# Homework Assignment – 9


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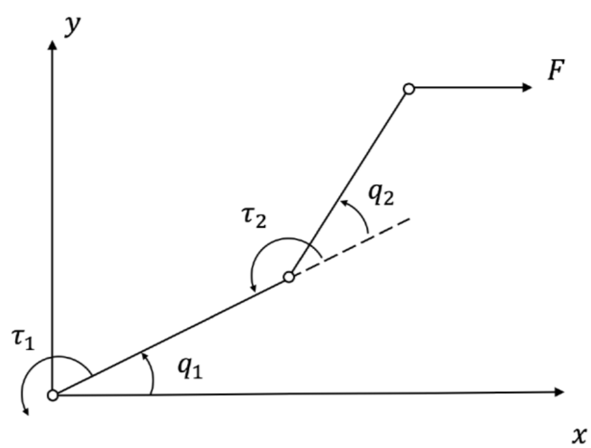
## Problem 1

**(a)**




**Ans – (a)**

Given: $F = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

Let us assume,

Length of the Link – 1 → $a_1$

Length of the Link – 2 → $a_2$

$$Joint\ Torque, \tau = J^T F$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & a_1 c_1 + a_2 c_{12} \\ -a_2 s_{12} & a_2 c_{12} \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} a_1(s_1 - c_1) + a_2(s_{12} - c_{12}) \\ a_2(s_{12} - c_{12}) \end{bmatrix}$$

**Therefore, Joint Torque :** $\boldsymbol{\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} a_1(s_1 - c_1) + a_2(s_{12} - c_{12}) \\ a_2(s_{12} - c_{12}) \end{bmatrix}}$

# Problem 1

**(b)**

**Ans – (b)**

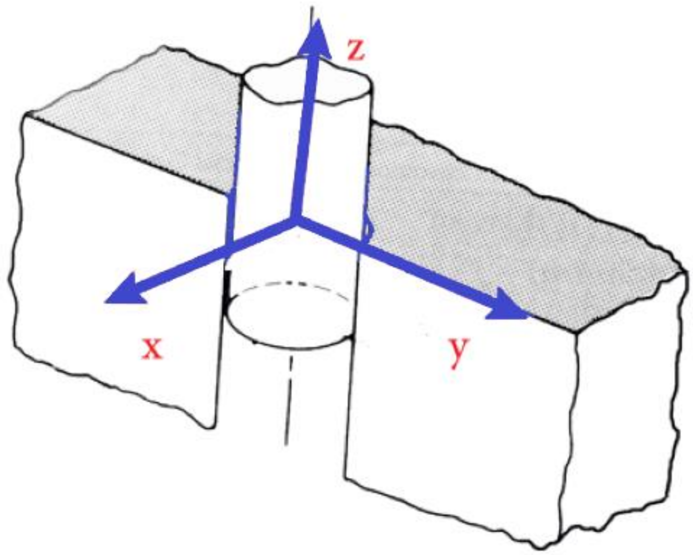



**Figure – 1b Compliance Frame for inserting a peg into a hole**

From the above Figure – 1b, we can deduce that:

Natural Constraints:

$v_x = 0$ $v_y = 0$ $w_x = 0$ $w_y = 0$ $f_z = 0$ $n_z = 0$

Artificial Constraints:

$v_z = V_z$ $w_z = W_z$ $f_x = F_x$ $f_y = F_y$ $n_x = N_x$ $n_z = N_z$

Here,

$V_z, W_z, F_x, F_y, N_x, N_z$ → depends on the configurations of the design

For the convenience, we consider:

$v_z = v_{slide}$ $w_z = 0$ $f_x = 0$ $f_y = 0$ $n_x = 0$ $n_z = 0$

# Problem 1

**(c)**

**Ans – (c)**

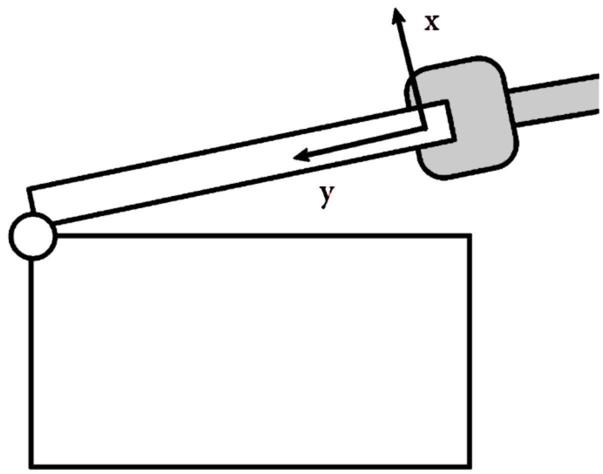



**Figure – 1c Compliance frame for opening a box with a hinged lid**

From the above Figure – 1c, we can deduce that:

Natural Constraints:

$v_y = 0$ $v_z = 0$ $w_x = 0$ $w_y = 0$ $f_x = 0$ $n_z = 0$

Artificial Constraints:

$v_x = V_x = W_z l$ $w_z = W_z$ $f_y = F_y$ $f_z = F_z$ $n_x = N_x$ $n_y = N_y$

Here,

$V_x, W_z, F_y, F_z, N_x, N_y$ → depends on the configurations of the design

$l$ → Length of the lid

For the convenience, we consider:

$v_x = w_{rotatel}$ $w_z = w_{rotate}$ $f_y = 0$ $f_z = 0$ $n_x = 0$ $n_y = 0$

# Problem 1

**(d)**

**Ans – (d)**

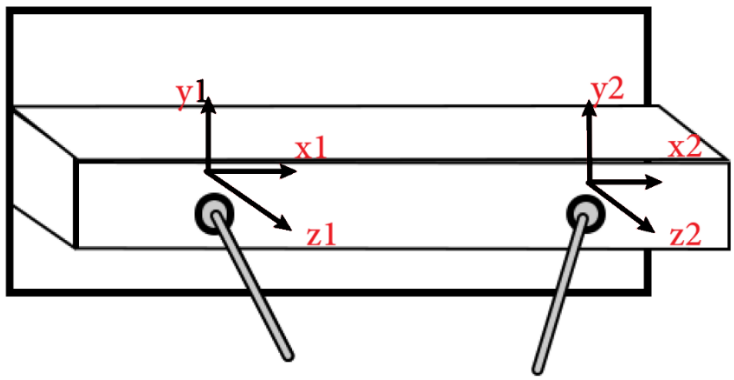



**Figure – 1d Compliance frame for opening a long two-handled drawer**

From the above Figure – 1d, we can deduce that:

Natural Constraints:

$v_{x1} = 0$ $v_{y1} = 0$ $w_{x1} = 0$ $w_{y1} = 0$ $w_{z1} = 0$ $f_{z1} = 0$

$v_{x2} = 0$ $v_{y2} = 0$ $w_{x2} = 0$ $w_{y2} = 0$ $w_{z2} = 0$ $f_{z2} = 0$

Artificial Constraints:

$v_{z1} = V$ $f_{x1} = F_{x1}$ $f_{y1} = F_{y1}$ $n_{x1} = N_{x1}$ $n_{y1} = N_{y1}$ $n_{z1} = N_{z1}$

$v_{z2} = V$ $f_{x2} = F_{x2}$ $f_{y2} = F_{y2}$ $n_{x2} = N_{x2}$ $n_{y2} = N_{y2}$ $n_{z2} = N_{z2}$

Here,

$V, F_{x1}, F_{x2}, F_{y1}, F_{y2}, N_{x1}, N_{x2}, N_{y1}, N_{y2}, N_{z1}, N_{z2}$ → depends on the configurations of the design

For the convenience, we consider:

$v_{z1} = v_{slide}$ $f_{x1} = 0$ $f_{y1} = 0$ $n_{x1} = 0$ $n_{y1} = 0$ $n_{z1} = 0$

$v_{z2} = v_{slide}$ $f_{x2} = 0$ $f_{y2} = 0$ $n_{x2} = 0$ $n_{y2} = 0$ $n_{z2} = 0$

# Problem 1

**(e)**

**Ans – (e)**

## 1. Turning a crank

- ❖ Tangent to Circle → Inertial
- ❖ Along the Crank → Capacitive

## 2. Inserting a peg in a hole

- ❖ Parallel to Hole → Inertial
- ❖ Perpendicular to Hole → Capacitive

## 3. Polishing the hood of a car

- ❖ Tangent to Hood → Inertial
- ❖ Normal to Hood → Capacitive

## 4. Cutting cloth

- ❖ Along the cutting direction → Resistive

## 5. Sheering a sheep

- ❖ Along the sheering direction → Resistive
- ❖ Normal to Sheep → Capacitive

## 6. Placing stamps on envelopes

- ❖ Tangent to Envelope → Inertial
- ❖ Normal to Envelope → Capacitive

## 7. Cutting meat

- ❖ Along the cutting direction → Resistive

# Problem 2

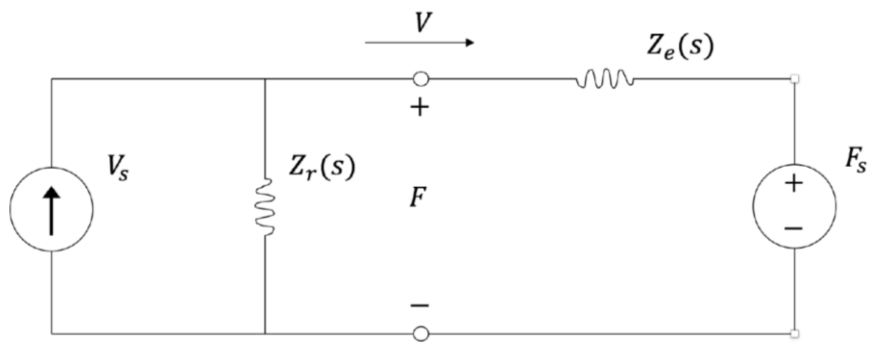



Figure 2: Inertial environment.

**Ans – 2**

**(i)** Given: $\boldsymbol{F_s = 0}$

$$V.Z_e(s) = V_s\left(\frac{1}{\frac{1}{Z_r(s)} + \frac{1}{Z_e(s)}}\right)$$

$$\boldsymbol{\frac{V}{V_s} = \frac{Z_r(s)}{Z_e(s) + Z_r(s)}} \qquad \ldots(\mathbf{1})$$

**(ii)**

$$\boldsymbol{V_s = \frac{V^d}{s}} \qquad \ldots(\mathbf{2})$$

$$E = V - V_s$$

Substituting value of V from Equation (1), we have

$$E = V_s\left(\frac{Z_r(s)}{Z_e(s) + Z_r(s)}\right) - V_s$$

Substituting value of $V_s$ from Equation (2), we have

$$E = \frac{V^d}{s}\left[\left(\frac{Z_r(s)}{Z_e(s) + Z_r(s)}\right) - 1\right]$$

$$\boldsymbol{E = \frac{V^d}{s}\left(\frac{-Z_e(s)}{Z_e(s)+Z_r(s)}\right)} \qquad \dots(3)$$

$$e_{ss} = \lim_{s\to 0} s\,E = 0$$

Substituting value of $E$ from Equation (3), we have

$$\boldsymbol{e_{ss} = \lim_{s\to 0} s\left[\frac{V^d}{s}\left(\frac{-Z_e(s)}{Z_e(s)+Z_r(s)}\right)\right] = 0}$$

Here, we know

For Inertial environment → $Z_e(0) = 0$

For Non-inertial robot → $Z_r(0) \neq 0$

Therefore,

$$\boldsymbol{e_{ss} = \lim_{s\to 0} V^d\left(\frac{-Z_e(0)}{Z_e(0)+Z_r(0)}\right) = 0}$$

**Hence, it is possible to conduct position control**.

**(iii)**

Given: $\boldsymbol{a_x = \ddot{x}}$

$$\boldsymbol{Z_r(s) = M_c s + Z_{rem}(s)}$$

$$\boldsymbol{a_x = \ddot{x}^d + \frac{1}{M_c}Z_{rem}(\dot{x}^d - \dot{x}) + \frac{1}{M_c}F}$$

Substituting the given value, we have

$$\ddot{x} = \ddot{x}^d + \frac{1}{M_c}Z_{rem}(\dot{x}^d - \dot{x}) + \frac{1}{M_c}F$$

$$\boldsymbol{M_c(\ddot{x} - \ddot{x}^d) + Z_{rem}(\dot{x} - \dot{x}^d) = F}$$

**Hence, position control can be used to specify a desired robot impedance.**