Luther College **CoCalc Module 6**
Math 200 - Fall 2024 **Due December 4**
Dr. Johnson

In this Module, will use algebraic and geometric methods to explore and describe how the behavior of a family of dynamical systems changes by different choices of a parameter C.

Create a file named Module6Assignment.sagews in your CoCalc folder. Use commenting to organize your work so that each question is clearly labeled, relevant questions are answered in the CoCalc file, and context is provided for each problem. You are welcome to work in pairs, but submit your own assignment, which will be graded directly from your CoCalc folder.

Bifurcation of Eigenvalues

Consider the matrix $A_C = \begin{bmatrix} .5 & 1 \\ C & 0 \end{bmatrix}$ and the corresponding dynamical system $\mathbf{x}_{n+1} = A\mathbf{x}_n$. We wish to classify all the types of behavior that can occur for different values of C. Specifically, find all values of C where "most" orbits in $\mathbb{R}^2$ Specifically, find all values of C where "most" orbits in $\mathbb{R}^2$

- spiral towards the origin,
- spiral around the origin in a elliptical path,
- spiral away from the origin and becomes unbounded in size,
- repel from the origin while asymptotically approaching some line,
- approach the origin along a line or some other path.,
- approach a fixed point other than the origin,
- exhibit some other type of behavior.

The above list is not intended to be a checklist, but a starting point to think about all types of behavior you might see.

(1) Start by finding all values of C where A_C has
   - 2 distinct real eigenvalues,
   - 1 distinct real eigenvalue,
   - 2 complex eigenvalues.

(2) In each case above, algebraically identify the values of C where the behavior of "most" orbits change. Consider when the size (absolute value or modulus) of each eigenvalue crosses an important threshold.

(3) Create a classification that identifies the behavior of a typical orbit for all values of C. For example: If C is in the interval ..., then orbits .... Explain why you expect to seee this type of behavior for typical orbits. (Connect to eigenvalues of A_C.)

(4) Illustrate that values of C in the the given ranges above, lead to orbits with the behavior you've described. Choose a specific C value to illustrate each case, and provide a image that follows along a typical orbit (or orbits) for the corresponding dynamical system . If relevant, include the graphs of any eigenspaces for the matrix A_C. Describe the key features seen in your typical orbit, and connect it to the classification in part (3). Do they match or not?