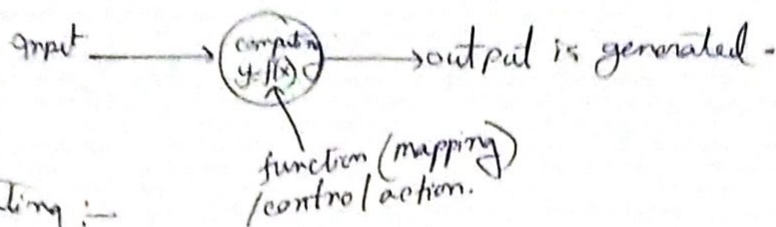


computing  $\Rightarrow$  process to complete a given goal oriented task.



features of computing :-

- 1) Control actions unambiguous and accurate.
- 2) Control actions should be easy to conceptualize, construct and operate.
- 3) should provide precise and optimized solution.

Hard computing  $\Rightarrow$  (L. A Zadeh).

primal procedure for solving engineering problems that can be <sup>formulated</sup> mathematically.  
 Input-output relationship expressed in terms of mathematical expression.  
 Control actions must be accurate, suitable to problem defined, stable, highly predictable.

examples  $\rightarrow$  shortest tour in a graph.  
 searching, sorting

soft computing example  $\rightarrow$  (Fuzzy logic) [values lies b/w 0 and 1].  
 $\rightarrow$  Human oriented.

example  $\rightarrow$  Micro controllers, Workstation.  
 Crisp logic [machine oriented] [values 0 or 1 nothing else].

GA (Genetic Algo)  $\rightarrow$  search based optimization algo based on principles of Genetic and Natural selection.

ANN  $\rightarrow$  Pattern recog, E-commerce, control systems.  
 [TSP, Image segmentation].

Hard Computing

1) Involves ~~more~~ binary logic, crisp systems and numerical analysis.

2) Exact input data required.

3) sequential computation follows.

4) produce precise results.

Soft Computing

1) involves fuzzy logic, genetic algo, neural networks.

2) Ambiguous and noisy data.

3) parallel computation follows.

4) produce approximate results.

Hybrid Computing

Autonomous robots.

(combination of)

conventional hard computing and soft computing.

Fuzzy logic  $\rightarrow$  cognitive process (ex: human brain).

mathematical tool dealing with uncertainty.

Crisp logic  $\rightarrow$  deals with crisp sets and Boolean algebra.

Is water colourless  $\begin{cases} \rightarrow \text{Yes} \\ \rightarrow \text{No} \end{cases}$

Is Ram honest  $\rightarrow$  Extremely honest  $\nabla$   
 $\rightarrow 0.80$   
 $\rightarrow 0.4$   
 $\rightarrow 0$

features  $\rightarrow$  1) flexible, easy to understand.  
2) It is used for helping the minimization of logics by human.  
3) allow user to build or create the functions which are non-linear of arbitrary complexity.

Crisp set:  $A \subset U$  (universe of discourse).

either  $x \in A$  or  $x \notin A$

total no of elements belonging to  $A$ , is  
cardinality of  $A$ .

~~crisp function~~ membership function of  $A$  -  
$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

Fuzzy allows partial membership.

$$A = \left\{ \sum_{i=1}^n \frac{\mu_A(x_i)}{x_i} \right\}, \text{ } n \text{ is finite} \rightarrow U \text{ is finite}$$

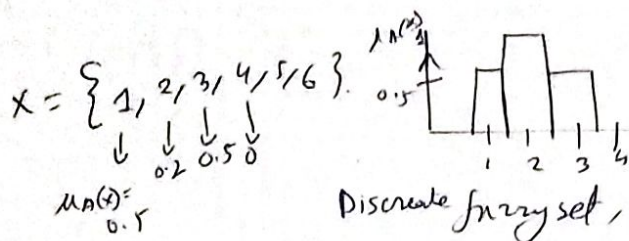
$\downarrow$   
Fuzzy set.

$$A = \left\{ \int \frac{x \mu_A(x)}{x} \right\} \rightarrow \text{continuous, finite.}$$

$\mu_A(x)$  is degree of membership of  $x$  in  $A$  and indicates that  $x$  belongs to  $A$ .

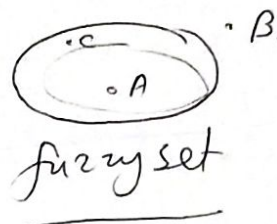
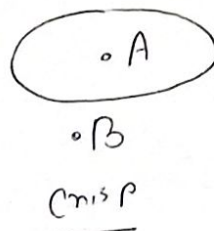


## Example - 1



Diff Crisp set and fuzzy set

- |                                    |  |
|------------------------------------|--|
| 1) Sharp boundaries by 0/1 and 1/1 | 1) fuzzy boundaries with membership value. |
| 2) some crisp set can be fuzzy     | 2) some fuzzy set can make crisp.          |
| 3) logic is bi-valued              | 3) logic is infinite-valued.               |



$$\mu_U(x) = 1 \text{ for all } x \in U.$$

↓  
universal fuzzy set.

$$\mu_\emptyset(x) = 0 \text{ for all } x \in U.$$

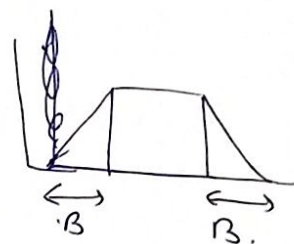
↓  
empty fuzzy set.

$$\mu_A(x) = \mu_B(x) \text{ for } x \in U.$$

↓  
equal fuzzy sets.

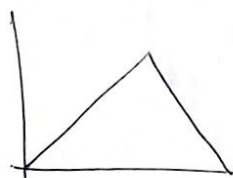
Boundary of A

$$\{x \in U \mid 0 < \mu_A(x) < 1\}$$



A convex fuzzy set 1) rules-

1) membership values monotonically increasing decreasing on increasing then decreasing.

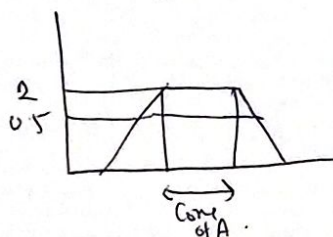


Convex

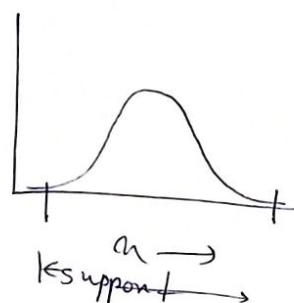


Non convex.

Core of A.  $\Rightarrow \{x \in U \mid \mu_A(x) = 1\}$



$$\text{Support of } A = \{x \in U \mid \mu_A(x) > 0\}$$



Height of  $A$  is defined as max value of membership functions of all elements contained in that set.

$$\text{Height}(A) = \{ \max(\mu_A(x) \mid x \in U) \}$$

It may not be 1 always, if  $\emptyset$  is non empty, height of fuzzy set is 1.



Normality:-

A fuzzy set  $A$  is defined as normal whose membership func has at least one element  $x$  in the universe such that:  $\mu_A(x) = 1$ .

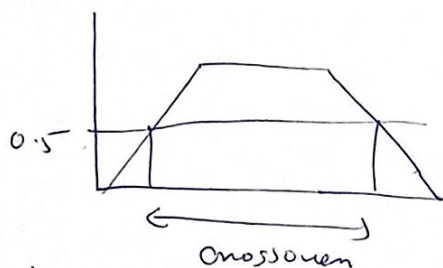
~~or~~  $\mu_A(x) < 1 \rightarrow$  subnormal.

height of fuzzy set is always 1 and less than 1 for subnormal.



Crossover points:-

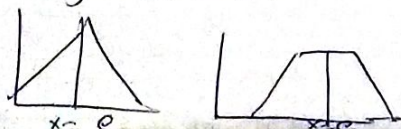
$$\text{Crossover}(A) = \{ x \in U \mid \mu_A(x) = 0.5 \}$$



Symmetry:-

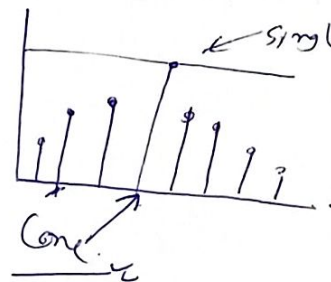
$$\mu_A(x+c) = \mu_A(x-c)$$

for all  $x \in U$ .



Fuzzy Singleton.

$$|A| = \{ x \in U \mid \mu_A(x) = 1 \}$$



Fuzzy Bandwidth.

Bandwidth(A)

$$= |x_1 - x_2| \text{ where } x_1, x_2 \in U \text{ } \mu_A(x_1) = \mu_A(x_2) = 0.5$$



$\alpha$  cut:-  $A_\alpha = \{x \mid \mu_A(x) \geq \alpha\}$ .

$A_\alpha = \{x \mid \mu_A(x) > \alpha\} \rightarrow$  strong  $\alpha$  cut.

If  $\alpha_1 > \alpha_2$  ,  $A_{\alpha_1} \subseteq A_{\alpha_2}$ .

Cardinality

Scalar  $|A| = \sum_{x \in U} \mu_A(x)$ .

Relative  $\|A\| = \frac{|A|}{|U|}$

$|A_3| = 0 + 0.2 \dots 3.2$

Fuzzy

$|AF| = \{( |A|, \alpha )\}$

$\|A_3\| = \frac{3.2}{10} = \frac{3.2}{9} = \dots$

Fuzzy logic vs Probability.

↓  
Identifies  
Partial truth

↓  
Identifies the  
Partial knowledge.

Takes degree  
of membership  
in a set  
based on  
math.

Take mathematical likelihood  
of events to occur.

Adv, Disadv →