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3D RECONSTRUCTION

COMPUTER VISION

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3D RECONSTRUCTION

1. THEORY

1.1 Let p_1 be the point in the image which has optical center as C_1 ②

Let p_2 be the point in the image which has optical center as C_2.

Now fundamental matrix F :

$$P_2^T F P_1 = 0 \rightarrow ①$$

Here we know that :

P_1 & P_2 are at origin

$$\Rightarrow P_1 = P_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

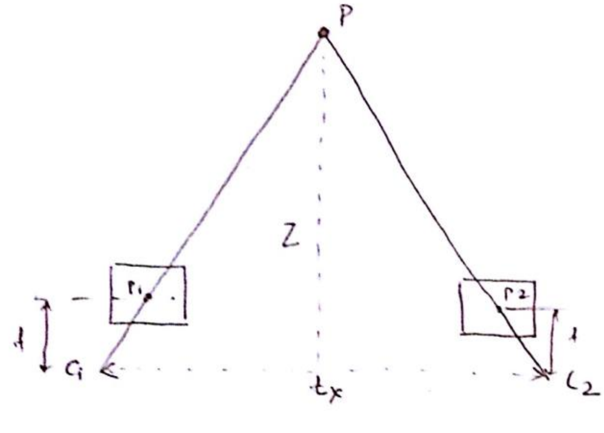
or, putting it in eq① we get

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}_{1\times 3} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}_{3\times 3} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_{3\times 1}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_{13} \\ f_{23} \\ f_{33} \end{bmatrix} = 0$$

or $\boxed{F_{33} = 0}$

1.2 ②



we know that $E = [T_x] R$

here $R = I_3$ (as it is the case of pure translation.)

$T = [-t_x, 0, 0]^T$ (translation parallel to x axis

$$\Rightarrow E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & t_x \\ 0 & -t_x & 0 \end{bmatrix}$$

Now, essential matrix contraint says that

$$P_2^T E P = 0 \rightarrow ①$$

$P_1 = [X_1\ Y_1\ Z] = \left[\frac{Z x_1}{f}, \frac{Z y_1}{f}, Z\right]$ { from figure $x = \frac{f X}{Z}$ }

& $P_2' = [X_2', Y_2', Z] = \left[\frac{Z x_2'}{f}, \frac{Z y_2'}{f}, Z\right]$

(3)

put values of P_1 & P_2 in eq (1) we get

$$\begin{bmatrix} \frac{zx_2}{f} & \frac{zy_2}{f} & z \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & t_x \\ 0 & -t_x & 0 \end{bmatrix} \begin{bmatrix} \frac{zx_1}{f} \\ \frac{zy_1}{f} \\ z \end{bmatrix} = 0$$

multiply the equat by $\frac{f}{z}$ we get

$$\begin{bmatrix} x_2 & y_2 & f \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & t_x \\ 0 & -t_x & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ f \end{bmatrix} = 0$$

$$\begin{bmatrix} x_2 & y_2 & f \end{bmatrix} \begin{bmatrix} 0 \\ t_x f \\ -t_x y_1 \end{bmatrix} = 0$$

or $\quad t_x f y_2 - t_x y_1 f = 0$

or $\quad \cancel{t_x} \cancel{f} y_2 = \cancel{t_x} y_1 \cancel{f}$

$$\Rightarrow \boxed{y_2 = y_1}$$

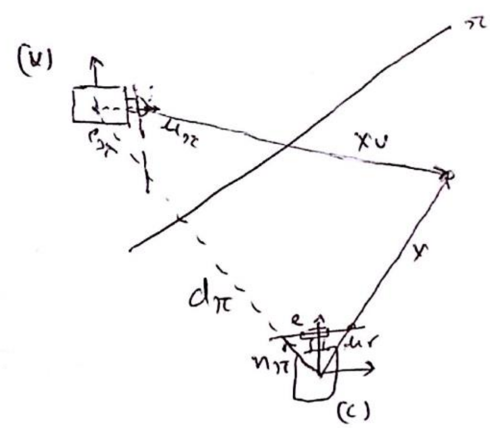
If a point (x_1, y_1) is given in the first image then we get a line parallel to x-axis in the second image.

1.3 (4)

Taken reference from (infact the entire concept)

Cata dioptric stereo with Planar Mirrors : Multiple view Geometry & Camera localization.

Martinelli, Scheggi, Morbidi, Prattichizzo



Here the key point tobe noted is

'The camera projection of the reflected points corresponds to the virtual camera projection of the real points

Reflected Essential matrix is given by $E^{(\pi)} = 2d_\pi[n_\pi]_x$

here $[n_\pi]_x$ is skew symmetric matrix

Let x & xv be 3-D coordinates of a point in camera from c & v as shown in figure

5

$$\Rightarrow \quad X_v = S^{[\pi]} X + 2 d_\pi n_\pi \qquad \text{(of the form } X_2 = R X_1 + T)$$

Now,

$$\lambda_\pi \tilde{u}_\pi = \lambda_\sigma S^{[\pi]} \tilde{u}_\sigma + 2 d_\pi n_\pi$$

here $\lambda_\pi, \lambda_\sigma$ are unknown depths.

or $\tilde{u}_\pi^T \left(2 d_\pi [n_\pi]_\times S^{[\pi]} \right) \tilde{u}_\sigma = 0$

$$E^{[\pi]} \triangleq 2 d_\pi [n_\pi]_\times \cdot S^{[\pi]} = 2 d_\pi (n_\pi)_\times (I - 2 n_\pi n_\pi^T) = 2 d_\pi (n_\pi)_\times$$

as both the camera's have same intrinsic matrix K which is given by $\begin{bmatrix} f_x & s & u_0 \\ 0 & f_y & v_0 \\ 0 & 0 & 1 \end{bmatrix}$

(u_0, v_0) camera's principal point coordinate, f_x, f_y (focal length along x & y)

$$\Rightarrow \quad F^{[\pi]} = K^{-T} E^{[\pi]} K^{-1}$$

$\therefore$ $F^{[\pi]}$ is skew symmetric

$$F^{[\pi]} + F^{[\pi]T} = 2 d_\pi \left(K^{-T} [n_\pi]_\times K^{-1} - K^{-T} [n_\pi]_\times K^{-1} \right) = 0$$

$\Downarrow$

left & right null space of F^π are equal.

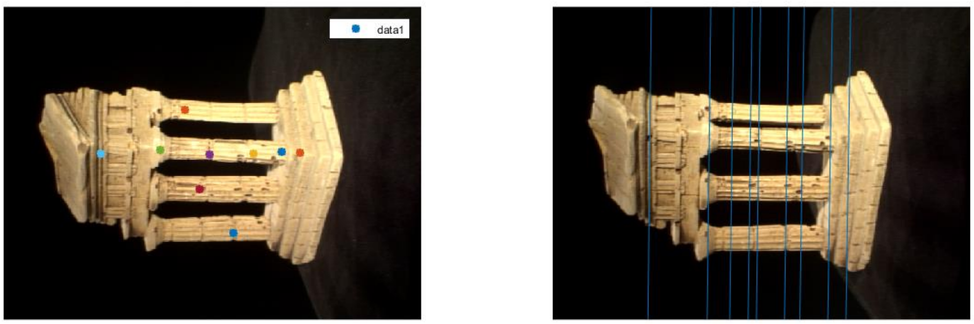
$\Rightarrow$ epipoles are equal $\hat{e} = \tilde{e}_\pi$

F^π has 2 DOF.

Note:- imaging geometry relating camera c & v corresponds to existing b/w 2 cameras undergoing a pure translation motion.

2.1 EIGHTPOINT

```
F =

   1.0e-03 *

   -0.0000    0.0001   -0.7164
   -0.0000    0.0000   -0.0059
    0.7006    0.0013    0.0622
```



2.2 SEVENPOINT

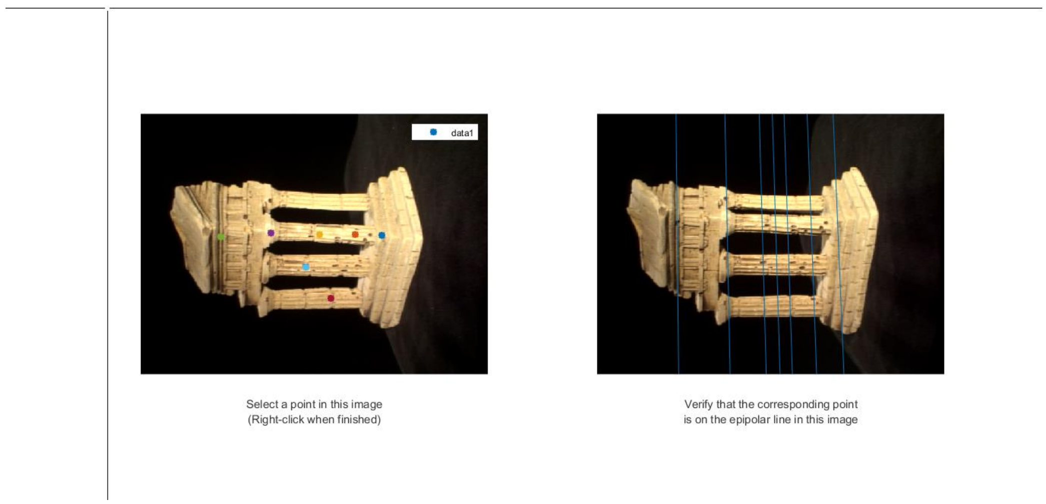
```
% Taken reference from https://www8.cs.umu.se/kurser/TDBD19/VT05/reconstruct-4.pdf
```

```
>> F{1}

ans =

   -0.0000    0.0000   -0.0043
    0.0000   -0.0000   -0.0000
    0.0041   -0.0001    0.0171
```

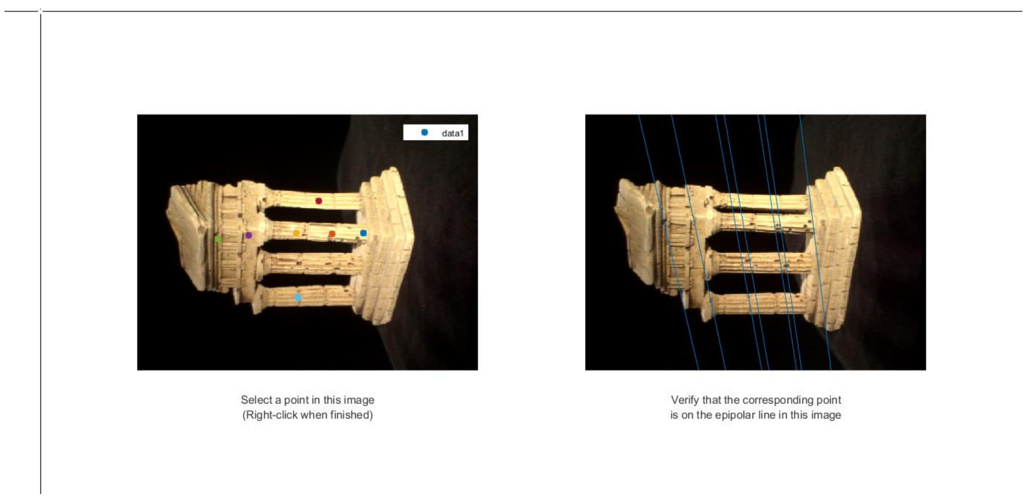
```
>> F{2}

ans =

   1.0e+09 *

    0.0000   -0.0001   -0.0414
   -0.0000    0.0000    0.0135
    0.0619    0.0099   -5.8544
```

```
>> F{3}

ans =

   1.0e+13 *

    0.0001   -0.0003   -0.0394
    0.0000    0.0001    0.1355
    0.0352   -0.0857   -5.3989
```

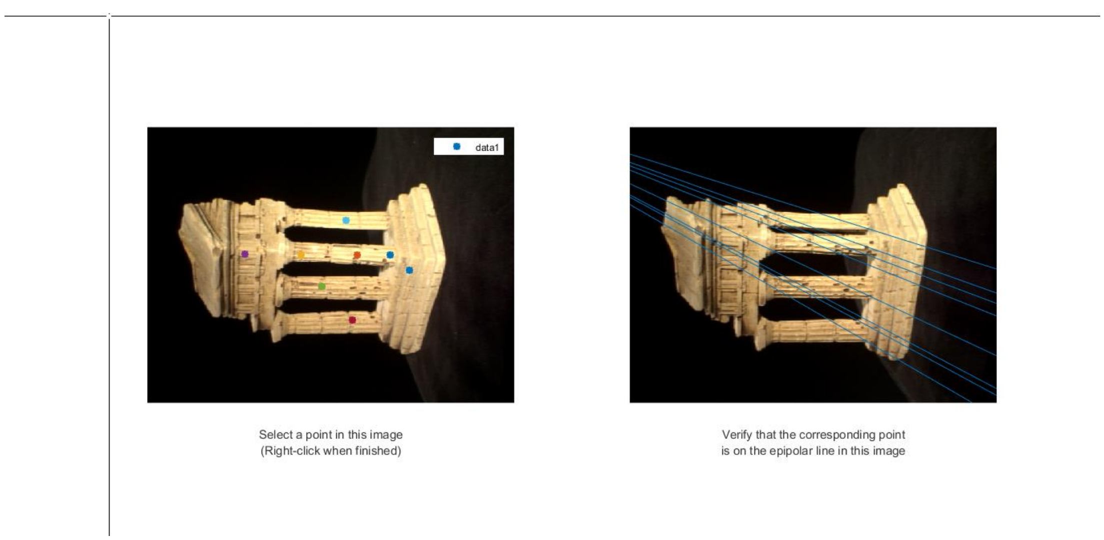
The images for these are attached in the serial order.


data1
Select a point in this image
(Right-click when finished)
Verify that the corresponding point
is on the epipolar line in this image

F{1}


data1
Select a point in this image
(Right-click when finished)
Verify that the corresponding point
is on the epipolar line in this image

F{2}


data1
Select a point in this image
(Right-click when finished)
Verify that the corresponding point
is on the epipolar line in this image

F{3}

2.X RANSACF

```
%Taken reference from Learning Computer Vision with OpenCV book
```

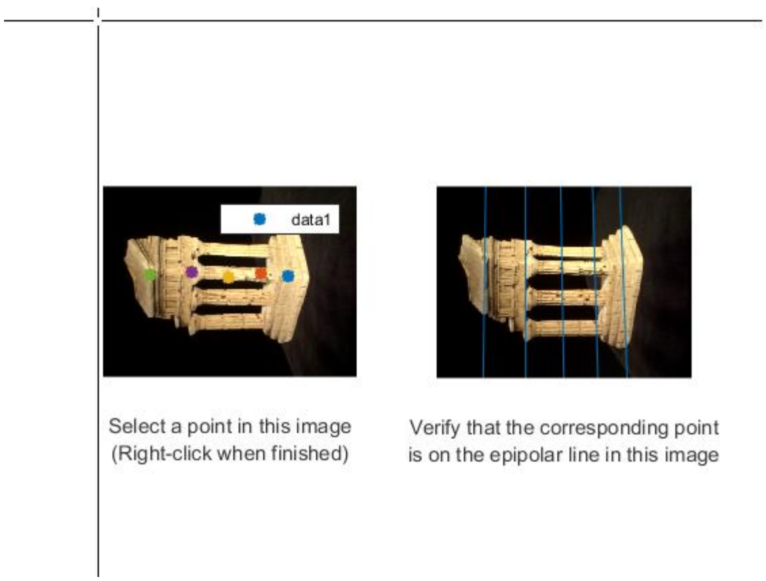
Error matrix used is Sampson distance:

The Sampson distance for this case is defined as:

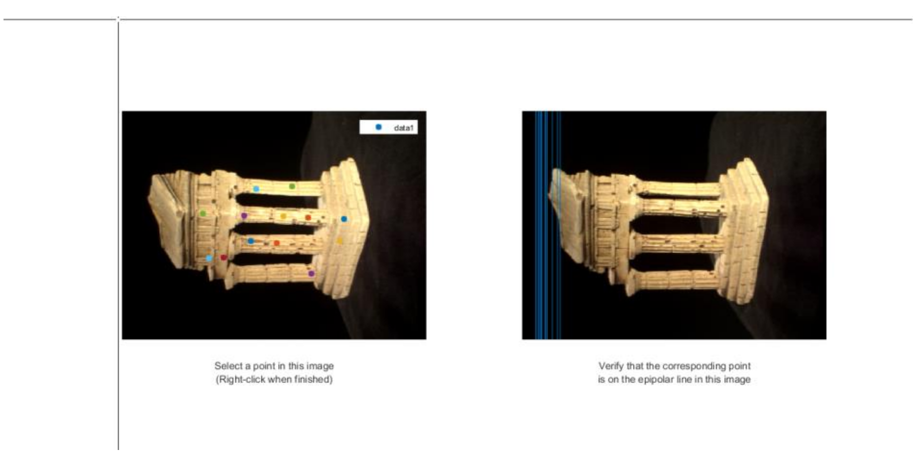
$$\frac{(\mathbf{x}'^{\top}\mathbf{F}\mathbf{x})^2}{(\mathbf{F}\mathbf{x})_1^2 + (\mathbf{F}\mathbf{x})_2^2 + (\mathbf{F}^{\top}\mathbf{x}')_1^2 + (\mathbf{F}^{\top}\mathbf{x}')_2^2}$$

Took the error threshold to be the following. Points having distance less than threshold value are considered as inliers. `error_thresh = 1e-3;`

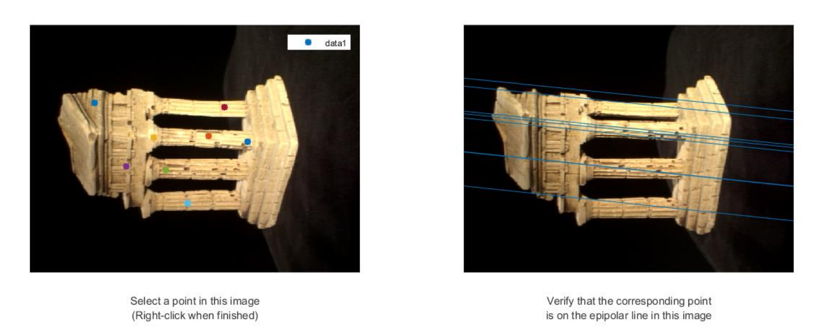
Output of Ransac:



Output of eight point in the noisy case



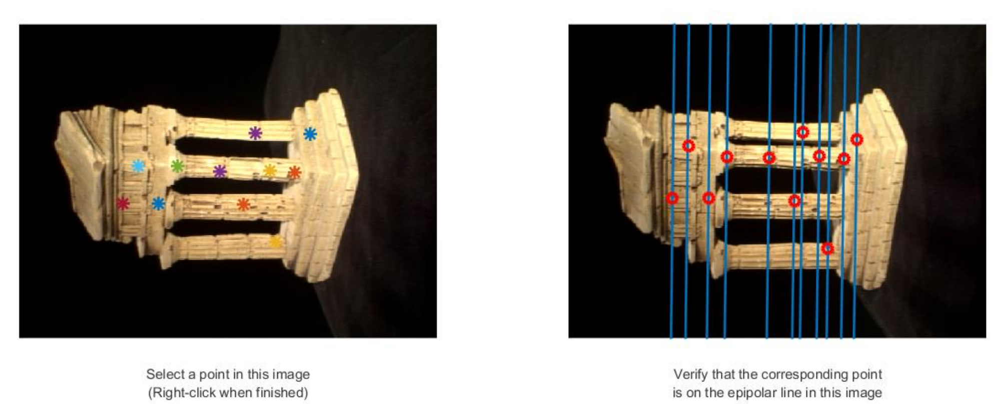
Output of Ransac in noisy case



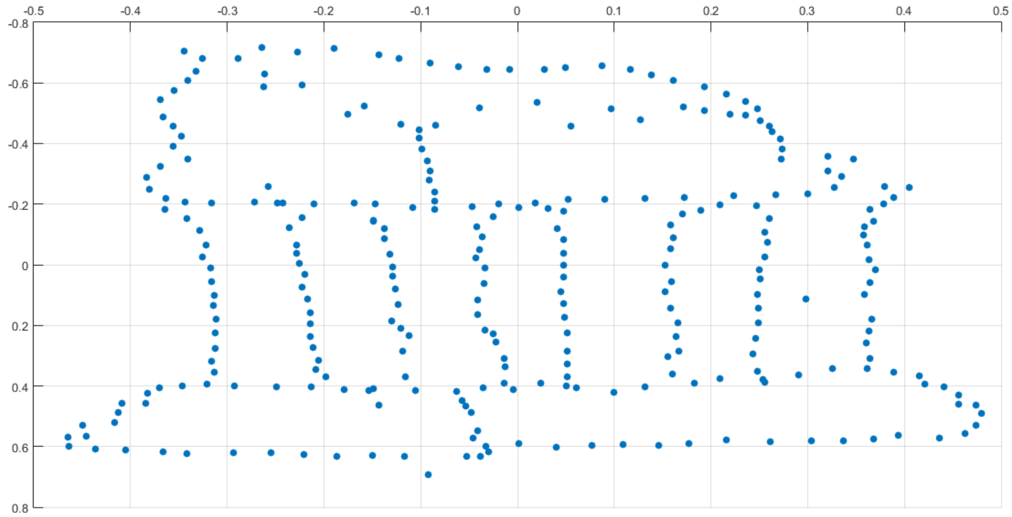
2.3 ESSENTIAL MATRIX

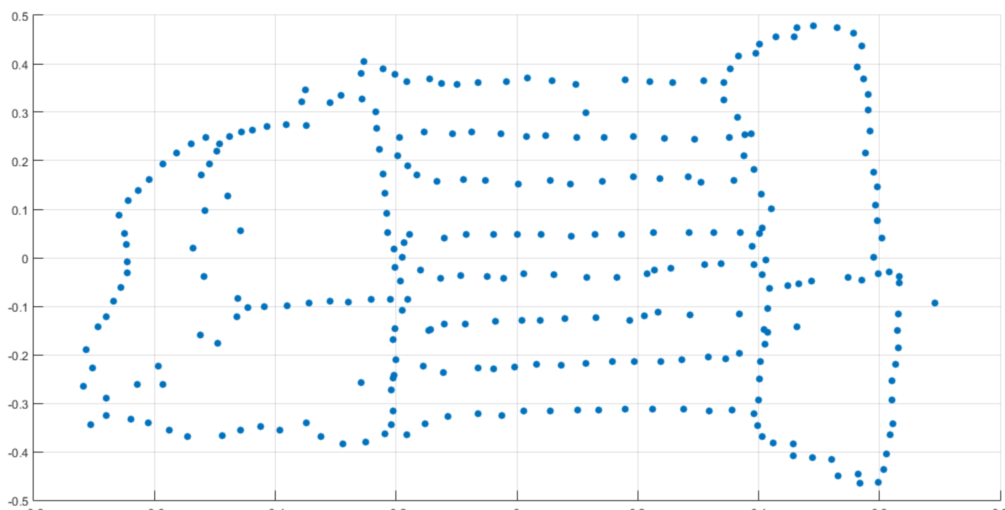
```
>> essentialMatrix( F, K1, K2 )

ans =

   -0.0287    0.2512   -1.0543
   -0.0245    0.0020   -0.0136
    1.0555    0.0522    0.0003
```

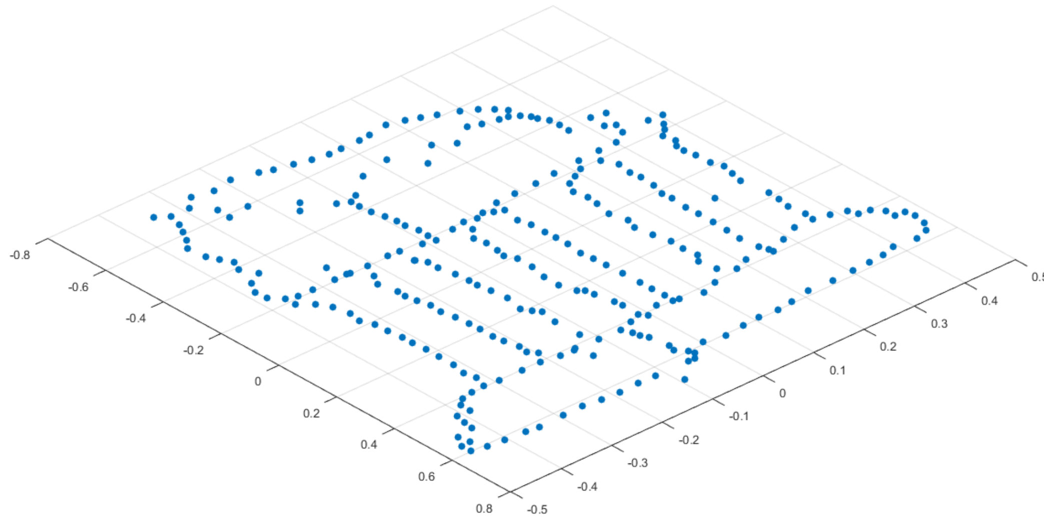
2.6 EPIPOLARCORRESPONDANCE

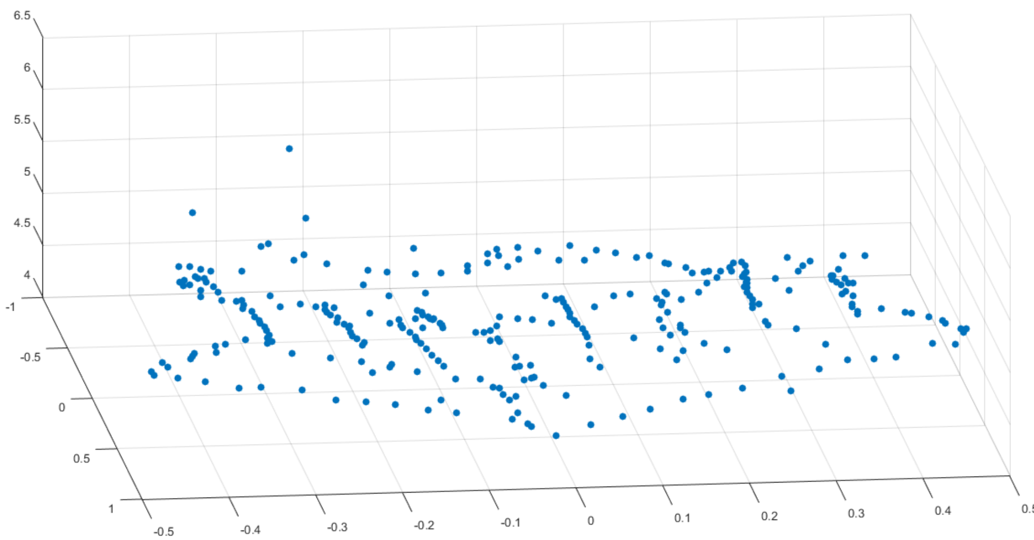


2.7 POINT CLOUD






-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5


6.5
6
5.5
5
4.5
4
-1
-0.5
0
0.5
1
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5