Definitions

A = number of possible playthroughs.

V = number of nodes in at least one valid path from node 0 to node N-1

E = number of edges in at least one valid path

N = number of all nodes

Upper bound for A is obviously E.

In fact, A = E when there are only 2 nodes.

Let's see what happens when a new node is added to this path.

To add a node and make it a part of a valid path, we need at least two edges;

one for going from an existing path to the new node,

and another for going from the new node back to an existing path.

Notice that a new node with only one edge will not make it a part of a new path.

So V <= (N)

Let's say the new node brings Enew number of edges. (ex> 4 new edges,)

Then (Enew - 1) new additional paths are available.

For example, Assume we add node B to A -> C

original A -> C number of path = 1

new : A -> B -> C, A->C number of path = 1 + (2-1) = 1 + 1 = 2

Note that there are (V-2) such new nodes.

Proof.

Assume that there was originally only node 0 and node N-1.

and all other nodes were added one by one.

A = E - sum(1)(from 1 to V-2)

= E - (V - 2)