CHAPTER 2

Introduction to Memristor

2.1 Memristor: A Memory Element

Most of us know there exists three basic circuit elements: A resistor (R), a capacitor (C) and an inductor (L). It is assumed that the real-world circuits can be modeled using only these elements with an ideal voltage source. These three basic two-terminal circuit elements are defined in terms of a relationship between two of the four fundamental circuit variables which are the current (i), the voltage (v), the charge (q) and the flux-linkage (Φ). There exists three differential equations that characterize the basic elements provide three relationships out of the six possible relationships between the four fundamental elements.

$$Resistance,\ R=\frac{dv}{di} \tag{2.1}$$

$$Capacitance,\ C=\frac{dq}{dv} \tag{2.2}$$

$$Inductance,\ L=\frac{d\varphi}{di} \tag{2.3}$$

The definition of current, i= dq/dt and the Lenz's law, v = $d\Phi/dt$, give two more relations between the four constituents. These five relations are shown in **Figure 2.1**. Only relationship that is missing is the relationship between q(t) and φ(t).

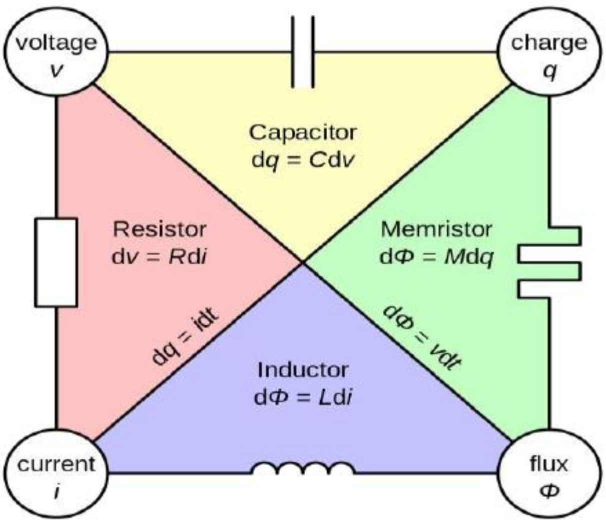

Figure 2.1: Relationships between the four fundamental circuit variables

In 1971 to find this missing case, Dr. Leon Chua postulated a new ideal element which was defined by the single-valued relationship dφ = Mdq must exist to relate between q(t) and φ(t) [1]. He named this element Memristor M, a short for memory-resistor. Thus a new two terminal basic circuit element was found apart from the three fundamental circuit elements. The Memristance of the Memristor is charge controlled (flux controlled) and the voltage across it is given by [1]:

$$\mathrm{v}(t) = \mathrm{M}(\mathrm{q}(t)i(t)) \tag{2.4}$$

$$\mathrm{M}(q) = \frac{d\Phi(q)}{dq} \tag{2.5}$$

Substituting the flux as the time integral of the voltage, and charge as the time integral of current, the more convenient forms are:

$$\mathrm{M}(\mathrm{q}(t)) = \frac{d\varphi(q)/dt}{dq/dt} = \frac{V(t)}{I(t)} \tag{2.6}$$

To relate the memristor to the resistor, capacitor, and inductor, it is helpful to isolate the term *M*(*q*), which characterizes the device, and write it as a differential equation.

TABLE 2.1: Characteristic property and differential equation of fundamental element of electrical circuit

Device	**Characteristic property (units)**	**Differential equation**
Resistor (R)	Resistance (V / A, or ohm, Ω)	R = dV / dI
Capacitor (C)	Capacitance (C / V, or farad)	C = dq / dV
Inductor (L)	Inductance (Wb / A, or henry)	$L = d\Phi_m / dI$
Memristor (M)	Memristance (Wb / C, or ohm)	$M = d\Phi_m / dq$

Thus M(q) has the unit of resistance, it is also called the incremental Memristance. It can be inferred that Memristance is charge-dependent resistance.

The most significant feature of a Memristor is its non-volatility: it retains its resistance after voltage across it is turned off. The Memristor is static if no current is applied. If $I(t) = 0$, we find $V(t) = 0$ and $M(t)$ is constant. This is the essence of the memory effect.

Due to its inability of fabrications it was not fabricated for more than three decades until the researcher of HP labs disclosed the discovery of the so-called “missing element”(i.e is memristor) in 2008. HP labs have full device characterization inclusive of basic theory and fabrication. These has therefore opened a new avenue of research by which novel circuits can be implemented to exploit the operational characteristics of this circuit element [5].

HP labs first realized the Memristor [9] which consisted of a thin film of thickness D (shown in Figure 2.2) with one layer of insulating TiO_2 and another layer of oxygen deficient TiO_{2-x} each sandwiched between platinum electrodes. The doped layer of TiO_{2-x} has significantly lower resistance compared to the layer of TiO_2 which is without doping. Applying a voltage across a Memristor causes the oxygen deficiencies in the TiO_{2-x} layer to migrate, and this changes the thickness of the oxygen deficient layer as well as the resistance of the Memristor device.

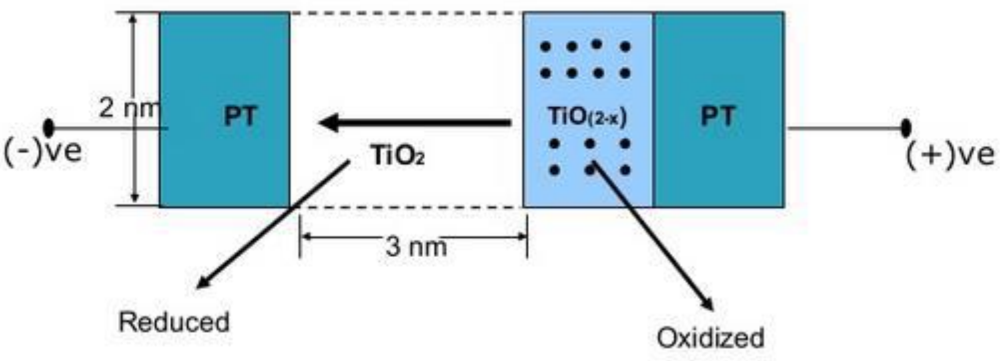

Figure 2.2: A single Memristor

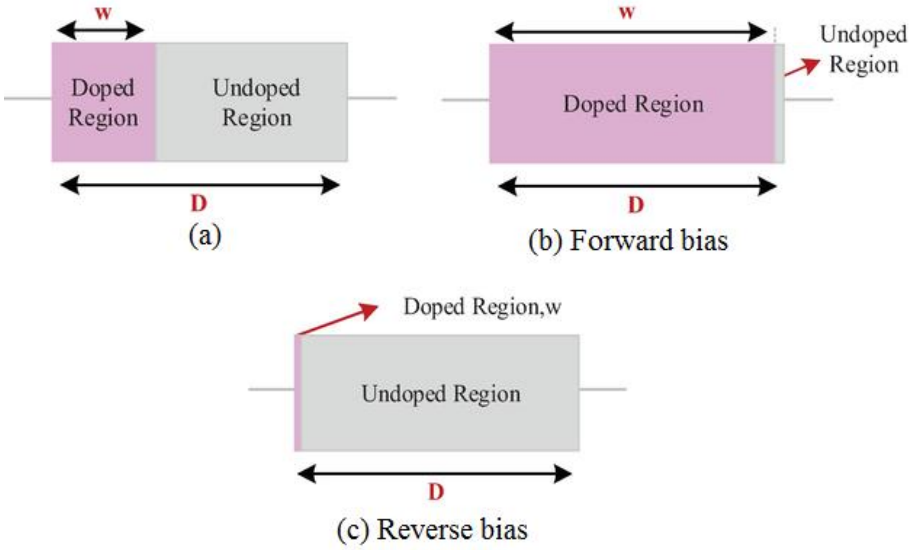

Figure 2.3: Function of memristor

Basic working principle of the Memristor has been shown in **Figure 2.3**. The resistance of the Memristor depends on the amount of charge flown through it. The Memristor is turned ON (resistance decreased) by channeling current through the Memristor with a forward bias that displaces oxygen vacancies (major charge carrier) drifting them into bulk layer line-up to create a conducting channel with lower resistivity than the bulk layer. This will convert the high resistance bulk into low resistance. On the other hand, the Memristor is turned OFF (resistance increased) by flowing current in the opposite direction with a reverse bias. The movement of oxygen vacancies in the opposite direction would cause disruption in the conduction channel converting the bulk layer again into a high resistance substance.

Thus Memristance can be altered between high and low by using alternate current direction, hence alternate voltage direction. Since the oxygen vacancies have a low mobility, they tend to stay in the same position after the voltage source is removed [2]. This phenomenon shows that the Memristor can be used as non-volatile memory device where the resistance of a Memristor is used to store information.

2.2 Memristor Modeling:

It is very important to have a Memristor model that can characterize the device properly for memristor based circuits and systems. As we know, all electric devices can be modeled by basic ideal components. Researchers have also tried to describe Memristor as combination of basic circuit components and derived Spice Model of Memristor to be used in analysis of Memristor based systems.

In 2008, HP Labs [9] published results for their previously mentioned Titanium based Memristor device and modeled it according to equations (2.7) through (2.9). They considered the Memristor device of thickness D and the thickness of the oxygen deficient titanium dioxide layer (TiO_{2-x}) defined a state variable w(t). They considered R_{OFF} and R_{ON} are the maximum and minimum possible resistances of the device respectively. Specifically, the region with high concentration of dopants (oxygen deficient titanium dioxide) having low resistance R_{ON}, and the remainder has a low dopant concentration and much higher resistance R_{OFF}. The Memristance value will depend on the ratio between the state variable w(t) and the total thickness D.

$$V(t) = [\, R_{ON} \frac{w(t)}{D} + R_{OFF} (1 - \frac{w(t)}{D} \,]\, I(t) \tag{2.7}$$

$$v_D = \frac{dw}{dt} = \frac{\mu_D R_{ON}}{D} I(t) \tag{2.8}$$

$$w(t) = \frac{\mu_D R_{ON}}{D} q(t) \tag{2.9}$$

Here, V(t) is the applied voltage across the Memristor and I(t) is the value of the Memristor current. As the value of $w(t)$ increases, it can be seen that the overall device resistance lowers. Inserting (2.9) into (2.7), the Memristance system can be characterized by:

$$M(q(t)) = R_{OFF} (1 - \frac{\mu_D R_{ON}}{D^2}) \tag{2.10}$$

For deriving equation (2.10), It has been assumed that $R_{OFF} >> R_{ON}$. The Memristance property of a device is dependent on the factor $1/D^2$, and that's why the Memristance property becomes significant in nanometer scale.

Figure 2.4 shows the simulation result for the model described by HP lab [9]. A simple sinusoid is applied to the Memristor and the current-voltage wave shapes with respect to time have been shown in **Fig 2.4(a).** For a single voltage loop, the I-V curve of the Memristor has been developed in LTSPICE. The I-V curve displays a pinched hysteresis loop that is one of the significant characteristic of Memristors. The hysteresis shows that the conductivity in a

Memristor is not only related to the voltage applied, but also to the previous value of the state variable $w(t)$. This hysteresis loop reflects the memory retaining property of the Memristor.

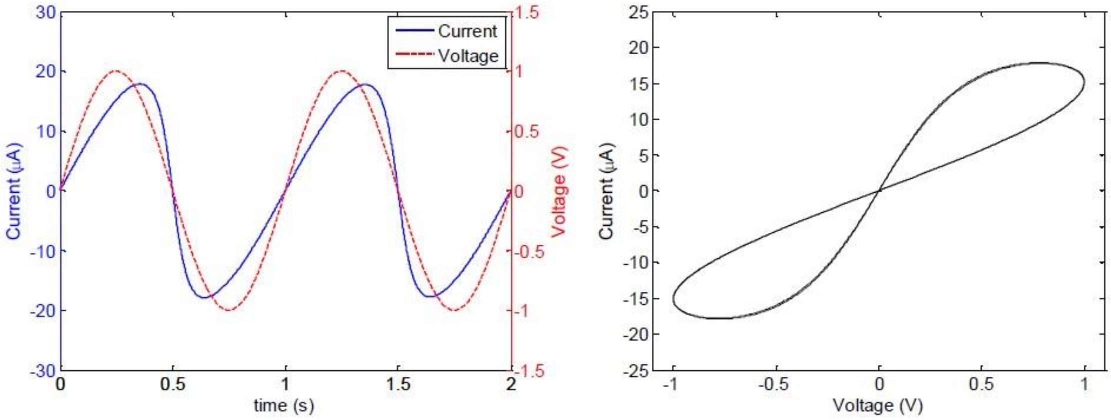

Figure 2.4: Simulation results for the HP Labs memristor with a sinusoidal input [12]. In this simulation: R_{ON}=10kΩ, R_{OFF}=100kΩ, $\mu_D=10^{-14}$ $m^2s^{-1}V^{-1}$, D=27nm, $x_0=0.1D$, and $V(t) = \sin(2\pi t)$

2.3 Memristor Spice Model

In the analysis of complex circuits having Memristor requires reliable simulation tools that are easy to implement and efficient enough for predicting the output behavior of the total system. The SPICE (Simulation Program with Integrated Circuit Emphasis) environment is a general-purpose simulator that is used in integrated circuit and board-level design to check the integrity of circuit designs and to predict circuit behavior. Simulation using SPICE helps to model any device and also enables the model to be used in large systems and give an overview of the circuit characteristics that is close to the actual.

So, before manufacturing an integrated circuit the best practice is to simulate it using SPICE simulators to verify its' operation. Till now a lot of modifications [16, 17, 26, 27] to the initial Memristor equations have been proposed and some SPICE models have also been developed for analyzing and simulating Memristor based systems. In this work, a SPICE model of bipolar Memristive system with threshold [28] has been used.

This model [28] was developed by Biolek et. el. and it provides a realistic description of bipolar Memristive device by taking into account boundary values of Memristance and threshold type switching behavior. In this model, the Memristance is considered as an internal state variable defining device state x≡R and the Memristance change when the absolute value of voltage across it is greater than a certain threshold voltage Vt. The device state can be defined by the following equations [28]:

$$I = x^{-1} V_M \tag{2.11}$$

$$\frac{dx}{dt} = f(V_M) W(x, V_M) \tag{2.12}$$

$$f(V_M) = (V_M - 0.5(|V_M + V_t| - |V_M - V_t|) \tag{2.13}$$

$$W(x, V_M) = \theta(V_M)\theta(R_{OFF} - x) + \theta(-V_M)\theta(x - R_{ON}) \tag{2.14}$$

Here θ(.) is the step function and β is a positive constant characterizing the rate of Memristance change when $V_M > V_t$ is the threshold voltage, and R_{ON} and R_{OFF} are limiting values of the Memristance. The role of the θ(.) function is to limit the Memristance value between the boundaries. The sketch of the function $f(V_M)$ is shown in **Figure 2.5** and this helps to understand the voltage threshold effect [28].

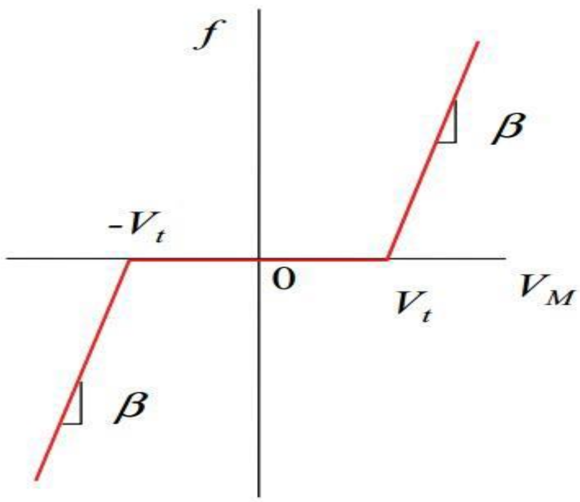

Figure 2.5: Sketch of the function $f(V_M)$ modeling the voltage threshold property [28]

Based on above equations, Biolek et. el. developed a SPICE model which is presented in Figure 2.6.

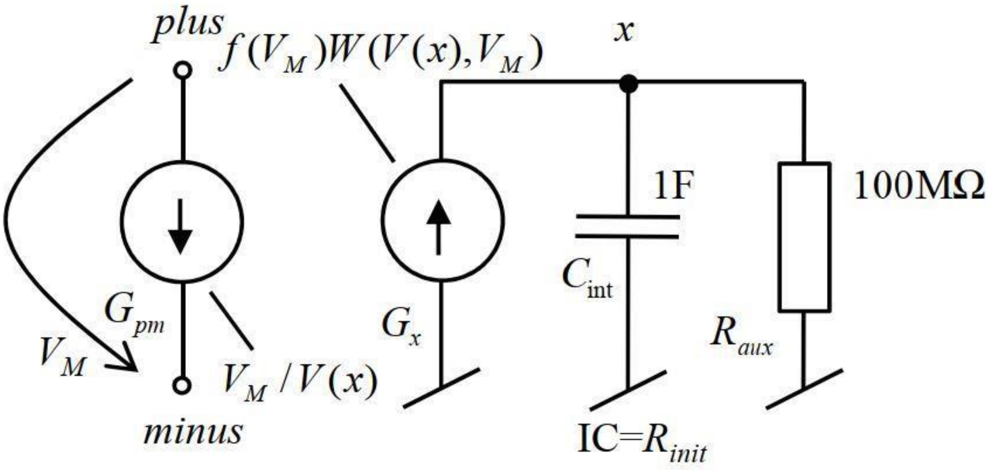

Figure 2.6: SPICE model of the Memristive device with threshold [15]

In this SPICE model, the derivative of the Memristance(x) according to Eq. 2.12 is modeled by the current of the voltage controlled source Gx, and its integral or the Memristance (in ohms) is equal to the voltage of the node x(in volts). According to Eq. 2.11, the Memristive port is modeled by the current source Gpm. Its current is computed as a ratio of the terminal voltage and the Memristance. Eq. 2.13 and 2.14 contain discontinuous function (step) and function with discontinuous derivatives (absolute value). It can be a source of serious convergence problems, especially for applications utilizing large-scale models. To avoid this convergence problem, Biolek et. el. [28] proposed smoothed function on sigmoid modeling of the step function according to the formula:

$$(x) = \frac{1}{1+e^{-x/b}} \tag{2.15}$$

Where b is a smoothing parameter and the smoothed function of the absolute value function can be expressed by [28]:

$$abs_s(x) = x([(x) - \theta_s(-x)] \tag{2.16}$$

By adjusting the b parameter, a proper tradeoff between the accuracy and reliability can be found when a convergence problem appears.

According to these equations a SPICE code was developed [28].

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**** Bipolar memristive system with threshold R2 ****
*D. Biolek, M. Di Ventra, Y. V. Pershin*
*Reliable SPICE Simulations of Memristors, Memcapacitors and
Meminductors, 2013*
*Code for PSpice and LTspice; tested with Cadence PSpice v. 16.3
and LTspice v. 4*
*****************************************************
***************
.subcktmemR_TH plus minus PARAMS:
+ Ron=1K Roff=25K Rinit=5K beta=1E13 Vt=1.5
*model of memristive port
Gpm plus minus value={V(plus,minus)/V(x)}
*end of the model of memristive port
*integrator model
Gx 0 x
value={fs(V(plus,minus),b1)*ws(v(x),V(plus,minus),b1,b2)*1p}
Raux x 0 1T
Cx x 0 1p IC={Rinit}
*smoothed functions
.param b1=10u b2=10u
.funcstps(x,b)={1/(1+exp(-x/b))}
.funcabss(x,b)={x*(stps(x,b)-stps(-x,b))}
.func fs(v,b)={beta*(v-0.5*(abss(v+Vt,b)-abss(v-Vt,b)))}
.func ws(x,v,b1,b2)={stps(v,b1)*stps(1-x/Roff,b2)+stps(-
v,b1)*stps(x/Ron-1,b2)}
.ends memR_TH
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Characteristics of this model, a Memristor using SPICE model with threshold have been simulated using a voltage source as **Figure 2.7**.

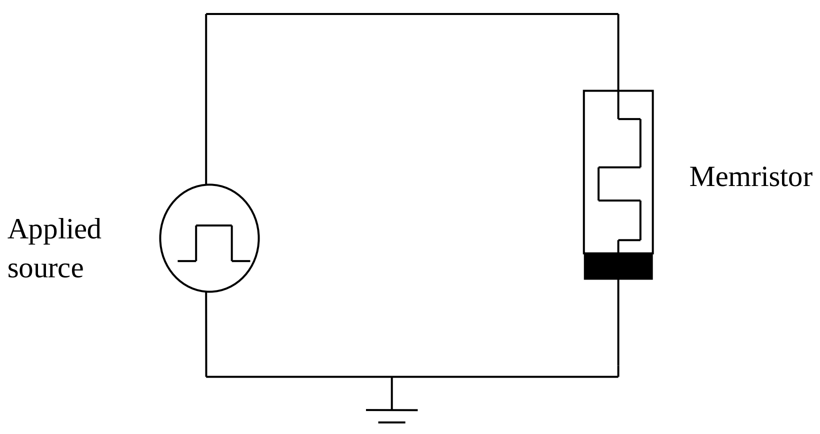

Figure 2.7: A Memristor driven by a voltage source

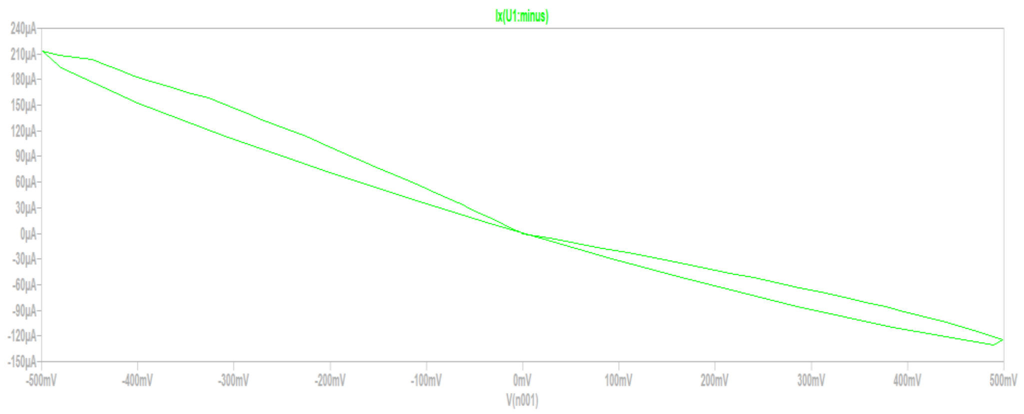

Figure 2.8: I-V curve of the Memristor

The Memristor SPICE model used in this thesis work, have the parameters R_{ON}=1KΩ ,R_{OFF}=25KΩ, β=10^{13} and threshold voltage of 1.5V. To observe the memory principle of the Memristor, the Memristor device driven by a pulse voltage source has been simulated in LTSPICE and the simulated result is shown in **Figure 2.9**.

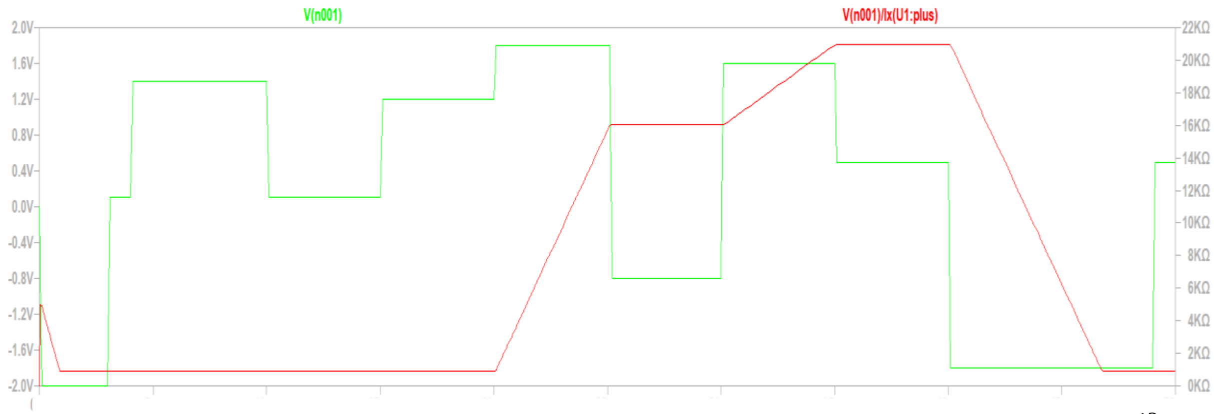

Figure 2.8: Simulation result for a Memristor having R_{ON}=1KΩ ,R_{OFF}=25KΩ, β=10^{13} and threshold voltage of 1.5V driven by a pulse voltage source

Observing from the figure that when a threshold voltage greater than 1.5V is applied to the Memristor it's resistance value will increased and the rate of increasing of resistance depend on the value of β. The value of Memristance increases as increasing the applied voltage greater than threshold and it is fixed when the value of Memristance reaches R_{OFF}=25KΩ. The most important noticeable characteristics from the figure is that the value of resistance was not changed after removing the applied voltage so the Memristance retain its previous resistance. By this supreme principle the Memristor can be used as a non-volatile memory. Similarly, if the reverse bias greater than threshold voltage is applied to the Memristor then its resistance will decreased.