Homework Assignment – 6 & 7

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Problem 1

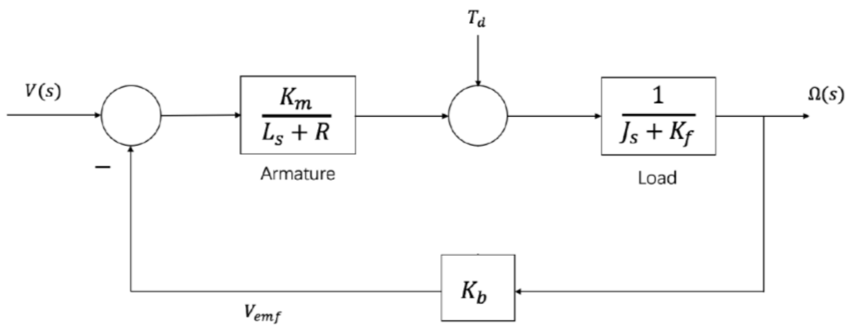


Figure 2: A simplified model of each DC motors with permanent magnets.

(a)

Ans – (a)

The determinant Δ from the Mason's Formula:

$$\Delta = 1 - \left[\frac{K_m}{Ls + R}\,\frac{1}{Js + Kf}(-K_b)\right]$$

Characteristic Polynomial for the simplified DC motors can be found out as follows:

$$Characteristic\ Polynomial\ \rightarrow\ 1 - \left[\frac{K_m}{Ls + R}\,\frac{1}{Js + Kf}(-K_b)\right] = 0$$

$$\rightarrow\ 1 - \left[-\left(\frac{K_m K_b}{Ls + R}\,\frac{1}{Js + Kf}\right)\right] = 0$$

$$\rightarrow\ 1 = -K_m K_b\left[\frac{1}{Ls + R}\,\frac{1}{Js + Kf}\right]$$

$$\rightarrow\ 1 = \left[\frac{-K_m K_b}{LJs^2 + LsK_f + RJs + RK_f}\right]$$

$$\rightarrow\ LJs^2 + LsK_f + RJs + RK_f = -K_mK_b$$

$$\rightarrow (LJ)s^2 + \left(LK_f + RJ\right)s + \left(RK_f + K_mK_b\right) = 0$$

Therefore,

$$\boldsymbol{Characteristic\ Polynomial \rightarrow (LJ)s^2 + \left(LK_f + RJ\right)s + \left(RK_f + K_mK_b\right) = 0}$$

Problem 1

(b)

Ans – (b)

The given parameters are as follows:

$R = 2.0$ $\quad$ $L = 0.5$ $\quad$ $K_m = 0.1$ $\quad$ $K_b = 0.1$

$J = 0.02$ $\quad$ $K_f = 0.2$ $\quad$ $T_d = 1$ $\quad$ $V(s) = 1$

(i) Let us consider a condition when T_d is not considered or $T_d = 0$:

$$\frac{\Omega(s)}{V(s)} = \frac{\dfrac{K_m}{Ls + R}\,\dfrac{1}{Js + Kf}}{1 + \left[\dfrac{K_mK_b}{Ls + R}\,\dfrac{1}{Js + Kf}\right]} \qquad \dots (i)$$

Assuming,

$$Armature\ Gain, L_a(s) = \frac{K_m}{Ls + R}$$

$$Load\ Gain, L_l(s) = \frac{1}{Js + Kf}$$

Now, equation (i) becomes,

$$\frac{\Omega(s)}{V(s)} = \frac{L_a(s)L_l(s)}{1 + [\,K_bL_a(s)L_l(s)]} \qquad \dots (ii)$$

$$\Omega(s) = \frac{L_a(s)L_l(s)}{1 + [\,K_bL_a(s)L_l(s)]}V(s) \qquad \dots (iii)$$

(ii) Let us consider a condition when $V(s)$ is not considered or $V(s) = 0$:

$$\frac{\Omega(s)}{T_d(s)} = \frac{L_l(s)}{1 + [\,K_b L_a(s) L_l(s)]} \quad \dots (iv)$$

$$\Omega(s) = \frac{L_l(s)}{1 + [\,K_b L_a(s) L_l(s)]} T_d(s) \quad \dots (v)$$

Therefore, Angular Velocity Response i.e. $\Omega(s)$ can be calculated using equation (iii) and equation (v).

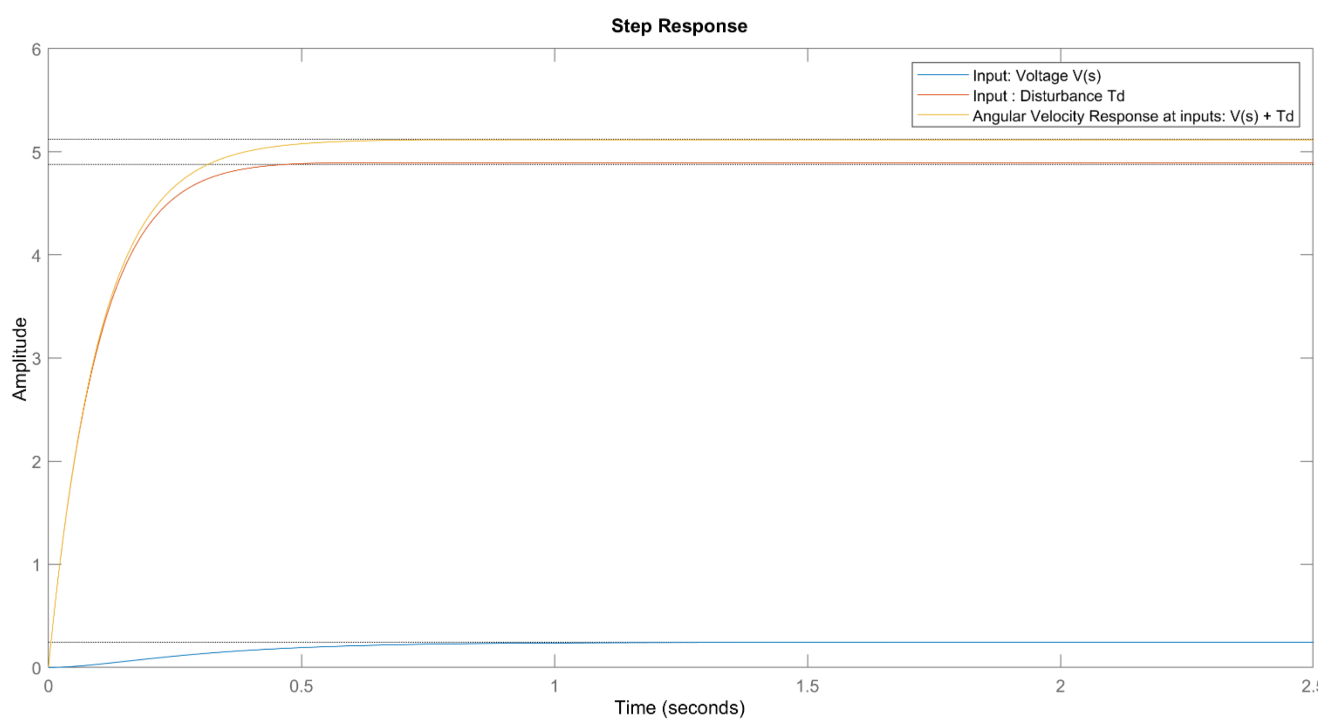
$$\boldsymbol{\Omega(s) = \frac{L_a(s) L_l(s)}{1 + [\,K_b L_a(s) L_l(s)]} V(s) + \frac{L_l(s)}{1 + [\,K_b L_a(s) L_l(s)]} T_d(s) \quad \dots (vi)}$$

Hence, the transform function can be constructed using given parameters and equation (vi).

$$Transform\ Function \rightarrow \Omega(s) = \frac{L_a(s) L_l(s)}{1 + [\,K_b L_a(s) L_l(s)]} V(s) + \frac{L_l(s)}{1 + [\,K_b L_a(s) L_l(s)]} T_d(s)$$

$$\boldsymbol{Transform\ Function \rightarrow \Omega(s) = \frac{10}{s^2 + 14s + 41} V(s) + \frac{50s + 200}{s^2 + 14s + 41} T_d(s) \dots (vii)}$$

Plot of the angular velocity response



Problem 1

(c)

Ans – (c)

Feed-Forward Control Plots:

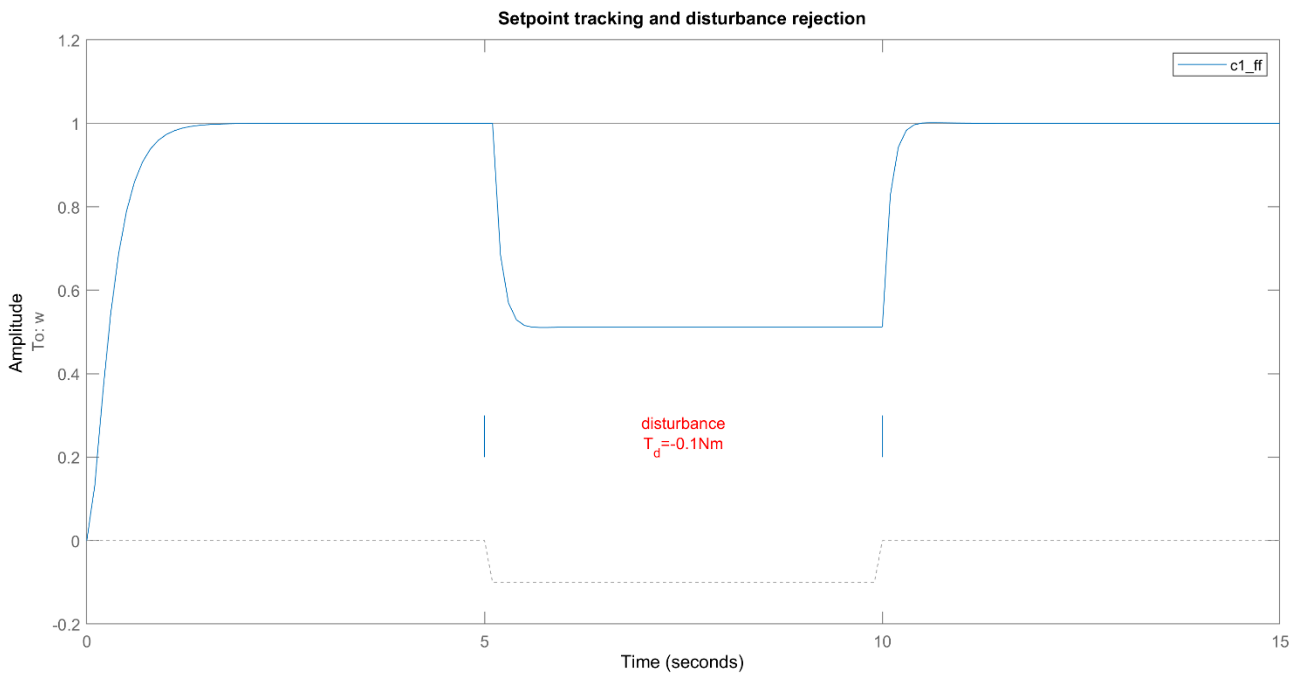


Figure-1

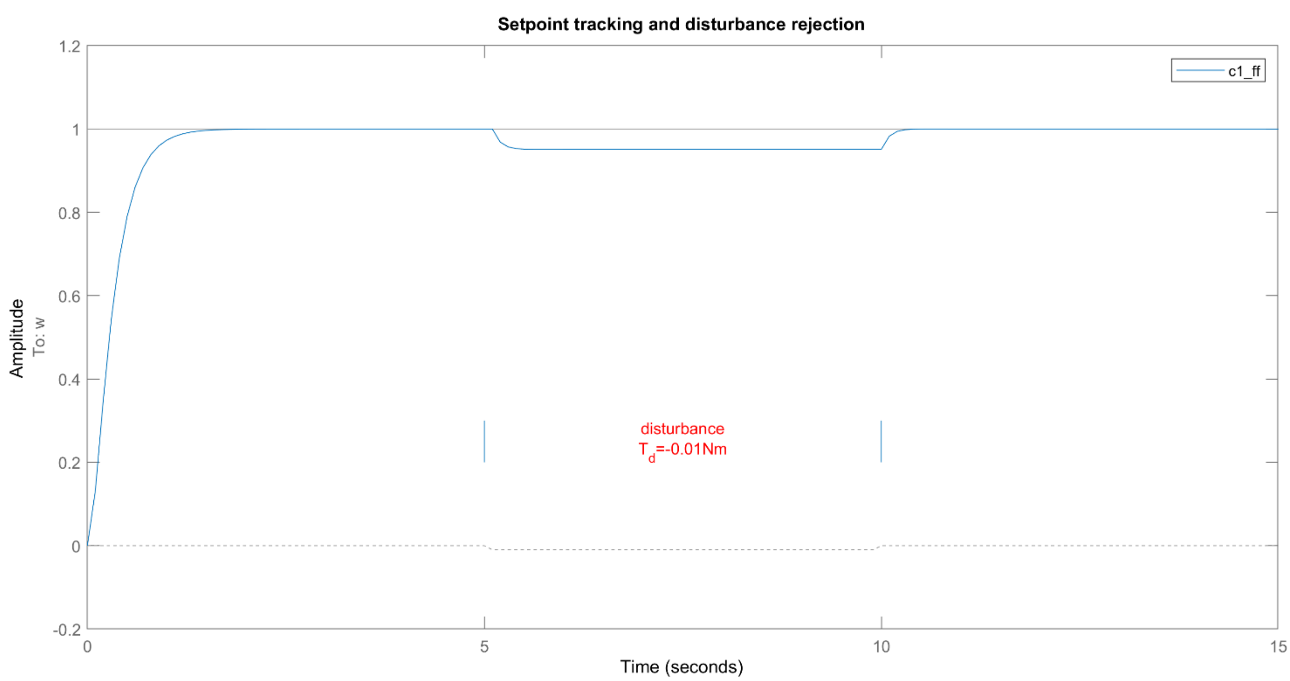


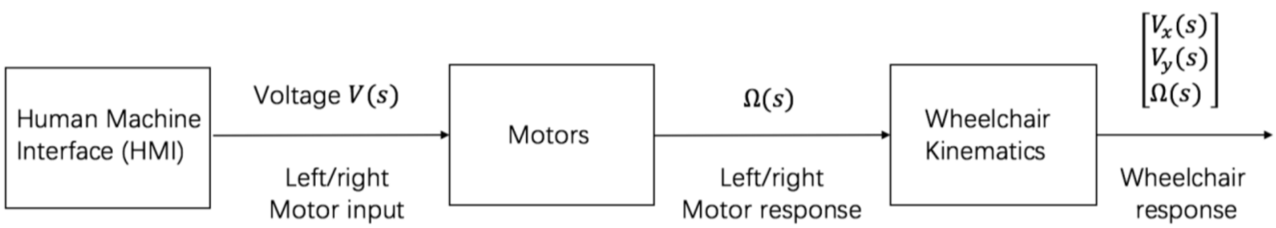
Figure-2

To minimize the speed variations induced by T_d disturbances, we can apply the following possible solutions:

- The feedforward control structure can be used as it minimizes the effect of the external disturbance T_d on the system as shown in the above figures. Therefore, improving the performance and control accuracy.
- The closed loop system can also be used as it helps in suppressing the effect of the external disturbance T_d. Thus, making the system independent of the variations. Therefore, closed loop systems tend to equalize inputs and outputs.
- The other possible solution is the PID controller.

Problem 2

(a)



Ans – (a)

$$\dot{\xi}_I = \begin{bmatrix} v_x(t) \\ v_y(t) \\ \omega(t) \end{bmatrix}_I = \begin{bmatrix} c_\theta & -s_\theta & 0 \\ s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \\ \frac{l}{2} & \frac{-l}{2} & 0 \end{bmatrix} \begin{bmatrix} r_R\omega_R(t) \\ r_L\omega_L(t) \\ 0 \end{bmatrix}$$

Given: $\theta = \frac{\pi}{4}$

$$\dot{\xi_I} = \begin{bmatrix} V_x(s) \\ V_y(s) \\ \Omega(s) \end{bmatrix}_I = \begin{bmatrix} \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{l}{2} & -\frac{l}{2} & 0 \end{bmatrix} \begin{bmatrix} r_R\Omega(s) \\ r_L\Omega(s) \\ 0 \end{bmatrix} \quad \ldots(viii)$$

Angular Velocity Response Model from equation (vi):

$$\Omega(s) = \frac{L_a(s)L_l(s)}{1 + [\, K_b L_a(s) L_l(s)]} V(s) + \frac{L_l(s)}{1 + [\, K_b L_a(s) L_l(s)]} T_d(s)$$

For simplification, let us assume

$$G_1(s) = \frac{L_a(s)L_l(s)}{1 + [\, K_b L_a(s) L_l(s)]}$$

$$G_2(s) = \frac{L_l(s)}{1 + [\, K_b L_a(s) L_l(s)]}$$

So, the above Angular Velocity Response Model equation can be reduced as:

$$\Omega_i(s) = G_1(s)V_i(s) + G_2(s)T_{d_i}(s) \quad \ldots(ix)$$

From (viii) and (ix), we can deduce

$$\dot{\boldsymbol{\xi}_I} = \begin{bmatrix} \boldsymbol{V_x(s)} \\ \boldsymbol{V_y(s)} \\ \boldsymbol{\Omega(s)} \end{bmatrix}_I = \begin{bmatrix} \frac{1}{2\sqrt{2}} r_R(G_1(s)V_R(s) + G_2(s)T_{d_R}(s)) + \frac{1}{2\sqrt{2}} r_L(G_1(s)V_L(s) + G_2(s)T_{d_L}(s)) \\ \frac{1}{2\sqrt{2}} r_R(G_1(s)V_R(s) + G_2(s)T_{d_R}(s)) + \frac{1}{2\sqrt{2}} r_L(G_1(s)V_L(s) + G_2(s)T_{d_L}(s)) \\ \frac{l}{2} r_R\left(G_1(s)V_R(s) + G_2(s)T_{d_R}(s)\right) - \frac{l}{2} r_L(G_1(s)V_L(s) + G_2(s)T_{d_L}(s)) \end{bmatrix}$$

In the above equation,

$T_{d_R}(s), T_{d_L}(s) \rightarrow$ *represents the torque subjected to the wheels*

$V_R(s)\,, V_L(s) \rightarrow$ *represents the voltages*