Presentation on

Inverse Matrix

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outline

- DEFINITION OF INVERSE MATRIX
- METHOD
- Crammer's Method
- GAUSS METHOD FOR INVERSION
- SHORTCUT METHOD

Inverse Matrix

As usual the notion of inverse matrix has been developed in the context of matrix multiplication .Every nonzero number possesses an inverse with respect to the operation 'number multiplication'

Definition:Let 'M' be any square matrix.An inverse matrix of 'M' is denoted by M^{-1} and is such a matrix that MM^{-1}=M^{-1}M=I_n

Matrix 'M' is said to be invertible if M^{-1} exists.Non-square matrices do not have inverse

Note:Not all square matrices have inverses.A square matrix which has an inverse is called invertible or nonsingular ,and a

Example:

M=$\begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$ and it's inverse is M^{-1}=$\begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$ since

$$\mathrm{M}M^{-1}=\begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$M^{-1}\mathrm{M} = \begin{bmatrix} -2 & 3 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Therefore, $\begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix} and \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$ are inverses of each other

METHOD

There are usually two methods to find the inverse of a matrix. These are:

(a)Crammer's Method

(b)Gauss Method

Crammer’s Method

Equation

$M^{-1}=\frac{1}{|M|}$(adj M)

Flowchart

- **Matrix M**
- Cofactor M[Cof(M)]
- Adjoint M[adj(M)]
- Inverse Matrix: M^{-1}

Example:

$A^{-1}=\frac{1}{detA}$(adjoint of A) or $A^{-1}=\frac{1}{detA}(cofactor\ matrix\ of\ A)^t$

Example: The following steps result in A^{-1} for A=$\begin{bmatrix}1 & 2 & 3\\0 & 4 & 5\\1 & 0 & 6\end{bmatrix}$

The cofactor matrix for A is $\begin{bmatrix}24 & 5 & -4\\-12 & 3 & 2\\-2 & -5 & 4\end{bmatrix}$,so the adjoint is $\begin{bmatrix}24 & -12 & -2\\5 & 3 & -5\\-4 & 2 & 4\end{bmatrix}$.since

det A=22,we get

$$A^{-1}=\frac{1}{22}\begin{bmatrix}24 & -12 & -2\\5 & 3 & -5\\-4 & 2 & 4\end{bmatrix}=\begin{bmatrix}\frac{12}{11} & \frac{-6}{11} & \frac{-1}{11}\\\frac{5}{22} & \frac{3}{22} & \frac{-5}{22}\\\frac{-2}{11} & \frac{1}{11} & \frac{2}{11}\end{bmatrix}$$

GAUSS METHOD FOR INVERSION

Use Gauss-Jordan elimination to transform [A][I] into [I][A^{-1}].

Example:The following steps result in $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1}$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{3}{2} & \frac{-1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & \frac{-1}{2} \end{bmatrix}$$

so we see that $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & \frac{-1}{2} \end{bmatrix}$

SHORTCUT METHOD

Shortcut for 2x2 matrices

For A=$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$,the inverse can be found using this formula:

$$A^{-1}=\frac{1}{detA}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Example:$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1} = \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & \frac{-1}{2} \end{bmatrix}$

Thank you