

Chapter: 12 - Schwarzschild geometry and black holes

A star is visible to us because light escapes from its surface. But, if a star is more compact than the sun, but with the same mass then it would have escape velocity larger than the speed of light. Then, the star would be dark, invisible, which is called black hole.

The size that makes a star invisible is given by. $R = \frac{2Gm}{c^2}$

where m is the mass and R is radius of a particle of the star.

conserved quantities: The trajectories of particles of a black holes will, allow us to see whether light rays are trapped or can escape.

The basic equations for orbits;

$$\text{particle: } \left(\frac{dr}{d\tau}\right)^2 = E^2 - \left(1 - \frac{2M}{r}\right) \left(1 + \frac{L^2}{r^2}\right);$$

$$\text{photon: } \left(\frac{dr}{d\lambda}\right)^2 = E^2 - \left(1 - \frac{2M}{r}\right) \frac{L^2}{r^2};$$

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Perihelion: 'Peri' means closest and 'helion' refers to the sun. In general it is kind of rare curve. Such a curve is similar to an ellipse curve which shifts a bit with each additional orbit. The perihelion shift for mercury

$$\Delta\theta = .43^\circ/\text{yr} = 43^\circ \text{ century}$$

Binary pulsars: A pulsar is an extra-terrestrial source of radiation that has a regular periodicity. Pulsars that orbit another significant physical object are referred as binary pulsars.

PSR B1509-58 was the first binary pulsar discovered by Russel A. Hulse and Joseph H. Taylor in 1974.

Post-newton: It is a correction to Newtonian motion in the limit of weak fields and slow motion. The post newtonian expression assumes not just weak gravitational fields but also slow velocities.

Gravitational deflection of light: It occurs when we treat ~~photon~~ photons in our discussion of orbits and their deflection from straight line motion as they pass through a gravitational field.

Gravitational lensing: Gravitational lensing happens that photons from the same star will travel

trajectories that pass on opposite sides with of the deflecting star and intersect each other after deflection. The star causing the light curve is accordingly called a gravitational lens. Gravitational lensing can detect dark matter.

*11.3 :-

Formation of black holes in general: A black hole can be formed by the death of a massive star. When such a star has exhausted the internal thermonuclear fuels in its core at the end of its life, the core becomes unstable, and gravitationally collapse inward upon itself, and the star's outer layers are blown away.

General properties of black holes:

- ① An isolated black hole should become stationary and it is not constantly disturbed by outside effects like accretion.
- ② If the black hole is not vacuum, its structure may be more general.
- ③ If gravitational collapse is nearly spherical then a stationary black hole of the Kerr type is left behind.

Real-black holes in astronomy: A typical stellar class of black hole has a mass between about 3 and 10 solar masses, supermassive black hole exist in the centre of most galaxies. Intermediate-mass black holes have masses between 100 and 1000 solar masses. No single star could ever form such a heavy black hole. Also there are situated where black holes are expected to be highly dynamical and there are more difficult to treat analytically.

Chapter-12: cosmology

Cosmology is the study of the universe as a whole its history, evolution, composition, dynamics. It also provides the starting point for the development of all the detailed small-scale structure that arose as the universe expanded, away from the Big-Bang.

*12.2:

The universe seem to be receding from us at a speed which is proportional to their distance from us. $v = H_0 d$ the recession

velocity is called the Hubble flow after it's discoverer Edwin Hubble.

Models of the universe: In cosmology, Copernican principle states that humans, on the earth or in the solar systems are not privileged observers of the universe, that observations from the Earth are representative of observations from the universe.

Beyond general relativity:

The new physics could take many different forms, some kind of collision with another brane might have triggered the Big-Bang. Brane is a physical object that generalize the motion of point particle to higher dimensions.

* 12.4 :

Physical cosmology:

Decoupling: The moment of decoupling defines the moment at which the cosmic microwave background radiation was created. This occurs at a rather small energy than 13 eV. at a temperature a bit below 1 eV. at a time when the universe was matter dominated.

Dark matter and galaxy formation: After decoupling refers to the epoch during which charged electrons and photons first become bound to form electrically neutral hydrogen atoms.

The early universe & Fundamental physics meets cosmology: If we take about the universe laws away from the Big-Bang, the temperature rises about 50 eV, at which nuclear reactions among photons and neutrons come into equilibrium with each other, and still the well understood physics can be applied in this average position in the universe.

The cosmological spacetime has the metric,

$$ds^2 = -dt^2 + A^2(t) \left[\frac{dx^2}{1 - kx^2} + r^2 d\Omega^2 \right]$$

This is called the Robertson-Walker metric.

Cosmography: It means the description of the expansion of the universe and its history. Hence, here we do not apply universe and the Einstein equation to explain the motion of the universe, instead we simply measure its expansion history.

12.3 :-

cosmological dynamics

it means understanding the expanding universe by applying Einstein equations. By studying the dynamics of Robertson-Walker universe, we get the dark energy density has to be exactly half of the matter energy density.

$$\therefore \rho_A = \frac{1}{2} \rho_0$$

Also, there is critical density which is the average density of matter required for the universe to just halt its expansion, but only after an infinite time. A universe with the critical density is said to be flat,

$$\rho_c = \frac{3H_0^2}{8\pi G}$$