AI1110 - ASSIGNMENT 4

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Outline

1. Question
2. Solution

Question

N.C.E.R.T Mathematics for class 12,Probability Chapter, Exercise 1, question 8

An instructor has a question bank consisting of 300 easy True/ False questions, 200 difficult True/ False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question, given that it is a multiple choice question?

Given Information

In the question, it was given that there were

1. 300 easy True/ False questions
2. 200 difficult True/False questions
3. 500 easy multiple choice questions
4. 400 difficult multiple choice questions

Additionally, it was also known that the random question picked was multiple choice question.
We have to find the probability that it will be an easy question, given that it is a multiple choice question.

Concepts

Formulas

$$\Pr(E) = \frac{n(E)}{n(S)} \tag{2.0.1}$$

$$\Pr(E|F) = \frac{\Pr(E,F)}{\Pr(F)} \tag{2.0.2}$$

Note:

We use the concept of Random Variables here. A random variable is a real valued function defined on the sample space. Since they are assigned to the outcome of the sample space, probabilities can be assigned to them.

Assigning Random Variables

Note:

This question has two simultaneous aspects to be covered:

1. Difficulty of the question ($Easy$, $tough$)
2. Type of Question (T/F, MCQ)

Therefore we shall use two different random variables X and Y.

Random Variable X : Assigned to the difficulty of the question

$X \in \{0, 1\}$.
If $X = 0$, the question is easy. Else difficult.

Random Variable Y: Assigned to the type of question asked

$Y \in \{0, 1\}$.
If $Y = 0$, the question is a multiple choice question. Else, it is a True/False question.

Probabilities using Random Variables

$$\Pr(X = k) = \begin{cases} \frac{8}{14}, & k = 0 \\ \frac{6}{14}, & k = 1 \end{cases} \tag{2.0.3}$$

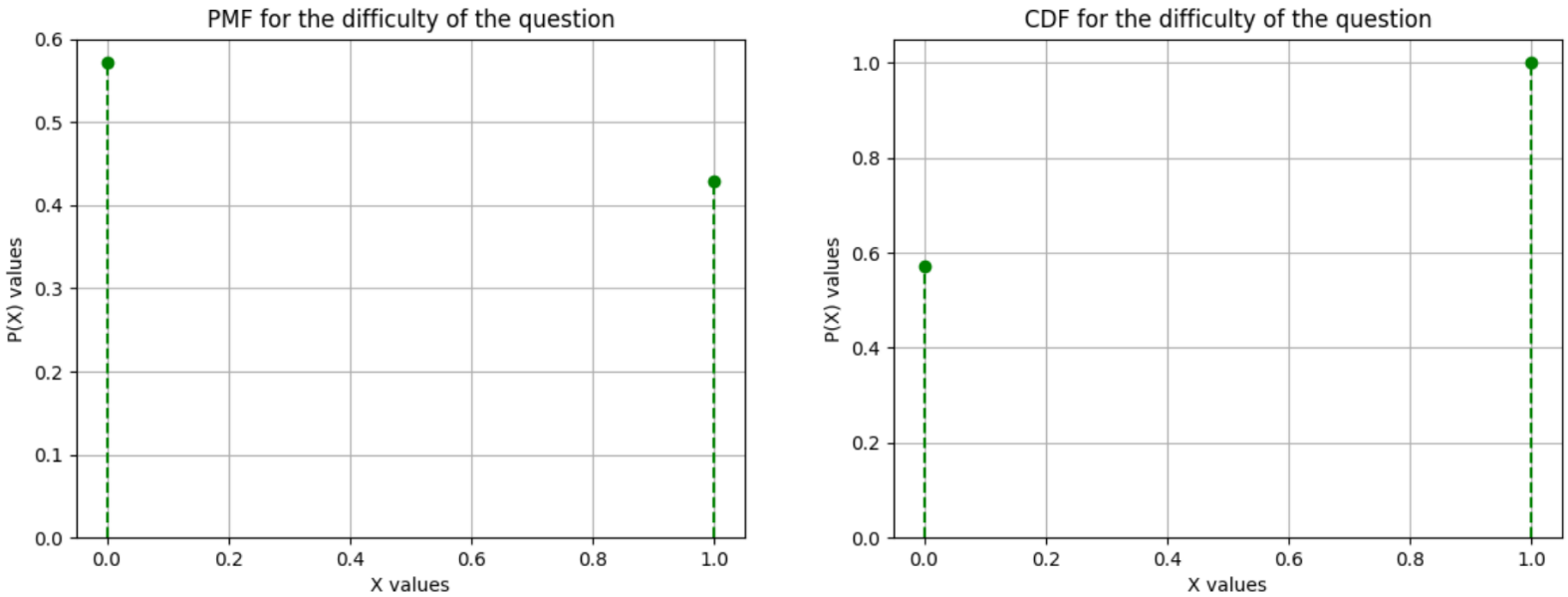

Figure: PMF and CDF

Probabilities using Random Variables

$$\Pr(Y = k) = \begin{cases} \frac{5}{14}, & k = 0 \\ \frac{9}{14}, & k = 1 \end{cases} \tag{2.0.4}$$

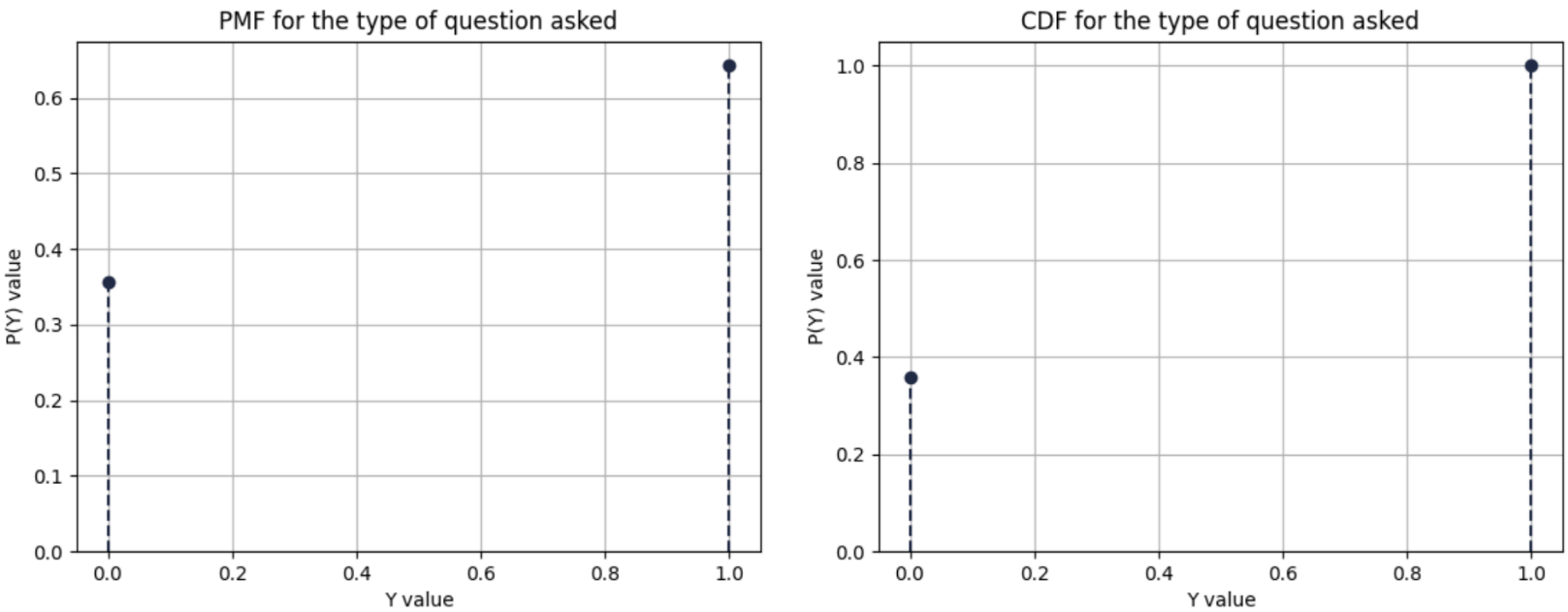

Figure: PMF and CDF

Solution

We require the probability that it is an easy question given that it is a Multiple Choice Question.
i.e., We require $\Pr(X = 0 \mid Y = 1)$. We know that

$$\Pr(X = 0) = \frac{8}{14} \tag{2.0.5}$$

$$\Pr(Y = 1) = \frac{9}{14} \tag{2.0.6}$$

$$\Pr(X = 0, Y = 1) = \frac{5}{14} \tag{2.0.7}$$

Using the general formula for conditional probability, we have

$$\Pr(X = 0 \mid Y = 1) = \frac{\Pr(X = 0, Y = 1)}{\Pr(Y = 0)} \tag{2.0.8}$$

Solution

Substituting the values, we have:

$$\implies \Pr(X = 0 \mid Y = 1) = \frac{5/14}{9/14} \tag{2.0.9}$$

$$\implies \Pr(X = 0 \mid Y = 1) = \frac{5}{9} \tag{2.0.10}$$

Answer

Therefore, the probability that the question will be an easy one, given that it is a multiple choice question is $\frac{5}{9}$.

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