AI1110 ASSIGNMENT 5

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***Abstract*—This document refers to the Example 10, Chapter 2, from the Papoulis and Pillai Probability, Random Variables and Stochastic Processes Text Book.**

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1. Question

In a fair die experiment, S represents the set consisting of the faces $f_1, f_2, f_3, f_4, f_5, f_6$. Determine the conditional probability of the event $\{f_2\}$, assuming the event $even$ occurred.

2. Formulas

$$\Pr(E) = \frac{n(E)}{n(S)} \tag{2.0.1}$$

$$\Pr(E|F) = \frac{\Pr(E,F)}{\Pr(F)} \tag{2.0.2}$$

3. Solution

Let X be a random variable which is used to denote the outcome of the dice. Then we have, $X \in \{f_1, f_2, f_3, f_4, f_6\}$ where in f_k, k denotes the outcome of the dice.

We must find the value of

$$\Pr(X = f_2 \mid X = f_2 \cdot f_4 \cdot f_6) \tag{3.0.1}$$

$$\Pr(X = k) = \begin{cases} \frac{1}{6}, & 1 \le k \le 6 \\ 0, & otherwise \end{cases} \tag{3.0.2}$$

$$\Pr(X = f_2 \mid X = f_2, f_4, f_6) = \frac{\Pr(X = f_2 \cdot f_2, f_4, f_6)}{\Pr(X = f_2, f_4, f_6)} \tag{3.0.3}$$

Since the events f_2, f_4, f_6 are mutually exclusive, we have

$$\Pr(X = f_2, f_4, f_6) = \Pr(X = f_2) + \Pr(X = f_4) + \Pr(X = f_6) \tag{3.0.4}$$

$$\implies \Pr(X = f_2, f_4, f_6) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} \tag{3.0.5}$$

$$\implies \Pr(X = f_2, f_4, f_6) = \frac{3}{6} = \frac{1}{2} \tag{3.0.6}$$

We see that $\{f_2\}$ is a subset of $\{f_2, f_4, f_6\}$. We know that if

$$A \subset B \ then, A \cdot B = A. \tag{3.0.7}$$

Therefore

$$\Pr(X = f_2 \cdot f_2, f_4, f_6) = \Pr(f_2) = \frac{1}{6} \tag{3.0.8}$$

Therefore, by substituting the results (3.0.6) and (3.0.8), in the equation (3.0.3), we have

$$\Pr(X = f_2 \mid X = f_2, f_4, f_6) = \frac{1/6}{1/2} \tag{3.0.9}$$

$$\implies \Pr(X = f_2 \mid X = f_2, f_4, f_6) = \frac{1}{6} \times \frac{2}{1} \tag{3.0.10}$$

$$\implies \Pr(X = f_2 \mid X = f_2, f_4, f_6) = \frac{1}{3} \tag{3.0.11}$$