Luther College **CoCalc Module 2**
Math 200 - Fall 2024 **Due September 27**
Dr. Johnson

Create a file named Module2Assignment.sagews in your CoCalc Module2 folder. Use commenting to organize your work so that each question is clearly labeled, relevant questions are answered in the CoCalc file, and context is provided for each problem. Expect to work in groups of two, but submit your own assignment, which will be graded directly from your CoCalc folder.

(1) Consider the two planes with equations $x + y - 3z = 3$ and $2x - 3y - z = 5$.
  (a) Compute the line of intersection and describe your answer in parametric vector form.
  (b) Create an image with both planes in blue, and the line of intersection in red.

(2) Let $\mathbf{u} = [2, 1]$ and $\mathbf{v} = [1, 1]$.
  (a) Create a coordinate grid that highlights the linear combinations of $\mathbf{u}, \mathbf{v}$ in standard position, with lines parallel to $\mathbf{u}$ in red and lines parallel to $\mathbf{v}$ in blue. Include copies of $\mathbf{u}, \mathbf{v}$ as vectors in standard position as arrows.
  (b) On this grid, include the points $O = (0, 0)$, $P = (3, 5)$, $Q = (5, 2)$, and $R = (-2, 4)$ in black.
  (c) Describe $\overrightarrow{OP}$, $\overrightarrow{PR}$, and $\overrightarrow{OR}$ as linear combinations of $\mathbf{u}, \mathbf{v}$.

(3) Let $\mathbf{u} = [3, -2, 1]$, $\mathbf{v} = [2, -2, -1]$.
  (a) Create a coordinate grid in $\mathbb{R}^3$ that highlights the linear combinations of $\mathbf{u}, \mathbf{v}$ in standard position, with lines parallel to $\mathbf{u}$ in red and lines parallel to $\mathbf{v}$ in blue. Include copies of $\mathbf{u}, \mathbf{v}$ as vectors in standard position as arrows.
  (b) Use this grid to determine if $\mathbf{w}_1 = [-6, 2, -7]$, and $\mathbf{w}_2 = [4, 5, 4]$ are linear combinations of $\mathbf{u}, \mathbf{v}$.
  (c) Create two vector equations to determine if $\mathbf{w}_1 = [-6, 2, -7]$, and $\mathbf{w}_2 = [4, 5, 4]$ are linear combinations of $\mathbf{u}, \mathbf{v}$. Use CoCalc to reduce the corresponding augmented matrices. Compare your answer with part (b).

(4) Let $\mathbf{u} = [1, 0, 1]$, $\mathbf{v} = [1, 1, 0]$, $\mathbf{w}_1 = [0, 1, 1]$, and $\mathbf{w}_2 = [0, -1, 1]$.
  (a) Are $\{\mathbf{u}, \mathbf{v}, \mathbf{w}_1\}$ linearly independent? Describe the vector equation you need to solve and identify the corresponding linear system. Use CoCalc to reduce the corresponding augmented matrix to answer the question.
  (b) Are $\{\mathbf{u}, \mathbf{v}, \mathbf{w}_2\}$ linearly independent? Describe the vector equation you need to solve and identify the corresponding linear system. Use CoCalc to reduce the corresponding augmented matrix to answer the question.
  (c) Create a new coordinate grid in $\mathbb{R}^3$ that highlights the linear combinations of $\mathbf{u}, \mathbf{v}, \mathbf{w}_1$ in standard position, with lines parallel to $\mathbf{u}$ in red, lines parallel to $\mathbf{v}$ in blue, and lines parallel to $\mathbf{w}_1$ in green.
    (i) Does this create a 3-dimensional grid?
    (ii) Make a prediction: Can you write every vector $\mathbf{b} = [b_1, b_2, b_3]$ in $\mathbb{R}^3$ as a linear combination of $\mathbf{u}, \mathbf{v}, \mathbf{w}_1$?
  (d) Create a new coordinate grid in $\mathbb{R}^3$ that highlights the linear combinations of $\mathbf{u}, \mathbf{v}, \mathbf{w}_2$ in standard position, with lines parallel to $\mathbf{u}$ in red, lines parallel to $\mathbf{v}$ in blue, and lines parallel to $\mathbf{w}_1$ in green.
    (i) Does this create a 3-dimensional grid? What caused this to be different than in (d)?
    (ii) Make a prediction: Can you write every vector $\mathbf{b} = [b_1, b_2, b_3]$ in $\mathbb{R}^3$ as a linear combination of $\mathbf{u}, \mathbf{v}, \mathbf{w}_1$?