Prime Number: positive number with only 2 factors [1 and itself]

Different

Ex: 2, 3, ~~4~~, 5, ~~6~~, 7, ~~8~~, ~~9~~, ~~10~~, 11, ~~12~~, 13

1, 2, n

1: 1 (one factor) Not Prime & Not composite

Numbers with >2 factors: Composite Number

11 – 20: 11, 13, 17, 19 [4]

Question: Find no. of factors/divisors of N

N = 12 {1, 2, 3, 4, 6, 12} 6

N = 16 {1, 2, 4, 8, 16} 5

Brute Force:

factors of N: [1 ..... N]

```
count = 0
for(i=1; i<=N; i++){
    if(N%i == 0){
        count++;
    }
}
return count;
```

T.C: O(N)

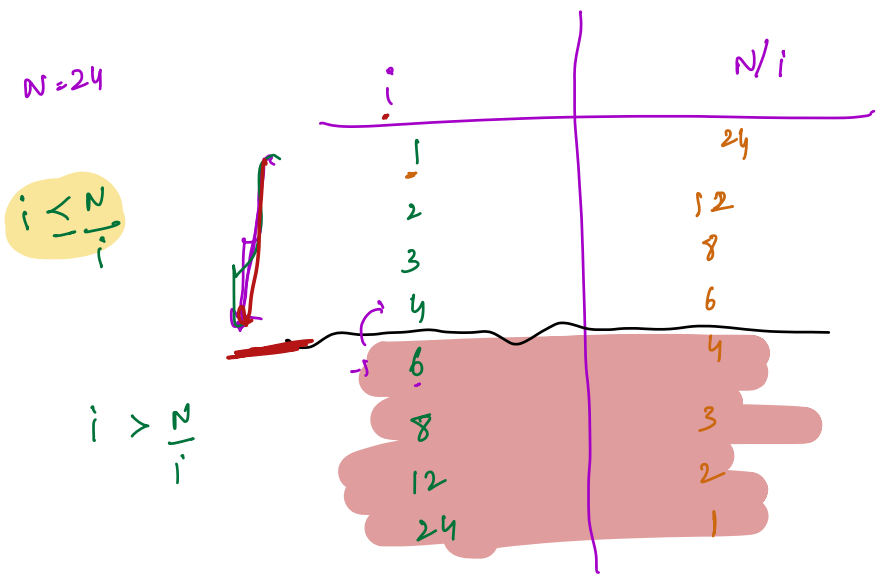
S.C: O(1)

Efficient Approach

24 : 3, $\frac{24}{3}$

4, $\frac{24}{4}$

If i is a factor, $\frac{N}{i}$

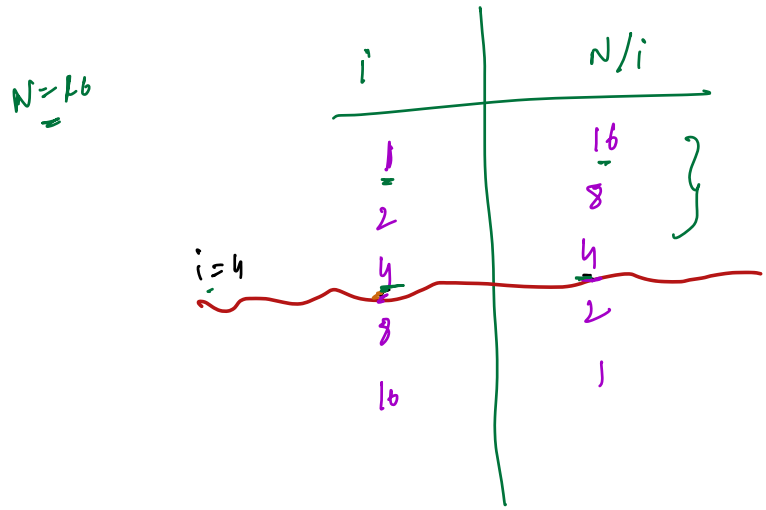


$i \le \frac{N}{i} \Rightarrow i^2 \le N \Rightarrow i \le \sqrt{N}$

i : [1, √N]
T.C : O(√N)
S.C : O(1)

```
ans = 0;
for(i=1; i*i <= N ; i++){
    if(N%i == 0){
        ans += 1 ✓
        if(i != N/i){
            ans += 1
        }
    }
}
return ans;
```

// i, $\frac{N}{i}$ are factors



Question: Check if N is prime

Factors: 1, N

If there is no factor in the range $[2, \sqrt{N}]$, its a prime

```
if(i==1) return false
for(i=2; i*i <= N; i++) {
    if(N%i == 0) {
        return false;
    }
}
return true;
```

T.C: $O(\sqrt{N})$

S.C: $O(1)$

Question: Given N, find out all prime numbers in the range 1 to N

N = 10 : [2, 3, 5, 7]

N = 15 : [2, 3, 5, 7, 11, 13]

Brute Force:

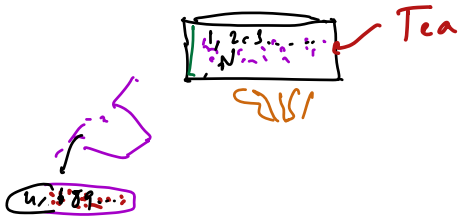
```
for (i=1;  i<=N; i++){
    if (isPrime(i)){
        ans.add(i);
    }
}
```

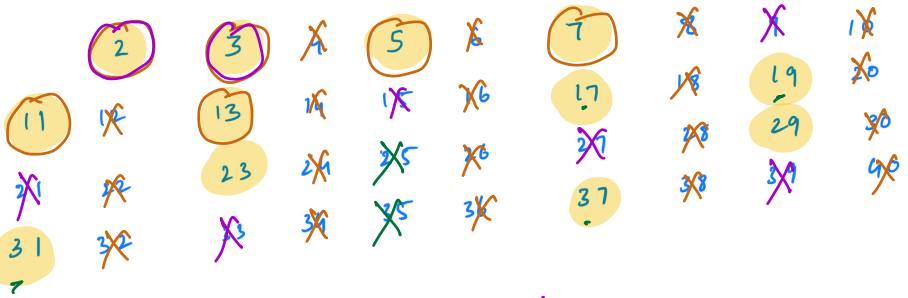
N × √N

T.C : $O(N\sqrt{N})$

S.C : $O(1)$

Sieve of Eratosthenes





boolean prime[N+1] = { True }

i = 3

j = 6, 9, 12, 15, 18

i = 2

j = 4, 6, 8, 10...

```
for(i=2; i<=N; i++){
    if(prime[i] == true){
        for(j=2*i; j<=N; j+=i)
            prime[j] = false;
    }
}
```

T.C:

N

$i=2$	$\frac{N}{2}$
$i=3$	$\frac{N}{3}$
$i=4$	0
$i=5$	$\frac{N}{5}$
$i=6$	0
$i=7$	$\frac{N}{7}$

$$= \left(\frac{N}{2} + \frac{N}{3} + \frac{N}{5} + \frac{N}{7} + \frac{N}{11} + \cdots\right)$$

$$= N\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{11} + \cdots\right)$$

AP ✗
GP ✗
HP ✗

Sum of reciprocals of Prime Numbers in $[1-N]$

$$X < \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \cdots \frac{1}{N}\right)$$ ✓ use

$$\int_2^N \frac{1}{x}\,dx = \log_e^N \Rightarrow \log N$$

T.C : $O(N \cdot \log N)$

$\frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{11} + \frac{1}{13} + \cdots\cdots = \log(\log N)$

Precise T.C : $O(N \cdot \log(\log(N)))$

$\downarrow$

$O(N \cdot 5) \simeq O(N)$

$N = 2^{32} \simeq 2 \times 10^9$

$\log_2^N : \log_2^{2^{32}} = 32$

$\log_2(\log_2^N) = \log_2^{32} : 5$

Optimization :

$i = 7$

7×2 cut by 2

7×3 cut by 3

7×4 cut by 2

7×5 cut by 5

7×6 cut by 2

7×7 cut by 7

$j = i \times i$

$i = 5$

$i \times i$

2×5 cut by 2

3×5 cut by 3

4×5 cut by 2

5×5 cut by 5

```
for(i=2; i*i <= N ; i++){
    if(prime[i] == true){
        for(j=i*i ; j<=N ; j+=i)
            prime[j] = false;
    }
}
```

$i*i \le N$

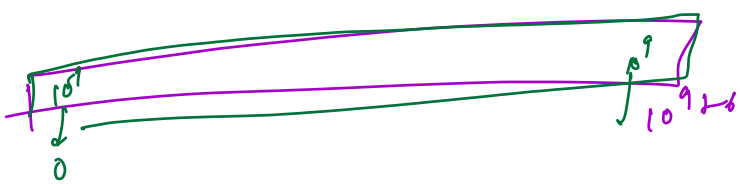
$i \le \sqrt{N}$

$j = i*i > N$

T.C:

Segmented Sieve

Prime nos in $10^9, 10^9 + 10^6$



Question: Find No. of factors of all Numbers from 1 to N

N = 6

1	2	3	4	5	6
1	2	2	3	2	4

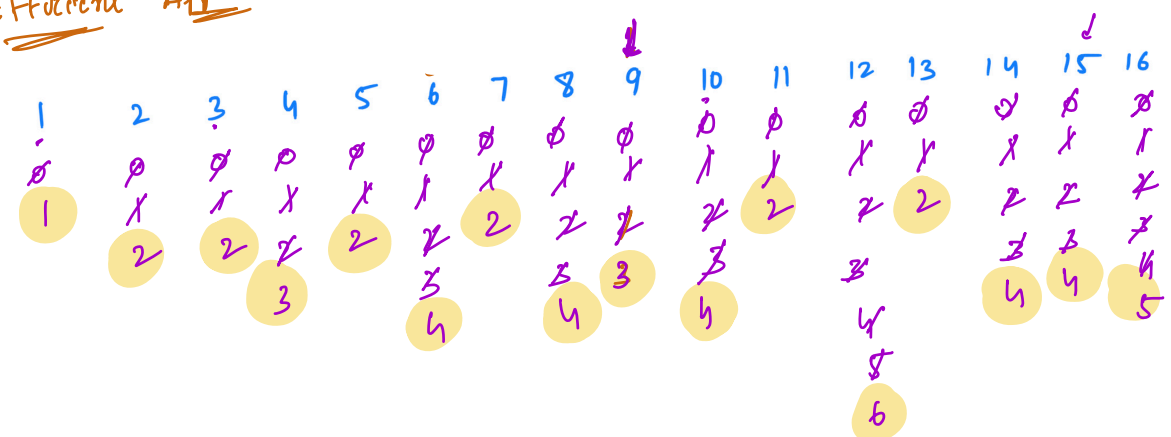
[1 2 2 3 2 4]

Brute Force:

```
for (i=1; i<=N; i++) {
    findFactors(i)  ⟶ √N
}
```

T.C: $O(N \cdot \sqrt{N})$

Efficient Approach:



```
factors[N+1] = {0}
for(i=1; i<=N; i++){
    for(j=i; j<=N; j+=i){
        factors[j]++;
    }
}
```

$i=1 \Rightarrow N$

$i=2 \Rightarrow \frac{N}{2}$

$i=3 \Rightarrow \frac{N}{3}$

$i=4 \Rightarrow \frac{N}{4}$

$N\left[\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots\cdots\frac{1}{N}\right]$

$\log N$

⇒ T.C: $O(N \cdot \log N)$

S.C: $O(1)$

Question: Find the smallest Prime factor (SPF) of all numbers in [1, N]

	2	3	4	5	6	7	8	9	10
SPF:	2	3	2	5	2	7	2	3	2

3: 3 6 9 12

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
SPF:	~~1~~	~~2~~ 2	~~3~~ 3	~~4~~ 2	~~5~~ 5	~~6~~ 2	~~7~~ 7	~~8~~ 2	~~9~~ 3	~~10~~ 2	~~11~~ 11	~~12~~ 2	~~13~~ 13	~~14~~ 2	~~15~~ 3	~~16~~ 2

```
int spf[N+1] = {-1};
for(i=2; i <= N; i++){
    if(spf[i] == -1){
        spf[i] = i;
        for(j=i*i; j <= N; j+=i){
            if(spf[j]==-1){
                spf[j] = i;
            }
        }
    }
}
return spf;
```

T.C: O(N log(log N))

S.C: O(1)

i = 7

7·1

7·2 ✗

7·3 ✗

7·4 ✗

7·5 ✗

7·6 ✗

7·7 ✓

7·8

spf[7] = 7

SPF: Find Prime factorisation of number

15 : 3, 5

24 : 2·2·2·3

N = 36 $\xrightarrow{2}$ 18 $\xrightarrow{2}$ 9 $\xrightarrow{3}$ 3 $\xrightarrow{3}$ 1

$N \xrightarrow{2} \frac{N}{2} \xrightarrow{2} \frac{N}{4} \xrightarrow{2} \frac{N}{8} \cdots \cdots 1 \Rightarrow O(\log N)$

$N \xrightarrow{2} < \frac{N}{2}$

Question: Find prime factors for all numbers in [1, N]

N = 10

2	3	4	5	6	7	8	9	10
[2]	[3]	[2]	[5]	[2,3]	[7]	[2]	[3]	[2,5]

2	3	4	5	6	7	8	9	10
[2]	[3]	[2]	[]	[2,3]	[]	[2]	[3]	[2]

T.C: O(N log log N)

S.C: O(1)

→ If we have to iterate over multiples of prime numbers ⇒ N log log N

→ If iterate over multiples of all numbers: N log N

36 → 18 → 9 → 3 → 1

36

2, 3, 4, ... }

$gcd(x, x+1) = gcd(x, x+1\%x) = gcd(x, 1) = 1$

Question: Are primes finite / infinite

Assumption: There are N prime numbers

$p_1, p_2, p_3, p_4, p_5 \ldots p_N$

$x = p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot p_5 \cdot p_6 \ldots p_N$

$Y = x+1 \quad (p_1 \cdot p_2 \cdot p_3 \cdot p_4 \ldots p_N + 1)$

$gcd(x, Y) = gcd(x, x+1) = 1$

X, Y do not have any common prime factors

None of $p_1, p_2, p_3 \ldots p_N$ are factors of N

Hence, Y has to be prime.