







Finding the angle using the Arc Length

Use the same formula to find the Angle that you would use to find the arc length and rearrange

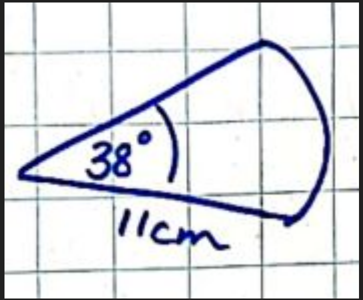
E.g

1. If the length of the arc AB is 12.56 cm and the radius is 12cm, find the angle
   - 12.56 = x/360 x (π x Diameter)
   - 12.56/π x 24 = x/360 (Rearrange)
   - x = 0.167 x 360 (Multiply out)
   - x = 60º

Sectors

You can find the area of a sector using the formula below

Example



- AoS = Angle/360 x (π x r^2)
- = 38/360 x (π x 11^2)
- = 40.1 cm^2

NOTE: Both Arc Length and Area of a Sector are **NOT** on the formula sheet. This means you must remember them without assistance.

Questions

Arcs & Sectors 1: Link (Ans)

Arcs & Sectors 2 (Past Papers): Link

Arcs & Sectors 3 (Past Papers): Link

Random Qs: Link

Mixed Qs: Link (Ans)

Averages & Standard Deviation

Interquartile Range

Most people should be well aware of these types of averages:

- Range: Highest - Lowest
- Mode: Most common value
- Median: Middle value
- Mean: Add all values and divide by number of values

There is also Interquartile Range

Interquartile Range measures the spread by finding the range of the middle 50% of the values

Example:

- 3, 4, 4, 5, 6, 7, 8
- Q1 = 4, Q2 (median) = 5, Q3 = 7
- IQR = Q3 - Q2
- = 7 - 4
- = 3

Standard Deviation

The standard deviation measures how far away, on average, each of the values are from the mean

Formula: $s = \sqrt{\frac{\sum (X-\bar{X})^2}{n-1}}$ (where n is the sample size)

- Σ: Total of (Sigma)
- n: Sample Size
- x̄: Mean

Standard Deviation

x	$x - \bar{x}$	$(x - \bar{x})^2$
10	-3	9
16	3	9
11	-2	4
15	2	4
13	0	0
	0 (This column should always add to 0)	$\Sigma(x - \bar{x})^2 = 26$

Follow along with this example

1. Calculate the mean and standard deviation of: 10, 16, 11, 15, 13

- Step 1: Calculate the mean
- (10+16+11+15+13)/5 = 13
- Step 2: Construct a table as shown to the left
- $s = \sqrt{\frac{\sum (X - \bar{X})^2}{n-1}}$
- s = sqrt(26/4) (sqrt = square root)
- s = <u>2.5</u>

Comparing Data

When comparing data, you can use the mean or standard deviation

The mean - measures the ‘average’ case. E.g “On average…”

Standard Deviation - Measures the spread

- Low s.d: high consistency
- High s.d: low consistency

Example: These are the first 6 scores for 2 people playing darts

Name 1	18	22	20	20	13	19
Name 2	3	28	6	30	1	30

Name 1 $\bar{x} = 19$

s.d = 2.4

Name 2 $\bar{x} = 18$

s.d = 16.4

Compare the mean and standard deviation

<u>On</u> <u>average</u>, Name 1 scored higher <u>because</u> the mean is higher

Name 1’s scores are <u>more</u> <u>consistent</u> <u>because</u> the standard deviation is lower.

Questions

Standard Deviation 1: Link (Ans)

Statistics 1 (Past Papers): Link

Statistics 2: Link

Mixed Qs: Link (Ans)

Random Qs: Link

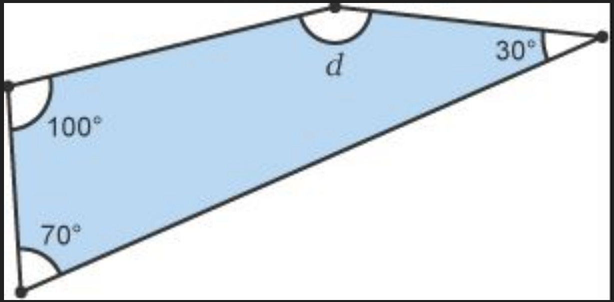
Angles in Shapes

Angles in Quadrilaterals

Angles in a quadrilateral always add up to 360º

Examples

1. Find angle d



- 100 + 30 + 70 = 200
- 360 - 200 = 160º

Angles in Circles

The diameter is always a right angled triangle

The radius always makes a tangent at a right angle

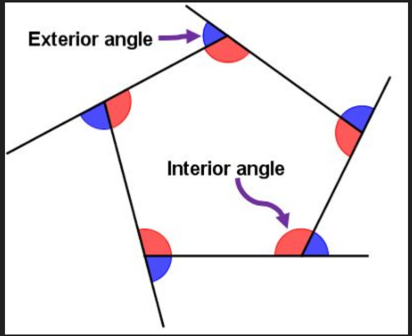
A triangle with 2 sides that are radii are isosceles

Always check for right angles and isosceles triangles

(That's literally all the notes I have for it)

Angles in Polygons

A polygon is a many sided shape e.g triangles, quadrilaterals, pentagon etc



Interior + exterior angle = 180º

For all polygons, the sum of the exterior angles is 360º

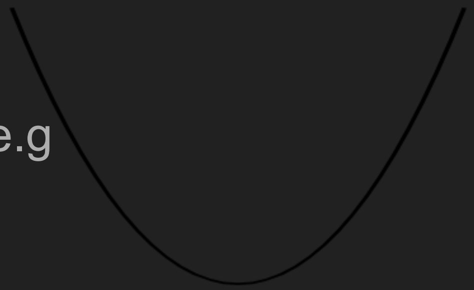
For an n sided polygon, the sum of the interior angle is (n - 2) x 180

Quadratic Formula

Solving Quadratics

A quadratic is an expression with x^2 as the highest power

If we were to sketch a quadratic it would be a parabola (curve) e.g



To solve a quadratic

- Make it equal 0
- Factorise
- Solve for x

1. $(x + 3)(x - 5) = 0$
   - $x + 3 = 0$ $x - 5 = 0$
   - $x = -3$ $x = 5$
2. $x^2 + 13x + 30 = 0$
   - $(x + 3)(x + 10) = 0$
   - $x + 3 = 0$ $x + 10 = 0$
   - $x = -3$ $x = -10$

Solving Quadratics Qs: Link (Ans)

Completing the Square

$(x + 4)^2 = x^2 + 8x + 16$

$(x - 7)^2 = x^2 - 14x + 49$

$(x + 2)^2 = x^2 + 4x + 4$

These are all complete squares

Double Squared

All Quadratics can be written in the form: $(x + \text{p})^2 + \text{q}$

Example

1. $x^2 + 8x + 9$
   - $= (x^2 + 8x + \mathbf{16}) + 9 - \mathbf{16}$ (Half x coefficient then square and subtract)
   - $= (x + 4)^2 - 7$

Graphs in Completed Square Form

Graphs in the form

$y = (x + p)^2 + q$

Have turning point

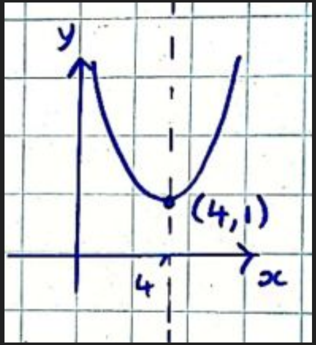
$(-p, q)$

And the equation of the axis of symmetry is

$x = -p$

Example

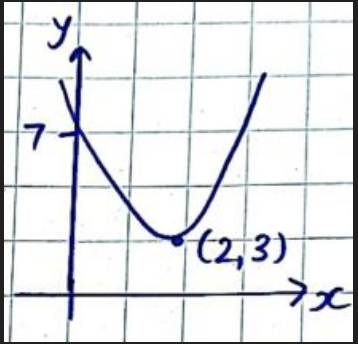
1. $y = (x - 4)^2 + 1$
   - Turning Point (4, 1)
   - Axis of Symmetry $x = 4$



We can make the sketch more accurate by finding the y intercept ($x = 0$)

Example

2. Sketch $y = (x - 2)^2 + 3$
   - Turning Point (2, 3)
   - y-intercept ($x = 0$)
   - $y = (0 - 2)^2 + 3$
   - $= 7$ (0, 7)



Questions

Completing the Square: Link (Ans)

Mixed Qs: Link (Ans)

The Quadratic Formula

The quadratic formula is simple enough, follow along with the example

1. Solve x^2 + 5x + 3 to 2 decimal places
- Step 1: Write out the Quadratic Formula
- $$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$
- Step 2: Sub in the values (If no value is assigned to the x, use 1)
- = -5 ± √5^2 - 4(1)(3) / 2(1)
- Step 3, simplify the values inside of the square root
- = -5 ± √25 - 12 / 2
- Step 4, do a calculation for both the subtract and the add
- = (-5 - 3.61) / 2
- = 4.31
- = (-5 + 3.61) / 2
- = -0.70

Questions

Quadratic Formula: Link (Ans)

Mixed Qs: Link (Ans)

Sketching Parabolas using factorisation

Example

1. Sketch a graph of $y = x^2 - 2x - 3$
   - Step 1: Find the roots ($y = 0$)
   - $x^2 - 2x - 3 = 0$
   - $(x + 1)(x - 3) = 0$
   - $x = -1$
   - $x = 3$

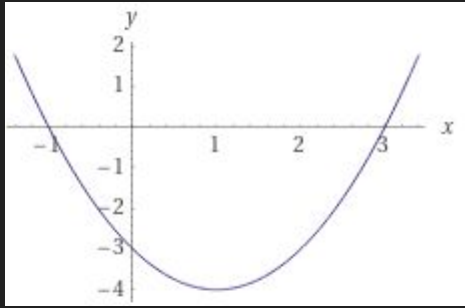
Step 2: Find the Turning Point (Halfway between the roots) At $x = 1$

- $y = x^2 - 2x - 3$
- $= (1)^2 - 2(1) - 3$
- $= (1, -4)$

Step 3: Find the y intercept ($x = 0$):

- $y = 0^2 - 2(0) - 3$
- $= -3$
- $\rightarrow (0, -3)$

Step 4: Sketch the Parabola



Graphs with Maximum Turning Points

Minimum Turning Points

- $y = x^2 + 3x + 2$
- $y = (x + 2)^2 - 5$
- $y = 3 - 4x + x^2$
- $y = 2x^2 + 5x - 8$

* x^2 is always positive

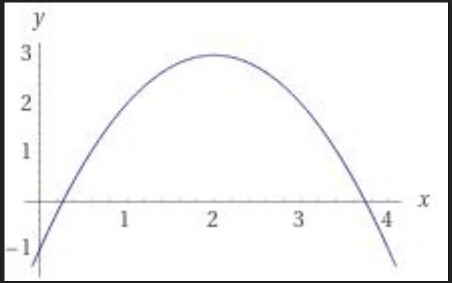
Maximum Turning Points

- $y = -x^2 + 3x + 2$
- $y = -(x + 2)^2 - 5$
- $y = 3 - 4x - x^2$
- $y = -2x^2 + 5x - 8$

* x^2 is always negative

Example

1. Sketch the graph of $y = 3 - (x + -2)^2$
   - Turning point = (2, 3)
   - Y-intercept ($x = 0$)
   - $y = 3 - (0 - 2)^2$
   - = (0, -1)



Quadratics in the form $y = kx^2$

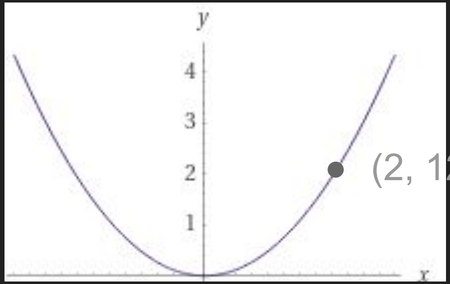
Graphs with the equation $y = kx^2$ have a turning point of (0,0)

k determines the width of the parabola

Example

1. This is the graph $y = kx^2$, find k

- $y = kx^2$
- $12 = k(2)^2$
- $12 = 4k$
- $k = 3$
- $y = 3x^2$



(2, 12)

The discriminant

Formula: $b^2 - 4ac$

We use the discriminant to determine the nature of the roots of a quadratic

- If $b^2 - 4ac$ is greater than 0, then the quadratic has 2 real roots
- If $b^2 - 4ac$ is equal to 0, then the quadratic has equal roots
- If $b^2 - 4ac$ is less than 0, then the quadratic has no real roots

Example

Determine the nature of the roots of these quadratics

1. $y = x^2 + 6x + 9$ (a = 1, b = 6, c = 9)
   - $= 6^2 - 4(1)(9)$
   - $= 36 - 36$
   - $= 0$
   - $B^2 - 4ac = 0$, so the quadratic has equal roots.

2. $y = x^2 + 5x + 6$ (a = 1, b = 5, c = 6)
   - $= 5^2 - 4(1)(6)$
   - $= 25 - 24$
   - $= 1$
   - $b^2 - 4ac > 0$, so the quadratic has 2 real roots

Questions

The Discriminant: Link (Ans)

The Discriminant (Past Papers): Link

Random Qs 1: Link

Random Qs 2: Link

Congratulations, you made it to the end and are now ready for your exam, make sure to keep revising using these slides and other sources given to you by your teacher and good luck!