

AI1110 ASSIGNMENT 8

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Abstract—This document gives solution to Chapter 7, Question 7.20 from the Papoulis and Pillai Probability, Random Variables and Stochastic Processes Text Book.

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Case 1: $x \leq y$ then,

$$\bar{A}_x B_Y = \{\text{all points in } (x, y)\} \quad (2.0.5)$$

$$\implies \Pr(\bar{A}_x B_Y) = (y - x)^n \quad (2.0.6)$$

Case 2: $x > y$ then,

$$\bar{A}_x B_Y = \{\phi\} \quad (2.0.7)$$

Therefore, we have $F_{XY}(x, y) = \Pr(A_X B_Y) =$

$$\begin{cases} y^n - (y - x)^n & x \leq y \\ y^n & x > y \end{cases}$$

1. QUESTION

We place at random n points in the interval $(0, 1)$ and we denote by x and y the distance from the origin to the first and last point respectively. Find $F(x)$, $F(y)$, $F(x, y)$.

2. ANSWER

The event $\tilde{x} \leq x$ occurs if there is at least one point in the interval $(0, x)$; The event $\tilde{y} \leq y$ occurs if all the points are in the interval $(0, \tilde{y})$.

Let $A_X =$ at least one point in $(0, x) = \tilde{x} \leq x$

Let $B_Y =$ no points in $(y, 1)$

$=$ all points in $(0, y) = \tilde{y} \leq y$

Hence for $0 \leq x \leq \tilde{1}$, $0 \leq y \leq 1$, we have:

$$F_X(x) = \Pr(A_X) = 1 - \Pr(\bar{A}_X) = 1 - (1 - x)^n \quad (2.0.1)$$

$$F_Y(y) = \Pr(B_Y) = y^n \quad (2.0.2)$$

Further more, we have

$$\left\{ \tilde{x} \leq x, \tilde{y} \leq y \right\} = A_x \cdot B_y \quad (2.0.3)$$

$$A_X B_Y + \bar{A}_X B_Y = B_Y \quad (2.0.4)$$