INSTRUCTOR: PROF. DEVA RAMANAN

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LUKAS KANADE

COMPUTER VISION

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LUKAS KANADE

1.1 HESSIAN

Soln. 1.1

$$\min_{u,v} J(u,v) = \sum_{(x,y)\in R_t} \left(I_{t+1}(x+u, y+v) - I_t(x,y)\right)^2 \rightarrow (1)$$

1) Brightness constancy contraint.

$$I_{t+1}(x+u, y+v) - I(x,y) = 0$$

2. Small motion (Taylor expansion of I

$$I_{t+1}(x+u, y+v) = I_{t+1}(x,y) + \frac{\partial I_{t+1}}{\partial x} u + \frac{\partial I_{t+1}}{\partial y} v$$

(taking till first order terms)

combining these 2

$$0 = I_{t+1}(x,y) + I_x u + I_y v - I_t(x,y) \quad \begin{cases} \text{where } I_x = \dfrac{\partial I_{t+1}}{\partial x} \\ I_y = \dfrac{\partial I_{t+1}}{\partial y} \end{cases}$$

$$\Rightarrow \left[I_{t+1}(x,y) - I_t(x,y)\right] + I_x u + I_y v = 0$$

$$\Rightarrow I_t + \nabla I . \langle u,v \rangle = 0$$

$$\text{or } -I_t = \nabla I . \langle u.v \rangle$$

This is of the form

$$\underset{\substack{A \\ n\times 2}}{\begin{bmatrix} I_x(p_1) & I_y(p_1) \\ I_x(p_2) & I_y(p_2) \\ \vdots & \vdots \\ I_x(p_n) & I_y(p_n) \end{bmatrix}} \times \underset{\substack{\Delta p \\ 2\times 1}}{\begin{bmatrix} u \\ v \end{bmatrix}} = \underset{\substack{b \\ n\times 1}}{\begin{bmatrix} I_t(p_1) \\ I_t(p_2) \\ \vdots \\ I_t(p_n) \end{bmatrix}} \rightarrow (4)$$

$\Rightarrow$ we need to minimize $\|A\Delta p - b\|^2$

or $\min_{\Delta p} (A\Delta p - b)^2$

multiplying eq (4) $A \Delta p = b$ by A^T we get :-

$$(A^T A)\, d = A^T b$$

here :- $$A^T A = \begin{bmatrix} \Sigma I_{xx} & \Sigma I_{xy} \\ \Sigma I_{yx} & \Sigma I_{yy} \end{bmatrix}$$

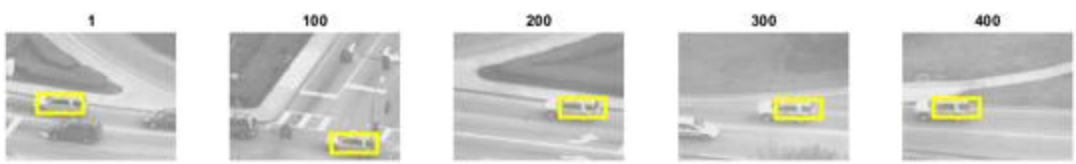
(b) conditions

(i) AA^T should be invertable

(ii) $\underbrace{\lambda_1 / \lambda_2}_{\text{eigen values}}$ should not be too large

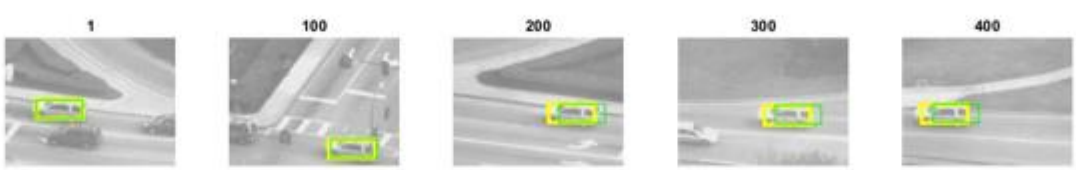
large λ_1 & λ_2 (both of them) relate to corner (like in harris corner detector).

1.3 LUKAS KANADE



1.4 LUKAS KANADE WITH TEMPLATE CORRECTION

It can be seen in the frames below that this is quite better than the previous case.



2.1 APPEARANCE BASIS

2.1

$$I_{t+1} = I_t + \sum_{c=1}^{k} w_c B_c$$

(Taken reference from Lukas-Kanade 20 years on paper) section 3.4

=> ~~Given~~ where $\{B_c\}_{c=1}^{k}$ is a set of k orthogonal image bases.

Here, the least square problem aims to minimize

$$\sum \left[I_{t+1} - I_t - \sum_{c=1}^{k} w_c B_c \right]^2$$

Now $I_t = T$ (template) (In the given question)
the it becomes $\quad$ & $I_{t+1} = I(w;p)$

$$\sum_x \left[I(W(x;p)) - T(x) - \sum_{c=1}^{m} w_c B_c(x) \right]^2$$

or

$$\| I(W(x;p)) - T(x) - \sum_{c=1}^{m} w_c B_c \|^2$$ { || is L2 norm euclidean.

Now as collection of B_c vectors are spanned by (B_c) & its orthogonal complement by span $(B_c)^{\perp}$

$$\Rightarrow \| I(W(x;p)) - T(x) - \sum_{c=1}^{m} w_c B_c \|^2_{span(B_c)} + \| I(W(x;p)) - T(x) - \sum_{c=1}^{m} w_c B_c \|^2_{span(B_c)^{\perp}}$$

with norm 2, ~~orthogo~~ orthogonal components in ↑ all directions

$$\| I(W(x;p)) - T(x) - \sum_{c=1}^{m} w_c B_c \|^2_{span(B_c)} + \| I(W(x;p)) - T(x) \|^2_{span(B_c)^{\perp}}$$

then minimize this w.r.t w.

does not depend on w
first minimize w.r.t p

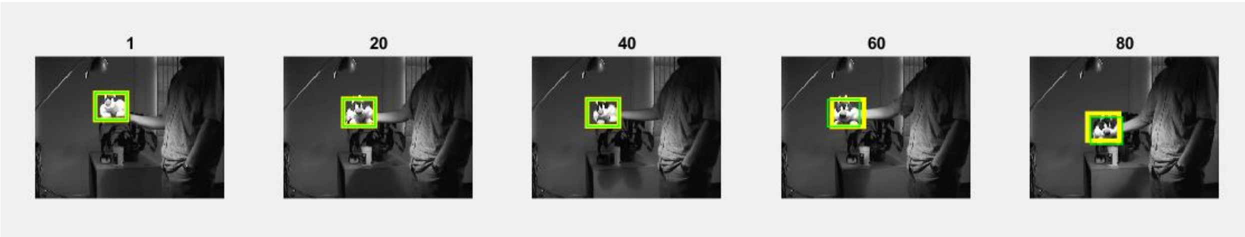
$\boxed{B_c \text{ are orthonormal}}$

$$w_c = \sum_x B_i(x) \cdot [I_{t+1}(w;p) - I(t)]$$

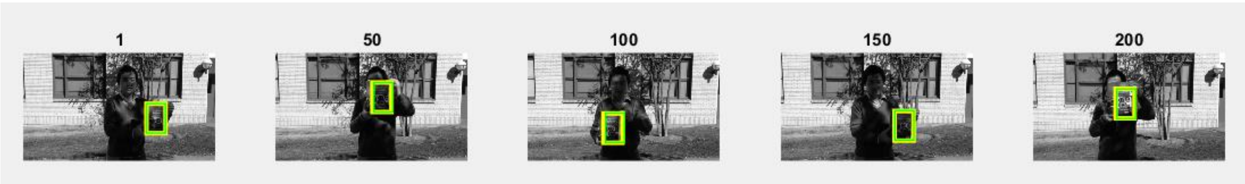
2.3 LUKAS KANADE BASIS

Yellow: Lukas Kanade
Green: Lukas Kanade Basis
Both are coming to the same location

Results when skipping 4 frames in between and output for frame number 1, 20, 40, 60 and 80.



Output for book:
For Frames 1,50, 100, 150 and 200



3.3 LUKAS KANADE AFFINE