

ASSIGNMENT 1

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Question: Find the value(s) of k for which the quadratic equation $x^2 + 4kx + k^2 - k + 2 = 0$ has equal roots.

Solution: A quadratic equation

$$ax^2 + bx + c = 0$$

has equal roots only if the discriminant

$$b^2 - 4ac = 0$$

Substituting $a = 1$, $b = 4k$, $c = k^2 - k + 2$,

$$(4k)^2 - 4(k^2 - k + 2)(1) = 0 \quad (1)$$

$$\implies 16k^2 - 4k^2 + 4k - 8 = 0 \quad (2)$$

$$\implies 12k^2 + 4k - 8 = 0 \quad (3)$$

$$\implies 3k^2 + k - 2 = 0 \quad (4)$$

On factorising, we get

$$3k^2 + 3k - 2k - 2 = 0 \quad (5)$$

$$\implies 3k(k + 1) - 2(k + 1) = 0 \quad (6)$$

$$\implies (k + 1)(3k - 2) = 0 \quad (7)$$

$$\implies k = -1 \text{ or } k = \frac{2}{3} \quad (8)$$

Check: if $k = -1$

$$x^2 + 4kx + k^2 - k + 2 = 0 \quad (9)$$

$$\implies x^2 - 4x + 4 = 0 \quad (10)$$

$$\implies (x - 2)^2 = 0 \quad (11)$$

$$\implies x = 2, 2 \quad (12)$$

Check: if $k = \frac{2}{3}$

$$x^2 + 4kx + k^2 - k + 2 = 0 \quad (13)$$

$$\implies x^2 + \frac{8}{3}x + \frac{16}{9} = 0 \quad (14)$$

$$\implies \left(x + \frac{4}{3}\right)^2 = 0 \quad (15)$$

$$\implies x = -\frac{4}{3}, -\frac{4}{3} \quad (16)$$

VERIFICATION USING GRAPHS:

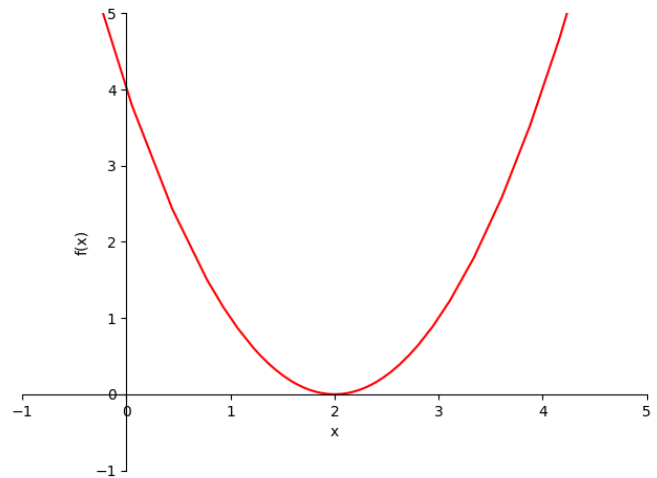


Fig. 1. Touches the x axis at (2, 0), has equal root "2"

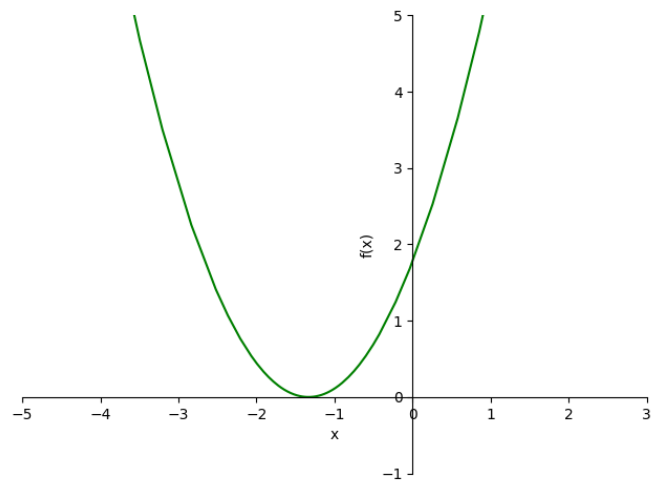


Fig. 2. Touches the x axis at $(-\frac{4}{3}, 0)$, has equal root, $-\frac{4}{3}$