

Harmony Series

Number Theory

Harmonic Series

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Date / /

① Number of divisors $\rightarrow a[i]$

$$a[1] = 1$$

$$a[2] = 2$$

$$a[3] = 2$$

$$a[4] = 3$$

$$a[5] = 2$$

$$a[6] = 4$$

$$a[7] = 2$$

$$a[8] = 4$$

$$a[9] = 3$$

$$a[10] = 4$$

① 1 divisor for 1 to 10

$\rightarrow 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$

$$② 2 \Rightarrow 2, 4, 6, 8, 10$$

$$③ 3 \Rightarrow 3, 6, 9$$

$$④ 4 \Rightarrow 4, 8$$

$$⑤ 5 \Rightarrow 5, 10$$

$$⑥ 6, 7, 8, 9, 10 \rightarrow \text{divisor}$$

② contribution technique
pre-calculation

③ Multiple

$$1 = \frac{10}{1} = 10 \quad 6 = \frac{10}{6} = 1$$

$$2 = \left\lfloor \frac{10}{2} \right\rfloor = 5 \quad 7 = \frac{10}{7} = 1$$

$$3 = \frac{10}{3} = 3 \quad 8 = \frac{10}{8} = 1$$

$$4 = \frac{10}{4} = 2 \quad 9 = \frac{10}{9} = 1$$

$$5 = \frac{10}{5} = 2 \quad 10 = \frac{10}{10} = 1$$

Rivotril

1 ଥର କାମ ସମ୍ପାଦନ $\frac{n}{1}$

2 ଥର କାମ ସମ୍ପାଦନ $\frac{n}{2}$

n ଥର କାମ ସମ୍ପାଦନ $\frac{n}{n}$

$$\left(\frac{n}{1} + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \dots + \frac{n}{n} \right)$$

$$= n \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n} \right) \rightarrow \text{Harmonic Series}$$

$$\sum_{k=1}^n \frac{1}{k}$$

$$H_n = \ln n + \cancel{\frac{1}{2n}} + \frac{\cancel{1}}{\cancel{2n}} - \cancel{\frac{1}{2n}}$$

small small small

→ complexity = $n \ln n$

$$\hookrightarrow O(n \ln(n))$$

$$\rightarrow n = 10^7$$

then

$$10^7 \cdot \ln(10^7)$$

$$= 10^7 \cdot 16 = 1.6 \times 10^8$$

```
#include<bits/stdc++.h>
using namespace std;
const int mx = 1e7+123;
int cnt[mx];

int main()
{
    int lim = 10;

    for ( int i = 1; i <= lim; i++ ) {        ///i = 1
        for ( int j = i; j <= lim; j += i ) {    ///j = 1
            cnt[j]++;                          ///cnt[1] = 1
        }
    }

    for(int i=1;i<=lim;i++) cout<<i<<" : "<<cnt[i]<<endl;

    return 0;
}
```

```
1 : 1
2 : 2
3 : 2
4 : 3
5 : 2
6 : 4
7 : 2
8 : 4
9 : 3
10 : 4
```

```
#include<bits/stdc++.h>
using namespace std;
const int mx = 1e3+123;
vector<int> d[mx];

int main()
{
    int lim = 10;
    for ( int i = 1; i <= lim; i++ )        ///i = 1
    {
        for ( int j = i; j <= lim; j += i )    ///j = 1
        {
            d[j].push_back(i);                ///cnt[1] = 1
        }
    }
    for(int i=1; i<=lim; i++)
    {
        cout<<i<<" : ";
        for(auto u : d[i]) cout<<u<<" ";
        cout<<endl;
    }

    return 0;
}
```

```
1 : 1
2 : 1 2
3 : 1 3
4 : 1 2 4
5 : 1 5
6 : 1 2 3 6
7 : 1 7
8 : 1 2 4 8
9 : 1 3 9
10 : 1 2 5 10
```

```

#include<bits/stdc++.h>
using namespace std;
const int mx = 1e7+123;
int sum_of_div[mx];
int main()
{
    int lim = 10;

    for ( int i = 1; i <= lim; i++ ) {        ///i = 1
        for ( int j = i; j <= lim; j += i ) {    ///j = 1
            sum_of_div[j] += i;                ///cnt[1] = 1
        }
    }

    for(int i=1;i<=lim;i++) cout<<i<<" : "<<sum_of_div[i]<<endl;

    return 0;
}

```

```

1 : 1
2 : 3
3 : 4
4 : 7
5 : 6
6 : 12
7 : 8
8 : 15
9 : 13
10 : 18

```

- 1) <https://cses.fi/alon/task/1713>
- 2) https://atcoder.jp/contests/abc172/tasks/abc172_d
- 3) <https://cses.fi/problemset/task/1082>

$$\sum_{k=1}^n k \times f(n)$$

n
1-n

5

1	2	3	4	5
1	2	2	3	2
1 + 9 + 6 + 12 + 10 = 38				

1)

```
/// *** --- || In the name of ALLAH || --- *** ///
```



```
#include<bits/stdc++.h>
using namespace std;

typedef long long ll;
typedef vector<int> vi;
typedef vector<ll> vl;
typedef vector<vi> vvi;
typedef vector<vl> vvl;
typedef pair<int,int> pii;
typedef pair<double, double> pdd;
typedef pair<ll, ll> pll;
typedef vector<pii> vii;
typedef vector<pll> vll;
typedef double dl;

#define endl '\n'
#define PB push_back
#define F first
#define S second
#define all(a) (a).begin(),(a).end()
#define rall(a) (a).rbegin(),(a).rend()
#define sz(x) (int)x.size()

const double PI = acos(-1);
const double eps = 1e-9;
const int inf = 2000000000;
const ll infLL = 9000000000000000000;
#define MOD 1000000007

#define mem(a,b) memset(a, b, sizeof(a) )
#define sqr(a) ((a) * (a))

#define optimize() ios_base::sync_with_stdio(0);cin.tie(0);cout.tie(0);
#define fraction() cout.unsetf(ios::floatfield); cout.precision(10); cout.setf(ios::fixed,ios::floatfield);
#define file() freopen("input.txt","r",stdin);freopen("output.txt","w",stdout);

#define dbg(args...) do {cerr << #args << " : "; faltu(args); } while(0)
void faltu ( ) {      cerr << endl;}
template < typename T, typename ... hello>void faltu( T arg, const hello &... rest) {cerr << arg << '
';faltu(rest...);}
```

```

ll gcd ( ll a, ll b ) { return __gcd ( a, b ); }
ll lcm ( ll a, ll b ) { return a * ( b / gcd ( a, b ) ); }

const int mx = 1e6+123;
int cnt[mx];

int main()
{
    optimize();

    int lim = 1e6;

    for ( int i = 1; i <= lim; i++ ) {
        for ( int j = i; j <= lim; j += i ) {
            cnt[j]++;
        }
    }

    int n;
    cin >> n;

    while ( n-- ) {
        int x;
        cin >> x;
        cout << cnt[x] << endl;
    }

    return 0;
}

```

2)

```

/// *** --- || In the name of ALLAH ||| --- *** ///

#include<bits/stdc++.h>
using namespace std;

typedef long long ll;
typedef vector<int> vi;
typedef vector<ll> vl;
typedef vector<vi> vvi;
typedef vector<vl> vvl;
typedef pair<int,int> pii;

```

```

typedef pair<double, double> pdd;
typedef pair<ll, ll> pll;
typedef vector<pii> vii;
typedef vector<pll> vll;
typedef double dl;

#define endl '\n'
#define PB push_back
#define F first
#define S second
#define all(a) (a).begin(),(a).end()
#define rall(a) (a).rbegin(),(a).rend()
#define sz(x) (int)x.size()

const double PI = acos(-1);
const double eps = 1e-9;
const int inf = 2000000000;
const ll infLL = 9000000000000000000;
#define MOD 1000000007

#define mem(a,b) memset(a, b, sizeof(a) )
#define sqr(a) ((a) * (a))

#define optimize() ios_base::sync_with_stdio(0);cin.tie(0);cout.tie(0);
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#define dbg(args...) do {cerr << #args << " : "; faltu(args); } while(0)
void faltu ( ) {      cerr << endl;}
template < typename T, typename ... hello>void faltu( T arg, const hello &... rest) {cerr << arg << '
';faltu(rest...);}

ll gcd ( ll a, ll b ) { return __gcd ( a, b ); }
ll lcm ( ll a, ll b ) { return a * ( b / gcd ( a, b ) ); }

const int mx = 1e7+123;
int cnt[mx];

int main()
{
    optimize();

    int lim = 1e7;

    for ( int i = 1; i <= lim; i++ ) {
        for ( int j = i; j <= lim; j += i ) {
            cnt[j]++;
        }
    }
}

```

```
}

int n;
cin >> n;

ll ans = 0;
for ( int i = 1; i <= n; i++ ) {
    ans += ( 1LL * i * cnt[i] );
}

cout << ans << endl;

return 0;
}
```