TUTorial - 4

Ans-1

$T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n)$, $a=3, b=2$

$C = \log_2 3 = 1.58$

$n^c = n^{1.58} \simeq n^2$

$f(n) = n^2$

By case 3 $f(n) > n^c$

$T(n) = \Theta(f(n)) = \Theta(n^2)$

Ans-2

$T(n) = 4T(n/2) + n^2$

$a=4, b=2, f(n) = n^2$

$n^c = n^{\log_2 4} = \underline{n^2}$

By case 2 $\rightarrow$ $f(n) = n^c$

$T(n) = \Theta(n^c \log n) = \underline{\Theta(n^2 \log n)}$

Ans-3

$T(n) = T(n/2) + 2^n$

$a=1, b=2, f(n) = 2^n$

$n^c = n^{\log_2 1} = 1$

By case $\rightarrow$ $f(n) > n^c$

$T(n) = \Theta(f(n)) = \underline{\Theta(2^n)}$

Ans-4

$T(n) = 2^n T(n/2) + n^n$

$a = 2^n, b=2, f(n) = n^n$

$n^c = n^{n \log_2 2} = \underline{n^n}$

By case $f(n) = n^c$

$T(n) = \Theta(n^c \log n)$

$T(n) = \underline{\Theta(n^n \log n)}$

Ans-5

$T(n) = 16T(n/4) + n$

$a = 16, \quad b = 4, \quad f(n) = n$

$n^c = n^{\log_b a} = n^2$

By case $\quad n^c > f(n)$

$T(n) = \Theta(n^c)$

$T(n) = \Theta(n^2)$

Ans-6

$T(n) = 2T(n/2) + n\log n$

$a = 2, \quad b = 2, \quad f(n) = n\log n$

$n^c = n^{\log_b a} = n$

By case $\quad f(n) > n^c$

$T(n) = \Theta(f(n))$

$T(n) = \Theta(n\log n)$

Ans-7

$T(n) = 2T(n/2) + n/\log n$

$a = 2, \quad b = 2, \quad f(n) = n/\log n$

$n^c = n^{\log_b a} = n$

By case $\quad n^c > f(n)$

$T(n) = \Theta(n)$

Ans-8

$T(n) = 2T(n/4) + n^{0.5}$

$a = 2, \quad b = 4, \quad f(n) = n^{0.5}$

$n^c = n^{\log_b a} = n^{0.5}$

By case $\quad f(n) > n^c$

$T(n) = \Theta(n^{0.5})$

Ans-9

$T(n) = 0.5T(n/2) + 1/n$

$a = 0.5,\ b = 2,\ f(n) = 1/n$

$n^c = n^{\log_2 0.5} = n^{-1} = 1/n$

By case $f(n) = n^c$

$T_n = \Theta(1/n \log n)$

Ans 10

$T(n) = 16T(n/4) + n!$

$a = 16,\ b = 4,\ f(n) = n!$

$n^c = n^{\log_4 4^2} = n^2$

By case $n^c < f(n)$

$T(n) = \Theta(n!)$

Ans-11

$T(n) = 4T(n/2) + \log n$

$a = 4,\ b = 2,\ f(n) = \log n$

$n^c = n^{\log_2 2^2} = n^2$

By case $n^c > f(n)$

$T(n) = \Theta(n^2)$

Ans-12

$T(n) = \sqrt{n}\, T(n/2) + \log n$

$a = n^{1/2},\ b = 2,\ f(n) = \log n$

$n^c = n^{\log_2 n^{1/2}} = n^{\frac{1}{2}\log n}$

By case $f(n) < n^c$

$T(n) = \Theta(n^{\frac{1}{2}\log n})$

Ans-13

$T(n) = 3T(n/2) + n$

$a = 3,\ b = 2,\ f(n) = n$

$n^c = n^{\log_2 3}$

By case $f(n) < n^c$

$T(n) = \Theta(n^{\log_2 3})$

Am-14

$T(n) = 3T(n/3) + \sqrt{n}$

$a = 3,\ b = 3,\ f(n) = \sqrt{n}$

$n^c = n^{\log_3 3} = n$

By case $n^c > f(n)$

$T(n) = \Theta(n)$

Am-15

$T(n) = 4T(n/2) + cn$

$a = 4,\ b = 2,\ f(n) = cn$

$n^c = n^2$

By case $n^c > f(n)$

$T(n) = \Theta(n^2)$

Am-16

$T(n) = 3T(n/4) + n\log n$

$a = 3,\ b = 4,\ f(n) = n\log n$

$n^c = n^{\log_4 3}$

By case $f(n) > n^c$

$T(n) = \Theta(n\log n)$

Am-17

$T(n) = 3T(n/3) + n/2$

$a = 3,\ b = 3,\ f(n) = n/2$

$n^c = n$

By case $n^c > f(n)$

$T(n) = \Theta(n)$

Am-18

$T(n) = 6T(n/3) + n^2\log n$

$a = 6,\ b = 3,\ f(n) = n^2\log n$

$n^c = n^{\log_3 6} = n^{1.63}$

By case $f(n) > n^c$

$T(n) = \Theta(f(n))$

$T(n) = \Theta(n^2\log n)$

Ans-19

$T(n) = 4T(n/2) + n/\log n$

$a = 4,\ b = 2,\ f(n) = n/\log n$

$n^c \cdot n^{2\log_2} = n^2$

By case $n^c > f(n)$

$T(n) = \Theta(n^2)$

Ans-20

$T(n) = 64T(n/8) - n^2 \log n$

$a = 64,\ b = 8,\ f(n) = -n^2 \log n = n^2 \log 1/n$

$n^c = n^2$

By case $n^c < f(n)$

$T(n) = \Theta(n^2 \log 1/n)$

Ans-21

$T(n) = 7T(n/3) + n^2$

$a = 7,\ b = 3,\ f(n) = n^2$

$n^c = n^{\log_3 7}$

By case $n^c < f(n)$

$T(n) = \Theta(n^2)$

Ans 22

$T(n) = T(n/2) + n(2 - \cos n)$

$a = 1,\ b = 2,\ f(n) = n(2 - \cos n)$

$n^c = n^0 = 1$

By case $f(n) > n^c$

$T(n) = \Theta(n(2 - \cos n))$