Luther College **CoCalc Module 4**
Math 200 - Fall 2024 **Due November 6**
Dr. Johnson

In this Module, we will visually explore the connection between eigenvectors/eigenvalues of a 2×2 matrix A and images of subspaces (lines through the origin) for the corresponding matrix map.

Create a file named Module4Assignment.sagews in your CoCalc Module4 folder. Use commenting to organize your work so that each question is clearly labeled, relevant questions are answered in the CoCalc file, and context is provided for each problem. You are welcome to work in pairs, but submit your own assignment, which will be graded directly from your CoCalc folder.

Part 1 Creating a "eigenpicture". An eigenpicture will contain 20-80 unit vectors in standard position with their images under $T(\mathbf{x}) = A\mathbf{x}$ attached to the terminal point of the unit corresponding unit vector.

(1) Start by a graphics object by naming 'image = Graphics()". Add the following graphics objects to "image" using "+="
  (a) The unit circle $x^2 + y^2 = 1$ using the parametric plot for $x(t) = \cos t$, $y(t) = \sin(t)$.
  (b) A set of $k = 20 - 80$ equally spaced unit vectors in standard position. Start by defining a variable for an angle θ and use it to define a rotation matrix $R(\theta)$. Then create parametric plots of rotations of the unit vector $\mathbf{e}_1$ using $x(t) = t$, $y(t) = 0$ rotated by angle $\theta = 2\pi M/k$ where $M = 0, 1, \ldots k$. I recommend creating a "for loop"
  for M in (0..k):
  image += parametric plot for line times rotation matrix for angle $\theta = 2\pi M/k$.
  (c) Images of those 20-80 unit vectors under a matrix map A (start with $A = I_2$), but attached to the terminal point of the unit corresponding unit vector. Choose a different color.

Part 2: Exploring Eigenvalues/Eigenvectors for specific matrix maps.

Let
- $A_1 = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$,
- $A_2 = \begin{bmatrix} 2 & 0 \\ -3/2 & 1/2 \end{bmatrix}$,
- $A_3 = \begin{bmatrix} 1/2 & 3/2 \\ 0 & -1 \end{bmatrix}$,
- $A_4 = \begin{bmatrix} 1 & 11 \\ 2 & 0 \end{bmatrix}$,
- $A_5 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$,
- $A_6 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$,
- $A_7 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$,
- $A_8 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

(2) For a 2×2 matrix A_i, create an eigenpicture. By visually inspecting the picture, approximately identify corresponding eigenvectors and eigenvalues for the matrix.
(3) After visually identifying the eigenvectors/eigenvalues, use the command $A.eigenvectors\_right()$ to compute the corresponding eigenvectors/eigenvalues and compare your answer.

(4) Go back to each of your eigenpictures in part (2), and add a portion of the eigenspaces $E_\lambda =$ span of the eigenvectors for eigenvalue λ to your eigenpicture. Do these eigenspaces match your prediction for eigenvectors in part (2)?