Lab 3

Lagrange interpolation

Using *the barycentric form* of the Lagrange interpolation polynomial, solve the following problems:

Problems:

1. The table below contains the population of the USA from 1930 to 1980 (in thousands of inhabitants):

1930	1940	1950	1960	1970	1980
123203	131669	150697	179323	203212	226505.

Approximate the population in 1955 and 1995.

2. Approximate $\sqrt{115}$ with Lagrange interpolation, using the known values for three given nodes.

3. Plot the graphics of the function $f : [0, 10] \to \mathbb{R}$, $f(x) = \frac{1+\cos(\pi x)}{1+x}$ and of the Lagrange interpolation polynomial that interpolates the function f at 21 equally spaced points in the interval $[0, 10]$.

Facultative:

1. Consider the function $f : [-\frac{\pi}{4}, \frac{\pi}{2}] \to \mathbb{R}$, $f(x) = \cos(x)$ and the given nodes $0, \frac{\pi}{4}, \frac{\pi}{3}$.

a) Plot the fundamental interpolation polynomials $\ell_i(x) = \frac{u_i(x)}{u_i(x_i)}$, $i = 0, .., m$.

b) Compute the value of Lagrange interpolation polynomial at $x = \frac{\pi}{6}$ using both the classical formula $(L_m f)(x) = \sum_{i=0}^{m} \ell_i(x) f(x_i)$ and the barycentric formula.

c) Plot the graphs of the function f and of the corresponding Lagrange interpolation polynomial.

d) Give two other sets of nodes in $[-\frac{\pi}{4}, \frac{\pi}{2}]$ and plot the correponding Lagrange interpolation polynomials.

2. a) Plot the graphs of the function $f : [-5,5] \to \mathbb{R}$, $f(x) = \frac{1}{1+x^2}$ and of the corresponding Lagrange interpolation polynomials of 4-th, 8-th and 14-th degrees.

b) Consider the Chebyshev zeros of the first kind $x_i = \cos(\frac{(2i-1)\pi}{2n}) \in [-1,1]$, $i = 1, ..., n$. Plot the same graphs as the ones from a) using 15 nodes obtained by linear transformation $\frac{1}{2}((b-a)x_i + a + b)$.

c) Consider the Chebyshev zeros of the second kind $x_i = \cos(\frac{i\pi}{n}) \in [-1,1]$, $i = 0, ..., n-1$. Plot the same graphs as the ones from a) using 15 nodes obtained by linear transformation $\frac{1}{2}((b-a)x_i + a + b)$.