Welcome to My Presentation

My Presentation Topic is

<u>Matrix Operations</u>

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Introduction to Matrices

- A *matrix* (plural matrices) is a rectangular array of numbers, symbols, or expressions, arranged in rows and columns.
- Matrices are commonly written in box brackets. The horizontal and vertical lines of entries in a matrix are called *rows* and *columns*, respectively.
- The size of a matrix is defined by the number of rows and columns that it contains. A matrix with m rows and n columns is called an $m×n$ matrix or m-by-n matrix, while m and n are called its *dimensions*.

The dimensions of the following matrix are 2×3 up(read “two by three”), because there are two rows and three columns..

$$A = \begin{pmatrix} 1 & 9 & -13 \\ 20 & 5 & -6 \end{pmatrix}$$

Operation on Matrices

Addition, subtraction and multiplication are the basic operations on the matrix. To add or subtract matrices, these must be of identical order and for multiplication, the number of columns in the first matrix equals the number of rows in the second matrix.

- Addition of Matrices
- Subtraction of Matrices
- Scalar Multiplication of Matrices
- Multiplication of Matrices

Addition of Matrices

- If $A[a_{ij}]_{m\,x\,n}$ and $B[b_{ij}]_{m\,x\,n}$ are two matrices of the same order then their sum $A + B$ is a matrix, and each element of that matrix is the sum of the corresponding elements. i.e. $A + B = [a_{ij} + b_{ij}]_{m\,x\,n}$

Consider the two matrices A & B of order 2 x 2.

Then the sum is given by:

$$A+B = \begin{pmatrix} a1 & b1 \\ c1 & d1 \end{pmatrix} + \begin{pmatrix} a2 & b2 \\ c2 & d2 \end{pmatrix} = \begin{pmatrix} a1+a2 & b1+b2 \\ c1+c2 & d1+d2 \end{pmatrix}$$

Properties of Matrix Addition

If A, B and C are matrices of same order, then

- **Commutative Law:** $A + B = B + A$
- **Associative Law:** $(A + B) + C = A + (B + C)$
- **Identity of the Matrix:** $A + O = O + A = A$, where O is zero matrix which is additive identity of the matrix.
- **Additive Inverse:** $A + (-A) = 0 = (-A) + A$, where $(-A)$ is obtained by changing the sign of every element of A which is additive inverse of the matrix.
- Then $\left.\begin{array}{l} A + B = A + C \\ B + A = C + A \end{array}\right\} \longrightarrow B = C$
- $tr\,(A \pm B) = \mathrm{tr}(A) \pm \mathrm{tr}(B)$
- If $A + B = 0 = B + A$, then B is called additive inverse of A and also A is called the additive inverse of A.

Example of Matrix Addition

- If $A = \begin{pmatrix} 6 & -10 \\ 4 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 8 & 3 \\ -6 & 2 \end{pmatrix}$ then A + B = ?

Solution :

$$A + B = \begin{pmatrix} 6 & -10 \\ 4 & 7 \end{pmatrix} + \begin{pmatrix} 8 & 3 \\ -6 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 6+8 & (-10)+3 \\ 4+(-6 & 7+2 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & -7 \\ -2 & 9 \end{pmatrix}$$

Answer

Subtraction of Matrices

- If A$[a_{ij}]_{m\,x\,n}$ and B$[b_{ij}]_{m\,x\,n}$ are two matrices of the same order then their subtract A - B is a matrix, and each element of that matrix is the sum of the corresponding elements. i.e. A - B = $[a_{ij} - b_{ij]}]_{m\,x\,n}$

Consider the two matrices A & B of order 2 x 2.

Then the sum is given by:

$$A+B = \begin{pmatrix} a1 & b1 \\ c1 & d1 \end{pmatrix} - \begin{pmatrix} a2 & b2 \\ c2 & d2 \end{pmatrix} = \begin{pmatrix} a1-a2 & b1-b2 \\ c1-c2 & d1-d2 \end{pmatrix}$$

Example of Matrix Subtraction

- If $A = \begin{bmatrix} 10 & -7 \\ 6 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ -3 & 4 \end{bmatrix}$ then A - B = ?

Solution :

$$A - B = \begin{bmatrix} 10 & -7 \\ 6 & 5 \end{bmatrix} - \begin{bmatrix} 6 & 8 \\ -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 10-6 & (-7)-8 \\ 6-(-3) & 5-4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -15 \\ 9 & 1 \end{bmatrix}$$

Answer

Scalar Multiplication of Matrices

- If $A = [a_{ij}]_{m \times n}$ is a matrix and k any number, then the matrix which is obtained by multiplying the elements of A by k is called the scalar multiplication of A by k and

  It is denoted k A thus if $A = [a_{ij}]_{m \times n}$

  Then $kA_{m \times n} = A_{m \times n}k = [ka_{i \times j}]$

- **Properties of Scalar Multiplication:** If A, B are matrices of the same order and λ and μ are any two scalars then;
  - $\lambda(A+B) = \lambda A + \lambda B$
  - $(\lambda+\mu)A = \lambda A + \mu A$
  - $\lambda(\mu A) = (\lambda\mu A) = \mu(\lambda A)$
  - $\lambda(-\lambda A) = -(\lambda A) = \lambda(-A)$
  - $tr(kA) = k\, tr(A)$

Example of Matrix Scalar Multiplication

- Given that $A = \begin{pmatrix} 10 & 6 \\ 4 & 3 \end{pmatrix}$, Lets find 2A = ?

Solution :

To find 2**A**, simply multiply each matrix entry by 2:

$$2A = 2.\begin{pmatrix} 10 & 6 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 2.10 & 2.6 \\ 2.4 & 2.3 \end{pmatrix}$$

$$= \begin{pmatrix} 20 & 12 \\ 8 & 6 \end{pmatrix}_{\text{Answer}}$$

Multiplication of Matrices

- If A and B be any two matrices, then their product AB will be defined only when the number of columns in A is equal to the number of rows in B.

If A = $[a_{ij}]_{m\,x\,n}$ and B = $[b_{ij}]_{n\,x\,p}$

then their product AB = C = $[c_{ij}]_{m\,x\,p}$ will be a matrix of order m x p

where

$$(AB)_{ij} = C_{ij} = \sum_{r=1}^{n} a_{ir} b_{rj}$$

Properties of matrix multiplication

- Matrix multiplication is not commutative in general, i.e. in general $AB \neq BA$.
- Matrix multiplication is associative, i.e. $(AB)C = A(BC)$.
- Matrix multiplication is distributive over matrix addition, i.e.

  $A.(B + C) = A.B + A.C$ and $(A + B)C = AC + BC$.
- If A is an $m \times n$ matrix, then $I_m A = A = A I_n$.
- The product of two matrices can be a null matrix while neither of them is null, i.e. if $AB = 0$, it is not necessary that either $A = 0$ or $B = 0$.
- If A is an $m \times n$ matrix and O is a null matrix then $A_{m \times n} . O_{n \times p} = O_{m \times p}$. i.e. the product of the matrix with a null matrix is always a null matrix.
- If $AB = 0$ (It does not mean that $A = 0$ or $B = 0$, again the product of two non-zero matrices may be a zero matrix).
- If $AB = AC$, $B \neq C$ (Cancellation Law is not applicable).
- $tr(AB) = tr(BA)$.
- There exist a multiplicative identity for every square matrix such $AI = IA = A$

Example of matrix Multiplication

- If $A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 1 & 4 \\ 0 & -1 & 3 \\ 2 & 7 & 5 \end{pmatrix}$, then find AB if possible.

Solution : Using matrix multiplication. Here, A is a 2 × 3 matrix and B is a 3 × 3 matrix, therefore, A and B are conformable for the product AB and it is of the order 2 × 3 such that

$$AB = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{pmatrix} \cdot \begin{pmatrix} 4 & 1 & 4 \\ 0 & -1 & 3 \\ 2 & 7 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 1.4+2.0+4.2 & 1.1+2(-1)+4.7 & 1.4+2.3+4.5 \\ 2.4+6.0+0.2 & 2.1+6(-1)+0.7 & 2.4+6.3+0.5 \end{pmatrix}$$

$$= \begin{pmatrix} 4+0+8 & 1-2+28 & 4+6+20 \\ 8+0+0 & 2-6+0 & 8+18+0 \end{pmatrix} = \begin{pmatrix} 12 & 27 & 30 \\ 8 & -4 & 26 \end{pmatrix}$$

Answer

Thank You