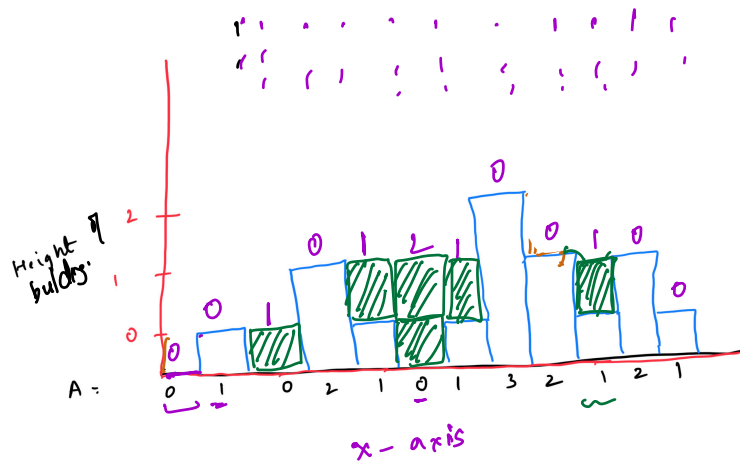


Question: rain water trapping

→ Given heights of N buildings

→ compute the amount of water that will be trapped

A: 0 1 0 2 1 0 1 3 2 1 2 1



width of building = 1 unit

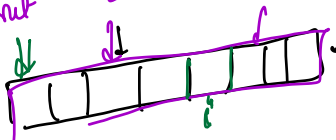
Ans = 6

w_i = Amount of water stored above building i

$$\text{Ans} = \sum w_i$$

Contribution

Construct rightMax $\Rightarrow O(N)$

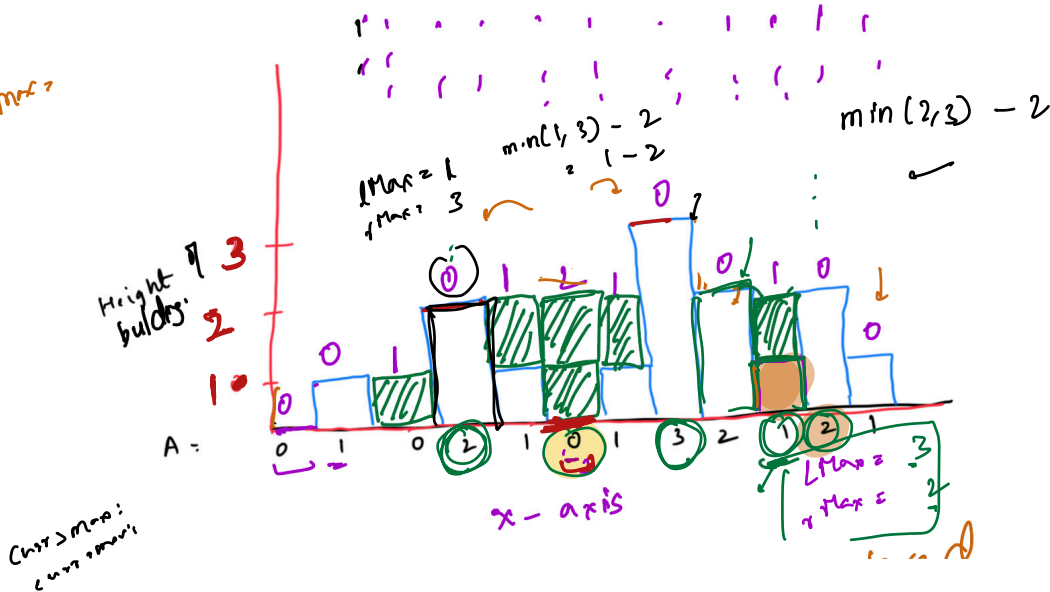


$\Rightarrow O(N)$

W_i

rightMax[i] = $\max(A[i], \dots, N-1)$

max



leftMax = 2 ✓
rightMax = 3 ✓

$\min(\text{leftMax}, \text{rightMax})$ units of water

$$W_i = \min(\text{leftMax}, \text{rightMax}) - h_i$$

Brute Force:

Simply iterate and get leftMax & rightMax

leftMax = $\max(0, \dots, i)$

rightMax = $\max(i, \dots, n-1)$

T.C: $O(N^2)$
S.C: $O(1)$

for (i=0; i<N; i++)
O(N) // Find leftMax
O(N) // Find rightMax
}

Efficient Approach

for every 'i', we need leftMax[i] and rightMax[i]

$$T.C: \begin{matrix} O(N) & + & O(N) & + & O(N) & \Rightarrow & O(N) \\ \downarrow & & \downarrow & & \downarrow \\ \text{leftMax} & & \text{rightMax} & & \text{finding ans} \end{matrix}$$

$$S.C: O(N) \Rightarrow \text{extra array}$$

$$T.C: O(N) \\ S.C: O(N)$$

```
ans = 0;
for (i = 0; i < N; i++) {
    ans += min [ leftMax[i], rightMax[i] ] - height[i]
}
```

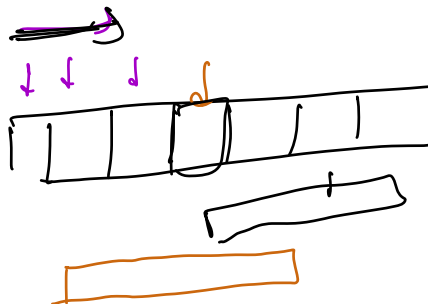
✓ Approach 3:

Carry forward leftMax as a single variable

$$T.C: O(N) + O(N)$$

$$S.C: O(N)$$

↓
extra array



leftMax = 0

Question: First missing non-negative integer

Given an unsorted array, find the smallest non-negative number missing from the array

A = 8 10 1 -3 2 -5 0

0 ✓
1 ✓
2 ✓
3 ✗

Ans = 3

Quiz: [1, 2, 5, 6, 4, 3]

0 ✓

Ans = 0

Quiz: 2 4 -1 -6 3 7 8 4 -3 0

0 ✓
1 ✗

Ans = 1

Quiz: { 3 1 5 0 2 4 }

Ans = 6

Brute Force :

check for 0, 1, 2, 3, ... —

min = 0

max = N

If 0 is not answer \Rightarrow 0 should be present

1 " " "

2 " " "

3 " " "

\Rightarrow 1 " " "

\Rightarrow 2 " " "

\Rightarrow 3 " " "

N-1 is not answer \Rightarrow N-1 should be present

Array Size : \boxed{N}

$[0, 1, 2, \dots, N-1]$

N is the first missing non-negative int

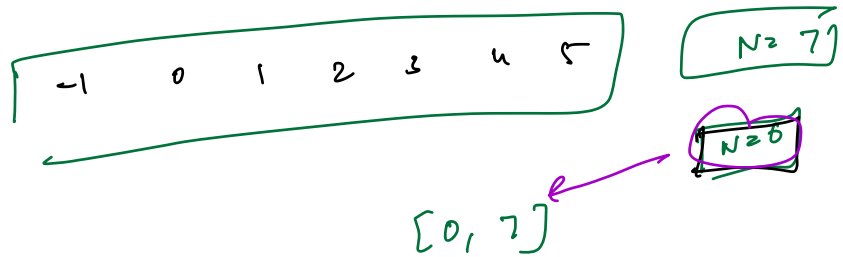
A = -3 -4 -5 -6

Ans = 0

A = 0 1 3 5 6

10^9 \Rightarrow faster upper bound $\boxed{2}$

Ans = $10^9 + 1$



for($i=0$; $i \leq N$; $i++$) {
 $O(N)$ // check if i exists in array
}

T.C: $O(N^2)$
S.C: $O(1)$

Approach 2: Hashset -

\Rightarrow insert all elements into set $\Rightarrow O(N)$

for($i=0$; $i \leq N$; $i++$) {
 $O(1)$ // check if i exists in array
}

T.C: $O(N) + O(N) = O(N)$
S.C: $O(N)$
Hashset

Approach 3:

Sort the array

A: $-5 \quad -3 \quad 0 \quad 1 \quad 2 \quad 8 \quad 10$

$x = 0 \neq 3$

Ans = 3

$$T.C: O(N \log N) + O(N) = O(N \log N)$$

$$S.C: O(1)$$

A = 0 1 2 3 4 5
 min = 0
 max = 5

A = [-2, 0, 1, 10000, 10000]

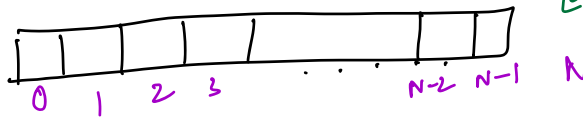
Efficient Approach

Hashset ?

0, 1, 2, 3, ..., N

[0 - N]

A =



N = 7
 A = [0, 7]
 5

Index Range: [0, N-1]

Let's consider only elements in the range

[0, N-1]

A =

	↓	↓	↓	↓			↓
	0	1	2	8	10	-5	6
	0	1	2	3	4	5	6

[0, N-1] = [0, 6]

N = 7
 ans: [0, N]

$A =$

0	1	6	1
0	1	2	3

$N = 4$

$[0-3]$

Swap only if they are not equal

$A =$

0	1	2	3	4
0	1	2	3	4

T.C: $O(N)$
 S.C: $O(1)$

7 mins

$A =$

0	1	2	3
0	1	2	3

 $\Rightarrow O(N)$

long

$[-2 \times 10^9, 2 \times 10^9]$

\downarrow
 $N:$

$[-\infty, \infty]$

Question: Max absolute difference ✓

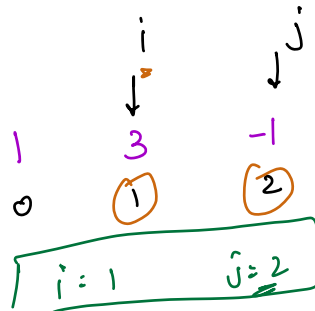
Given an array of N integers, find the maximum value of

$$F(i, j) = |arr[i] - arr[j]| + |i - j|$$

$0 \leq i, j \leq N-1$

$$[arr[i] - arr[j] + i - j]$$

Ex 1: $A =$



$$F(i, j) = |3 - (-1)| + |1 - 2|$$

\downarrow \downarrow
 4 $+1$ $=$ 5

$A[i], A[j]$

Ex 2:

$A =$

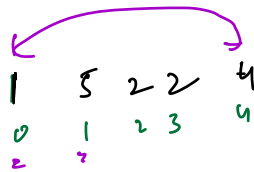
1 2 3 1
 0 1 2 3

$i=0$ $j=2$

$$|1 - 3| + |0 - 2|$$

$2 + 2 =$ 4

Ex 3:



$max = 5$
 $min = 1$

$$|5 - 1| + |1 - 4| =$$

5

$$|1 - 4| + |0 - 4| =$$

5

$i=0, j=4$

$$\frac{|A[i] - A[j]|}{|A[j] - A[i]|}$$

Brute Force

Consider all
 $F(i, j) =$

$$A = [1, 3, -1]$$

$N=3$

The pairs
 $|A[i] - A[j]| + |i - j|$

i	j	$F(i, j)$
0	0	$\Rightarrow 0$
0	1	$\Rightarrow 1 - 3 + 0 - 1 = 3$
0	2	$\Rightarrow 1 - (-1) + 0 - 2 = 4$
1	0	$\Rightarrow 3 - 1 + 1 - 0 = 3$
1	1	$\Rightarrow 0 + 0 = 0$
1	2	$\Rightarrow 3 - (-1) + 1 - 2 = 5$
2	0	$\Rightarrow -1 - 1 + 2 - 0 = 4$
2	1	$\Rightarrow -1 - 3 + 2 - 1 = 5$
2	2	$\Rightarrow 0 + 0 = 0$

Observations:

- 1) $F(i, i) = 0$
- 2) $F(i, j) = F(j, i)$

Consider

$$i < j$$

for ($i=0$; $i < N$; $i++$) {

for ($j=i+1$; $j < N$; $j++$) {

ans = max(ans, $F(i, j)$)

}

T.C: $O(N^2)$

S.C: $O(1)$

Solution:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\begin{aligned} x &= -2 \\ |x| &= -x \quad (-(-2)=2) \\ x &= 2 \\ |x| &= x \end{aligned}$$

$$F(i, j) = |A[i] - A[j]| + |i - j| \quad \text{if } i < j$$

$$F(i, j) = |A[i] - A[j]| + j - i$$

$$\begin{aligned} x &= i - j < 0 \\ |x| &= -x \end{aligned}$$

$$\begin{aligned} |i - j| &= -(i - j) \\ &= -i + j \end{aligned}$$

Case 1:

$$A[i] \leq A[j]$$

$$\begin{aligned} F(i, j) &= (A[j] - A[i]) + (j - i) \\ F(i, j) &= (A[j] + j) - (A[i] + i) \end{aligned}$$

$$\text{if } (A[i] > A[j])$$

only if $A[i] > A[j]$

$$\text{max1} = \max_{i, j} F(i, j)$$

2)

$$\begin{aligned} A &= \begin{matrix} 3 & 1 & -2 & 4 \\ 0 & 1 & 2 & 3 \end{matrix} \\ i &\geq 1, j \geq 2 \end{aligned}$$

Case 2: $A[i] > A[j]$

$$F(i, j) = A[i] - A[j] + j - i$$

$$F(i, j) = (A[i] - i) - (A[j] - j)$$

$$\text{max2} = \max_{i, j} F(i, j)$$

$$\text{ans} = \max(\text{max1}, \text{max2})$$

Case 1:

$$F(i, j) = (A[j] + j) - (A[i] + i)$$

$$\max_i = \max_{i, j} F(i, j)$$

$$X[k] = A[k] + k$$

$$F(i, j) = \left(\underset{\substack{\uparrow \\ \max}}{X[j]} - \underset{\substack{\downarrow \\ \min}}{X[i]} \right)$$

$$\max_i = \max_{i, j} (X[j] - X[i])$$

\downarrow max \downarrow min

A =

0 1 2 ✓
1 3 -1

if $X =$
 $O(N)$

1 4 1

$$\max_i =$$

$$(4 - 1)$$

$$= 3$$

$$\max = 4$$

$$\min = 1$$

Case 2:

$$A[i] > A[j]$$

$$F(i, j) = A[i] - A[j] + j - i$$

$$F(i, j) = (A[i] - i) - (A[j] - j)$$

$$\text{max2} = \max_{i, j} (F(i, j))$$

$$Y[k] = A[k] - k \quad \checkmark$$

$$F(i, j) = Y[i] - Y[j]$$

$$\text{max2} = \max_{i, j} (Y[i] - Y[j])$$

\downarrow \downarrow
max min

$$A = \begin{matrix} 1 & 3 & -1 \\ 0 & 1 & 2 \\ 1 & 2 & -3 \end{matrix}$$

$$Y = \begin{matrix} 1 & 2 & -3 \end{matrix}$$

$$\text{max} = 2$$

$$\text{min} = -3$$

$$\text{max2} = \text{max} - \text{min} \\ = 2 - (-3) \\ = \boxed{5}$$

$$\text{ans} = \max(\text{max1}, \text{max2})$$

$$= \max(3, 5)$$

$$\text{ans} = 5$$

Steps

- $O(N)$ 1) Construct array X
 $O(N)$ 2) Find \max & \min of X
 $O(1)$ 3) $\max1 = X_{\max} - X_{\min}$
 $O(N)$ 4) Construct array Y
 $O(N)$ 5) Find Y_{\max}, Y_{\min}
 $O(1)$ 6) $\max2 = Y_{\max} - Y_{\min}$
 $O(1)$ 7) $\text{ans} = \max(\max1, \max2)$

T.C: $O(N)$

S.C:

$X_{\max} = -\text{INF}, \quad X_{\min} = +\text{INF}$
 $Y_{\max} = -\text{INF}, \quad Y_{\min} = +\text{INF}$

for($i=0; i < N; i++$) {

$X_{\max} = \max(X_{\max}, A[i] + i)$

$X_{\min} = \min(X_{\min}, A[i] + i)$

$Y_{\max} = \max(Y_{\max}, A[i] - i)$

$Y_{\min} = \min(Y_{\min}, A[i] - i)$

}

return $\max(X_{\max} - X_{\min}, Y_{\max} - Y_{\min})$

T.C: $O(N)$
S.C: $O(1)$

Case 1:

$$A[i] \leq A[j]$$

Doubts

$$F(i, j) = A[j] - A[i] + j - i$$

$$F(i, j) = (A[j] + j) - (A[i] + i)$$

$$\max_i = \max_{i, j} F(i, j)$$

z)

$$A[j] > A[i]$$

$$X[k] = A[k] + k$$

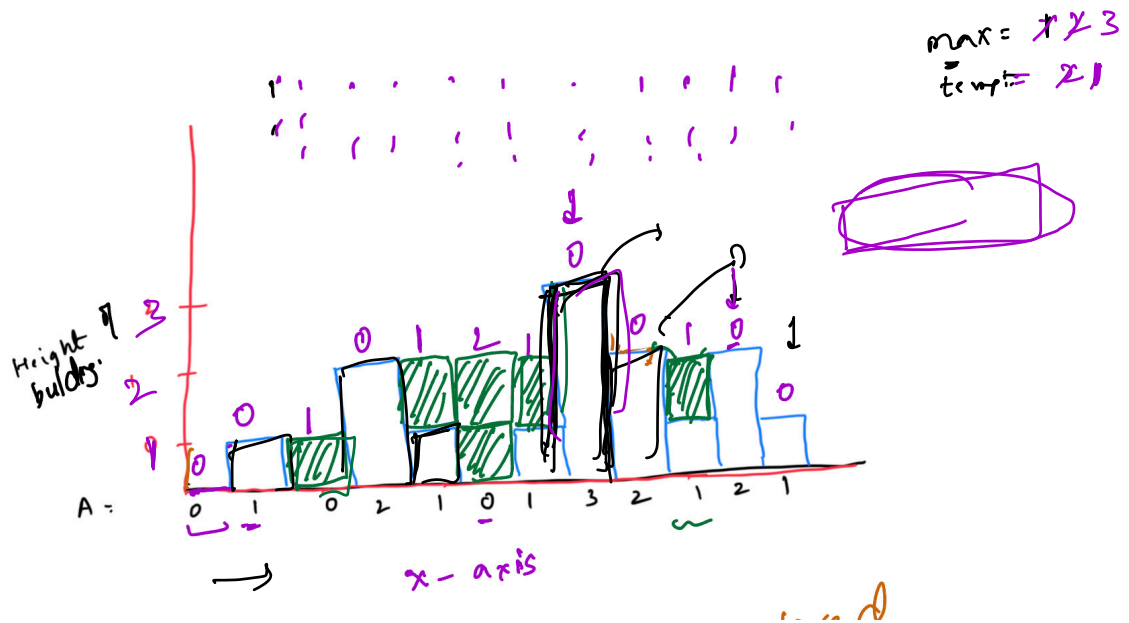
$$F(i, j) = X[j] - X[i]$$

$$i < j$$

$$\max(X, Y)$$

$$Y > X$$

$$A[i] > A[j]$$



$$2m - 3z$$

$$2m?$$

$$(2^m -$$

$$2m - m = temp$$

$$temp = n$$

$$max < 2max - 2$$

$$(a, b)$$