Control Systems
Assignment 1

Tushar More
EE19BTECH11025
Dept. of Electrical Engg.,
IIT Hyderabad.

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Problem

Find the transfer function, $G(s) = X_3(s)/F(s)$, for the translational mechanical system shown in the given figure.

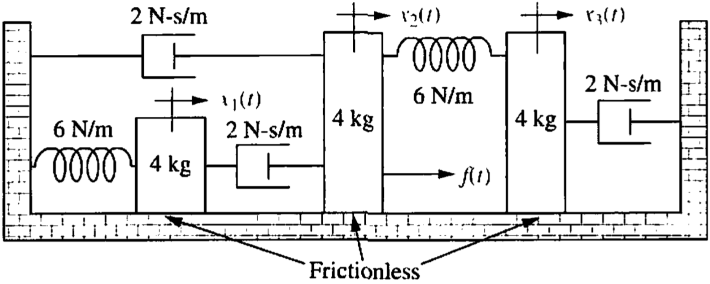

Figure: Problem Diagram

Solution

The degrees of freedom for the system is 3 because each of the three masses can be moved in horizontal direction while other masses being at rest.
The **Displacement** and **Force** in translational mechanical system is analogous to **Charge** and **Voltage** in Electrical System. The equations for the three displacements x_1, x_2 and x_3 will be same as the equations resulting from mesh analysis in Electrical networks.

For x_1 equation is of form,

$$\left[\text{Sum of impedances attached to motion at } x_1\right] X_1(s) \text{ - } \left[\text{Sum of impedances between } x_1 \text{ and } x_2\right] X_2(s) \text{ - } \left[\text{Sum of impedances between } x_1 \text{ and } x_3\right] X_3(s) = \left[\text{Sum of applied forces at } x_1\right]$$

At x_1, there is a mass of 4Kg, a spring with spring constant of 6N/m and a viscous damper of coefficient 2Ns/m. Impedances related to these are $4s^2$, 6 and 2s in laplacian domain. Between x_1 and x_2, impedance is 2s. There is no impedance connected between x_1 and x_3.
Similarly, we can write for equations for x_2 and x_3.

System of Equations

For x_1, equation is,

$$(4s^2 + 2s + 6)X_1(s) - 2sX_2(s) - 0X_3(s) = 0 \tag{3.1}$$

For x_2, equation is,

$$-2sX_1(s) + (4s^2 + 4s + 6)X_2(s) - 6X_3(s) = F(s) \tag{3.2}$$

For x_3, equation is,

$$0X_1(s) - 6X_2(s) + (4s^2 + 2s + 6)X_3(s) = 0 \tag{3.3}$$

Solving the system of equations for $X_3(s)$,

$$X_3(s) = \frac{\begin{vmatrix} 4s^2+2s+6 & -2s & 0 \\ -2s & 4s^2+4s+6 & F(s) \\ 0 & -6 & 0 \end{vmatrix}}{\begin{vmatrix} 4s^2+2s+6 & -2s & 0 \\ -2s & 4s^2+4s+6 & -6 \\ 0 & -6 & 4s^2+2s+6 \end{vmatrix}} \tag{3.4}$$

$$X_3(s) = \frac{-6F(s)(4s^2+2s+6)}{(4s^2+2s+6)(36+4s^2-(4s^2+2s+6)(4s^2+4s+6))} \tag{3.5}$$

So the transfer function $G(s) = X_3(s)/F(s)$ is

$$G(s) = \frac{3}{2s(4s^3 + 6s^2 + 13s + 9)} \tag{3.6}$$