Agenda

1) Numbers system & conversions
2) Negative Numbers
3) Data Type ranges
4) Bitwise operators & properties
5) check if ith bit is set
6) count no of set bits
7) Unique Number

Number Systems

$$\underbrace{\underset{\underset{10^4}{\downarrow}}{3}\ \underset{\underset{10^3}{\downarrow}}{4}\ \underset{\underset{10^2}{\downarrow}}{\boxed{7}}\ \underset{\underset{10^1}{\downarrow}}{\boxed{8}}\ \underset{\underset{10^0}{\downarrow}}{\boxed{9}}}_{\text{Base value}} = 3\times10^4 + 4\times10^3 + 7\times10^2 + 8\times10^1 + 9\times10^0$$

Base: 10 [Decimal Number System]

Unique Values of any digit: [0 - 9]

Octal Number System

Base: 8

Unique values: [0 - 7]

$$(\underset{\underset{8^3}{\downarrow}}{4}\ \underset{\underset{8^2}{\downarrow}}{5}\ \underset{\underset{8^1}{\downarrow}}{1}\ \underset{\underset{8^0}{\downarrow}}{7})_8 = 4\times8^3 + 5\times8^2 + 1\cdot8^1 + 7\cdot8^0$$

Ternary Number System

Base : 3

Unique values : [0-2]

Binary Number System

Base : 2

Unique values : [0, 1]

Binary to Decimal

$(1\,0\,1\,0\,1)_2 \Rightarrow 1\cdot2^4 + 1\cdot2^2 + 1\cdot2^0$

$2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$= 16 + 4 + 1 = \underline{21}$

$(1\,0\,1\,1\,0)_2 = 1\cdot2^4 + 1\cdot2^2 + 1\cdot2^1$

$2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$= 16 + 4 + 2 = 22$

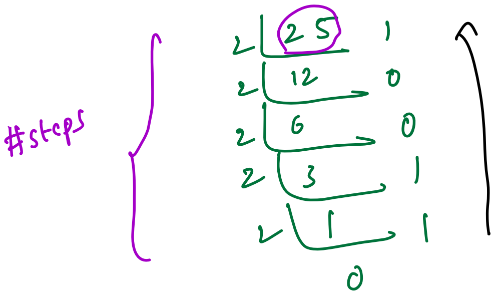
Decimal to Binary

Ex1: 28

2 | 28 0
2 | 14 0
2 | 7 1
2 | 3 1
2 | 1 1
0

= (1 1 1 0 0)

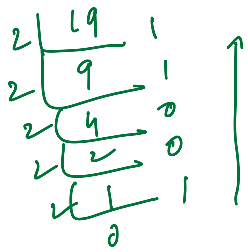
Ex2:



$\underbrace{N \to \frac{N}{2} \to \frac{N}{4} \to \frac{N}{8} \to \cdots}_{\log_2^N} \; 1$

$\boxed{110011}$

Ex3:



$\boxed{10011}$

Quiz: Min no of bits required to represent N

4 : 100 => 8 { 0 0 0 0 0 0 1 0 0 }

6 : 110 => {

Min bits = $O(\log_2^N)$

Negative Numbers

$$10: \quad \begin{array}{cccccccc} 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{array}$$

2's complement form

1) 1's complement (Toggle all the bits)
2) Add 1 to it

$$\begin{array}{lcccccccc} 10: & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ \hline \text{step 1:} & 1 & 1 & 1 & 1 & 0 & 1 & \boxed{\overset{1}{0}} & \boxed{\begin{matrix}1\\+1\end{matrix}} \\ \hline -10 \ \text{step 2:} & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ \hline \end{array}$$

$$\begin{array}{lcccccccc} 4: & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ & 1 & 1 & 1 & 1 & 1 & \overset{1}{0} & \overset{1}{1} & 1 \\ & & & & & & & & +1 \\ \hline -4: & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ \hline \end{array}$$

MSB

$$-10: \quad \begin{array}{cccccccc} 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ -2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

$= (-2^7) + 2^6 + 2^5 + 2^4 + 2^2 + 2^1 = 246$

$= -128 + 118 = \boxed{-10}$

1 0 0 0 0

4 3 2 1 0

$2^4 = 16$

0 1 1 1 1

$= 2^3 + 2^2 + 2^1 + 2^0$

$= 8 + 4 + 2 + 1$

$= 15$

2^0

2^{N-1} 2^1 2^0

1 0 0 0 0 0

N bits

Value : 2^{N-1}

2^{N-1} 2^{N-2} 2^2 2^1 2^0

[0 1 1 1 1 1 1 1]

N bits

$= 2^0 + 2^1 + 2^2 + \dots \dots 2^{N-2}$

[0 . . . . N-2]

$a = 1$

$r = 2$

$K = N-1$

$$S_K = \frac{a(r^K - 1)}{r-1}$$

$$= \frac{1 \cdot (2^{N-1} - 1)}{2-1}$$

$$= \boxed{2^{N-1} - 1}$$

$a_0 - a_7$: {0, 1}

1	1	1	1	1	1	1	1
a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0

$$-2^7 \cdot a_7 + 2^6 \cdot a_6 + 2^5 \cdot a_5 + 2^4 \cdot a_4 + 2^3 \cdot a_3 + 2^2 \cdot a_2 + 2^1 \cdot a_1 + 2^0 \cdot a_0$$

$$-2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0 =$$

$$-128 + 127 = \boxed{-1}$$

Unsigned Integer

✓

Base value of MSB: Positive

Signed Integer

Base value of MSB: Negative

Ranges of datatype

Unsigned Integer

$$\text{Min} = \underset{2^{N-1}}{\underset{\downarrow}{\underline{0}}}\ \ \underline{0}\ \ \underline{0}\ \ \underline{0}\ \cdots\cdots\ \underset{2^{2}}{\underset{\downarrow}{\underline{0}}}\ \ \underset{2^{1}}{\underset{\downarrow}{\underline{0}}}\ \ \underset{2^{0}}{\underset{\downarrow}{\underline{0}}}\ \Rightarrow 0$$

$$\text{Max} = \underset{2^{N-1}}{\underset{\downarrow}{\underline{1}}}\ \ \underset{2^{N-2}}{\underset{\downarrow}{\underline{1}}}\ \ \underset{2^{N-3}}{\underset{\downarrow}{\underline{1}}}\ \ \underline{1}\ \cdots\cdots\ \underset{2^{2}}{\underset{\downarrow}{\underline{1}}}\ \ \underset{2^{1}}{\underset{\downarrow}{\underline{1}}}\ \ \underset{2^{0}}{\underset{\downarrow}{\underline{1}}}\ = 2^N - 1$$

$$2^0 + 2^1 + 2^2 + \cdots\ 2^{N-1} =$$

$a = 1$

$r = 2$

$K = N$

$$= \frac{1(2^N - 1)}{2-1}$$

$$\boxed{\text{Range}: [0, 2^N - 1]}$$

$N = 32$

$[0, 2^{32} - 1]$

Signed Integer

Min : $\underset{2^{N-1}}{\underline{1}}\ \underline{0}\ \underline{0}\ \underline{0}\ \cdots\cdots\ \underset{2^2}{\underline{0}}\ \underset{2^1}{\underline{0}}\ \underset{2^0}{\underline{0}} = \boxed{-2^{N-1}}$

Max : $\underset{2^{N-1}}{\underline{0}}\ \underset{2^{N-2}}{\underline{1}}\ \underline{1}\ \underline{1}\ \cdots\cdots\ \underset{2^2}{\underline{1}}\ \underset{2^1}{\underline{1}}\ \underset{2^0}{\underline{1}}$

$= 2^0 + 2^1 + 2^2 + \cdots\cdots 2^{N-2}$

$a = 1$

$r = 2$

$K = N-1$

$$= \frac{1(2^{N-1} - 1)}{2-1}$$

$$= 2^{N-1} - 1$$

Range of N bit signed Integer $\left[-2^{N-1},\ 2^{N-1} - 1\right]$

$2^{10} = 1024 \approx (10^3)$ Approx

$(2^{30}) = (10^3)^3 = 10^9$

$2^{31} = 2 \times 10^9$

$(2^{10})^3 = (10^3)^3$

$2^{31} = 2 \cdot 2^{30}$
$\downarrow$
$2 \cdot 10^9$

$N = 32$: $\left[-2^{31},\ 2^{31} - 1\right]$

$= \left[-2 \times 10^9,\ 2 \times 10^9 - 1\right]$

$N = 64$ $[-2^{63}, 2^{63} - 1]$

$\downarrow$

$[-10^{18}, 10^{18}]$

Bitwise Operators

OR	(|)
AND	(&)
XOR	(^)
NOT	(~)
Left Shift	(<<)
Right shift	(>>)

O(1)

A	B	A & B	A | B	A ^ B
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

If all bits are set, then it is 1. Else 0

If atleast one bit is set, it is 1. Else 0

If we odd no of set bits, it is 1. Else 0

int n = 5 & 8 = $\boxed{0}$

5 :	0	1	0	1
& 8 :	1	0	0	0
	0	0	0	0

$13 \wedge 10$

13 : 1 1 0 1

^ 10 : 1 0 1 0

0 1 1 1 = 0111 ⇒ $\boxed{7}$

11 | 9

11 : 1 0 1 1

9 : 1 0 0 1

1 0 1 1 ⇒ $\boxed{11}$

NOT [Toggle]

A	~A
0	1
1	0

Left Shift

MSB will overflow

5 : 0 0 0 0 0 1 0 1 ⇒ 5

5<<1 = 0 0 0 0 1 0 1 0 ⇒ 10

5<<2 = 0 0 0 1 0 1 0 0 ⇒ 20

×2

2

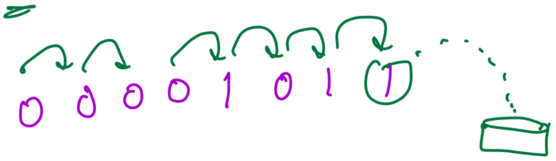
$5 << i = 5 \times 2^i$

$N << i = N \times 2^i$

100000 ⇒ 010000..

Right Shift

11: 0 0 0 0 1 0 1 1



11>>1: 0 0 0 0 0 1 0 1 ⇒ 5 ↩ /2

11>>2: 0 0 0 0 0 0 1 0 ⇒ 2 ↩ /2

11>>3: 0 0 0 0 0 0 0 1 ⇒ 1

$$N >> i = \frac{N}{2^i}$$

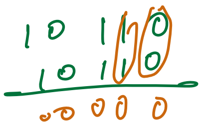
Properties

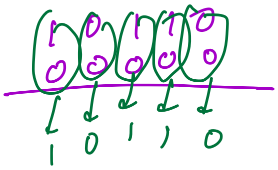
$a \mid a = a$

$a \,\&\, a = a$

$a \wedge a = 0$

$a \wedge 0 = a$





Commutative Property

$a \& b = b \& a$

$a \mid b = b \mid a$

$a \wedge b = b \wedge a$

Associative Property

$$a \wedge b \wedge c = \left.\begin{array}{l} (a \wedge b) \wedge c \\ (a \wedge c) \wedge b \\ (b \wedge c) \wedge a \end{array}\right\} \text{Same}$$

$\boxed{\text{7 mins break}}$

Question: Given an integer, check if ith bit is set or not

14 : 000000 0 1 1 1 0
3 2 1 0

$i < \log_2 N$

i = 1 → YES
i = 3 → YES
i = 0 → NO
i = 10 → 0

Even

N = 1 1 1 0 ⇒ (N & 1 == 1)
3 2 1 0

0th bit is set

else 0th bit is unset

$$\begin{array}{r} 1\ 1\ 1\ 0 \\ 0\ 0\ 0\ 1 \\ \hline 0\ 0\ 0\ 0 \end{array}$$

LSB → 0 [Even Num]
LSB → 1 [odd Num]

jth

N : 0 0 0 0 1 0 1 0 ⇒ 0 0 0 0 0 0 0 1
7 6 5 4 3 2 1 0

Sol1:

((N >> i) & 1)

Sol2:

$$\begin{array}{rl} N= & 0\,0\,0\,0\ 1\ 0\ 1\ 0 \\ \& & 0\,0\,0\,0\ 1\ 0\ 0\ 0 \Rightarrow (1 << i) \\ \hline & 0\,0\,0\,0\ 1\ 0\ 0\ 0 \end{array}$$

ith

N = 0 1 0 0 1 1 0 1

Mask = 0 0 0 1 0 0 0 0

0 0 0 0 0 0 0 0

```
if( N & (1<<i)) == 0){
    "UNSET"
}
else
    "SET"
```

$\boxed{T.C: \ O(1)}$

Question: Count no. of set bits

N = 10 => 1010 => 2

N = 15 => 1111 => 4

Approach1:

ceil $\lfloor \log_2 N \rfloor$

C++, Java: 32 bits

```
X {
    for(i=0; i<32; i++){
        if(isSetBit(N, i)){
            count++;
        }
    }
    return count;
```

X in Python

T.C: O(1) ?

#iters = No. of bits of N => $\boxed{O(\log N)}$

Python: $\boxed{10^{100}}$

Approach2:

N = 0 1 1 0 0 1 1 $\boxed{0}$

N>>1: 0 0 1 1 0 0 1 $\boxed{1}$

N>>1: 0 0 0 1 1 0 0 $\boxed{1}$

count = 1 +

```
count = 0;
while ( N != 0 ){
    if( N&1 == 1 )
        count++;
    N = N >> 1;
}
```

$\Rightarrow N, \frac{N}{2}, \frac{N}{4}, \frac{N}{8} \ldots\ldots 0$

$\log_2^N$

N = 10000 => 5

N = 01111 => 4

T.C: $O(\# \text{bits})$

$= O(\log_2^N)$

1000000

0100000000

000000000 1

while(N>0)

N = 10000000 => -1

N&1

N = 1010

0001

0000

(N&1) == N

Approach3:

	N	(N-1)	N & (N-1)
5:	101	100	100
9:	1001	1000	1000
4:	0100	0011	0
8:	1000	0111	0
10:	1010	1001	1000
12:	1100	1011	1000

$a \,\&\, (\sim a) = 0$

N&(N-1) would unset the right-most set bit

#set bits

1011010
↓ N&(N-1)
1011000
↓ N&(N-1)
1010000
↓ N&(N-1)
1000000
↓
000000

cout = ~~1~~ ~~2~~ ~~3~~ 4

```
count = 0
while( N! = 0){
        N =     N & (N-1);
        count++;
}
return   count.
```

(N > 0)

T. C: O(#set bits.) => O($\log_2^N$)

N & (N-1)

N = 16   1 0000 => 0

N = 15   0 1111 -> 0 110 -> 0100 -> 0010

N = 11111 =>

Question: Check if N is a power of 2

N = 0001 00000

N-1 = 0000 11111

N & N-1 = 000000

```
if( N & (N-1) = = 0){
    "power of 2"
}
```

Question : All numbers occur even no of times and exactly one number occurs odd no. of times.

→ Find the no. which occurs odd no of times

A : 2 8 3 1 2 2 3 2 8 1 1

$XOR(A) = 2^\wedge 8^\wedge 3^\wedge 1^\wedge 2^\wedge 2^\wedge 3^\wedge 2^\wedge 8^\wedge 1^\wedge 1 =$

$\underbrace{x^\wedge x}_{0}{}^\wedge \underbrace{x^\wedge x}_{0} \ldots \ldots$ even times $=$

$0^\wedge 0^\wedge 0^\wedge \ldots 0^\wedge 0 = 0$

$\underbrace{y^\wedge y}_{0}{}^\wedge \underbrace{y^\wedge y}_{0} \ldots y$ odd times $= y$

$0^\wedge 0^\wedge 0^\wedge 0^\wedge y$

$a^\wedge b^\wedge c = a^\wedge c^\wedge b$

$XOR(A) = 2^\wedge 8^\wedge 3^\wedge 1^\wedge 2^\wedge 2^\wedge 3^\wedge 2^\wedge 8^\wedge 1^\wedge 1 =$

$= (2^\wedge 2^\wedge 2^\wedge 2)^\wedge (3^\wedge 3)^\wedge (8^\wedge 8)^\wedge (1^\wedge 1^\wedge 1)$

$0 \;^\wedge\; 0 \;^\wedge\; 0 \;^\wedge\; 1 = \boxed{1}$

```
XOR=0
for(i=0; i<N; i++){
    XOR= XOR^ A[i];
}
return XOR;
```

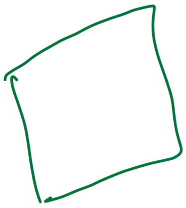
T.C : O(N)

S.C : O(1)

Question: All numbers occurs thrice except one number which occur only once. Find this number which occurs once [constant space]

T.C: O(N)

S.C: O(1)



$\Rightarrow \#\text{iters} = O(\log N + \underline{K})$ $\Rightarrow O(\log_2 N)$

$O(\log N + K)$

$\Rightarrow$ 15 iterations

$\log_2(1000) = \boxed{9}$

$\boxed{\log_2 10^5}$ = 15 (O(1))

$\log_c a^b = b \cdot \log_c a$

$= \log_2 10^5 = 5 \cdot \log_2 10$

$= \log_2 10 = 3.xxx$