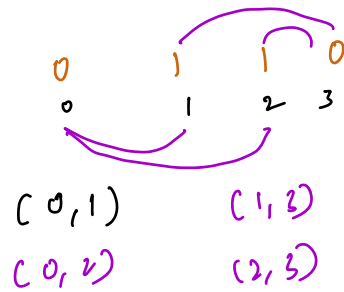


Question: No. of pairs with XOR value as 1
 in a binary [0/1] array [i < j]
 $(i, j) == (j, i)$

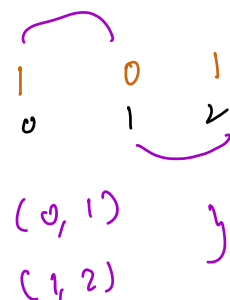
Ex:



$$x \oplus y = 1$$

} [4]

Ex2:



2 pairs

Brute Force:

```

ans = 0;
for (i = 0; i < N; i++) {
    for (j = i + 1; j < N; j++) {
        if (A[i] ^ A[j] == 1)
            ans++;
    }
}
return ans;

```

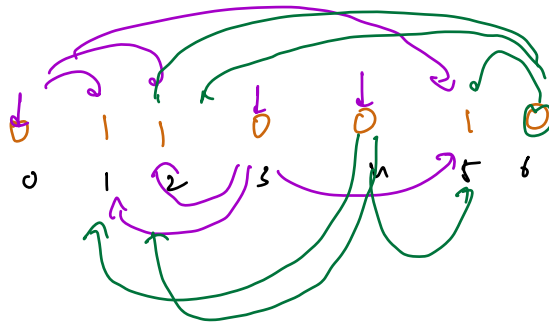
T.C: $O(N^2)$
 S.C: $O(1)$

Efficient Approach

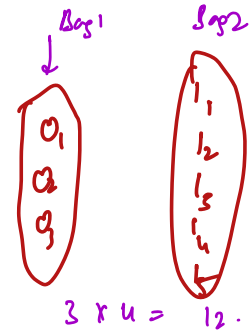
Find no. of $(0,1)$ pairs in the array as their XOR value will be 1

$a \oplus b = 1$
 a, b are different bits

Ex:



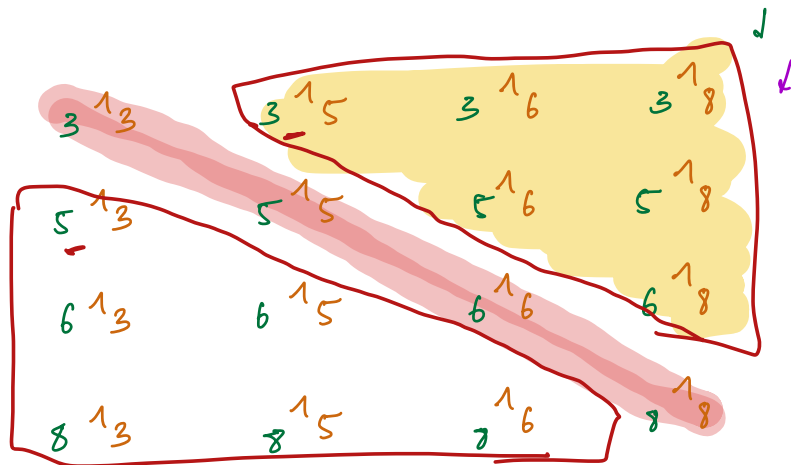
$$3 + 6 + 6 + 3 = 12$$



Ans = #0's \times #1's

Question: Find sum of XOR of all the pairs $[N^2 \text{ pairs}]$ of (i, j)

A: 3 5 6 8
0 1 2 3



pairs : $O(N^2)$

Brute Force

ans = 0;

for (i = 0; i < N; i++) {

for (j = 0; j < N; j++) {

ans += A[i] ^ A[j];

}

return ans;

Observations

1) $A[i] ^ A[i] == 0;$

2) $A[i] ^ A[j] == A[j] ^ A[i]$

T.C: $O(N^2)$
S.C: $O(1)$

 $O(N^2)$

Handwritten notes showing a binary addition example and a complexity analysis.

Binary Addition:

Input numbers (in binary):

- 2: 0010
- 3: 0011
- 5: 0101
- 6: 0110
- 8: 1000

Sum (in binary): 100010

Complexity Analysis:

The complexity is $O(N^2)$.

The number of set bits is 4.

The calculation is:

$$= 32 + 24 + 12 + 8 = 76 \times 2 = 152$$

Naive Approach:

iterate over all the pairs & find the no. of set bits at i th position

\downarrow
 $O(N^2) \times \log(\text{MAX})$

Approach 2:

Task: Find the no. of set bits at every i th position

0th bit: 0 1 1 0 0 $\Rightarrow 2 \times 3 = \boxed{6}$

	3 ↓	2nd ↓	1st ↓	0th ↓
2 :	0	0	1	0
3 :	0	0	1	1
5 :	0	1	0	1
6 :	0	1	1	0
8 :	1	0	0	0
<div> <div>1 x 1 2³</div> <div>3 x 2 2²</div> <div>2 x 3 2¹</div> <div>3 x 2 2⁰</div> </div>				
] x 2				

$\log(\text{MAX})$

```

ans = 0
for (i = 0; i < 32; i++) {
    count = 0;
    for (j = 0; j < N; j++) {
        if (A[j] & (1 << i) != 0)
            count++;
    }
    ans = (count) * (#1's) * (N - count) * (#0's) * (1 << i)
}
return ans * 2;

```

T.C: $O(N \cdot \log(\text{MAX}))$
 S.C: $O(1)$

$$1 \ll i = 2^i$$

Question: different Bits sum Pairwise

$F(X, Y)$ = No. of different corresponding bits

$$F(2, 7) = \begin{array}{c} \downarrow + \downarrow + \downarrow \\ \begin{array}{|c|c|c|} \hline 0 & 1 & 0 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \end{array} = 2$$

Find $\sum F(A_i, A_j)$ for pairs (i, j)

$A = \quad \quad 1 \quad \quad 3 \quad \quad 5$

$F(1, 1)$	$F(1, 3)$	$F(1, 5)$
$F(3, 1)$	$F(3, 3)$	$F(3, 5)$
$F(5, 1)$	$F(5, 3)$	$F(5, 5)$

Brute force:

- consider all the pairs
- for each pair, find no. of different bits

T.C: $O(N^2 \times \log(\text{Max}))$

S.C: $O(1)$

Efficient Approach

A: 1 3 5 2 7

	2	1	0
1:	0	0	1
3:	0	1	1
5:	1	0	1
2:	0	1	0
7:	1	1	1

3x2 2x3 n x 1

$$6 + 6 + 4 = 16 \times 2 = \boxed{32}$$

$$T.C: O(\log(M \times N) \times N)$$

$$S.C: O(1)$$

Question: Given an array of N positive integers, find the max **AND** value of any pair $\max(A[i] \& A[j])$ where $i \neq j$

$A = \quad 27 \quad \quad 18 \quad \quad 20$

$$\begin{array}{r} 27: \quad 1 \ 1 \ 0 \ 1 \ 1 \\ 18: \quad 1 \ 0 \ 0 \ 1 \ 0 \\ \hline 1 \ 0 \ 0 \ 1 \ 0 \Rightarrow 18 \end{array}$$

$$\begin{array}{r} 27: \quad 1 \ 1 \ 0 \ 1 \ 1 \\ 20: \quad 1 \ 0 \ 1 \ 0 \ 0 \\ \hline 1 \ 0 \ 0 \ 0 \ 0 \Rightarrow 16 \end{array}$$

$$\begin{array}{r} 18: \quad 1 \ 0 \ 0 \ 1 \ 0 \\ 20: \quad 1 \ 0 \ 1 \ 0 \ 0 \\ \hline 1 \ 0 \ 0 \ 0 \ 0 \Rightarrow 16 \end{array}$$

11 mins

Brute Force

consider all pairs

T.C: $O(N^2)$

S.C: $O(1)$

Approach 2:

A =

	26	13	23	28	27	7	25
	h	3	2	1	0		
26:	1	1	0	1	0		
13:	0	1	1	0	1		
23:	1	0	1	1	1		
28:	1	1	1	0	0		
27:	1	1	0	1	1		
7:	0	0	1	1	1		
25:	1	1	0	0	1		
	1	1	0	1	0		

⇒ []

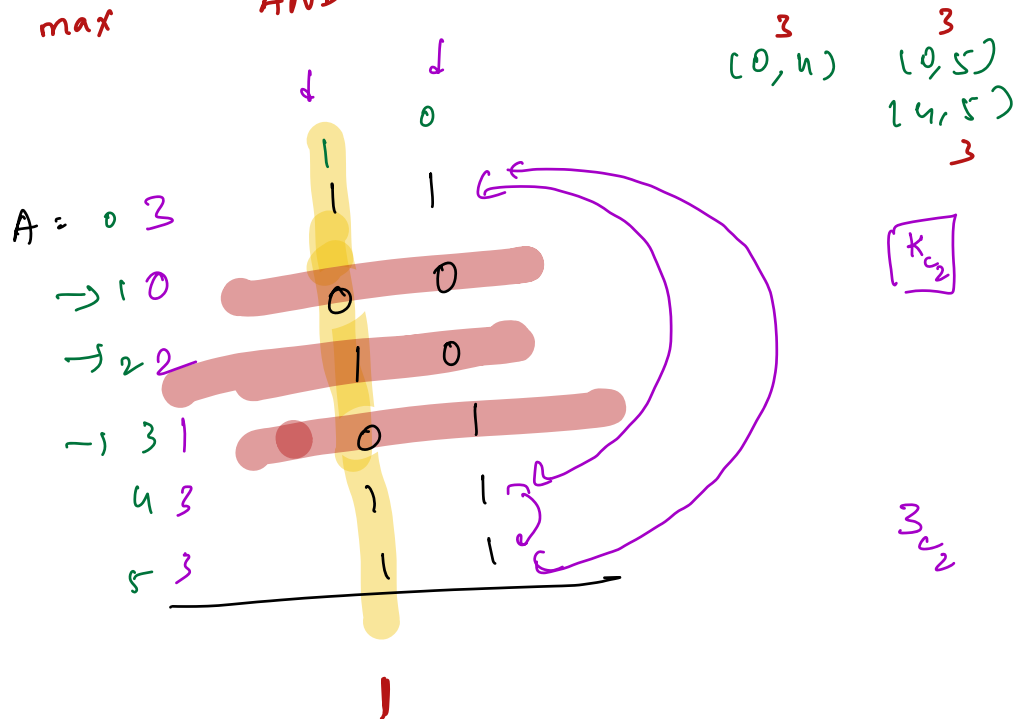
```

ans = 0;
for (i = log(MAX) + 1; i ≥ 0; i--) {
    cnt = 0;
    for (j = 0; j < N; j++) {
        if (A[j] & (1 << i) != 0)
            cnt++;
    }
    if (cnt ≥ 2) {
        ans = ans | (1 << i);
        for (k = 0; k < N; k++) {
            if (A[k] & (1 << i) == 0)
                A[k] = 0;
        }
    }
}
return ans;

```

Total T.C: $O(N \times \log(\text{MAX}))$
 S.C: $O(1)$

Quesn: Find No. of pairs with this
 max AND value.



Quesn: Find max $(A[i] \& A[j] \& A[k] \& A[l])$
 where $i \neq j \neq k \neq l$

$$N = 4$$

$$\begin{matrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \\ 0 & 0 \end{matrix}$$

$$\left. \begin{matrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \\ 0 & 0 \end{matrix} \right\} 1+1+2+1 = \boxed{5}$$

$$T.C: O(N \times 10^N)$$

$$\boxed{7}^{2^2-1}$$

$$x=3$$

	2	1	0
0 :	0	0	0
1 :	0	0	1
2 :	0	1	0
3 :	0	1	1
4 :	1	0	0
5 :	1	0	1
6 :	1	1	0
7 :	1	1	1

$$[1 \rightarrow 2^x - 1]$$

$$\left(\frac{2^x}{2} \right) \times x$$

$$\Rightarrow 4 \times 3 \geq 12$$

$$\begin{matrix} 4 & 4 & 4 \end{matrix}$$

8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1

$$N \geq 11 \rightarrow 8 \rightarrow 11$$

$$12 + 4$$

0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1

$$N = 4$$

$$0 - 2^n - 1$$

$$31^{32}$$