AI1110 ASSIGNMENT 8

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***Abstract*—This document gives solution to Chapter 7, Question 7.20 from the Papoulis and Pillai Probability, Random Variables and Stochastic Processes Text Book.**

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1. Question

We place at random n points in the interval $(0, 1)$ and we donate by x and y the distance from the origin to the first and last point respectively. Find $F(x)$, $F(y)$, $F(x, y)$.

2. Answer

The event $\underset{\sim}{x} \leq x$ occurs if there is at least one point in the interval (0,x); The event $\underset{\sim}{y} \leq y$ occurs if all the points are in the interval(0, y).

Let A_X = at least one point in (0, x) = $\underset{\sim}{x} \leq x$
Let B_Y = no points in (y, 1)
= all points in (0,y) = $\underset{\sim}{y} \leq y$

Hence for $0 \leq x \leq 1$, $0 \leq y \leq 1$, we have:

$$F_X(x) = \Pr\left(A_X\right) = 1 - \Pr\left(\bar{A_X}\right) = 1 - (1 - x)^n \tag{2.0.1}$$

$$F_Y\left(y\right) = \Pr\left(B_Y\right) = y^n \tag{2.0.2}$$

Further more, we have

$$\left\{\underset{\sim}{x} \leq x, \underset{\sim}{y} \leq y\right\} = A_x \cdot B_y \tag{2.0.3}$$

$$A_X \ B_Y + \bar{A_X} B_Y = B_Y \tag{2.0.4}$$

Case 1: $x \leq y$ then,

$$\bar{A_x} B_Y = \{all\ points\ in\ \ (x, y)\} \tag{2.0.5}$$

$$\implies \Pr\left(\bar{A_X} B_Y\right) = (y - x)^n \tag{2.0.6}$$

Case 2: $x > y$ then,

$$\bar{A_X} B_Y = \{\phi\} \tag{2.0.7}$$

Therefore, we have $\mathrm{F}_{XY}\left(x, y\right) = \Pr\left(A_X B_Y\right) = \begin{cases} y^n - (y-x)^n & x \leq y \\ y^n & x > y \end{cases}$