Manual Assignment

Tushita Sharva - CS21BTECH11022

Contents

1. Uniform Random Numbers

Let U be a uniform random variable between 0 and 1.

1.1 Generate 10^6 samples of U using a C program and save into a file called uni.dat .
Solution: Download the following files and execute the C program. Header file:

```
https://github.com/TushitaSharva/PRV_2022/blob/main/Manual/Codes/Src.h
```

Code for executing C file:

```
https://github.com/TushitaSharva/PRV_2022/blob/main/Manual/Codes/Q1−1.c
```

When executed, the 10^6 random numbers will be generated and be sent into to new file called "uni.dat".

1.2 Load the uni.dat file into python and plot the empirical CDF of U using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \leq x) \tag{1.0.1}$$

Solution: The following code plots Fig. 1.2

```
https://github.com/TushitaSharva/PRV_2022/blob/main/Manual/Codes/Q1−2.py
```

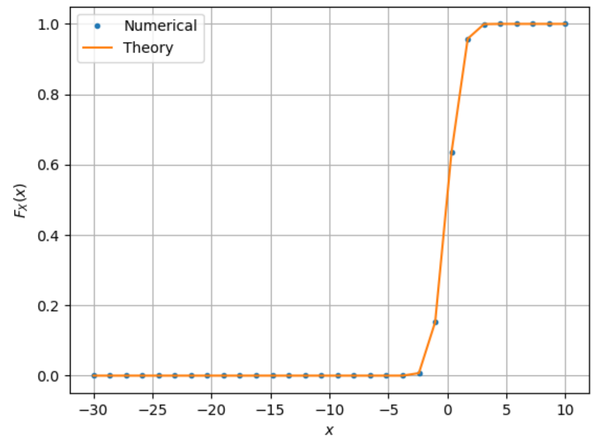

Fig. 1.2. The CDF of U

1.3 Find a theoretical expression for $F_U(x)$.
Solution: Given U is a uniform Random Variable

$$p_U(x) = 1 \text{ for} \tag{1.0.2}$$

$$F_U(x) = \int_{-\infty}^{\infty} p_U(x)dx \tag{1.0.3}$$

$$\boxed{\implies F_U(x) = \begin{cases} 0 & x \leq 0 \\ x & 0 < x < 1 \\ 1 & x \geq 1 \end{cases}} \tag{1.0.4}$$

1.4 The mean of U is defined as

$$E[U] = \frac{1}{N}\sum_{i=1}^{N} U_i \tag{1.0.5}$$

and its variance as

$$\text{var}[U] = E[U - E[U]]^2 \tag{1.0.6}$$

Write a C program to find the mean and variance of U.
Solution: Download the following files and execute the C program. Header file:

```
https://github.com/TushitaSharva/PRV_2022/blob/main/Manual/Codes/Src.h
```

C file:

https://github.com/TushitaSharva/PRV_2022/blob/main/Manual/Codes/Q1−4.c

1.5 Verify your result theoretically given that

$$E\left[U^k\right] = \int_{-\infty}^{\infty} x^k dF_U(x) \tag{1.0.7}$$

Solution:

$$\text{var}\left[U\right] = E\left[U - E\left[U\right]\right]^2 \tag{1.0.8}$$

$$\implies \text{var}\left[U\right] = E\left[U^2\right] - E\left[U\right]^2 \tag{1.0.9}$$

$$E\left[U\right] = \int_{-\infty}^{\infty} x dF_U(x) \tag{1.0.10}$$

$$E\left[U\right] = \int_{0}^{1} x \tag{1.0.11}$$

$$\implies \boxed{E\left[U\right] = \frac{1}{2}} \tag{1.0.12}$$

$$E\left[U^2\right] = \int_{-\infty}^{\infty} x^2 dF_U(x) \tag{1.0.13}$$

$$E\left[U^2\right] = \int_{0}^{1} x^2 dF_U(x) \tag{1.0.14}$$

$$\implies E\left[U^2\right] = \frac{1}{3} \tag{1.0.15}$$

$$\implies \boxed{\text{var}\left[U\right] = \frac{1}{12} = 0.0833} \tag{1.0.16}$$

2. Central Limit Theorem

2.1 Generate 10^6 samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \tag{2.0.1}$$

using a C program, where $U_i, i = 1, 2, \dots, 12$ are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat

Solution: Download the following files and execute the C program. Header file:

https://github.com/TushitaSharva/PRV_2022/blob/main/Manual/Codes/Src.h

C file:

https://github.com/TushitaSharva/PRV_2022/blob/main/Manual/Codes/Q2−1.C

2.2 Load gau.dat in python and plot the empirical CDF of X using the samples in gau.dat. What properties does a CDF have?

Solution: The CDF of X is plotted in Fig 2.2 using the code below

https://github.com/TushitaSharva/PRV_2022/blob/main/Manual/Codes/Q2−2.py

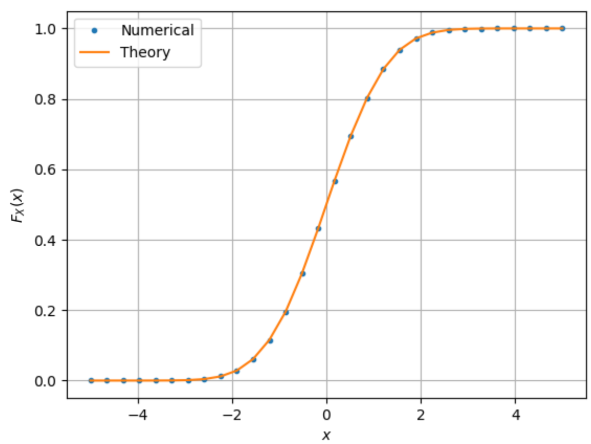

Fig. 2.2. The CDF of X

Some of the properties of CDF

a) $\lim_{x\to\infty} F_X(x) = 1$
b) $F_X(x)$ is non decreasing function.
c) Symmetric about one point.

2.3 Load gau.dat in python and plot the empirical PDF of X using the samples in gau.dat. The PDF of X is defined as

$$p_X(x) = \frac{d}{dx} F_X(x) \tag{2.0.2}$$

What properties does the PDF have?

Solution: The PDF of X is plotted in Fig. 2.3 using the code below

https://github.com/TushitaSharva/PRV_2022/blob/main/Manual/Codes/Q2−3.py

Some of the properties of the PDF:

a) Symmetric about $x = \mu$ in this case

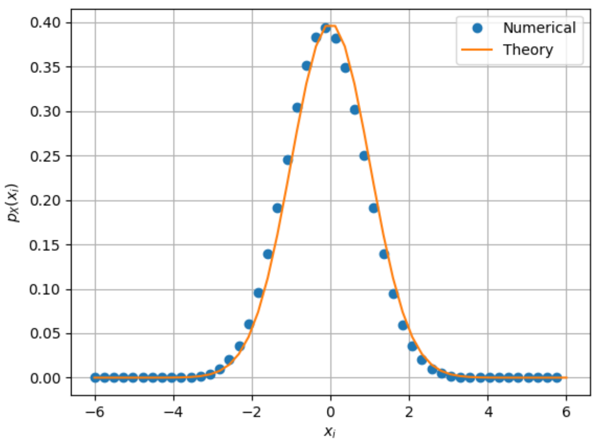

Fig. 2.3. The PDF of X

b) Decreasing function for $x > \mu$ and increasing for $x < \mu$ and attains maximum at $x = \mu$
c) Area under the curve is unity.

2.4 Find the mean and variance of X by writing a C program.
Solution: Download the following files and execute the C program. Header file:

```
https://github.com/TushitaSharva/
    PRV_2022/blob/main/Manual/Codes/
    Src.h
```

C file:

```
https://github.com/TushitaSharva/
    PRV_2022/blob/main/Manual/Codes/
    Q2−4.c
```

2.5 Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty, \tag{2.0.3}$$

repeat the above exercise theoretically.

Solution:

1) CDF is given by

$$F_X(x) = \int_{-\infty}^{\infty} p_X(x)dx \tag{2.0.4}$$

$$\boxed{F_X(x) = 1} \tag{2.0.5}$$

2) Mean is given by

$$E(x) = \int_{-\infty}^{\infty} x p_X(x)dx \tag{2.0.6}$$

$$\implies \boxed{E(x) = 0} \tag{2.0.7}$$

3) Variance is given by

$$\text{var}\left[U\right] = E(U^2) - (E(U))^2 \tag{2.0.8}$$

$$\implies \boxed{\text{var}\left[U\right] = \sqrt{2}} \tag{2.0.9}$$

3. From Uniform to Other

3.1 Generate samples of

$$V = -2\ln\left(1 - U\right) \tag{3.0.1}$$

and plot its CDF.
Solution: Download the following file and execute the C program.

```
https://github.com/TushitaSharva/PRV_2022/
    blob/main/Manual/Codes/Q3−1.c
```

The CDF of V is plotted in Fig. 3.1 using the code below

```
https://github.com/TushitaSharva/
    PRV_2022/blob/main/Manual/Codes/
    Q3−1.py
```

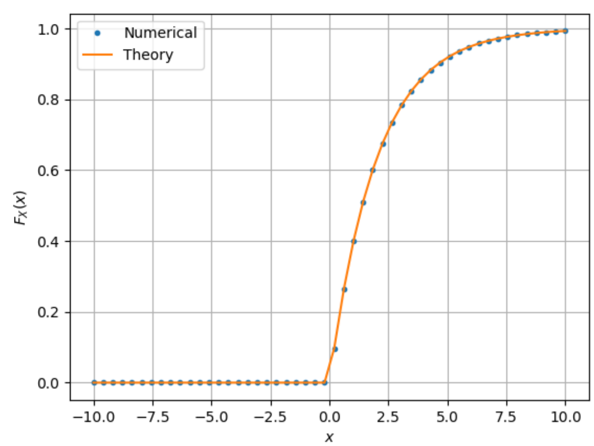

Fig. 3.1. The CDF of X

3.2 Find a theoretical expression for $F_V(x)$.
Solution: If Y = g(X), we know that $F_Y(y) = F_X(g^{-1}(y))$, here

$$V = -2\ln\left(1 - U\right) \tag{3.0.2}$$

$$1 - U = e^{\frac{-V}{2}} \tag{3.0.3}$$

$$U = 1 - e^{\frac{-V}{2}} \tag{3.0.4}$$

$$F_V(x) = F_U(1 - e^{\frac{-x}{2}}) \tag{3.0.5}$$

$$\implies F_V(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{\frac{-x}{2}} & x \geq 0 \end{cases} \tag{3.0.6}$$