

Agenda

- 1) Flip
- 2) Search for elements in row-wise & col-wise sorted matrix
- 2) Merge Overlapping Intervals
- 4) Kadane's AlgoC max sum subarray

↓
Question:

Flip

Given a binary string (0's, 1's), maximize the count of 1's in the string by flipping any substring.

S = 1 0 0 1 0 0 1 1 1 1 0 1

0 1 2 3 4 5 6 7 8 9 10 11

max sum = 3
start = 1, end = 5

sum = 1 1 1 1 1

[4, 5] = 9
[1, 5] = 10 ✓
[1, 10] =

S =

1 1 1 1 1 1
0 1 2 3 4 5

⇒

Ans = 6

Brute Force
 Consider all the substrings : $\frac{N(N+1)}{2}$ ✓
 we flip the substring & count #1s : $O(N)$

(i, j)
 (i, j)
 Total T.C : $O(N^3)$
 S.C : $O(1)$
 orig ones = 5
 max ones = 5

Approach 2:

A =
 1 1 0 1 0 1 1 0
 0 1 2 3 4 5 6 7
 $i=0, j=1$
 count 0 = 0
 count 1 = 0
 $i=0, j=0, 1, 2, \dots, 7$
 $5-1+0 = 4$

$[0, 0]$
 $[0, 1]$

flip
 1 0 1 1 0 0 1 1
 # ones = 5 (p)
 # zeros = 3 (q)

z = total #1s in the string

$$X - p + q$$

#ones in orig substring

$$p, q$$

// ones orig
ans;

for (i=0; i<N; i++) {
 count0 = 0, count1 = 0;

for (j=i; j<N; j++) {
 if (s[j] == '0') count0++;

 else count1++;

 ans = max(ans, ones-orig - count1 + count0;

 // ans + count0 - count1

}
return ans;

T.C: $O(N^2)$

S.C: $O(1)$

Approach 3:

We want a substring with

- a) More no. of 0's (#zeros) - (+1)
b) less no. of 1's (#ones) - (-1)

maxim (#zeros - #ones) ✓

[1 0 0 1 0 1 0 0]
diff = (-1) + 1 + 1 + (-1) + 1 - 1 + 1 + 1
= $\lfloor \frac{2}{2} \rfloor \rightarrow$ maximised

Convert

0's	\Rightarrow	+
1's	\Rightarrow	-

$$C = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 \end{bmatrix}$$

=> find the maximum sum subarray from this

Kadane's Algo: $O(N)$

S.C: $O(1)$

Ans: $\boxed{\text{maxSum}} + \text{\# orig-ons}$

Question:
sorted

Given matrix

a row-wise & col-wise search for a given element

A =

$K = 14 \rightarrow \text{True}$
 $K = 22 \rightarrow \text{False}$

$\log N$ ↓	$\log N$ ↓	$\log N$ ↓	$\log N$ ↓	$K = 18$
5	10	15	20	✓ $\rightarrow \log m$
6	12	20	23	✓ $\rightarrow \log m$
7	14	21	30	✓ $\rightarrow \log m$
17	26	33	48	✓ $\rightarrow \log m$
4×4				$N \times M$

Brute Force:

iterate over entire matrix

for (i = 0 \rightarrow N) {
 for (j = 0 \rightarrow M) {
 if (A[i][j] == K)
 True
 }
}

T.C: $O(N \times M)$
S.C: $O(1)$

Approach 2:

Binary Search on Rows



$\Rightarrow O(\log N)$
Total T.C: $O(N \log M)$

T.C: $O(N \cdot \log M)$

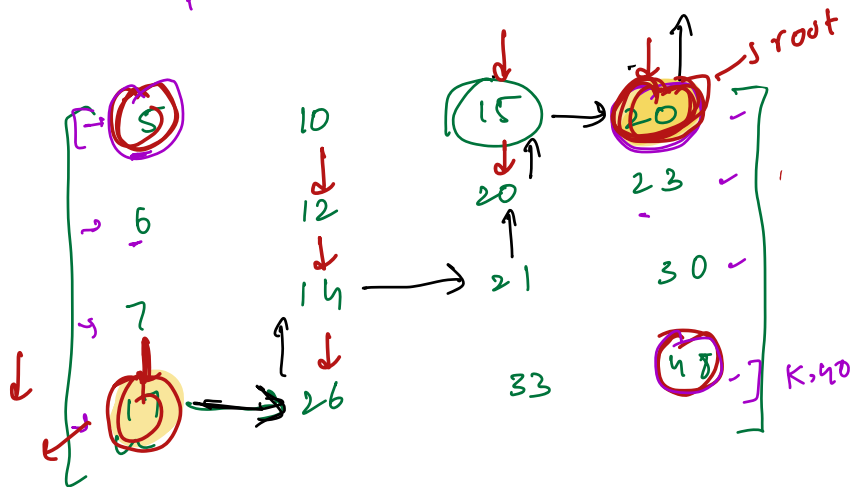
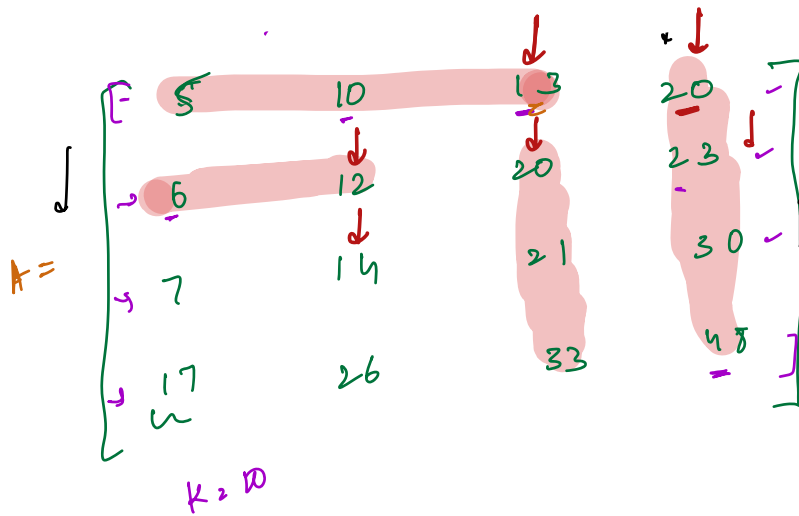
Binary search on col

$O(M \cdot \log N)$

Approach 3:

Approach 4:

$$K = 14$$



$$i < 0 \text{ || } j \geq N$$

$$T.C: O(M+N)$$

$$(0, M-1) \rightarrow (N-1, 0)$$

$N+M$ cells

B.S.T

giving



9:20 AM IST

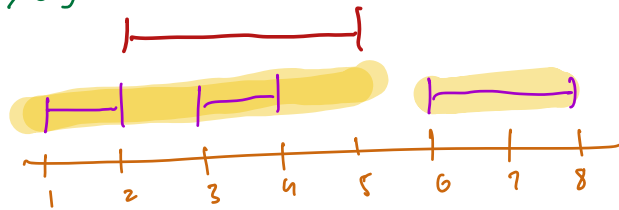
Question: Merge Overlapping Intervals

→ Given non-overlapping interval in a sorted

order
→ Given a new interval. Add this new interval to the existing ones & return the new set of intervals

A: [1, 2] [3, 4] [6, 8]

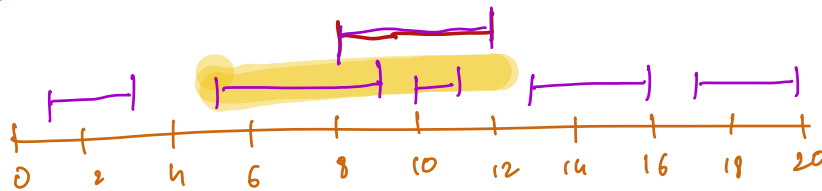
I = [2, 5]



Ans = [[1, 5], [6, 8]]

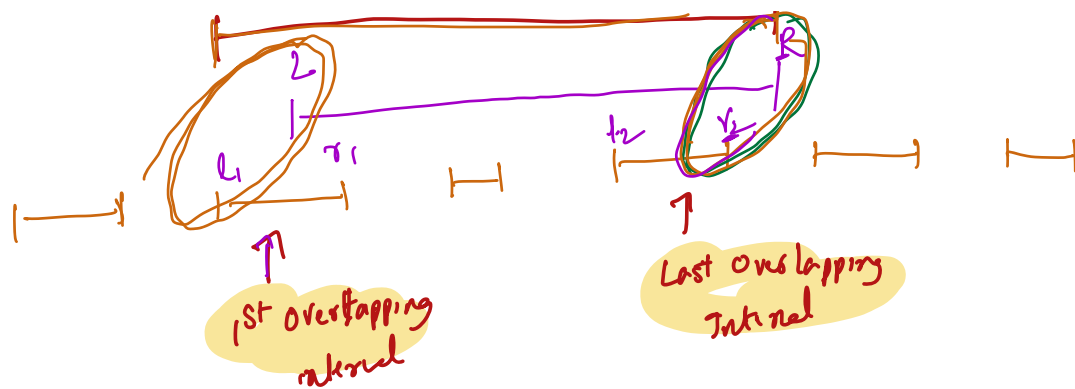
A = [1, 3] [5, 9] [10, 11] [13, 16] [17, 20]

I = [8, 12]



Ans = [[1, 3], [5, 12], [13, 16], [17, 20]]

Solution:

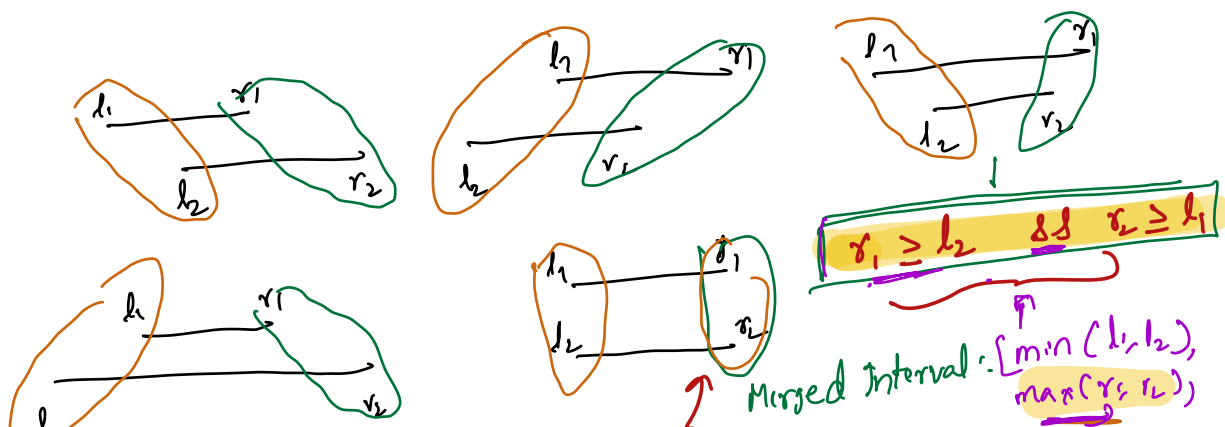


- Find 1st overlapping interval ✓
- Find last overlapping interval ✓
- Merge the interval

2 intervals
 (l_1, r_1)

(l_2, r_2)

$\{r_1 \geq l_2 \text{ and } r_2 \geq l_1\}$



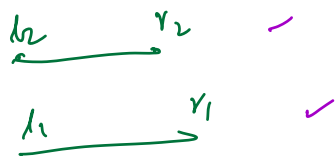
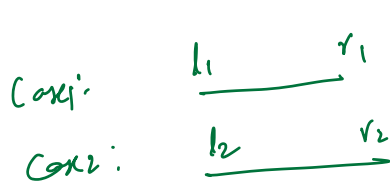
Observations:

- 1st interval has to stop after the 2nd interval starts $r_1 \geq l_2$
- 2nd interval has to stop after the 1st interval starts $r_2 \geq l_1$

No Overlap

(l_1, r_1)

(l_2, r_2)



$if (l_2 > r_1 || l_1 > r_2)$

→ for (i=0; i<N; i++) {
 if (isOverlap (Interval[i], I) {
 first-overlap = i;
 break;
 }
}

for (i=N-1; i>=0; i--) {
 if (isOverlap (Interval[i], I) {
 last-overlap = i;
 break;
 }
}

Merged Interval: → $O(1)$

First-overlap = (l, r_1)
Last-overlap = (l_2, r_2)
Interval = $[L, R]$

$[\min(l_1, L), \max(r_2, R)]$

⇒

A = $[[1,3] [5,9] [10,11] [13,16] [17,20]]$ → $O(N)$

I = $[8,12]$

A' = $[[1,3] [5,12] [13,16] [17,20]]$

output

S.C: $O(1)$

T.C: $O(N)$
S.C: $O(1)$

Kadane's Algo ✓

→ Given an array, find the max sum subarray

Ex1: $A = 1 \ 2 \ 3 \ 4 \ -10$
Ans = 10
[1]
[1, 2]
[1, 2, 3]
[1, 2, 3, 4] $\Rightarrow [4]$

Ex2: $-2 \ 1 \ -3 \ 4 \ -10 \ 2 \ 1 \ -5 \ 4$
Ans = 6
[4, -1, 2, 1] $\Rightarrow [4] \Rightarrow 4$
[4, -1] $\Rightarrow 3$
[4, -1, 2] $\Rightarrow 5$
[4, -1, 2, 1] $\Rightarrow 6$

Ex3: $-7 \ -10 \ -3 \ -5$
Ans = -3

Brute Force:

Consider all subarrays: $\frac{N(N+1)}{2}$

+
Carry Forward Approach

T.C: $O(N^2)$

maxSum = -INF;

for (i=0; i < N; i++) {
 sum = 0;

 for (j=i; j < N; j++) {

 // (i, j) represents subarray

 sum += A[j];

 maxSum = max(maxSum, sum);

 }

}

return maxSum

T.C: $O(N^2)$

S.C: $O(1)$

Approach 2: Kadane's Algo

[1, 2, 3, 4, 5] \Rightarrow

[1]	\Rightarrow	1
[1, 2]	\Rightarrow	3
[1, 2, 3]	\Rightarrow	6
[1, 2, 3, 4]	\Rightarrow	10
[1, 2, 3, 4, 5]	\Rightarrow	15

} Prefix

Let's assume A[i...j] is the max
sum subarray.

Claim: No prefix of the subarray $A[i \dots j]$ would have a negative sum

Proof: Let's assume, there is a prefix of $A[i \dots j]$ which has a negative sum

$$\begin{aligned} \text{sum}(A[i \dots j]) &= \text{sum}(A[i \dots k]) + \text{sum}(A[k+1 \dots j]) \\ \text{sum}(A[i \dots j]) &= \underbrace{-10}_{\text{prefix}} + \text{sum}(A[k+1 \dots j]) \end{aligned}$$

$$\text{sum}(A[k+1 \dots j]) = \underbrace{\text{sum}(A[i \dots j])}_{\text{max}} + 10$$

$A =$

↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
1	-2	1	-3	4	-1	2	1	-5	4
0	1	2	3	4	5	6	7	8	9

sum = ~~0~~ ~~1~~ ~~-1~~ ~~0~~ ~~1~~ ~~0~~ ~~4~~ ~~3~~ ~~8~~ ~~6~~ ~~1~~ ~~8~~ ~~15~~

maxSum = ~~1~~ ~~4~~ ~~8~~ ~~15~~

[]

[1, -2]

[4, -1]

└─┘

$A =$

↓	↓	↓	↓	↓	↓
5	6	7	9	1	4

sum = ~~0~~ ~~5~~ ~~11~~ ~~18~~ ~~29~~ ~~32~~

└─┘

A = ↓ ↓ ↓ ↓ ↓
 -7 -4 -2 -6 -5

sum = 0 → 0 → -1 → -2 → -6 → -5
 maxSum = -INF → -1 → -4 → -2

maxSum = -INF;

sum = 0;

for (i = 0; i < N; i++) {

sum += A[i];

maxSum = max(maxSum, sum);

if (sum < 0) {

sum = 0;
 start = i + 1;

}

return maxSum;

T.C: $O(N)$

S.C: $O(1)$

+1 = 50
 -1 = 1

[10001] ⇒ sum = 2
 [0110001100]
 100111011

June 22 Advanced

1/2 ⇒ 8 months