

DAA- Assignment

① void funct(int n)
{
 int j=1, i=0;
 while (i < n)
 {
 i += j;
 j++;
 }
}

for j=1 i=1
 j=2 i=1+2
 j=3 i=1+2+3 } m levels

$$\therefore 1+2+3+\dots + n$$

$$1+2+3+\dots + m < n$$

$$\frac{m(m+1)}{2} < n$$

$$m \approx \sqrt{n}$$

by summation method

$$1 = 1+1+\dots - \sqrt{n} \text{ times}$$

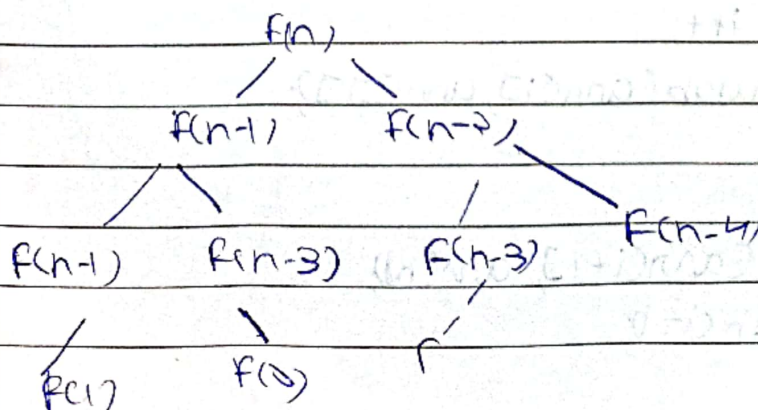
$$\therefore T(n) = \sqrt{n}$$

② for Fibonacci series

$$F(n) = F(n-1) + F(n-2)$$

$$F(0) = 0$$

$$F(1) = 1$$



at every function call we get two function calls
for n levels

We have $= 2 \times 2 \dots n$ times

$$T(n) = 2^n$$

minimum Space: - Considering recursive
Stack:-

no of calls $\text{max} = n$

for each call we have space completely $O(1)$

$$\therefore T(n) = O(n)$$

③ a) $n \log n$

quick sort (int arr[], int l, int h)

{ if (l < h)

{ int pi = partition (arr, l, h);

function (arr, l, pi-1)

func (arr, pi+1, h);

}

}

int partition (int arr[], int l, int h)

{ int pi = arr[h];

int i = (l-1);

for (int j = l; j <= h; j++)

{ if (arr[j] < pi)

{ i++;

swap (arr[i], arr[j]);

}

}

swap (arr[i+1], arr[h]);

return i+1;

}

(b) n^3

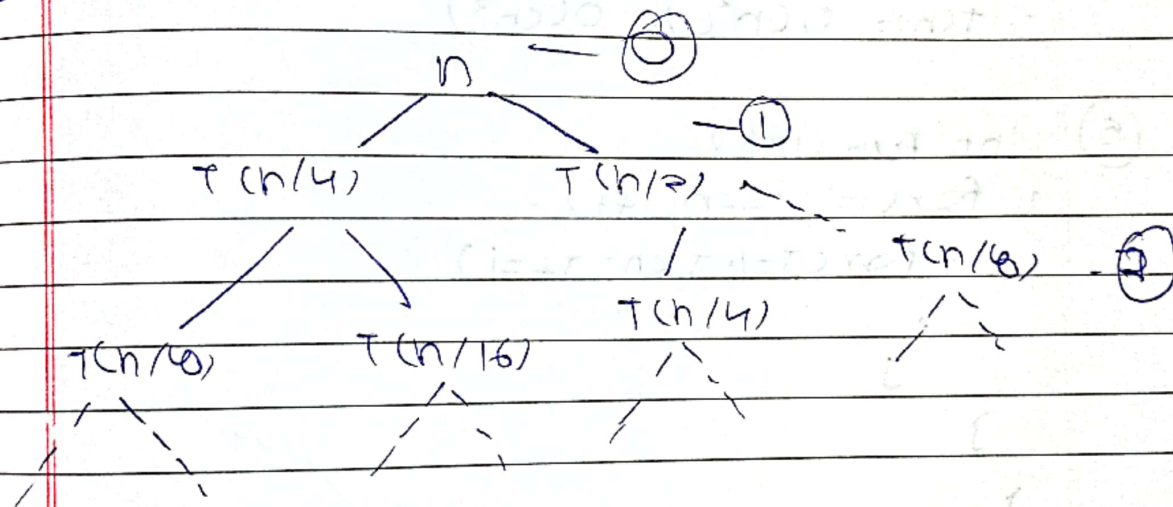
multiplication of two square matrix

```
for (i=0; i<n; i++)
{
    for (j=0; j<n; j++)
    {
        for (k=0; k<n; k++)
        {
            res[i][j] += a[i][k] * b[k][j];
        }
    }
}
```

(c) $\log(\log n)$

```
for (l=2; l<n; l=l+1)
{
    c++;
}
```

(4) $T(n) = T(n/4) + T(n/2) + cn^2$



At level $\rightarrow O(n^2)$

$$1 \rightarrow \frac{n^2}{4} + \frac{n^2}{2^2} = \frac{5n^2}{16}$$

$$2 \rightarrow \frac{n^2}{8^2} + \frac{n^2}{16^2} + \frac{n^2}{4^2} + \frac{n^2}{8^2} = \left(\frac{5}{16}\right)^2 n^2 c$$

A.S.

$$\text{max level} = \frac{n}{2^k} = d = k = \log_2 n$$

$$\therefore T(n) = \left[n^2 + \left(\frac{5}{16}\right)n^2 + \left(\frac{5}{16}\right)^2 + \dots + \left(\frac{5}{16}\right) \log^2 n \right]$$

$$T(n) = cn^2 \left[1 + \frac{5}{16} + \left(\frac{5}{16}\right)^2 + \dots + \left(\frac{5}{16}\right) \log n \right]$$

$$\Rightarrow T(n) = cn^2 \times \left[\frac{1 - \left(\frac{5}{16}\right) \log n}{1 - \frac{5}{16}} \right]$$

$$= cn^2 \times \frac{1}{5} \left(1 - \left(\frac{5}{16}\right) \log n \right)$$

$$\therefore T(n) = O(n^2) = O(n^2)$$

⑤

```

int fun(int)
{
    for(i=1; i<=n; i++)
    {
        for(j=1; j<=i; j++)
        {
            // ...
        }
    }
}

```

for i	j
1	1
2	1+3+5
3	1+4+7
4	1+5+9
...	...
n	(n)

$$n \sum_{i=1}^{n-1} \frac{1}{i}$$

$$\therefore T(n) = \frac{n-1}{2} + \frac{(n-1)}{3} + \frac{(n-1)}{4} + \dots + \frac{(n-1)}{n}$$

$$\Rightarrow T(n) = n \left[\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right] - 1 \times n \left[\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right]$$

$$= n \log n - \log n$$

$$= T(n) = O(n \log n)$$

⑥ for (i=2 ; i <= n ; i = pow(i,k))
 O(1)

for $\Rightarrow i$

2^1

2^k

2^{k^2}

2^{k^3}

1

2^{k^m}

where, $2^{k^m} \leq n$

$$k^m = \log_2 n$$

$$m = \log_k \log_2 n$$

$$\sum_{i=1}^m 1$$

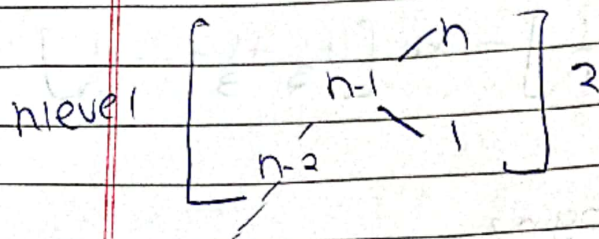
$$i=1$$

$$\Rightarrow 1 + \dots + n \text{ times } = T(n) = O(\log_k \log_2 n)$$

PS

② given algo divides array in 99% & 1% part

$$T(n) = T(n-1) + O(1)$$



'work' is done at each level for merging

$$T(n) = [T(n-1) + T(n-2) + \dots + T(1) + O(1)] \times n$$

$$\Rightarrow n \times n$$

$$T(n) = O(n^2)$$

lowest high $n=2$

highest high $n=n$

$$\therefore \text{diff} = n-2 \because (n>1)$$

③ considering for large value of 'n'

⇒

$$(a) \log \log n < \log n < (\log n)^2 < \sqrt{n} < n < n \log n < \log(n!) < n^2 < 2^n < 4^n < 2^{2^n}$$

$$(b) 1 < \log \log n < \sqrt{\log n} < \log n < \log^2 n < 2 \log n < n < n \log n < 2n < 4n < \log(n!) < n^2 < n < 2 \log^2 n < 5n$$

$$(c) 9n < \log_6 n < \log_2 n < 5n < n \log_6 n < n \log_2 n < \log(n!) < 8n^2 < 7n^3 < n! < 6^n$$