

Harmony Series

Number Theory

Harmonic Series

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① Numbers of divisors $\rightarrow a[n] \rightarrow$

② কোন সংখ্যার দিভাগীর সংখ্যা

$\rightarrow 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$

$$a[1] = 1$$

$$a[2] = 2$$

$$a[3] = 2$$

$$a[4] = 3$$

$$a[5] = 2$$

$$a[6] = 4$$

$$a[7] = 2$$

$$a[8] = 4$$

$$a[9] = 3$$

$$a[10] = 4$$

$$\textcircled{2} \Rightarrow 2, 4, 6, 8, 10$$

$$\textcircled{3} \Rightarrow 3, 6, 9$$

$$\textcircled{4} \Rightarrow 4, 8$$

$$\textcircled{5} \Rightarrow 5, 10$$

$$\textcircled{6}, 7, 8, 9, 10$$

④ contribution technique
pre-calculation

$$\frac{1}{1} \quad \frac{1}{2} \quad \frac{1}{3} \\ \Delta = 4$$

⑤ কোন সংখ্যার মুল কৌণ্ডী?

$$1 = \frac{10}{1} = 10 \quad | \quad 6 = \frac{10}{6} = 1$$

$$2 = \left\lfloor \frac{10}{2} \right\rfloor = 5 \quad | \quad 7 = \frac{10}{7} = 1$$

$$3 = \frac{10}{3} = 3 \quad | \quad 8 = \frac{10}{1} = 1$$

$$4 = \frac{10}{4} = 2 \quad | \quad 9 = \frac{10}{1} = 1$$

$$5 = \frac{10}{5} = 2 \quad | \quad \frac{10}{10} = \frac{10}{10} = 1$$

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$$1 \text{ to } n \text{ are terms } \frac{n}{1}$$

$$2 \text{ to } n \text{ are terms } \frac{n}{2}$$

$$n \text{ to } 1 \text{ are terms } \frac{n}{n}$$

$$\left(\frac{n}{1} + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \dots + \frac{n}{n} \right)$$

$$= n \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n} \right) \rightarrow \text{Harmonic Series}$$

$$\sum_{k=1}^n \frac{1}{k}$$

$$H_n = \ln n + \cancel{\gamma} + \cancel{\frac{1}{2n}} - \cancel{\frac{1}{3n}}$$

$$\rightarrow \text{complexity} = n \ln n$$

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$$\hookrightarrow O(n \ln(n)) \rightarrow n = 10^7$$

then

$$10^7 \cdot \ln(10^7) \\ = 10^7 \cdot 16 = 1.6 \times 10^8$$

```
#include<bits/stdc++.h>
using namespace std;
const int mx = 1e7+123;
int cnt[mx];

int main()
{
    int lim = 10;

    for ( int i = 1; i <= lim; i++ ) {      ///i = 1
        for ( int j = i; j <= lim; j += i ) {  ///j = 1
            cnt[j]++;
        }
    }

    for(int i=1;i<=lim;i++) cout<<i<<" : "<<cnt[i]<<endl;

    return 0;
}
```

```
1 : 1
2 : 2
3 : 2
4 : 3
5 : 2
6 : 4
7 : 2
8 : 4
9 : 3
10 : 4
```

```
#include<bits/stdc++.h>
using namespace std;
const int mx = 1e3+123;
vector<int> d[mx];

int main()
{
    int lim = 10;
    for ( int i = 1; i <= lim; i++ )      ///i = 1
    {
        for ( int j = i; j <= lim; j += i )  ///j = 1
        {
            d[j].push_back(i);           ///cnt[1] = 1
        }
    }

    for(int i=1; i<=lim; i++)
    {
        cout<<i<<" : ";
        for(auto u : d[i]) cout<<u<<" ";
        cout<<endl;
    }

    return 0;
}
```

```
1 : 1
2 : 1 2
3 : 1 3
4 : 1 2 4
5 : 1 5
6 : 1 2 3 6
7 : 1 7
8 : 1 2 4 8
9 : 1 3 9
10 : 1 2 5 10
```

```
#include<bits/stdc++.h>
using namespace std;
const int mx = 1e7+123;
int sum_of_div[mx];
int main()
{
    int lim = 10;

    for ( int i = 1; i <= lim; i++ ) {      ///i = 1
        for ( int j = i; j <= lim; j += i ) {  ///j = 1
            sum_of_div[j] += i;                ///cnt[1] = 1
        }
    }

    for(int i=1;i<=lim;i++) cout<<i<<" : "<<sum_of_div[i]<<endl;

    return 0;
}
```

1	:	1
2	:	3
3	:	4
4	:	7
5	:	6
6	:	12
7	:	8
8	:	15
9	:	13
10	:	18

- 1) <https://cses.fi/alon/task/1713>
 - 2) https://atcoder.jp/contests/abc172/tasks/abc172_d
 - 3) <https://cses.fi/problemset/task/1082>

$$\sum_{k=1}^n k \times f(k)$$

1)

```

ll gcd ( ll a, ll b ) { return __gcd ( a, b ); }
ll lcm ( ll a, ll b ) { return a * ( b / gcd ( a, b ) ); }

const int mx = 1e6+123;
int cnt[mx];

int main()
{
    optimize();

    int lim = 1e6;

    for ( int i = 1; i <= lim; i++ ) {
        for ( int j = i; j <= lim; j += i ) {
            cnt[j]++;
        }
    }

    int n;
    cin >> n;

    while ( n-- ) {
        int x;
        cin >> x;
        cout << cnt[x] << endl;
    }

    return 0;
}

```

2)

```

/// *** --- ||      In the name of ALLAH      ||| --- *** ///

```

```

#include<bits/stdc++.h>
using namespace std;

typedef long long ll;
typedef vector<int> vi;
typedef vector<ll> vl;
typedef vector<vi> vvi;
typedef vector<vl> vvl;
typedef pair<int,int> pii;

```

```

typedef pair<double, double> pdd;
typedef pair<ll, ll> pll;
typedef vector<pii> vii;
typedef vector<pll> vll;
typedef double dl;

#define endl '\n'
#define PB push_back
#define F first
#define S second
#define all(a) (a).begin(),(a).end()
#define rall(a) (a).rbegin(),(a).rend()
#define sz(x) (int)x.size()

const double PI = acos(-1);
const double eps = 1e-9;
const int inf = 2000000000;
const ll infLL = 90000000000000000000000000000000;
#define MOD 1000000007

#define mem(a,b) memset(a, b, sizeof(a) )
#define sqr(a) ((a) * (a))

#define optimize() ios_base::sync_with_stdio(0);cin.tie(0);cout.tie(0);
#define fraction() cout.unsetf(ios::floatfield); cout.precision(10); cout.setf(ios::fixed,ios::floatfield);
#define file() freopen("input.txt","r",stdin);freopen("output.txt","w",stdout);

#define dbg(args...) do {cerr << #args << " : "; faltu(args); } while(0)
void faltu () { cerr << endl;}
template < typename T, typename ... hello>void faltu( T arg, const hello &... rest) {cerr << arg << '\
';faltu(rest...);}

ll gcd ( ll a, ll b ) { return __gcd ( a, b ); }
ll lcm ( ll a, ll b ) { return a * ( b / gcd ( a, b ) ); }

const int mx = 1e7+123;
int cnt[mx];

int main()
{
    optimize();

    int lim = 1e7;

    for ( int i = 1; i <= lim; i++ ) {
        for ( int j = i; j <= lim; j += i ) {
            cnt[j]++;
        }
    }
}

```

```
}

int n;
cin >> n;

ll ans = 0;
for ( int i = 1; i <= n; i++ ) {
    ans += ( 1LL * i * cnt[i] );
}

cout << ans << endl;

return 0;
}
```