

Lab 4

1. Using the data from the following table

x	1	1.5	2	3	4
$\lg x$	0	0.17609	0.30103	0.47712	0.60206

approximate $\lg 2.5$ and $\lg 3.25$ using the Newton interpolation polynomial. Estimate the maximum interpolation error $E = \max |f(y_i) - (N_4 f)(y_i)|$, with $y_i = \frac{i}{10}$, for $i = \overline{10, 35}$.

2. To investigate the relationship between yield of potatoes, y , and level of fertilizer, x , an experimenter divided a field into 5 plots of equal size and applied differing amounts of fertilizer to each. The recorded data are given in the table (in pounds).

x	1	2	3	4	5
y	22	23	25	30	28

- a) According to Newton interpolation polynomial, approximate how many pounds of potatoes are expected from a plot to which 2.5 pounds of fertilizer had been applied.

- b) Plot the data given in the table and the corresponding Newton interpolation polynomial.

3. Consider the function $f : [0, 6] \rightarrow \mathbb{R}$, $f(x) = e^{\sin x}$ and 13 equidistant interpolation points. Plot the interpolation points, the function f and the Newton interpolation polynomial $N_{12}f$.

4. Approximate $\sqrt{115}$ with precision $\varepsilon = 10^{-3}$, using Aitken's algorithm.

Facultative:

1. Consider the function $f : [-5, 5] \rightarrow \mathbb{R}$, $f(x) = \sin x$ and 20 equidistant interpolation points. Plot the interpolation points, the function f and the Lagrange interpolation polynomial obtained using Aitken's algorithm with precision $\varepsilon = 10^{-3}$.

2. Consider the function $f : [-1, 1] \rightarrow \mathbb{R}$ and $n = 10$ nodes in the interval $[-1, 1]$.

- a) Compute the Lebesgue function $g(x) = \sum_{i=1}^n |l_i(x)|$ at a given point x , where $l_i(x)$, $i = 1, \dots, n$ are the fundamental Lagrange polynomials.

- b) Plot the function g for n equidistant nodes in $[-1, 1]$.
- c) Plot the function g for n Chebyshev nodes of the first kind, $x_i = \cos(\frac{(2i-1)\pi}{2n}) \in [-1, 1]$, $i = 1, \dots, n$.
- d) Plot the function g for n Chebyshev nodes of the second kind, $x_i = \cos(\frac{i\pi}{n}) \in [-1, 1]$, $i = 0, \dots, n-1$.