ASSIGNMENT 2

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Download python code for this question from:

https://github.com/TushitaSharva/PRV_2022/tree/main/ASSIGNMENT_2/codes

Download Latex code from:

https://github.com/TushitaSharva/PRV_2022/tree/main/ASSIGNMENT_2/latex

1. Question

Evaluate:

$$\int \frac{x^3+5x^2+4x+1}{x^2}\,dx$$

2. Solution

Formulas required:

1) $\frac{a^m}{a^n} = a^{(m-n)}$

2) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$

3) $\int \frac{1}{x}\,dx = \log x + c$

The same expression can be re-written as

$$\int \frac{x^3}{x^2} + \frac{5x^2}{x^2} + \frac{4x}{x^2} + \frac{1}{x^2}\,dx \tag{2.0.1}$$

Using the formula (1),

$$\int x + 5 + \frac{4}{x} + \frac{1}{x^2}\,dx \tag{2.0.2}$$

$$\implies \int x + \int 5\left(x^0\right) + \int \frac{4}{x} + \int \frac{1}{x^2}\,dx \tag{2.0.3}$$

Using formula (3) for the third algebraic term, we get:

$$\int \frac{4}{x}\,dx = 4\int \frac{1}{x} \tag{2.0.4}$$

$$\implies 4\log x + c \tag{2.0.5}$$

Using formula (2) for rest of the terms, we get

$$\int x\,dx = \frac{x^{(1+1)}}{1+1} + c = \frac{x^2}{2} + c \tag{2.0.6}$$

$$\int 5x^0\,dx = 5 \times \frac{x^{(0+1)}}{0+1} + c = 5x + c \tag{2.0.7}$$

$$\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{x^{-1}}{-1} + c = \frac{-1}{x} + c \tag{2.0.8}$$

Substituting them all in the equation given, we get:

$$\int \frac{x^3+5x^2+4x+1}{x^2}\,dx = \frac{x^2}{2} + 5x + 4\log x + \frac{-1}{x} + c$$

Final Answer:

$$\frac{x^2}{2} + 5x + 4\log x + \frac{-1}{x} + c$$