Assignment #1

TUSHAR GOYAL
A20449419
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1) Homogenous Coordinate Representation (2D) :

Let $x = (2,3)$ — a point in 2D Cartesian coordinate

• moving from Cartesian to homogenous coordinate

$$2D \rightarrow 2DH$$
$$(x,y) \leftrightarrow (x,y,1)$$

So, $x_H = (2,3,1)$

• To find another point in 2D homogenous coordinate that is equivalent to the same point x,

we can use equivalence class formula

$$(x,y,1) \leftrightarrow (kx, ky, k)$$

therefore $(2,3,1) \leftrightarrow (3\times 2, 3\times 3, 3\times 1)$ {taking $k=3$}

$$= (6,9,3)$$

i.e., $x_H' = (6,9,3)$

• moving back to Cartesian coordinate from homogenous coordinates we need to divide x, y by w.

$$2DH \leftrightarrow 2D$$
$$(x,y,w) \leftrightarrow (x/w, y/w, 1)$$

therefore

$x = \left(\frac{2}{1}, \frac{3}{1}\right) = (2,3)$ → for x_H

$x' = \left(\frac{\cancel{6}^{2}}{\cancel{3}}, \frac{\cancel{9}^{3}}{\cancel{3}}\right) = (2,3)$ → for x_H'

Hence, Both x_H and x_H' represent the same point $(2,3)$ in Cartesian coordinate.

2) Homogeneous Coordinate Representation (3D) →

Given: $y = (4,5,6)$ → a point in 3D cartesian coordinate

we apply the same rules,

- Converting it to homogeneous coordinate

$$3D \rightarrow 3DH$$
$$(x,y,z) \leftrightarrow (x,y,z,1)$$

Therefore

$$Y_H = (4,5,6,1)$$

- Scaling it by a factor of 2, i.e., $k=2$

$$Y_H' = (4\times2,\ 5\times2,\ 6\times2,\ 1\times2)$$
$$(8,10,12,2)$$

$$\left[\ (x,y,z,1) \rightarrow (kx,ky,kz,k)\ \right.$$

Converting Y_H' back to Cartesian coordinates,

$$3DH \rightarrow 3D$$
$$(x,y,z,w) \leftrightarrow \left(\frac{x}{w},\frac{y}{w},\frac{z}{w},\frac{1}{}\right)$$

$$Y' = \left(\frac{\cancel{8}^{4}}{\cancel{2}},\frac{\cancel{10}^{5}}{\cancel{2}},\frac{\cancel{12}^{6}}{\cancel{2}}\right)$$
$$= (4,5,6).$$

• Hence, $y' = y$

It also signifies that scaling of homogeneous coordinates preserves the original point in Cartesian space.

3) Affine Transformation in 2D →

Given: point, $P = (1,2)$ in 2D Cartesian coordinate

$P_H = (1,2,1)$ —— (i)

• Scaling by a factor of 3

$$S(3) = \begin{bmatrix} S & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

• Rotation (counter-clockwise by 45°)

$$R(45) = \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Translate it by (2, 3).

$$T(2,3) = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

The combined matrix transformation

$$M = T \cdot R \cdot S$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3/\sqrt{2} & -3/\sqrt{2} & 0 \\ 3/\sqrt{2} & 3/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3/\sqrt{2} & -3/\sqrt{2} & 2 \\ 3/\sqrt{2} & 3/\sqrt{2} & 3 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$ (multiplying it by P_n)

$$P_n' = \begin{bmatrix} 3/\sqrt{2} - \frac{6}{\sqrt{2}} + 2 \\ 3/\sqrt{2} + 2 \times 3/\sqrt{2} + 3 \\ 1 \end{bmatrix} = \begin{bmatrix} \left(\frac{-3+2\sqrt{2}}{\sqrt{2}}\right) \\ \left(\frac{3+3\sqrt{2}}{\sqrt{2}}\right) \\ 1 \end{bmatrix}$$

4) Given: $q = (3,4)$ $\longrightarrow$ $q_H = (3,4,1)$

- first reverse the translation, $T(5,5)$, we translate it by $(-5,$

$\therefore$ $T(-5,-5) = \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}$

- To reverse the rotation of $30°$ (counter-clockwise), $R(30°)$, we can rotate it clockwise by $30°$.

$\therefore$ $R(-30) = \begin{bmatrix} \cos(-30) & -\sin(-30) & 0 \\ \sin(-30) & \cos(-30) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

- For reversing the scaling of 2, we scale it by $1/2$. So the transformation matrix is,

$S(1/2) = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now, the combined inverse transformation matrix is given by

$$M^{-1} = S^{-1} \cdot R^{-1} \cdot T^{-1}$$

Here we have, $S(1/2)$ $R(-30°)$ $T(-5,-5)$

$$= \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.87 & 0.5 & -6.83 \\ -0.5 & 0.87 & -1.83 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.4330127 & 0.25 & -3.415 \\ -0.25 & 0.43 & -0.915 \\ 0 & 0 & 1 \end{bmatrix}$$

Now,

$$M^{-1} \times q_n$$

$$= \begin{bmatrix} 0.43 & 0.25 & -3.42 \\ -0.25 & 0.43 & -0.92 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} \begin{bmatrix} -1.12 \\ 0.07 \\ 1 \end{bmatrix}$$

$\therefore$ Point q after applying inverse $(-1.12, 0.07)$